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HEAT GENERATION EFFECTS ON SECOND GRADE FLUID

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HEAT GENERATION EFFECTS ON SECOND GRADE FLUID FLOW

Various substances like such as penetrating muds, clay coatings, oils, grease, molten polymers and emulsions are regarded as non-Newtonian fluids. A single model cannot be employed to justify properties of non-Newtonian fluids completely, they are more complicated compared to Newtonian fluids and this makes classification of non-Newtonian fluids tough. The present chapter is concerned with the study of heat generation/absorption effect on unsteady natural convective MHD Second grade fluid flow past an oscillating vertical plate in presence of thermal radiation and chemical reaction.

3.1 Introduction of the problem:

On broader sense, classification is given by (i) shear stress depending only on the rate of shear; (ii) relation between shear stress and shear rate depends on time; (iii) the visco-elastic fluids, which possess both elastic and viscous properties. Thus different non-Newtonian models are proposed.

In the problems having slower flow in viscoelastic sense, the second order constitutive equations are used in calculations.

The equation of motion of incompressible second grade fluids, in general, is of higher order than the Navier–Stokes equation. The Navier–Stokes equation is a second-order partial differential equation, but the equation of motion of a second-order fluid is a third-order partial differential equation. A marked difference between the case of the Navier–Stokes theory and that for fluids of second grade is that, ignoring the non-linearity in the Navier–Stokes equation does not lower the order of the equation, however, ignoring the higher order non-linearities in the case of the second grade fluid, reduces the order of the equation. Tan and Masuoka [160] considered the Stokes' first problem for a second grade fluid and Rashidi et al. [78] dealt with squeezing flow of a second-grade fluid. Hayat et al. [149] studied MHD flow of second grade fluid in porous channel whereas Hatat et al. [146] solved MHD transient rotating flow of second grade fluid. Hayat et al. [152] derived influenced of heat transfer in second grade fluid.

In a comprehensive sort of industrial applications, necessary temperature modifications around the adjoining fluid and the surface does exist. These involve the consideration of heat generation that effects heat transfer. The presence and inference of heat generation gained substantial interest, largely as it bears importance in engineering systems, such as thermal insulation, cooling of atomic reactors and geothermal supplies etc.

3.2 Novelty of the chapter:

Purpose of this chapter is to investigate exact solution of heat generation/absorption effect on unsteady natural convective MHD Second grade fluid flow. Such study may find application in fire dynamics in insulations and geothermal energy systems etc.

3.3 Mathematical Formulation of the Problem:

Fig. 1 gives sketch of the physical problem. Coordinate system is selected in such a way that $x' - axis$ is taken as the wall which is in the vertical direction and $y' - axis$ is horizontal direction.

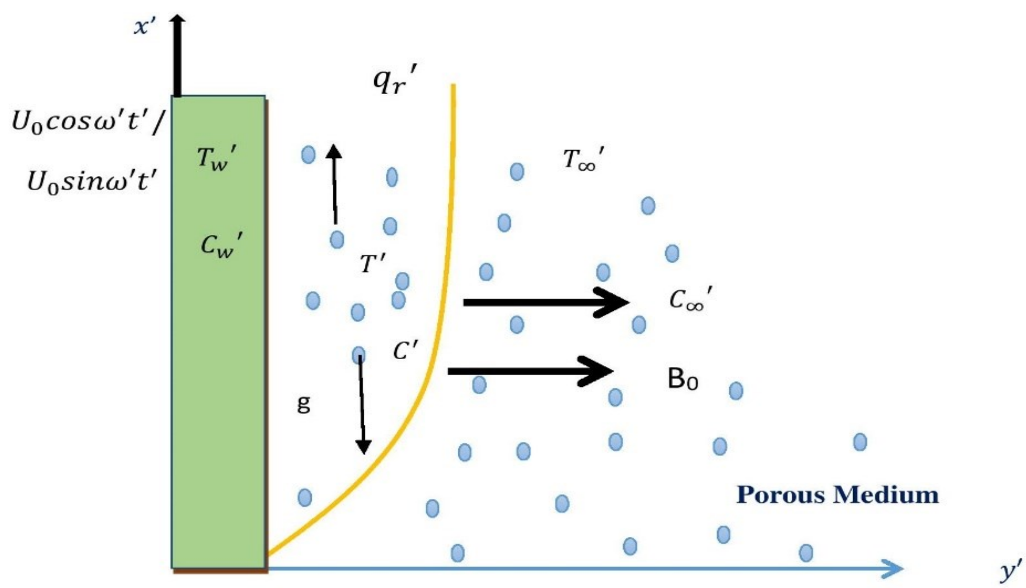

Figure 3.1: Physical sketch of the problem

As described in that figure, there exists a magnetic field with strength B_0 in transverse direction to the flow. Initially, at time $t' \leq 0$, both the fluid and the plate are at rest having a constant temperature T'_∞ and the surface concentration is assumed to be C'_∞ respectively. At time $t' > 0$, the temperature of the plate is either increased or decreased to $T'_\infty + (T'_w - T'_\infty)\ {t'}/{t_0}$ when $t' \leq\ t_0$. For $t' > t_0$, it is maintained constant T'_w. Mass transfer level at the wall surface is elevated or reduced to $C'_\infty + (C'_w - C'_\infty)\ {t'}/{t_0}$ when $t' \leq\ t_0$. For $t' > t_0$ it is maintained constant C'_w. Viscous dissipation effect, induced magnetic and electrical field effects are neglected. In MHD flow one of the body force term is the Lorentz force. The formula of Lorentz force is $J \times B = \sigma {B_0}^2 V$, where B is the total magnetic field, J is the current density, σ is electrical conductivity of the fluid and V is the velocity vector field.

Governing equations of Boussinesq's approximation under above assumptions are as follows.

$$\frac{\partial u'}{\partial t'} = \left(\nu + \frac{\alpha_1}{\rho}\frac{\partial}{\partial t'}\right)\frac{\partial^2 u'}{\partial y'^2} + g\beta'_T(T' - T'_\infty) - \frac{\sigma {B_0}^2}{\rho}u' - \frac{\emptyset}{k_1'}\left(\nu + \frac{\alpha_1}{\rho}\frac{\partial}{\partial t'}\right)u' + g\beta'_C(C' - C'_\infty) \tag{3.1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r'}{\partial y'} + \frac{Q_0}{\rho c_p}(T' - T'_\infty) \tag{3.2}$$

$$\frac{\partial C'}{\partial t'} = D_M\frac{\partial^2 C'}{\partial y'^2} - k_2'(C' - C'_\infty) \tag{3.3}$$

With following initial and boundary conditions:

$$u' = 0,\quad T' = T'_\infty,\quad C' = C'_\infty;\ as\ \ y' \geq 0\ and\ \ t' \leq 0$$

$$u' = \frac{U_0 \cos\omega' t'}{U_0 \sin\omega' t'\ as}\ t' > 0\ and\ y' = 0\ ,$$

$$T' = \begin{cases} T'_\infty + (T'_w - T'_\infty)\,{t'}/{t_0} & if\ \ 0 < t' < t_0 \\ T'_w & if\ \ \ t' \geq t_0 \end{cases},$$

$$C' = \begin{cases} C'_\infty + (C'_w - C'_\infty)\,{t'}/{t_0} & if\ \ 0 < t' < t_0 \\ C'_w & if\ \ \ t' \geq t_0 \end{cases};\ y' = 0$$

$$u' \to 0, T' \to T'_\infty,\ \ C' \to C'_\infty;\ as\ \ y' \to \infty\ and\ t' \geq 0 \tag{3.4}$$

Using the Rosseland approximation [127], the radiative heat flux term is given by.

$$q_r' = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{3.5}$$

Where σ^* and k^* are Stefan Boltzmann constant and mean absorption coefficient respectively. Assuming that the temperature difference between the fluid within the boundary layer and free stream is small, so T'^4 can be expressed as a linear function of the temperature, we expand T'^4 about T_∞' about Taylor's series and neglecting higher order terms, we get

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{3.6}$$

Thus we have

$$\frac{\partial q_r'}{\partial y'} = -\frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{3.7}$$

Using equations (3.6) and (3.7) in equation (3.3), we get

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho c_p}\frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2} + \frac{Q_0}{\rho c_p}(T' - T'_\infty) \tag{3.8}$$

Introducing the following dimensionless quantities:

$$y = \frac{U_0 y'}{v}, u = \frac{u'}{U_0}, t = \frac{t' U_0^2}{v},\ \ \theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)},\ C = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)}$$

Using equation (3.8) and dimensionless quantities, equations (3.1 – 3.4) becomes

$$\frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} - c\frac{\partial u}{\partial t} - bu + Gr\,\theta + Gm\,C = 0 \tag{3.9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1+R}{Pr}\frac{\partial^2 \theta}{\partial y^2} + H\theta \tag{3.10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - krC \tag{3.11}$$

With initial and boundary conditions

$$u = \theta = C = 0, \qquad y \geq 0, t \leq 0$$

$$u = cos\omega t/sin\omega t,\ \theta = \begin{cases} t, & 0 < t \leq 1 \\ 1 & t > 1 \end{cases},\ C = \begin{cases} t, & 0 < t \leq 1 \\ 1 & t > 1 \end{cases} \quad at\ \ y = 0,\ t > 0$$

$$u \to 0, \theta \to 0, C \to 0 \quad at\ \ y \to \infty,\ t > 0 \tag{3.12}$$

Where,

$$\alpha = \frac{U_0^2 \alpha_1}{\rho\, v^2}, Gr = \frac{g v \beta'_T (T'_w - T'_\infty)}{U_0^3}, M^2 = \frac{\sigma B_0^2\, v}{\rho U_0^2}, \frac{1}{k_1} = \frac{v^2\, \emptyset}{U_0^2 k'_1},$$

$$Gm = \frac{v\, g\beta'_C (C'_w - C'_\infty)}{U_0^3}, P_r = \frac{\rho v C_p}{k}, R = \frac{16\sigma^* T'^3_\infty}{3k\, k^*}, H = \frac{Q_0 v}{U_0^2 \rho C_p}, Sc = \frac{v}{D_M},$$

$$Kr = \frac{v\, k_2'}{U_0^2}, c = 1 + \frac{\alpha}{k_1}, b = M^2 + \frac{1}{k_1}$$

3.4 Solution of the Problem:

Exact solution for fluid velocity; Temperature and Concentration is obtained for equations (3.9) to (3.11) with initial and boundary condition (3.12) using the Laplace transform technique.

3.4.1 Solution of the Problem for ramped wall temperature and ramped surface concentration:

$$\theta(y,t) = f_7(y,t,-H,a_1) - f_7(y,t-1,H,a_1)H(t-1) \quad (3.13)$$

$$C(y,t) = f_7(y,t,Kr,a_2) - f_7(y,t-1,Kr,a_2)H(t-1) \quad (3.14)$$

$$u(y,t) = [g_1(y,t) + g_2(y,t,-H,a_1) - g_2(y,t-1,-H,a_1)H(t-1) + g_3(y,t,Kr,a_2) + g_3(y,t-1,Kr,a_2)H(t-1)] \quad (3.15)$$

3.4.2 Solution of the Problem for isothermal temperature and ramped surface concentration:

In order to understand effects of ramped temperature of the plate on the fluid flow, we must compare our results with isothermal temperature. In this case, the initial and boundary conditions are the same excluding Eq. (3.12) that becomes $\theta = 1$ at $y = 0, t \geq 0$.

$$\theta(y,t) = f_3(y,t,-H,a_1) \quad (3.16)$$

$$C(y,t) = f_7(y,t,Kr,a_2) - f_7(y,t-1,Kr,a_2)H(t-1) \quad (3.17)$$

$$u(y,t) = [g_1(y,t) + g_4(y,t,-H,a_1) + g_3(y,t,Kr,a_2) + g_2(y,t-1,Kr,a_2)H(t-1)] \quad (3.18)$$

Where

$$f_1(y,t,a) = L^{-1}\left(\frac{e^{\sqrt{\frac{c\,s+b}{\alpha\,s+1}}}}{s+a}\right) \quad (3.19)$$

$$f_2(y,t) = \frac{c}{\alpha} e^{-t/\alpha} \int_0^\infty erfc\left(\frac{y}{2\sqrt{z}}\right) e^{-c\,z/\alpha} I_0\left(\frac{2}{\alpha}\sqrt{(c-\alpha b)zt}\right) dz + \frac{b}{\alpha}\int_0^\infty \int_0^t erfc\left(\frac{y}{2\sqrt{z}}\right) e^{\frac{-cz+s}{\alpha}} I_0\left(\frac{2}{\alpha}\sqrt{(c-\alpha b)zs}\right) ds\, dz \quad (3.20)$$

$$f_3(y,t,a,b) = \frac{1}{2}\left[e^{-y\sqrt{b}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{bt}\right) + e^{y\sqrt{b}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{bt}\right)\right] \quad (3.21)$$

$$f_4(t) = a_9 + a_{10}\, e^{b_1 t} + a_{11}\, e^{b_2 t} \quad (3.22)$$

$$f_5(t) = a_{12} + a_{13}\, e^{b_3 t} + a_{14}\, e^{b_4 t} \quad (3.23)$$

$$f_6(y,t,a,b) = f_2(y,t) - f_3(y,t,a,b) \quad (3.24)$$

$$f_7(y,t,a,b) = \frac{1}{2}\left[\left(t - \frac{y}{2\sqrt{b}}\right) e^{-y\sqrt{b}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{bt}\right) + \left(t + \frac{y}{2\sqrt{b}}\right) e^{y\sqrt{b}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{bt}\right)\right] \quad (3.25)$$

$$f_8(t) = a_{15}\, e^{b_1 t} + a_{16}\, e^{b_2 t} \quad (3.26)$$

$$g_2(y,t,a,b) = \int_0^t f_6(y,u,a,b) f_4(t-u)\, du \quad (3.27)$$

$$g_3(y,t,a,b) = \int_0^t f_6(y,u,a,b) f_5(t-u)\, du \quad (3.28)$$

$$g_4(y,t,a,b) = \int_0^t f_6(y,u,a,b) f_8(t-u)\ du \quad (3.29)$$

3.4.3 Nusselt Number, Sherwood Number and Skin friction:

Expressions of Nusselt Number Nr, Sherwood Number Sh and Skin friction τ are calculated from equations (3.13 – 3.18) using the relation

$$N_r = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0},\ s_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \ and\ \tau_w(t) = -\tau(y,t)\ at\ y=0, \tau(y,t) = \left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial y}\bigg|_{y=0} \quad (3.30)$$

3.4.3.1 For ramped wall temperature and ramped surface concentration:

$$Nr = I_7(t,-H,a_1) - f_7(t-1,H,a_1)H(t-1) \quad (3.31)$$

$$Sh = I_7(t,Kr,a_2) - f_7(t-1,Kr,a_2)H(t-1) \quad (3.32)$$

$$\frac{\partial u}{\partial y}\bigg|_{y=0} = [I_9(t) + I_{10}(t,-H,a_1) - I_{10}(t-1,-H,a_1)H(t-1) + I_{11}(t,Kr,a_2) + I_{11}(t-1,Kr,a_2)\ H(t-1)] \quad (3.33)$$

3.4.3.2 For isothermal temperature and ramped surface concentration:

$$Nr = I_3(t,-H,a_1) \quad (3.34)$$

$$Sh = I_7(t,Kr,a_2) - I_7(t-1,Kr,a_2)H(t-1) \quad (3.35)$$

$$\frac{\partial u}{\partial y}\bigg|_{y=0} = [I_9(t) + I_{12}(t,-H,a_1) + I_{11}(t,Kr,a_2) + I_{11}(t-1,Kr,a_2)H(t-1)] \quad (3.36)$$

3.5 Results and Discussion:

Graphs for the fluid velocity, temperature and concentration for several values of Second grade fluid with diffusivity α, Magnetic field parameter M, thermal radiation parameter R, chemical reaction parameter Kr and Heat generation/absorption parameter H described in Figures. 3.2–3.10. Figure 3.2 describes effect of thermal diffusivity α on velocity for constant and variable wall temperature. It is seen that velocity falls with increment in values of α. It is also observed that, the boundary layer thickness appraises with reduction in diffusivity.

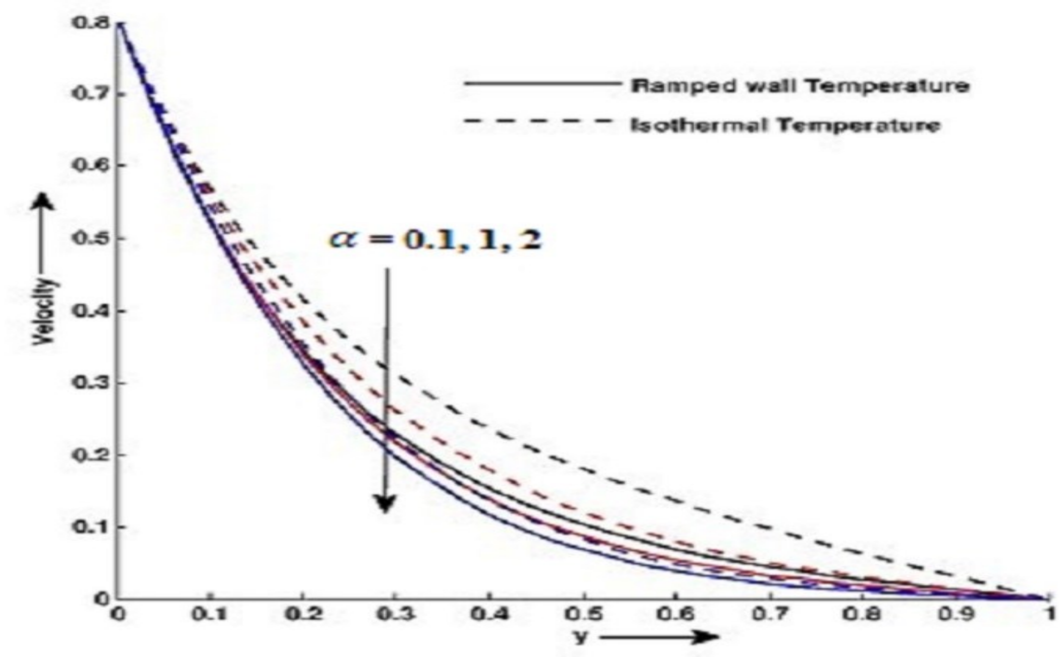

Figure 3.2: Velocity profile for different values of y and α at $k = 0.8, M = 5, Pr = 7,$ $Sc = 0.66, Gm = 4, Gr = 5, Kr = 5, H = 3, R = 5$ and $t = 0.4$

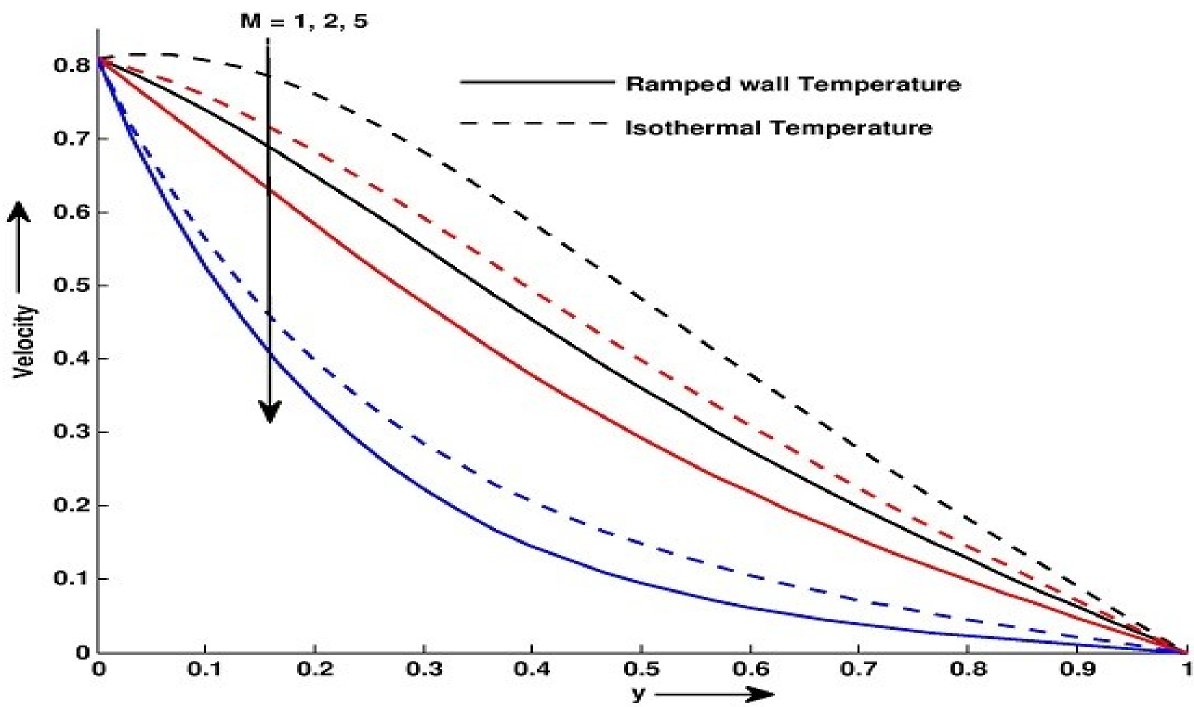

Figure 3.3: Velocity profile for different values of y and M at $k = 0.8, \alpha = 0.5, Pr = 7,$ $Sc = 0.66, Gm = 4, Gr = 5, Kr = 5, H = 3, R = 5$ and $t = 0.4$

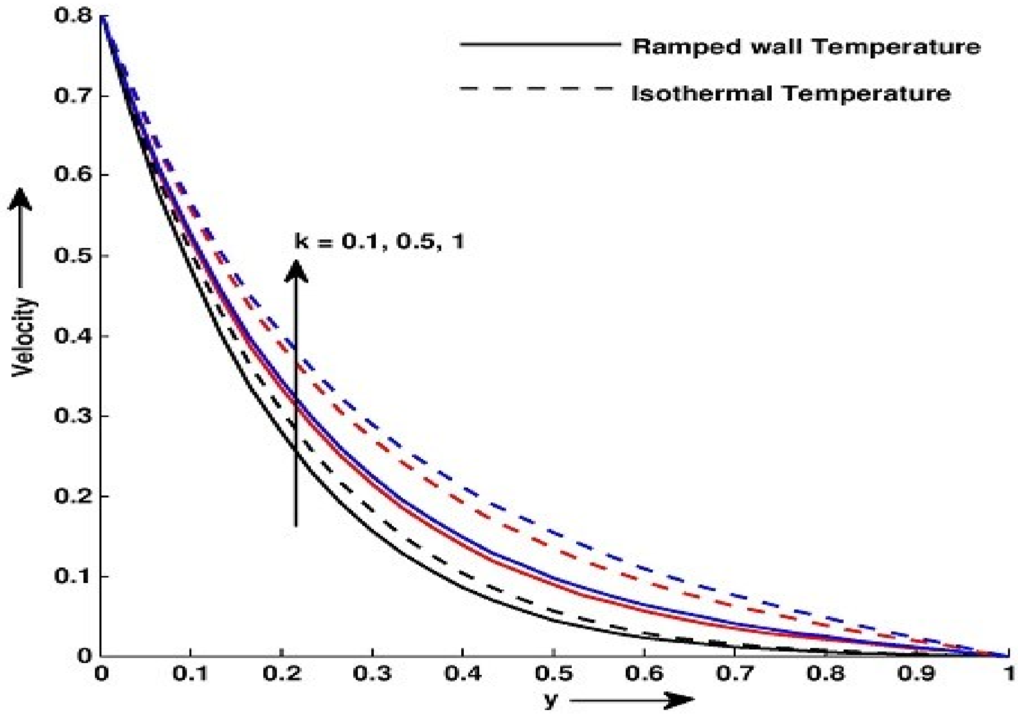

Figure 3.4: Velocity profile for different values of y and k at $M = 5, \alpha = 0.5, Pr = 7,$ $Sc = 0.66, Gm = 4, Gr = 5, Kr = 5, H = 3, R = 5\ and\ t = 0.4$

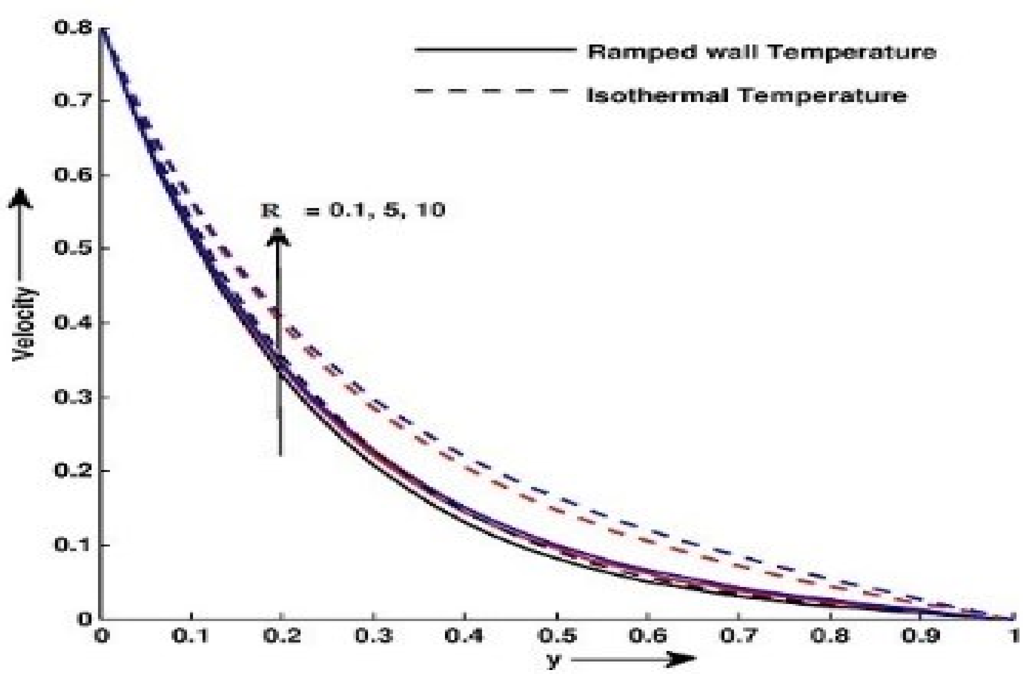

Figure 3.5: Velocity profile for different values of y and R at $M = 5, \alpha = 0.5, Pr = 7,$ $Sc = 0.66, Gm = 4, Gr = 5, Kr = 5, H = 3, k = 0.8\ and\ t = 0.4$

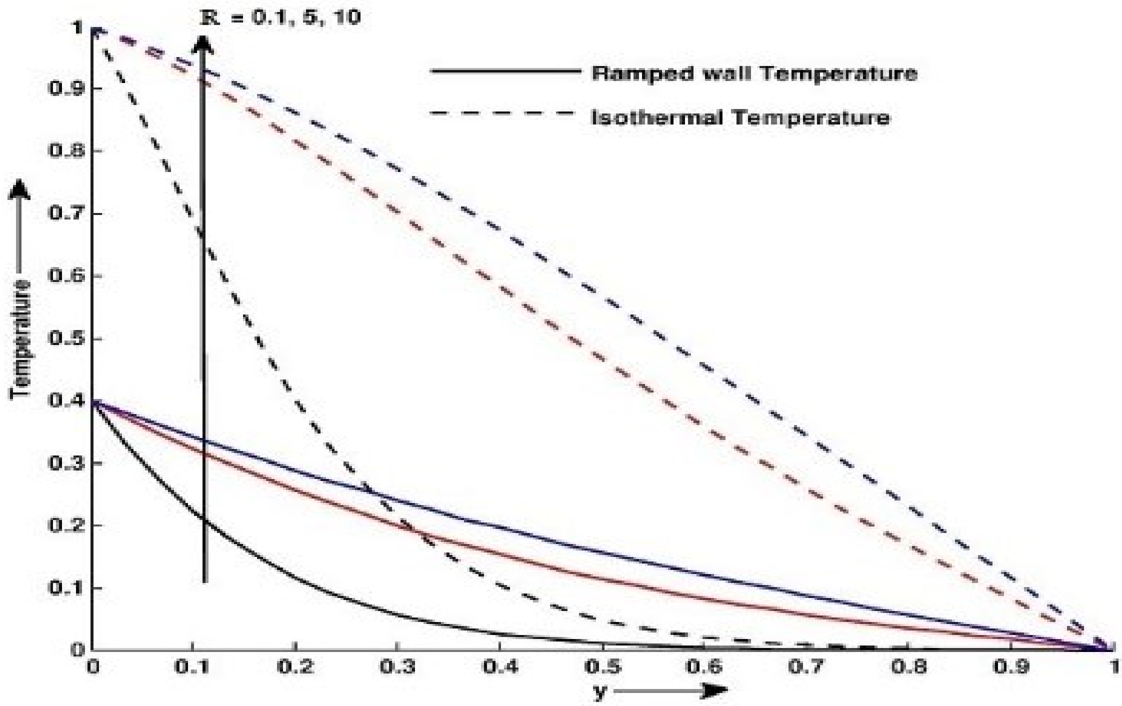

Figure 3.6: Temperature profile for different values of y and R at $Pr = 7, H = 3$ and $t = 0.4$

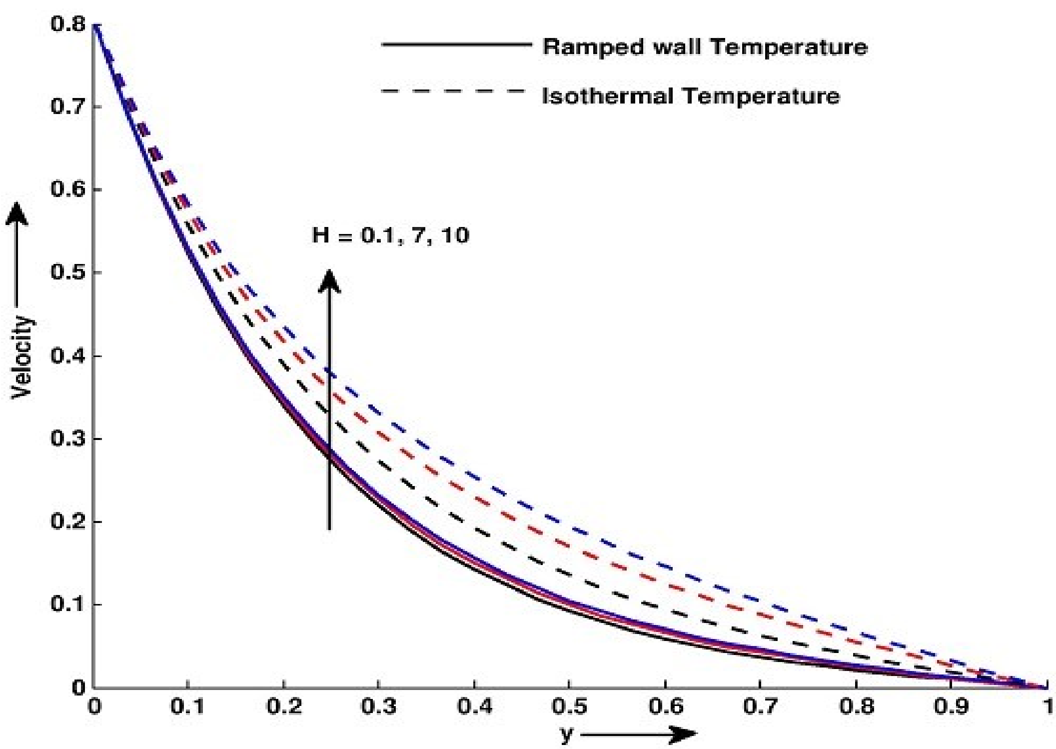

Figure 3.7: Velocity profile for different values of y and H at $M = 5, \alpha = 0.5, \Pr = 7,$ $Sc = 0.66, Gm = 4, Gr = 5, Kr = 5, R = 5, k = 0.8$ and $t = 0.4$

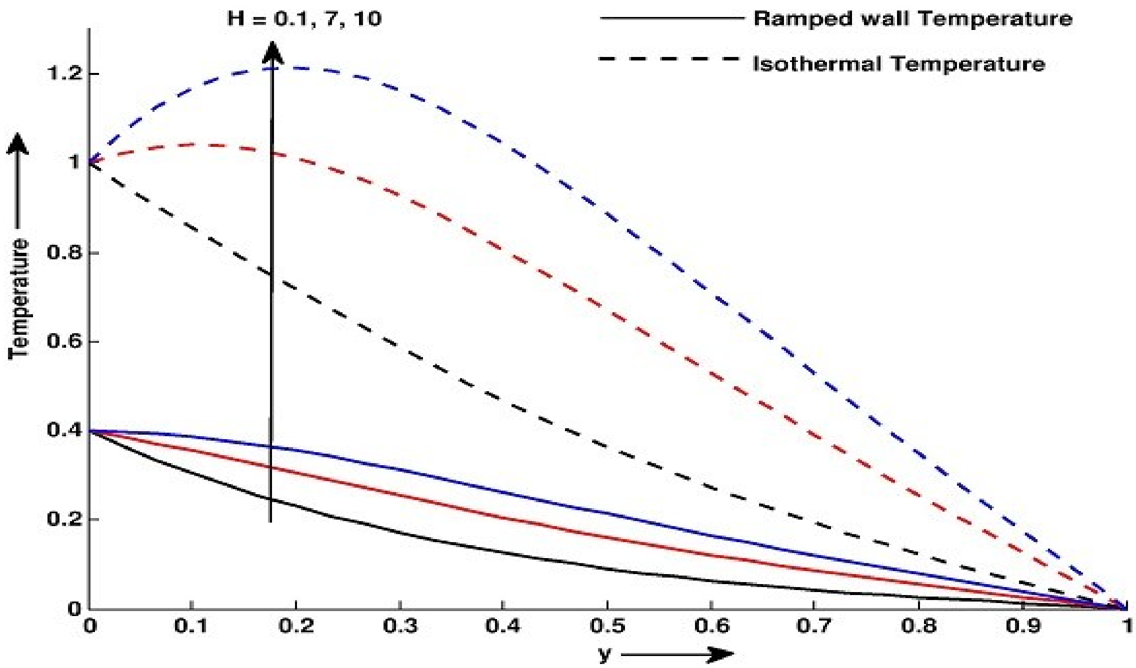

Figure 3.8: Temperature profile for different values of y and H at $Pr = 7, R = 5$ and $t = 0.4$

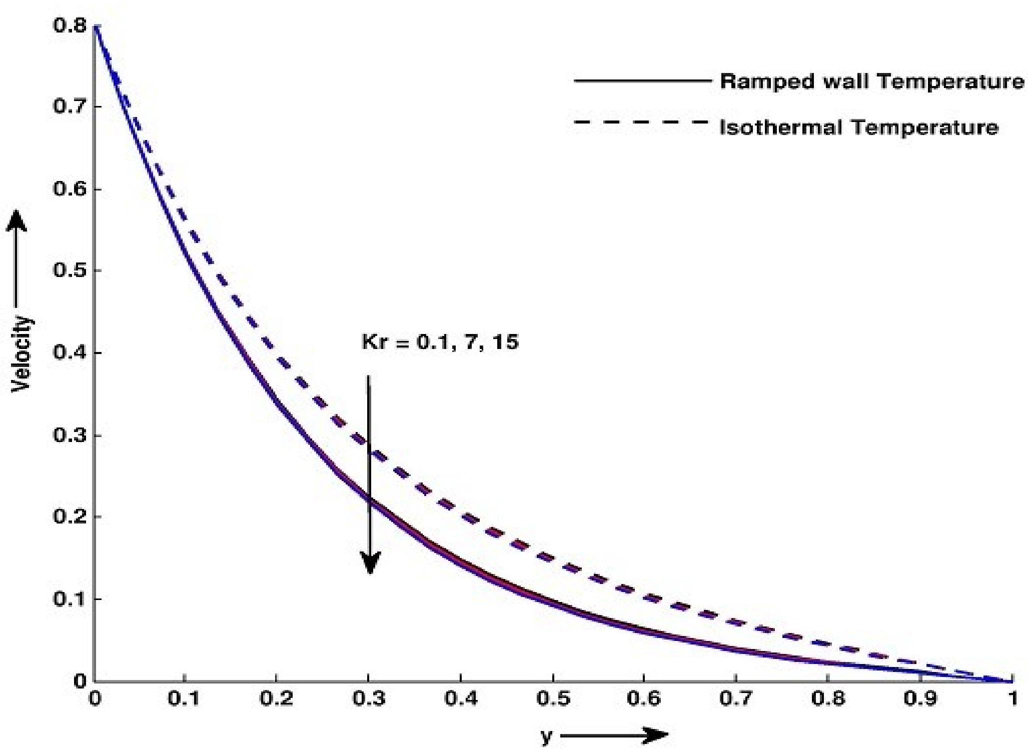

Figure 3.9: Velocity profile for different values of y and Kr at $M = 5, \alpha = 0.5, Pr = 7,$ $Sc = 0.66, Gm = 4, Gr = 5, H = 3, R = 5, k = 0.8$ and $t = 0.4$

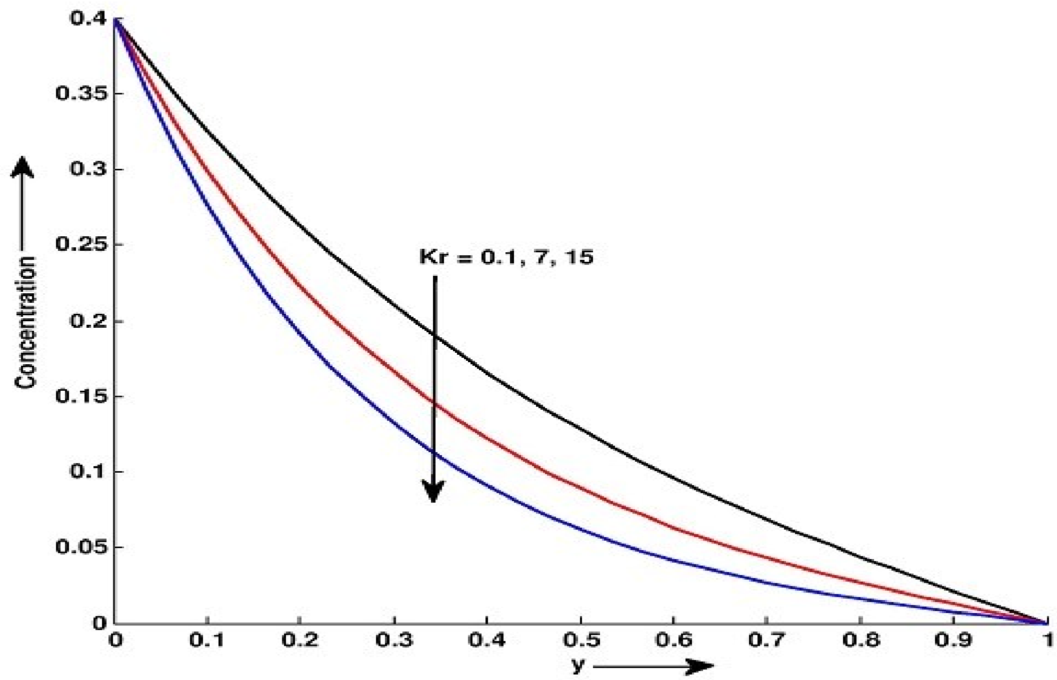

Figure 3.10: Concentration profile for different values of y and Kr at $Sc = 0.66, and\ t = 0.4$.

Figure 3.3 shows that Magnetic field parameter has negative impact on velocity for both thermal situations. This is due to Lorentz force on the fluid at boundary. Figure 3.4 reflects that in constant or variable temperature, Permeability of porous medium improves with velocity at entire boundary. Figure 3.5 and Figure 3.6 are about radiative impact on velocity and temperature for both isothermal and ramped thermal conditions. It is derived that velocity and temperature have positive correlation with thermal radiation. Temperature's correlation is obvious whereas increase in velocity with radiation is due to generation of heat, bond holding components of particles are broken. Figure 3.7 and Figure 3.8 are graphs of heat generation/absorption coefficient H on velocity and temperature. In both figures positive sign reflects the heat generation and negative sign means heat absorption. Heat generation obviously increases temperature which eventually increases flow of the fluid. So, if parameter of heat source is increased, there will be sudden rise in temperature. Results are very much supported physically as heat generation at the surface will increase porosity which rises fluid flow. Chemical reaction has a reverse impact on velocity and concentration for both thermal conditions as shown in Figure 3.9 and Figure 3.10. This means that when we increase values of Kr, buoyancy effect is reduced which eventually reduces concentration. Hence, flow is reduced.

Table 3.1 and table 3.2 validates Nusselt number and Sherwood number derived for different values by the values derived by seth et al. [24] and [25] respectively.

Table 3.1: Comparison of Nusselt number with Ref. [24] at $Pr = 0.71$

R	$\emptyset = -H/Pr$	t	Nusselt Number Nu for ramped temp. Ref [24]	Nusselt Number Nu for ramped temp.	Nusselt Number Nu for isothermal temp. Ref [24]	Nusselt Number Nu for isothermal temp.
2	3	0.3	0.38368	0.3837	0.89492	0.8949
2	3	0.5	0.55828	0.5583	0.85907	0.8591
2	3	0.7	0.72887	0.7289	0.84872	0.8487
2	1	0.5	0.44983	0.4498	0.56755	0.5675
2	3	0.5	0.55828	0.5583	0.85907	0.8591
2	5	0.5	0.65207	0.6521	1.09210	1.0921
2	3	0.5	0.55828	0.5583	0.85907	0.8591
4	3	0.5	0.43244	0.4324	0.66543	0.6654
6	3	0.5	0.36548	0.3655	0.56239	0.5624

Table 3.2: Comparison of Sherwood Number with Ref. [25]

t	Kr	Sc	Sherwood Number Sh for ramped temp. Ref [25]	Sherwood Number Nu for ramped temp.	Sherwood Number Sh for isothermal temp. Ref [25]	Sherwood Number Nu for isothermal temp.
0.3	0.2	0.22	0.295649	0.2956	0.525702	0.5257
0.5	0.2	0.22	0.386593	0.3866	0.428415	0.4284
0.7	0.2	0.22	0.463189	0.4632	0.379505	0.3796
0.3	2.0	0.22	0.344659	0.3447	0.839945	0.8399

0.5	2.0	0.22	0.488076	0.4881	0.785973	0.7860
0.7	2.0	0.22	0.625355	0.6254	0.757863	0.7579
0.3	5.0	0.22	0.416933	0.4169	1.1897	1.1897
0.5	5.0	0.22	0.628694	0.6287	1.12945	1.1294
0.7	5.0	0.22	0.838894	0.8389	1.09522	1.0952

3.6 Conclusion:

The objective of this research is to obtain analytical solution for MHD flow in oscillating vertical plate through porous medium of second grade fluid and observe radiation, heat generation or absorption and chemical reaction effects. Results are derived for constant and variable temperature of the surface. Graphical description is done for important parameters behaviors on velocity, temperature and concentration.

Key remarks for the conclusions can be summarized as follows.

- Velocity, temperature and concentration in constant temperature and constant surface temperature is more than those in variable temperature and variable surface concentration.
- Magnetic field parameter M, second grade parameter α and chemical reaction parameter Kr have retarding effects with velocity.
- Thermal radiation parameter R, permeability of porous medium k and heat generation parameter H have positive impacts with velocity.
- Temperature of the fluid has increase tendency with heat generation parameter H and thermal radiation parameter R.
- Concentration profile decreases if there is increment in chemical reaction parameter Kr.