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STUDY OF SORET EFFECT ON MHD FLOW OF NON-NEWTONIAN FLUID

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STUDY OF SORET EFFECT ON MHD FLOW OF NON-NEWTONIAN FLUID

The importance of heat transfer problem comprising non-Newtonian fluid has developed noticeably due to vide range of applications of non-Newtonian fluids. The indulgent of physics in such fluid flows have instant effects on polymer treating, varnish, ink-jet painting, micro fluids, geological flows, colloidal suspensions, liquid crystals, animal blood, turbulent shear flow etc. Non-Newtonian flow embrace shear-thinning, shear thickness, viscoelasticity etc. This motivates this chapter to include the study of non-Newtonian flows

2.1 Introduction of the problem:

The attentiveness in heat transfer problem comprising non-Newtonian fluid has developed substantially as such fluids have applications in almost all industrial and engineering processes. Distinctive non-Newtonian flow comprise shear-thinning, viscoelasticity and shear thickness etc. Non - Newtonian fluid flow posts exceptional challenges to the engineers, scientists, mathematicians and physicists. Hayat at el. [151] have derived solution for mixed convective MHD flow of peristalsis of Non-Newtonian nanomaterial with zero mass flux conditions. Hayat at el. [148] have studied Non-Newtonian fluid. Asghar at el. [129] have studied MHD flow of Non-Newtonian fluid in specific condition. Unlike Newtonian fluids, a single model is not sufficient to describe all characteristics of non-Newtonian fluid flow. It is known that the prevailing equations of non-Newtonian fluids are extremely non-linear and of higher order. Thus it is observed that the exact solutions are challenging. Governing equations are linearized and system of partial differential equations have been transformed into ordinary differential equations and then analytical solutions may be obtained [38 - 41]. The porous medium has a vital role in terms of controlling momentum and heat transfer in the boundary layer flow. In understanding of this, several eminent scientists are attracted towards exploration of porous medium and its effects on Newtonian and non-Newtonian fluids.

In most of the research works, the thermal-diffusion effects are insignificant due to smaller order of magnitude. However, an fascinating macroscopically phenomenon known as Soret effects have important effects on the thermal-diffusion processes. The combined buoyancy effect escalates due to the deviation of density with temperature and henceforward the temperature is influenced.

Consideration of thermal radiation in heat and mass transfer problems have attracted researchers due to its applications in advanced energy conversion systems operating at high temperature such as gas turbines, propulsion devices for aircraft, satellites and space vehicles, nuclear power plants etc

2.2 Novelty of the chapter:

The purpose of this chapter is to investigate impact of magnetic field on fluid flow, heat and mass transfer. Novelty of this chapter is analytic study of MHD non-Newtonian fluid flow with ramped boundary conditions. The derived ordinary differential equations are solved using the Laplace transform. The effects of the pertinent parameters governing the problem are discussed.

2.3 Mathematical Formulation of the Problem:

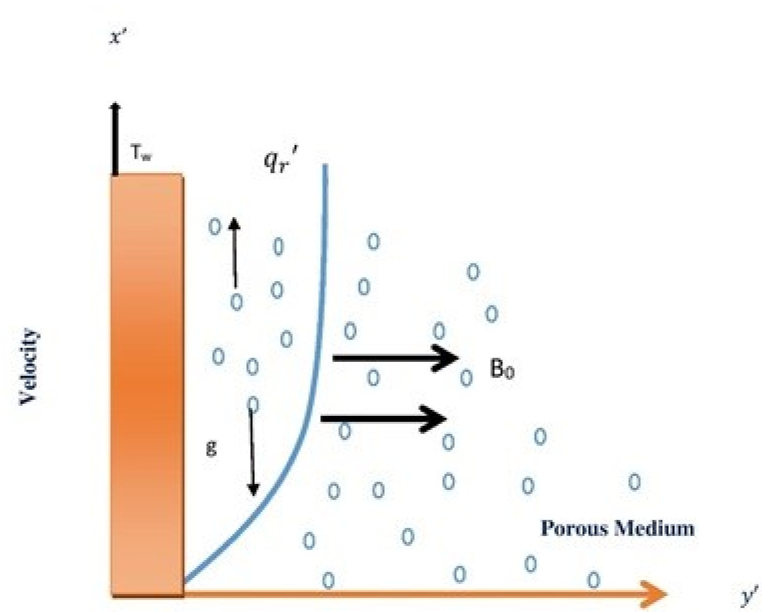

Figure 2.1: Physical sketch of the problem

Sketch of physical problem is drawn in Figure 2.1. Axes are chosen as follows. $x' - axis$ which is drawn vertically is the wall and $y' - axis$ is drawn horizontally. As shown in that figure, magnetic field of strength B_0 is in opposite direction to fluid flow. When time $t' \leq 0$, plate and fluid are stationary having surface concentration C'_∞ and constant temperature of fluid and the plate is assumed to be T'_∞. During time between $0 < t' \leq t_0$ velocity, temperature and concentration are $u'_\infty + (u'_w - u'_\infty)\ {t'}/{t_0}$, $T'_\infty + (T'_w - T'_\infty)\ {t'}/{t_0}$ and $C'_\infty + (C'_w - C'_\infty)\ {t'}/{t_0}$ respectively, whereas for time $t' > t_0$, they remain constant u'_w, T'_w and C'_w respectively. Effect of viscous dissipation induced by magnetic and electrical field are neglected. Here flow is considered as one dimensional laminar flow, and the fluid is incompressible Non-newtonian fluid. The equations which are governed for all these assumptions, are derived using Boussinesq's approximation. They are as follows.

$$\frac{\partial u'}{\partial t'} = \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{\mu\phi}{\rho k_1'} u' + g\beta'_T (T' - T'_\infty) + g\beta'_C (C' - C'_\infty) \tag{2.1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r'}{\partial y'} \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + D_T \frac{\partial^2 T'}{\partial y'^2} - k_2' (C' - C'_\infty) \tag{2.3}$$

With following initial and boundary conditions:

$$u' = 0,\ T' = T'_\infty,\ C' = C'_\infty;\ as\ \ y' \geq 0\ and\ \ t' \leq 0,$$

$$u' = \begin{cases} u'_\infty + (u'_w - u'_\infty)\, {t'}/{t_0} & if\ \ 0 < t' < t_0 \\ u'_w & if\ \ \ t' \geq t_0 \end{cases};\ \ as\ \ t' > 0\ and\ y' = 0\ ,$$

$$T' = \begin{cases} T'_\infty + (T'_w - T'_\infty)\, {t'}/{t_0} & if\ \ 0 < t' < t_0 \\ T'_w & if\ \ \ t' \geq t_0 \end{cases};\ \ as\ \ t' > 0\ and\ y' = 0\ ,$$

$$C' = \begin{cases} C'_\infty + (C'_w - C'_\infty)\, {t'}/{t_0} & if\ \ 0 < t' < t_0 \\ C'_w & if\ \ \ t' \geq t_0 \end{cases};\ \ as\ \ t' > 0\ and\ y' = 0\ ,$$

$$u' \to 0, T' \to T'_\infty,\ \ C' \to C'_\infty;\ as\ \ y' \to \infty\ and\ t' \geq 0 \tag{2.4}$$

Since the fluid is Non-Newtonian, by Rosseland approximation [127]

$$\frac{\partial q_r'}{\partial y'} = -4a^*\sigma^*(T'^4_\infty - T'^4) \tag{2.5}$$

Where σ^* and a^* are Stefan Boltzmann constant and absorption coefficient respectively.

We expand T'^4 in Taylor's series about T'_∞ and neglecting higher order terms, we get

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{2.6}$$

Inputting (2.6) in (2.5) and subsequently putting the resultant in (2.2) we get

$$\frac{\partial T'}{\partial t'} = \frac{k_4}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} 16a^*\sigma^* T'^3_\infty (T' - T'_\infty) \tag{2.7}$$

Introducing the following dimensionless quantities:

$$y = \frac{y'}{U_0 t_0},\ \ u = \frac{u'}{U_0},\ \ t = \frac{t'}{t_0}, \theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, C = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)},\ \ Gr = \frac{\nu g\beta'_T(T'_w - T'_\infty)}{U_0^3}$$

$$Gm = \frac{vg\beta'_C(C'_w - C'_\infty)}{U_0^3}, M^2 = \frac{\sigma B_0^2 v}{\rho U_0^2}, P_r = \frac{\rho v C_p}{k}, R = \frac{16\, a\, \sigma v^2 T'^3_\infty}{\mathrm{k}\, U_0^2},\ Sc = \frac{v}{D_M}$$

$$Kr = \frac{vk'_2}{U_0^2},\ Sr = \frac{D_T(T'_w - T'_\infty)}{v(C'_w - C'_\infty)} \tag{2.8}$$

Substituting (2.7) and (2.8) in the equations (2.1), (2.2) and (2.3) and dropping out the " ′ " notation (for simplicity) from all variables we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{k_1}\right)u + G_r\theta + G_m C \tag{2.9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1+R}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{2.10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} + Sr\frac{\partial^2 \theta}{\partial y^2} - krC \tag{2.11}$$

With initial and boundary condition

$$u = \theta = C = 0\,, y \geq 0, t \leq 0\,,$$

$$u = \begin{cases} t, & 0 < t \leq 1 \\ 1 & t > 1 \end{cases}, \quad at\ \ y = 0, \quad t > 0$$

$$\theta = \begin{cases} t, & 0 < t \leq 1 \\ 1 & t > 1 \end{cases}, \ at\ \ y = 0,\ t > 0$$

$$C = \begin{cases} t, & 0 < t \leq 1 \\ 1 & t > 1 \end{cases} at\ \ y = 0,\ t > 0\,,$$

$$u \to 0, \theta \to 0, C \to 0 \quad at\ \ y \to \infty,\ t > 0 \tag{2.12}$$

2.4 Solution of the Problem:

Found analytic solutions for fluid velocity; temperature and concentration. They are obtained from equations (2.9) to (2.11) with initial and boundary conditions (2.12) using the Laplace transform technique.

2.4.1 Solution of the problem for ramped velocity, ramped wall temperature and ramped surface concentration:

$$\theta(y,t) = f_7(y,t) - f_7(y,t-1)H(t-1) \tag{2.13}$$

$$C(y,t) = [g_4(y,t) - g_5(y,t)] - [g_4(y,t-1) - g_5(y,t-1)]H(t-1) \tag{2.14}$$

$$u(y,t) = f_2(y,t) - f_2(y,t-1)H(t-1) + h_1(y,t) - h_1(y,t-1)H(t-1) \tag{2.15}$$

2.4.2 Solution of the problem for constant velocity, ramped temperature and ramped surface concentration:

In order to understand effects of ramped velocity of the fluid flow, we must compare our results with constant velocity. In this case, initial and boundary conditions are same as Eq. (2.12) except only $u = 1$ for $y = 0,\ t > 0$. Velocity $u(y,t)$ found for this case using Laplace Transform will be

$$u(y,t) = f_1(y,t) + h_1(y,t) - h_1(y,t-1)H(t-1) \tag{2.16}$$

Where,

$$h_1(y,t) = g_1(y,t) + g_2(y,t) + g_3(y,t) \tag{2.17}$$

$$g_1(y,t) = a_{29}f_1(y,t) + a_{30}f_2(y,t) + a_{31}f_3(y,t) + a_{32}f_4(y,t) + a_{33}f_5(y,t) \tag{2.18}$$

$$g_2(y,t) = a_{34}f_6(y,t) - a_{17}f_7(y,t) - a_{31}f_8(y,t) + a_{28}f_9(y,t) \tag{2.19}$$

$$g_3(y,t) = a_{35}f_{10}(y,t) + a_{20}f_{11}(y,t) + a_{32}f_{12}(y,t) + a_{25}f_{13}(y,t) \tag{2.20}$$

$$g_4(y,t) = a_{37}f_{10}(y,t) - a_{36}f_{13}(y,t) \tag{2.21}$$

$$g_5(y,t) = a_{36}f_6(y,t) - a_{36}f_9(y,t) \tag{2.22}$$

$$f_1(y,t) = \frac{1}{2}\left[e^{-y\sqrt{a_1}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{a_1 t}\right) + e^{y\sqrt{a_1}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{a_1 t}\right)\right] \tag{2.23}$$

$$f_2(y,t) = \frac{1}{2}\left[\left(t - \frac{y}{2\sqrt{a_1}}\right) e^{-y\sqrt{a_1}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{a_1 t}\right) + \left(t + \frac{y}{2\sqrt{a_1}}\right) e^{y\sqrt{a_1}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{a_1 t}\right)\right] \tag{2.24}$$

$$f_3(y,t) = \frac{e^{a_8 t}}{2}\left[e^{-y\sqrt{\frac{1}{a_8}(a_1+a_8)}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(a_1+a_8)t}\right) + e^{y\sqrt{\frac{1}{a_8}(a_8-a_1)}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(a_1-a_8)t}\right)\right] \tag{2.25}$$

$$f_4(y,t) = \frac{e^{a_{13} t}}{2}\left[e^{-y\sqrt{\frac{1}{a_{13}}(a_1+a_{13})}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(a_1-a_{13})t}\right) + e^{y\sqrt{\frac{1}{a_{13}}(a_1-a_{13})}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(a_1-a_{13})t}\right)\right] \tag{2.26}$$

$$f_5(y,t) = \frac{e^{a_5 t}}{2}\left[e^{-y\sqrt{\frac{1}{a_5}(a_1+a_5)}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(a_1+a_5)t}\right) + e^{y\sqrt{\frac{1}{a_5}(a_5-a_1)}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(a_1-a_5)t}\right)\right] \tag{2.27}$$

$$f_6(y,t) = erfc\left(\frac{1}{2}\sqrt{\frac{1}{a_2 t}}\, y\right) \tag{2.28}$$

$$f_7(y,t) = \left(\frac{y^2}{2a} + t\right) erfc\left(\frac{y}{2\sqrt{a_2 t}}\right) - \frac{y}{2}\frac{\sqrt{t}}{\sqrt{a_2 \pi}} e^{-\frac{y^2}{4a_2 t}} \tag{2.29}$$

$$f_8(y,t) = \frac{e^{bt}}{2}\left[e^{-y\sqrt{\frac{a_8}{a_2}}}\, erfc\left(\frac{y\sqrt{a_2}}{2\sqrt{t}} - \sqrt{a_8 t}\right) + e^{y\sqrt{\frac{a_8}{a_2}}}\, erfc\left(\frac{y\sqrt{a_2}}{2\sqrt{t}} + \sqrt{a_8 t}\right)\right] \tag{2.30}$$

$$f_9(y,t) = \frac{e^{bt}}{2}\left[e^{-y\sqrt{\frac{a_5}{a_2}}}\, erfc\left(\frac{y\sqrt{a_2}}{2\sqrt{t}} - \sqrt{a_5 t}\right) + e^{y\sqrt{\frac{a_5}{a_2}}}\, erfc\left(\frac{y\sqrt{a_2}}{2\sqrt{t}} + \sqrt{a_5 t}\right)\right] \tag{2.31}$$

$$f_{10}(y,t) = \frac{1}{2}\left[e^{-y\sqrt{kr\,Sc}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{kr\,t}\right) + e^{y\sqrt{kr\,Sc}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{kr\,t}\right)\right] \tag{2.32}$$

$$f_{11}(y,t) = \frac{1}{2}\left[\left(t - \frac{y\sqrt{Sc}}{2\sqrt{kr}}\right)e^{-y\sqrt{Sc\,kr}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{kr\,t}\right)\right.$$
$$\left. + \left(t + \frac{y\sqrt{Sc}}{2\sqrt{kr}}\right)e^{y\sqrt{Sc\,kr}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{kr\,t}\right)\right] \tag{2.33}$$

$$f_{12}(y,t) = \frac{e^{-a_{13}t}}{2}\left[e^{-y\sqrt{Sc(kr-3)}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(kr-a_{13})t}\right)\right.$$
$$\left. + e^{y\sqrt{Sc(kr-a_{13})}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(kr-a_{13})t}\right)\right] \tag{2.34}$$

$$f_{13}(y,t) = \frac{e^{a_5 t}}{2}\left[e^{-y\sqrt{Sc(kr+a_5)}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(kr+a_5)t}\right)\right.$$
$$\left. + e^{y\sqrt{Sc(kr+a_5)}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(kr+a_5)t}\right)\right] \tag{2.35}$$

2.5 Results and Discussion:

Inspected fluid velocity, temperature and concentration for various parameters like Magnetic field parameter, Grashof number, mass Grashof number, thermal radiation parameter, chemical reaction parameter, Soret effect and Prandtl number. For plotting graphs as shown in figures 2.2 – 2.13, change is made in values of one parameter at a time and all remaining parameters remain fixed.

The Figure 2.2 shows effect of Magnetic field parameter M on velocity profiles for both velocity conditions. It is seen that velocity decreases with increment in M values. Lorentz force induced on boundary is the reason for such decrement. Permeability of porous medium k has positive impact on both velocity conditions which is reflected in Figure 2.3. Figure2.4 indicates that different values of Schmidt number Sc have reciprocal effect on concentration. Figure 2.5 and Figure 2.6 are about effects of radiation R on velocity and temperature for both ramped and constant velocity conditions. It can be observed that temperature and velocity have affirmative correlation. The increment in temperature with increment of radiation is natural and when we increase radiation, particles which holds the bonding cracks, which induces velocity.

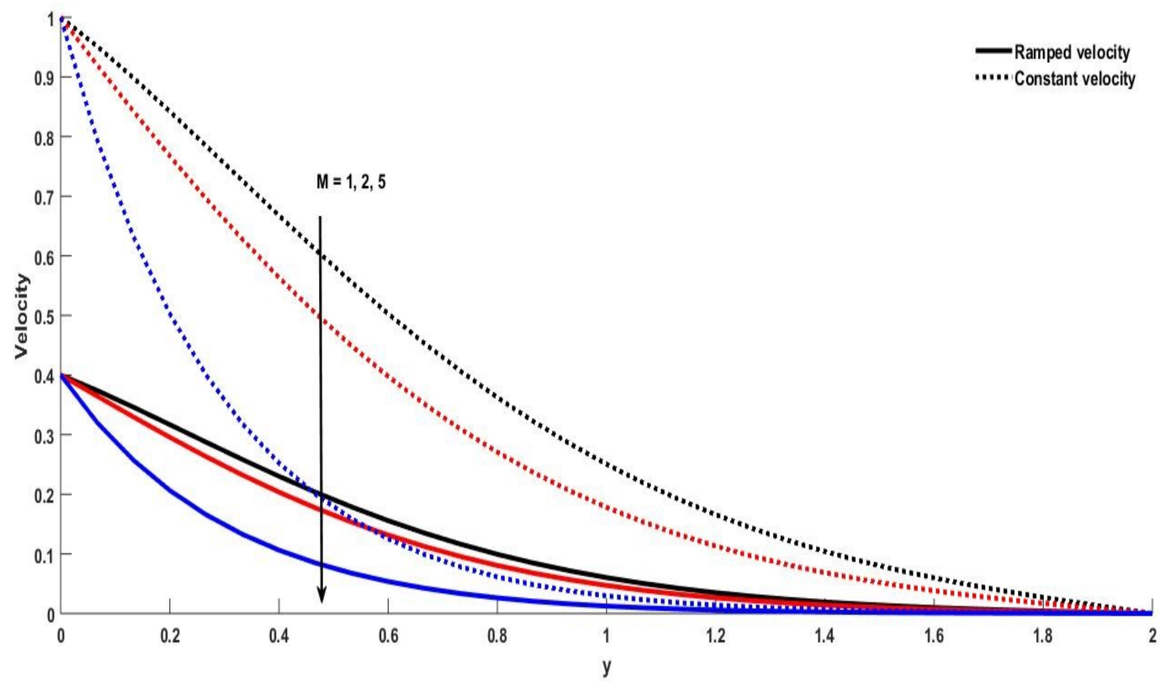

Figure 2.2: Velocity profile for different values of y and M for $k = 0.8, Pr = 10,$
$Sc = 6.2, Gr = 3, Gm = 2, R = 5, kr = 10, Sr = 5$

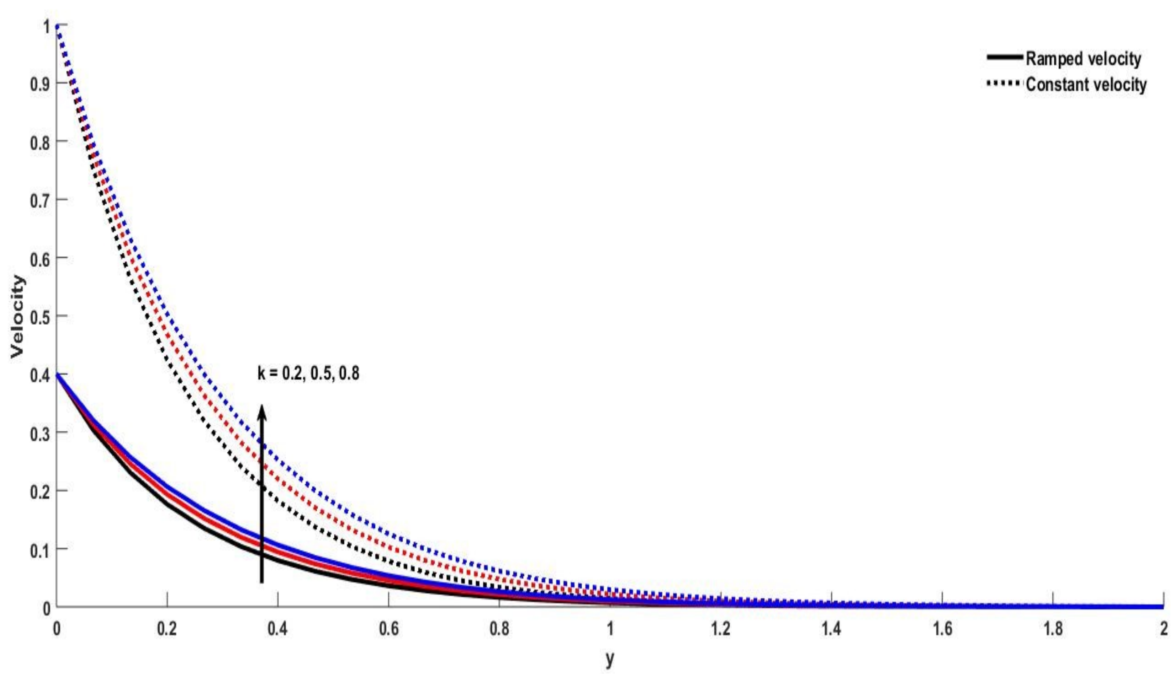

Figure 2.3.: Velocity profile for different values of y and k for $M = 5, Pr = 10,$
$Sc = 6.2, Gr = 3, Gm = 2, R = 5, kr = 10, Sr = 5$

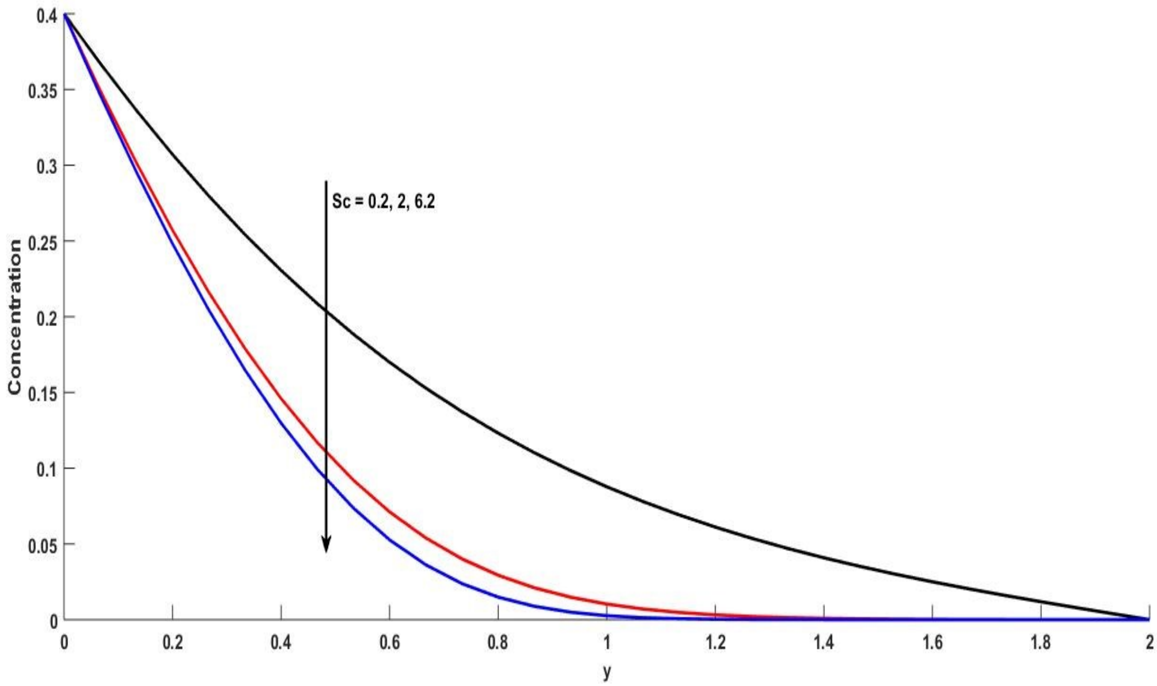

Figure 2.4: Velocity profile for different values of y and Sc for $M = 5, k = 0.8,$
$Pr = 10, Gr = 3, Gm = 2, R = 5, kr = 10, Sr = 5$

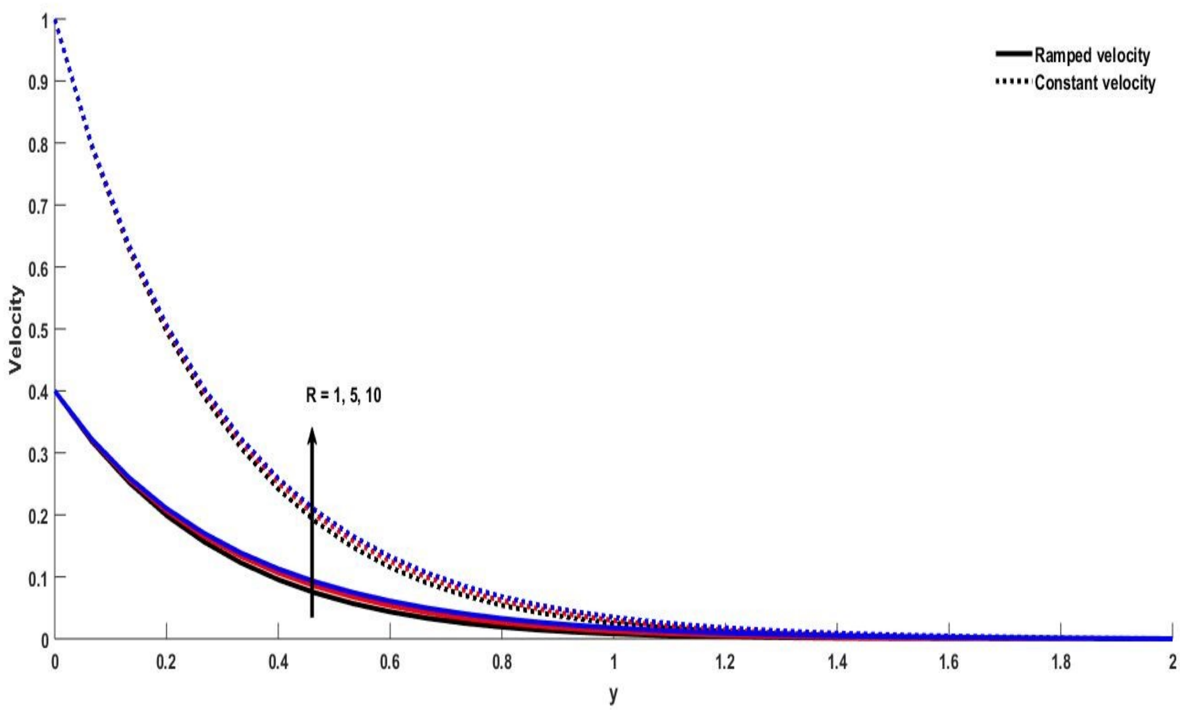

Figure 2.5: Velocity profile for different values of y and R for $M = 5, k = 0.8, Pr = 10,$
$Sc = 6.2, Gr = 3, Gm = 2, kr = 10, Sr = 5$

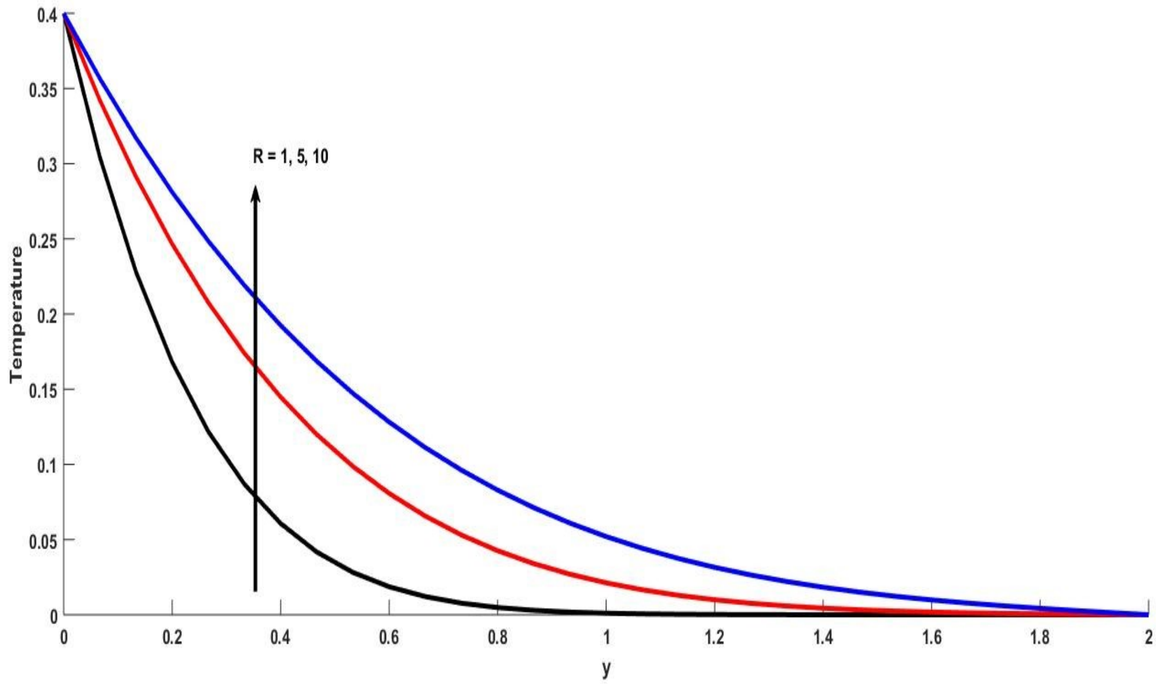

Figure 2.6: Temperature profile for different values of θ and R for $M = 5, k = 0.8$, $Pr = 10, Sc = 6.2, Sr = 3, Gm = 2, kr = 10, Sr = 5$

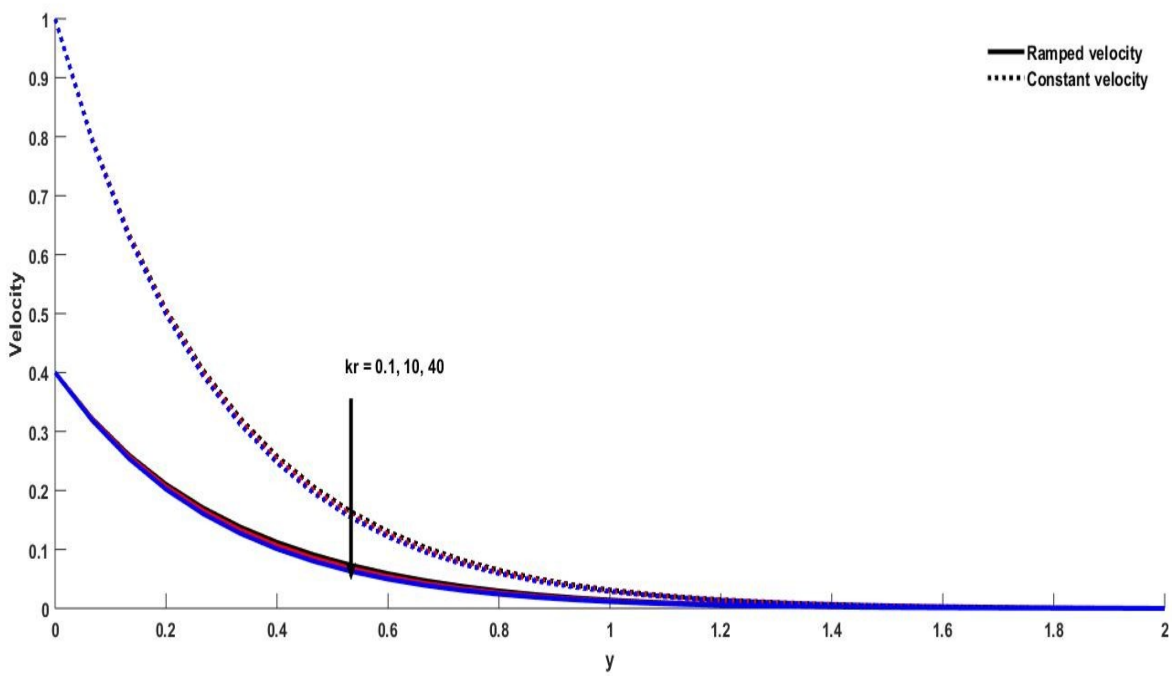

Figure 2.7: Velocity profile for different values of y and Kr for $M = 5, k = 0.8$, $Pr = 10, Sc = 6.2, Gr = 3, Gm = 2, kr = 10, Sr = 5$

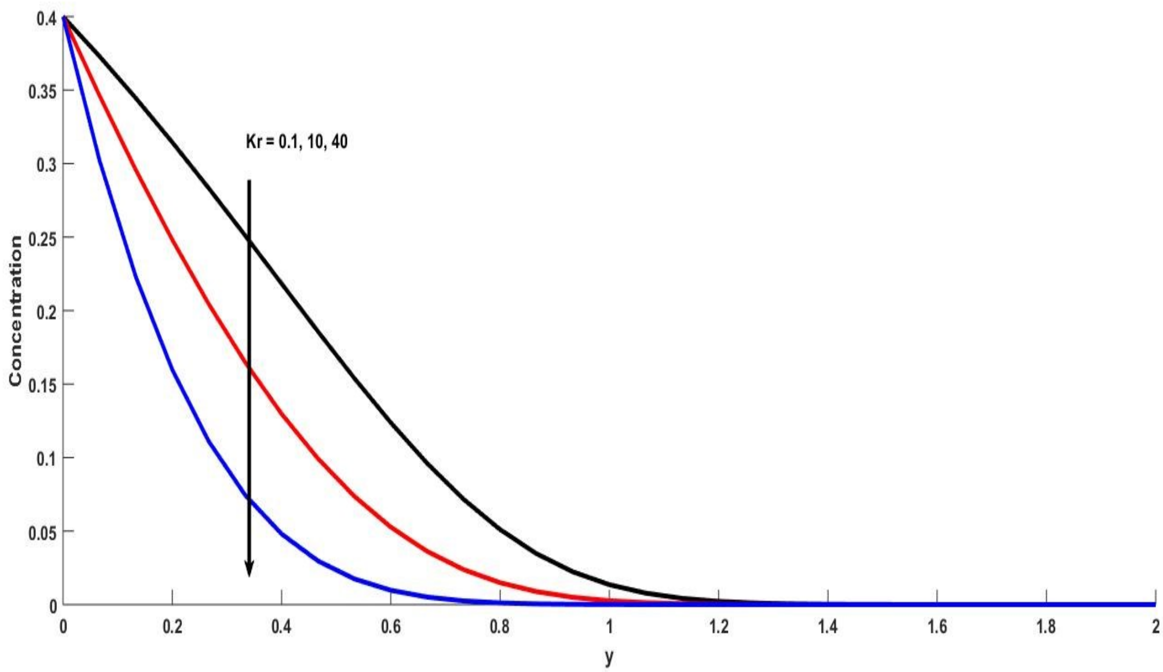

Figure 2.8: Concentration profile for different values of C and Kr for $M = 5, k = 0.8,$ $Pr = 10, Sc = 6.2, Gr = 3, gm = 2, R = 5, Sr = 5$

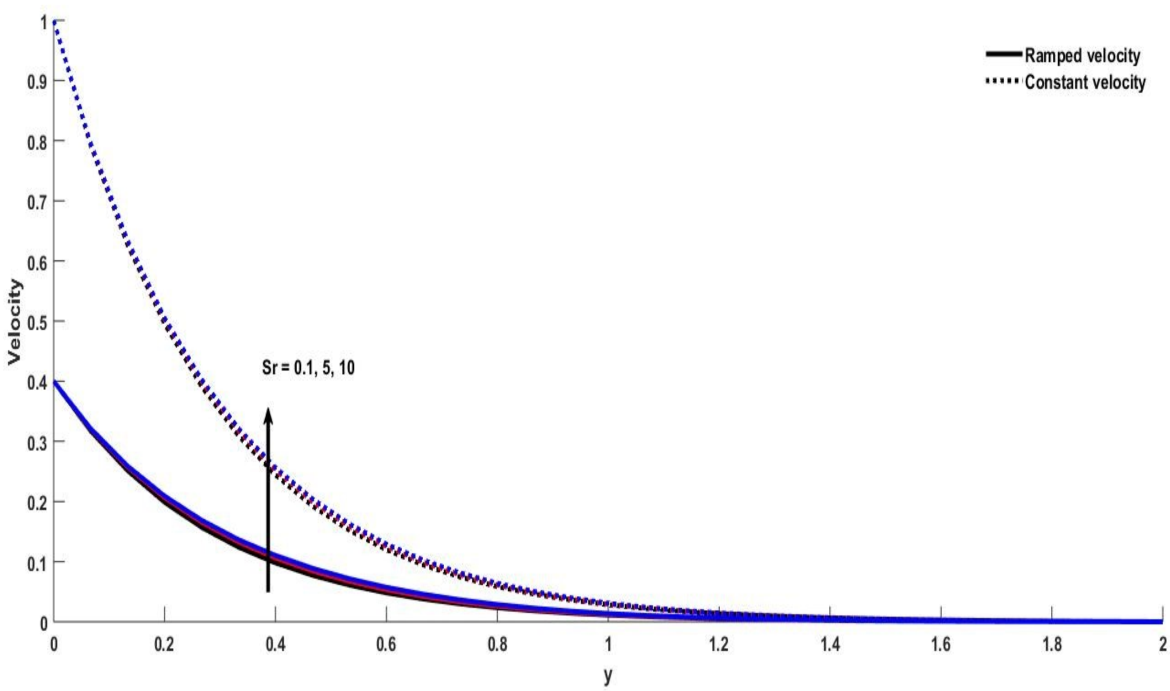

Figure 2.9: Velocity profile for different values of y and Sr for $M = 5, k = 0.8, Pr = 10,$ $Sc = 6.2, Gr = 3, Gm = 2, R = 5, kr = 10$

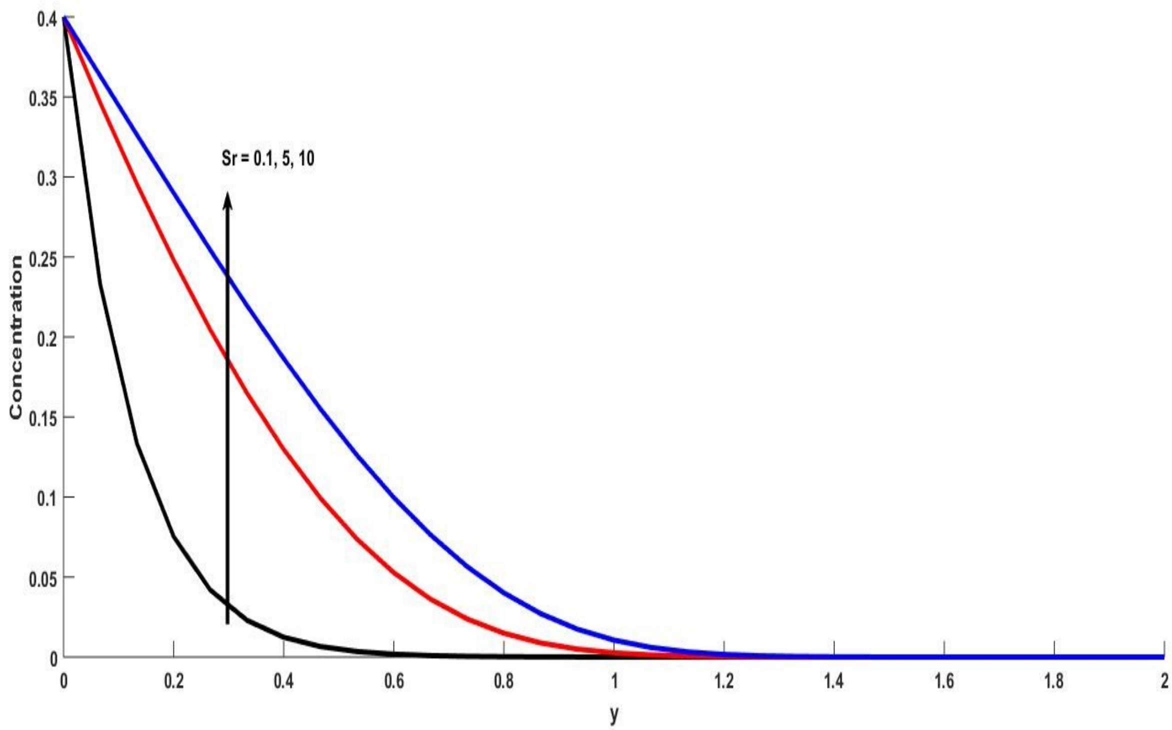

Figure 2.10: Concentration profile u for different values of C and Sr for $M = 5, k = 0.8,$ $Pr = 10, Sc = 6.2, Gr = 3, Gm = 2, R = 5, kr = 10$

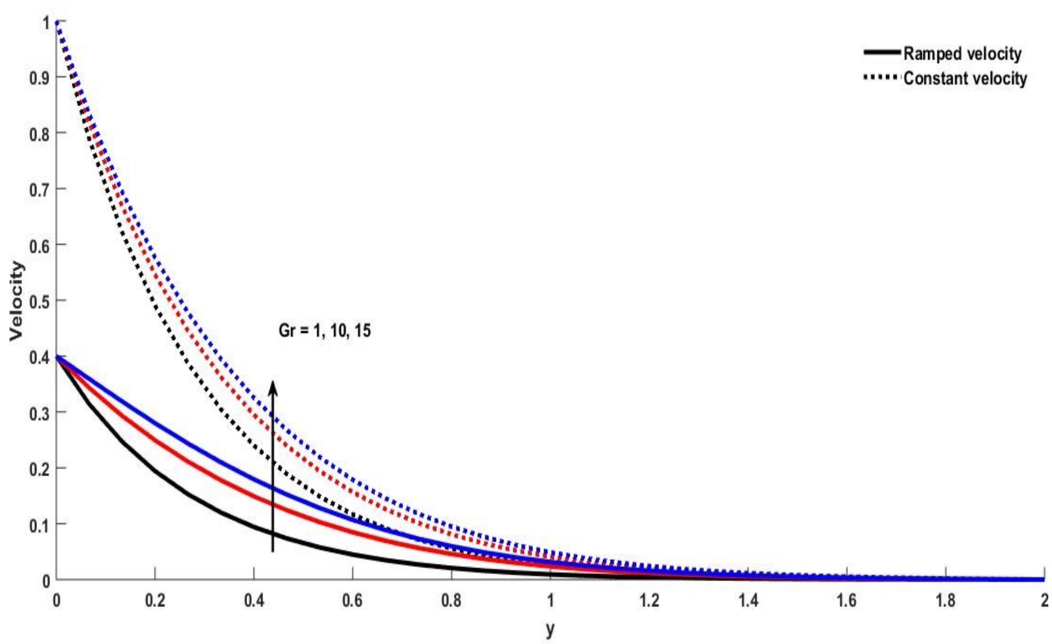

Figure 2.11: Velocity profile for different values of y and Gr for $M = 5, k = 0.8, Pr = 10,$ $Sc = 6.2, Gm = 2, R = 5, kr = 10, Sr = 5$

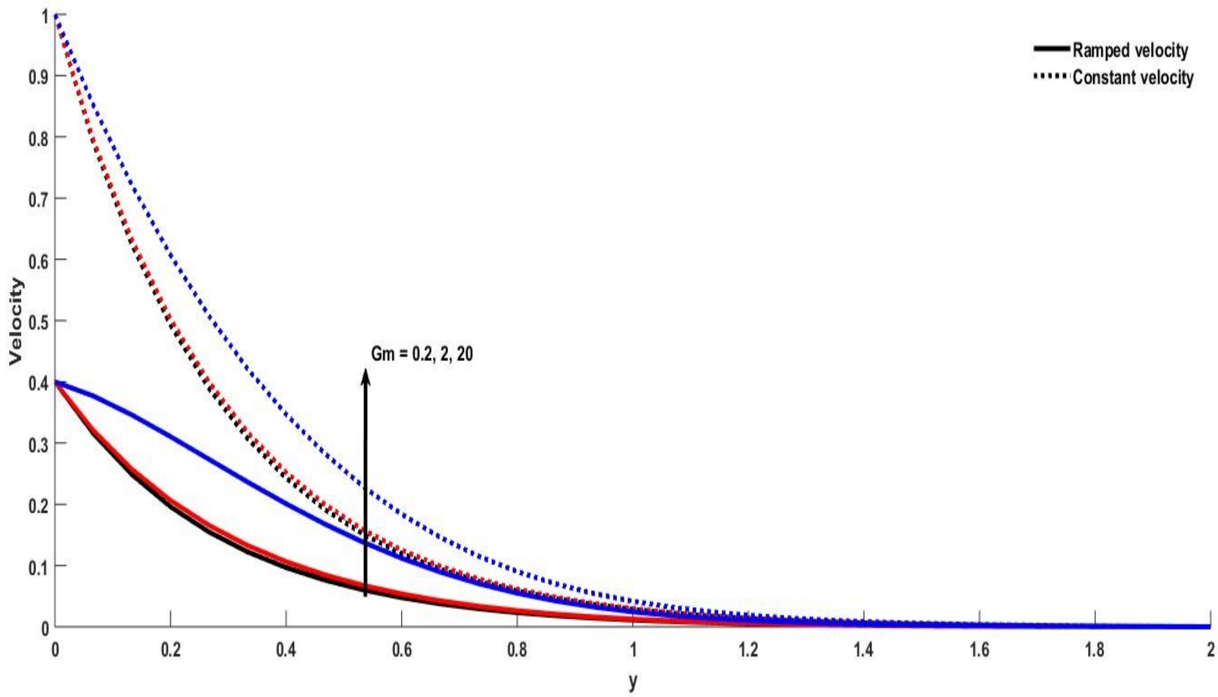

Figure 2.12: Velocity profile for different values of y and Gm for $M = 5, k = 0.8,$
$Pr = 10, Sc = 6.2, Gr = 3, R = 5, kr = 10, Sr = 5$

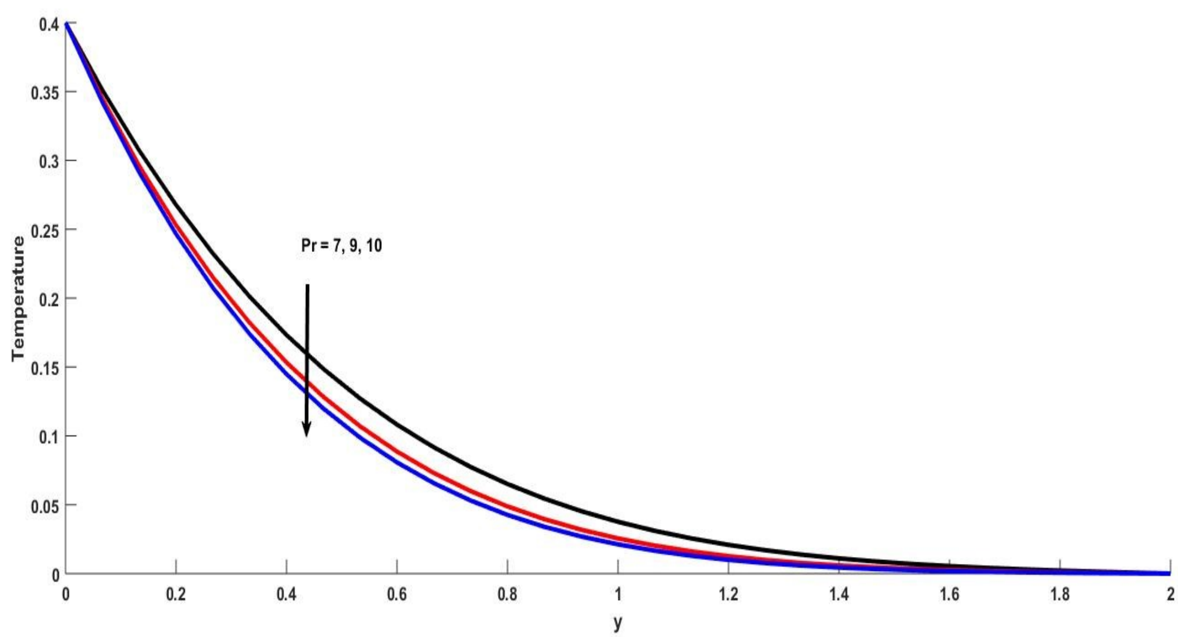

Figure 2.13: Temperature profile for different values of θ and Pr for $M = 5, k = 0.8,$
$Sc = 6.2, Gr = 3, Gm = 2, R = 5, kr = 10, Sr = 5$

The results of Chemical reaction parameter Kr on velocity and concentration profiles are shown in Figure 2.7 and Figure 2.8 respectively. It is evident from graphs that Kr limits both the velocity and concentration. Buoyancy force on particle is the reason for such fall. It is seen from Figure 2.9 and Figure 2.10 that when we increase Sr, velocity and concentration also increased. Effects of Grashof number Gr and mass Grashof number Gm has positive impact on velocity as described in Figure 2.11 and Figure2.12. Figure 2.13 shows that increment in Pr will reduce the temperature.

2.6 Conclusion:

The concept of this research is to get analytical solution for MHD flow of Non-Newtonian fluid passing through a vertical plate with ramped fluid velocity and observe various effects like effects of radiation, chemical reaction and Soret effect. Results are derived for constant and variable velocity.

Key remarks for the conclusions can be summarized as:

- Magnetic field parameter M, and chemical reaction parameter Kr have impeding effects with constant or ramped velocity.
- Permeability of porous medium k and Thermal radiation parameter R have affirmative correlation with velocity.
- Temperature of the fluid also have positive impact on thermal radiation parameter R but, negative impact with Pr.
- Concentration profile decreases if there is increment in chemical reaction parameter Kr.