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INTRODUCTION AND FUNDAMENTAL CONCEPTS

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In this chapter some preliminary definitions and basic terminologies related to this research are written.

1.1 Fluids:

The type of matter which needs some support to reminisce its shape as well as size is called fluid. In other words fluid is a material which flows or have capacity of flowing. Natural examples of fluid are air and water.

1.2 Classification of fluid:

The fluids are divided into two categories. Newtonian fluids and non-Newtonian fluids.

1.2.1 Newtonian fluid:

Fluid in which stress tensor and rate of strain tensor are linearly related (as shown in figure 1.1) are known as Newtonian fluid. Couple of examples of Newtonian fluids are Gasoline, glycerin.

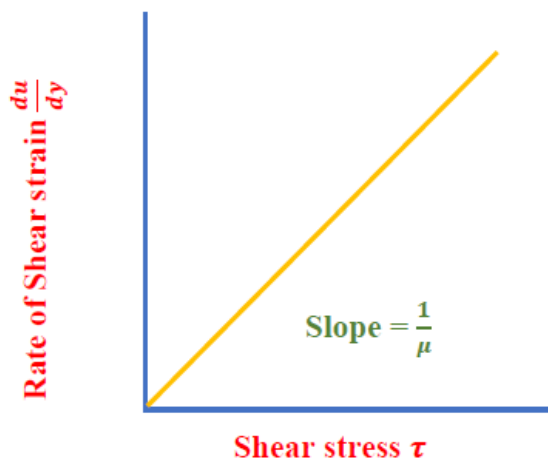


Figure 1.1 Newtonian fluid

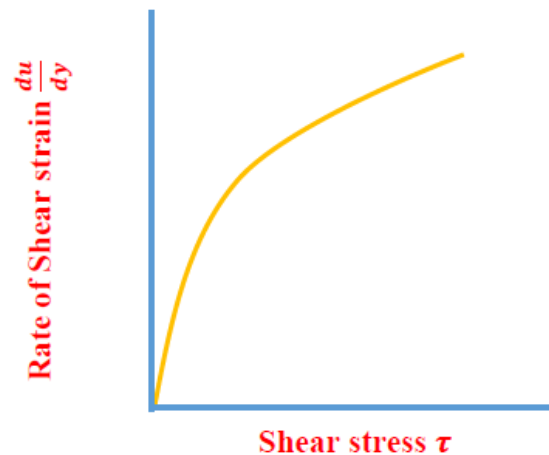


Figure 1.2 Non-Newtonian fluid

1.2.2 Non-Newtonian fluid:

Fluids in which relation of stress tensor and rate of strain tensor are not linearly related (as shown in figure 1.2) are known as Non-Newtonian fluids. Few examples of non – Newtonian fluids are custard, honey, shampoo, paint.

This research is focused on non – Newtonian fluids.

1.3 Types of non – Newtonian fluids:

Non-Newtonian fluids are classified in three types. Time dependent, Time independent, Elastic viscous fluids.

1.3.1 Time dependent fluids:

In the fluid, if relation between shear stress and rate of shear strain depends on time, then such fluids are called time dependent fluids. In such fluids viscosity of the fluid varies with time. Yoghurt, paint, gypsum paste are few examples of time dependent fluids.

1.3.2 Time independent fluids:

In the fluid, if relation between shear stress and rate of shear strain does not depend on time, then such fluids are called time independent fluids. In such fluids viscosity of the fluid does not vary with time. Water, oil etc. are time independent fluids.

1.3.3 Elastic viscous fluids:

The fluids which have viscous properties as well as elasticity are called elastic viscous fluids. Such fluids have more viscous properties but, also have partial elastic properties after deformation. Amorphous polymers, semi crystalline polymers, biopolymers are few examples of elastic viscous fluids.

1.4 Casson fluid

The fluid in which viscosity is assumed to be infinitely high when shear strain is zero and when viscosity is zero rate of shear strain is assumed to be infinity, is called Casson fluid. Most common examples of Casson fluids are honey, jelly, concentrated fruit juices etc. It possess yield stress and has a great importance in biomechanics and polymer processing industries. Casson fluid model was presented by Casson [102] for the prediction of the flow behavior of pigment – oil suspensions.

1.5 Second Grade fluid

A fluid whose stress tensor is the sum of all tensors that can be formed from the velocity field with up to two derivatives, much as a Newtonian fluid is formed from derivatives up to first order. The constitutive equation of a second grade fluid is a linear relation between the stress and the first Rivlin-Ericksen tensor. This constitutive equation has three coefficients and is used for fluids of the visco-elastic type. The governing differential equations of a second grade fluid are of higher

order than the Navier-Stokes equations and thus, in general, one needs conditions in addition to the usual adherence boundary condition.

1.6 Nanofluid

Performance and compactness of engineering equipment such as heat exchangers, nuclear reactors and electronic devices can be upgraded, if thermal conductivity of conventional fluids such as oil, water and ethylene glycol mixture is improved. Pioneering technique to improve thermal conductivity of conventional fluids, is uniform and stable suspension of nanoparticles in the fluid. Suspension of copper or alumina nanoparticles in water are examples of nanofluids. Other metallic, nonmetallic, and polymeric particles can also be added into fluids to form nanofluids. Suspension of small amount of nanoparticles in conventional fluids, increases thermal conductivity significantly.

1.7 Types of Fluid flow:

1.7.1 Steady and unsteady fluid flow

If any properties of fluids like temperature, density, pressure, velocity etc. remains unchanged with change in time, then flow is called steady with respect to that property. Means in steady flow, various fields are functions of only space co – ordinates and independent of time. Conversely, for properties of fluids varies with respect to time also, then flow is called unsteady flow. Means in unsteady flow, various fields are not only functions of space co – ordinates but also time.

1.7.2 Compressible or Incompressible fluid flow

By changing the pressure on the fluid flow, if density of the fluid changes; then fluid is called compressible and if density remains unchanged then flow is called incompressible. Most of the liquids are incompressible and many gases are compressible.

1.7.3 Rotational and Irrotational flow

If we put some object in the flow and if that object spins, then flow is considered as rotational if it doesn't spin then flow is considered irrotational. If u is a velocity vector function of fluid and if $\text{curl } u = \vec{0}$ then flow is irrotational.

1.8 Magnetohydrodynamics flow

To study many natural occurrences like swimming of fishes, flying of birds the bra Fluid dynamics is an important science used to solve many natural phenomena such as flying of birds, swimming of fishes and the development of weather conditions to be studied technically.

The study of the laws that govern the conversion of energy from one form to another, the direction in which the heat will flow, and the availability of energy to do work is the subject the Thermodynamics. The study of charge particle in motion, the forces created by electric and magnetic field, and the relationship between them give rise to the subject Electrodynamics. The collective effects of these three significant branches of science namely, Fluid dynamics, Thermodynamics and Electrodynamics give rise to the topic Magneto-fluid dynamics (MFD) which in the form of definition read as "The science of motion of electrically conducting fluid in the presence of a magnetic field". It has two subtopics: Magnetohydrodynatnics (MHD) and Magnetogasdynatnics (MGD). MHD deals with electrically conducting liquids whereas MGD deals with ionized compressible gases.

Magnetohydrodynamics (often referred to as MHD) deals with the dynamics of fluids having non-negligible electrical conductivity which interact with a magnetic field. As a result of motion of an electrically conducting fluid in the presence of a magnetic field, electric currents are induced in the fluid. An electrically conducting fluid moving in presence of a magnetic field (transverse) experiences a force called the Lorentz force. This force has a tendency to modify the initial motion of the conducting fluid. Moreover, the induced currents generate their own magnetic field, which is added to the primitive magnetic field. Thus there is an interlocking between the motion of the conductor and the electromagnetic field. MHD has several applications, namely, application in fusion research, in the field of engineering, in MHD accelerator and power generator and in causing delay in the transition from laminar to turbulent flow. First theory of laminar flow of an electrically conductive liquid in a homogenous magnetic field was introduced in 1937 by Hartman [52]. The magnetohydrodynamics is a combination of three words. Magneto means magnetic field, hydro stands for liquid and dynamics for movement. Study of motion of electrically conducting fluids in which current is induced by magnetic field, is known as magnetohydrodynamics (MHD). Examples of such fluids are plasma, electrolytes, salt water and liquid metals. Motion of a conducting fluid across magnetic field generates electric currents, which modify magnetic field; and at the same time electric currents react with magnetic field to produce

a body force, which in turn modifies the motion (Figure 1.3). This study was primarily inspired by geophysical and astrophysical problems and by problems associated with fusion reactor.

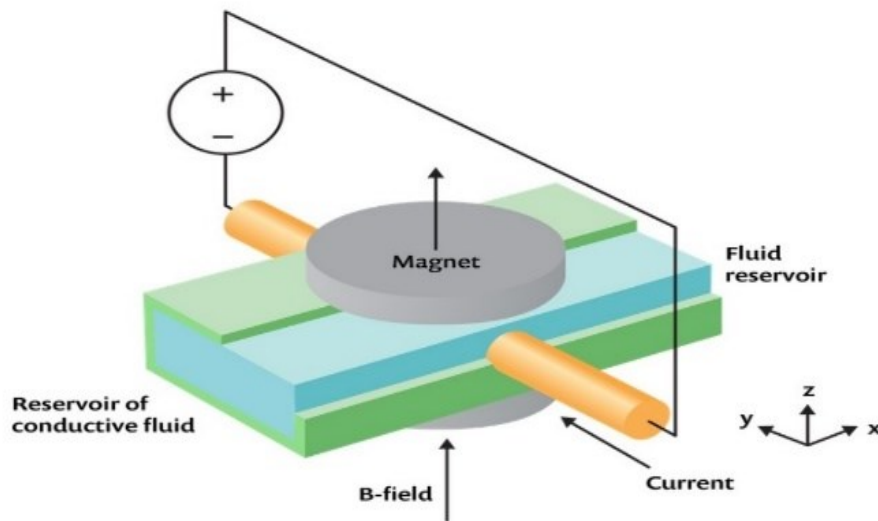


Figure 1.3: Induced current in a moving conductive fluid in the presence of a magnetic field

1.9 Heat transfer

Study of heat transfer includes thermal energy, where transfer is a result of temperature variance. In studying heat transfer, understanding of temperature distribution in a system is necessary. Heat flow takes place whenever there is a temperature gradient in a system. Once temperature distribution is known, heat flux, which is amount of heat transfer per unit area per unit time is obtained from rule connecting heat flows to temperature gradient. There are three fundamental modes of transfer of heat, which are conduction, convection and radiation.

1.10 Mass Transfer

When fluid's concentration changes in the system, Mass transfer is obtained. The alteration in concentration is a driving force for the transfer of mass. It always occurs from higher concentration to lower concentration. Visible effects of this phenomena are measurable at least on the solar surface.

The applications of mass transfer process are vital in several fields of Science, Technology and Engineering.

Mass transfer in nature are found in formation of smoke, clouds' evaporation, fog dispersion etc.

1.11 Porous Medium

Body of unconsolidated ordinary sand is the most common example of porous medium. Porous medium is present in such immeasurable holes of different shapes and sizes including “pore space” or slits among the distinct solid particles of sand. Moreover, each hole is connected by confined channels to other holes, the whole making a totally interconnected network of openings which form the channels through which the contained fluid may flow. It is the whole several interconnected of the minute openings that characterizes the ideal “porous media” assumed in this work.

1.12 Soret effect

When heat and mass transfer in a moving fluid occurs together, there is strong connection between potentials and fluxes. It is observed that temperature difference creates mass fluxes which represents the thermal-diffusion effect or Soret effect. This name Soret is named after the scientist Charles Soret.

Thermal diffusion disrupts the mixture arrangement's equality, due to which concentration is improved and temperature is reduced.

1.13 Dufour effect

Energy flux because of change in concentration of mass is called Dufour effect. In many flow problems potential of chemical differs, and this drive the flow of heat, this process is called Dufour process. This process is reciprocal phenomenon of Soret effect. The name is Dufour is named after Swiss physicist L. Dufour.

1.14 Chemical reaction

In systems where heat transfer and mass transfer occur, distribution rates can be reformed by chemical reaction. The chemical reaction is dependent on the mixed or identical reaction. Particularly, a reaction is called a first order if the rate of reaction is directly proportional to the

concentration. In nature, the existences of pure water or air are not possible. Some external mass either may be existing naturally or mixed with air or water. The existences of external mass source some kind of chemical reaction. The study of such type of chemical reaction procedures is beneficial for improving a number of chemical technologies, such as food processing, polymer production and manufacturing of ceramics or glassware. In chemical reaction problems, K_r is chemical reaction parameter. If $K_r > 0$, then it is a destructive chemical reaction (means endothermic, i.e., heat is absorbed) if $K_r < 0$, then it characterizes a propagative chemical reaction (means exothermic, i.e., heat is generated). In many chemical reaction processes, there is chemical reaction between external mass and the fluid in which the plate is moving. These processes take place in several engineering applications such as manufacturing of ceramics, food processing and polymer production.

1.15 Laplace transform technique (LTT)

Many ideas of classical analysis needed their sources in study of physical problems important to boundary value problems. Study for a solution of this initial boundary values problems leads to discovery of new mathematical tool - tools that are currently of huge practice in pure and applied mathematics, and other engineering branches, is Laplace transform.

The branches of science and engineering in which Laplace transform technique are used for solving linear system of partial differential equations with constant coefficients and ordinary differential equations in which coefficients are variables or simultaneous ordinary differential equations. Laplace transform technique can also be applied in mechanics (dynamics and statics), electrical circuits, to analysis characteristic of beam and several partial differential equations subject to initial and boundary conditions etc. Thus, it can be understood that Laplace transform has its remarkable applications in many branches of pure and applied mathematics.

1.15.1 Laplace transforms technique in MHD

The physical aspects of any fluid flow is expressed in terms of system of partial differential equation with initial and boundary condition, in which Laplace transform technique can be used properly; as it is art of substituting governing equations of fluid flow with numbers and proceeding these numbers in space and / or time into an ordinary differential equation, which can be solved by established rules and, then Inverse Laplace transforms techniques are useful to get required

results. This method is perfectly fitted for unsteady free convective MHD problems through porous medium.

1.16 Homotopy analysis method (HAM)

Two continuous functions from one topological space to another are called homotopic if one can be continuously deformed into the other, such a deformation is called a homotopy between the two functions.

The basic idea of HAM method [122] is to produce a succession of approximate solutions that tend to exact solution of the problem. Presence of auxiliary parameters and functions in approximate solution, results in production of a family of approximate solutions, rather than a single solution produced by traditional perturbation methods.

The general approach used by HAM is to solve non-linear equation,

$$\mathcal{N}(u(t)) = 0, \quad t > 0, \quad (1.01)$$

where \mathcal{N} is a nonlinear operator and $u(t)$ is unknown function of independent variable t .

1.16.1 Zero-order deformation equation

Let $u_0(t)$ denote an initial guess of exact solution of Equation (1.01), $\hbar \neq 0$ an auxiliary parameter, $H(t) \neq 0$ auxiliary function and \mathcal{L} an auxiliary linear operator with property,

$$\mathcal{L}(f(t)) = 0 \text{ when } f(t) = 0. \quad (1.02)$$

The auxiliary parameter \hbar , auxiliary function $H(t)$, and auxiliary linear operator \mathcal{L} play important roles within HAM to adjust and control convergence region of solution series. Liao [122] constructs, using $q \in [0, 1]$ as an embedding parameter, so - called zero-order deformation equation,

$$(1 - q)\mathcal{L}[\Phi(t; q) - u_0(t)] = q\hbar H(t)\mathcal{N}[\Phi(t; q)], \quad (1.03)$$

where $\Phi(t; q)$ is solution which depends on $\hbar, H(t), \mathcal{L}, u_0(t)$ and q . When $q = 0$, zero-order deformation Equation (1.03) becomes,

$$\Phi(t; 0) = u_0(t), \quad (1.04)$$

when $q = 1$, since $\hbar \neq 0$ and $H(t) \neq 0$, then Equation (1.04) reduces to,

$$\mathcal{N}[\Phi(t; 1)] = 0. \quad (1.05)$$

So, $\Phi(t; 1)$ is exactly solution of nonlinear Equation (1.01). Expanding $\Phi(t; q)$ in Taylor's series with respect to q , we have

$$\Phi(t; q) = u_0(t) + \sum_{m=1}^{\infty} q^m u_m(t), \quad (1.06)$$

where,

$$u_m(t) = \frac{1}{m!} \frac{\partial^m \Phi(t; q)}{\partial q^m} \Big|_{q=0}. \quad (1.07)$$

If power series (1.06) of $\Phi(t; q)$ converges at $q = 1$, then we get following series solution,

$$u(t) = u_0(t) + \sum_{m=1}^{\infty} u_m(t), \quad (1.08)$$

where terms $u_m(t)$ can be determined by so-called high-order deformation equations which are described below.

1.16.2 High-order deformation equation

Define vector,

$$\vec{u}_n = \{u_0(t), u_1(t), u_2(t), \dots, u_n(t)\}. \quad (1.09)$$

Differentiating Equation (1.03) m times with respect to embedding parameter q , then setting $q = 0$ and dividing them by $m!$, we have so-called m^{th} -order deformation equation,

$$\mathcal{L}[u_m(t) - \chi_m u_{m-1}(t)] = \hbar H(t) R_m(\vec{u}_m, t), \quad (1.10)$$

where

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & \text{otherwise} \end{cases} \quad (1.11)$$

$$R_m(\vec{u}_{m-1}, t) = \frac{1}{(m-1)!} \frac{\partial^{m-1} \mathcal{N}[\Phi(t; q)]}{\partial q^{m-1}} \Big|_{q=0}. \quad (1.12)$$

For any given nonlinear operator \mathcal{N} , term $R_m(\vec{u}_{m-1}, t)$ can be easily expressed by Equation (1.12). Thus, we can gain $u_1(t), u_2(t), \dots$ by means of solving linear high-order deformation Equation (1.10) one after other in order. m^{th} -order approximation of $u(t)$ is given by,

$$u(t) = \sum_{k=0}^m u_k(t). \quad (1.13)$$

Liao [122] points out that so-called generalized Taylor's series provides a way to control and adjust convergence region through an auxiliary parameter \hbar such that homotopy analysis method is particularly suitable for problems with strong nonlinearity.

1.16.3 Convergence analysis

One of chief aims of HAM method is to produce solutions that will converge in a much larger region than solutions obtained with traditional perturbation methods. Solutions obtained using this method depend on our choice of linear operator \mathcal{L} , auxiliary function $H(t)$, initial approximation $u_0(t)$ and value of auxiliary parameter \hbar .

Choice of base functions influence convergence of solution series significantly. For example, solution may be expressed as a polynomial or as a sum of exponential functions. It is expected that, base functions that more closely mimic behavior of actual solution should provide much better results than base functions whose behavior differs greatly from behavior of actual solution. Choice of a linear operator, auxiliary function, and initial approximation often determines base functions present in solution. Having selected a linear operator, auxiliary function, and an initial approximation, deformation equations can be developed and solved in series solution. Solution obtained in this way, still contains auxiliary parameter \hbar . This solution should be valid for a range of values of \hbar . In order to determine optimum value of \hbar , \hbar curves of solution are plotted. These curves are obtained by plotting partial sums $u_m(t)$ or their first few derivatives evaluated at a particular value of t against parameter \hbar . As long as equation (1.01) with given initial or boundary conditions has a unique solution, partial sums and their derivatives will converge to correct solution for all values of \hbar for which solution converges. Which means that \hbar curves will be essentially horizontal over range of \hbar for which solution converges. As long as, \hbar is chosen in this horizontal region, solution must converge to actual solution of equation (1.01).

1.17 Dimensionless parameters

Dimensionless parameters help us to understand physical importance of a particular phenomenon. Basic equations are made dimensionless using certain dependent or independent characteristic values. Some of dimensionless parameters used in thesis are clarified below.

1.17.1 Thermal Grashof number (Gr)

Thermal Grashof number (Or Grashof number) is the ratio of buoyancy to viscous force acting on a fluid. It often arises in study of situations involving free convection. Its expression is

$$Gr = \frac{g\beta_T L^3 (T_w - T_0)}{\nu^2} \quad (1.14)$$

1.17.2 Mass Grashof number (Gm)

The ratio of mass buoyancy force to hydrodynamics viscous force acting on a fluid is known as Mass Grashof number. It often arises in study of situations involving free convection and it is expressed by

$$Gm = \frac{g\beta_C L^3 (C_w - C_0)}{\nu^2} \quad (1.15)$$

1.17.3 Prandtl number (Pr)

It is defined as ratio of momentum and thermal diffusivity.

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \quad (1.16)$$

1.17.4 Schmidt number (Sc)

It is defined as ratio of momentum and mass diffusivity.

$$Sc = \frac{\nu}{D} \quad (1.17)$$

1.17.5 Magnetic parameter or Hartmann number (M)

It is defined as ratio of electromagnetic force to viscous force. It measures relative importance of drag forces resulting from magnetic induction and viscous forces in flow.

$$M = \frac{\sigma B^2 L^2}{\nu \rho} \quad (1.18)$$

1.17.6 Soret Number (Sr)

It is noticed that, mass fluxes can also be created by temperature gradients and this embodies thermal diffusion (Soret) effect. Soret number is represented by

$$Sr = \frac{D_T}{D_m} \quad (1.19)$$

1.17.7 Reynolds number (Re)

It is used to identify different flow behaviors like laminar or turbulent flow. It measure ratio of inertial force and viscous force. Mathematically,

$$Re = \frac{\rho u^2}{\frac{\mu u}{L^2}} \Rightarrow Re = Lu/\nu \quad (1.20)$$

At low Reynolds number, laminar flow arises, where viscous forces are dominant whereas at high Reynolds number, turbulent flow arises, where inertial forces are dominant.

1.17.8 Brownian diffusion coefficient (D_B)

Brownian diffusion occurs due to continuous collision between molecules and nanoparticles of fluid. Brownian diffusion coefficient D_B is given by

$$D_B = \frac{K_B T C_c}{3\pi\mu d p} \quad (1.21)$$

where K_B and C_c represent Boltzmann constant and correction factor respectively.

1.17.9 Thermophoresis diffusion coefficient (D_T)

Thermophoresis diffusion occurs when particles diffuse due to effect of temperature gradient. It is given by

$$D_T = \frac{-v_{th}T}{\nabla T} \quad (1.22)$$

where v_{th} and ∇T denote thermophoretic velocity and temperature gradient respectively.

1.17.10 Skin friction coefficient (C_f)

It occurs between solid and fluid surface through which motion of fluid becomes slow. Skin friction coefficient can be defined as,

$$C_f = -\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (1.23)$$

1.17.11 Nusselt number (Nu)

It is temperature gradient at surface. Its expression is

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \quad (1.24)$$

1.17.12 Sherwood number (Sh)

Sherwood number represents concentration gradient at surface.

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (1.25)$$

1.18 Review of relevant literature

The study of MHD flow of Non-Newtonian fluid is important phenomena in science and technology fields. In this thesis, study of one, two and three dimensional MHD flow of different types of Non-newtonian fluids likes, Casson fluid, Second grade fluid, viscoelastic fluid and Hybrid Nano-fluid with heat and mass transfer are discussed. The governing equations are convert in system of Linear or Non-linear partial differential equations. So, analytical methods or Numerical methods have applied for solving governing equations. Some of them (related to the thesis) are briefly reviewed here.

Fluid dynamics is main branch of science which is used to solve many natural phenomena such as flying of birds, swimming of fishes and the development of weather conditions to be studied technically [55]. The study of charge particle in motion, the forces created by electric and magnetic field, and the relationship between them give rise to the subject Electrodynamics. The collective effects of these three significant branches of science namely, Fluid dynamics, Thermodynamics and electrodynamics give rise to the topic Magneto-fluid dynamics (MFD) which in the form of definition read as The science of motion of electrically conducting fluid in the presence of a magnetic field. The study of Magnetohydrodynamics (MHD) flow of Non-Newtonian fluid has various application in science and engineering fields. The set of equations that describe MHD are a combination of the Navier Stokes and Maxwell's equations. Research works in the magneto hydrodynamics have been advanced significantly during the last few decades in natural sciences and engineering disciplines after the pioneer work of Hartmann [52] in liquid metal duct flows under the strong external magnetic field. Recently, the study of MHD flow done by Farman et al. [11], Kataria and Patel [42].

In modern engineering, many characteristics of flow are not understandable with the Newtonian fluid model. Hence non-Newtonian fluids theory has become useful. Non-Newtonian fluid exerts non-linear relationships between the shear stress and rate of shear strain. It has an extensive variety

of applications in engineering and industry, especially in the extraction of crude oil from petroleum products. Casson fluid is one of such fluids. Casson fluid model was introduced by Casson [102] for the prediction of the flow behavior of pigment-oil suspensions.

Second grade fluids can model many fluids such as dilute polymer solutions, slurry flows and industrial oils. Tan and Masuoka [160] examined the Stokes' first problem for a Second grade fluid and Hayat et al. [152] considered with unsteady stagnation point flow of Second grade fluid with variable free stream. Viscoelastic fluid behavior is a type of non-Newtonian fluid formed by a viscous component and an elastic one. Examples of viscoelastic fluids are paints, some biological fluids, and DNA suspensions, etc. A number of features make the viscoelastic fluids very interesting and of industrial importance. A proper understanding of viscoelasticity is key for industrial applications. Study of Viscoelastic fluids are common in very important applications discussed by Rashidi et al. [99], Turkyilmazoglu [155] and Khan et al. [162].

The study of heat transfer in boundary layer flows has many engineering applications such as in the design of thrust bearings and radial diffusers, in transpiration cooling, in drag reduction and in thermal recovery of oil. In the past many authors have studied a wide variety of flow situations. Fakour et al. [61] discussed micropolar fluid flow and heat transfer in a channel with permeable walls whereas, Ebrahimi et al. [65] studied heat transfer of fourth-grade fluid flow in the plane duct under an externally applied magnetic field with convection on walls. Recently, Nayak et al. [76], Larimi et al. [77], Sheikholeslami et al. [95], Turkyilmazoglu [96], Sheikholeslami and Seyednezhad [97], Sheikholeslami et al. [132] considered heat transfer effects on MHD flow of different fluid with different physical conditions. The convective heat transfer phenomena in nature are often attended by mass transfer. Convective mass transfer process creates the support of various procedures in the chemical engineering. This appears like sufficient purpose to contain mass transfer in heat convection as well. An analogy happens between convective mass transfer and convective heat transfer. This analogy is educationally actual significant because it provides a chance to organize the understanding of heat transfer and to learn mass transfer with the least memorization. Öztop et al. [30] studied natural convection in three-dimensional partially open enclosures. Zhou et al. [51] discussed design of microchannel heat sink with wavy channel and its time-efficient optimization with combined RSM and FVM methods whereas, Kundu [59] obtained exact solution of propagation of heat in a biological tissue subject to different surface conditions

for therapeutic applications. Recently, Rashidi et al. [81] and Turkyilmazoglu [154] considered MHD flow with heat and mass transfer.

The restriction of the conventional fluids to expedite cooling/heating rates give rise to exploration of nanofluids. Generally, water based single phase nanofluids containing nanoparticles such as CuO or Al₂O₃ are discussed. Enhancement of heat transfer is beneficial in engineering and actual world problems. To achieve this, experiments considering composite nanoparticles in place of single nanoparticle based nanofluids are performed. Consequently, investigators are fascinated towards heat transfer properties of composite nanofluids. Thermal conductivity of nanofluids is high, which is a motivation in this area, thus many investigators are doing research intensively. Choi and Eastman [130] were probably the first to employ a mixture of nanoparticles and base fluid that such fluids were designated as “nano-fluids”. Recently, many researcher like Öztop [31], Khan [47], Waini et al. [49], Oztop and Abu-Nada [46], Jusoh et al. [54], Shirvan et al. [57], Hatami et al. [62], Abolbashari et al. [66] discussed MHD flow of nanofluid with different types of physical conditions. Owing to extraordinary characteristics, recent works [10, 12, 15 – 20] are dedicated to nano-fluids. Hatami et al. [69 - 71] considered natural convection heat transfer of MHD nanofluids. Sheikholeslami [83] obtained the solution of CuO-water nanofluid free convection in a porous cavity whereas, Sheikholeslami and Oztop [84] studied MHD free convection of nanofluid in a cavity with sinusoidal walls by using CVFEM. Khashi et al [105] illustrated three-Dimensional hybrid nanofluid flow and heat transfer past a permeable stretching/shrinking Sheet. Sheikholeslami et al. [133-134] studied free/force convection MHD nanofluid considering MFD viscosity effect over a stretched surface while Abdellahoum et al. [8] discussed turbulent forced convection of nanofluid. Hayat et al. [136-144] described effects of magnetic field on different types of nanofluid flow. Miroshnichenko et al. [159] obtained results of MHD natural convection in a partially open trapezoidal cavity filled with a nanofluid whereas, Shah et al. [164] influence of Cattaneo-Christov model on Darcy-Forchheimer flow of Micropolar Ferrofluid over a stretching/shrinking sheet. The radiation effects become much vital when the difference between the surface and the ambient temperatures is extensive [127]. It has wide applications in manufacturing industries, such as the design of reliable equipment, nuclear plants, gas turbines, power plants, and various propulsion devices for aircraft and missiles. Further, the radiation effects on MHD convective flow problems are more significant in electrical power generation, solar power technology, and astrophysical ground. Recently, Patel and Mittal [44],

Sheikholeslami et al. [88], Sheikholeslami et al. [92] and Sheikholeslami et al. [94] studied thermal radiation effects on MHD flow of nanofluid. Pal et al. [109] considered thermal radiation and Ohmic dissipation effects on MHD Casson nanofluid flow over a vertical non-linear stretching surface using scaling group transformation. Heat generation/absorption played important role in MHD flow with heat and mass transfer. The study of MHD flow in the presence of heat generation and absorption done by Miroshnichenko et al. [100], Nandkeolyar et al. [111] and Thumma et al. [153]. Kandasamy et al. [58] considered thermophoresis and Brownian motion effects on MHD boundary-layer flow in the presence of thermal stratification due to solar radiation. The porous medium has a vital role in terms of controlling momentum and heat transfer in the boundary layer flow. In understanding of this, several eminent scientists are attracted towards exploration of porous medium and its effects on Newtonian and non-Newtonian fluids. Seyf et al. [29] obtained analytical solution of fluid Flow in porous media with injection/suction, whereas Xu et al. [32] illustrate effects of thermal radiation on nanofluids in porous media. Kataria and Mittal [34] obtained numerical solution of three dimensional nanofluid flow in a rotating system through porous medium. Recently, Sheremet et al. [63], Aleem et al. [64], Sheikholeslami et al. [85], Sheikholeslami [89], Sheikholeslami and Shehzad [91] and Ghasemi et al. [112] discussed MHD flow in porous medium. Study of free/force convection MHD flow of different types of physical conditions through porous medium are discussed in Rassoulinejad-Mousavi et al. [114], Rassoulinejad-Mousavi and Yaghoobi [115], Rassoulinejad-Mousavi and Abbasbandy [116-117] and Samiulhaq et al. [131]. Rashidi et al. [79] obtained analytical solutions of steady MHD convective and slip flow due to a rotating disk with viscous dissipation and Ohmic Heating. Rashidi et al. [98] considered effect of solid surface structure on the condensation flow of argon in rough nanochannels with different roughness geometries using molecular dynamics simulation. Ramped velocity is helpful to diagnose, establish treatment, determine prognosis and evaluate functioning of cardiovascular system. Some contributions regarding TT may be found in the works of Bruce [2] Myers and Bellin [101], Astrand and Rodahl [107]. Mohammadian et al. [113] studied thermal management improvement of an air-cooled high-power lithium-ion battery by embedding metal foam. Abbasbandy et al. [119] obtained numerical and analytical solutions for falkner-skam flow of MHD Oldroyd-B fluid. Silva et al. [135] studied velocity and inclination in the ramp protocol for the treadmill ergometer.