SYNOPSIS

OF THE THESIS ENTITLED

MATHEMATICAL ANALYSIS OF MAGNETOHYDRODYNAMICS FLOW OF NON-NEWTONIAN FLUID

SUBMITTED FOR THE AWARD OF

THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

MATHEMATICS

TO

THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA

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VADODARA-390 002

APRIL 2021

Fluids can be defined as substances that flow under the application of shear stress. Fluids include liquids and gases. Basically, in the study of science, fluids are divided into two broad groups, Newtonian and Non-Newtonian. Fluids which obey the Newton's law of viscosity are called Newtonian fluids like water, benzene, alcohol & hexane etc. Fluids which do not obey the Newton's law of viscosity are called Non-Newtonian fluids like pastes, gels, polymer solutions, Casson fluids, Nano fluids, second grade fluids, micropolar fluids etc. Due to increasing significance, application of Non-Newtonian fluid is required in engineering.

Second grade fluid is a Non-Newtonian fluid where the stress tensor is the sum of all tensors that can be formed from the velocity field with up to two derivatives. The second grade fluids can model many fluids such as dilute polymer solutions, slurry flows and industrial oils. Tan and Masuoka [1] consider the Stokes' first problem for a second grade fluid and Rashidi et al. [2] deals with squeezing flow of a second grade fluid, whereas Hayat et al. [3] deals with unsteady stagnation point flow of second grade fluid with variable free stream. Hameed et al. [4] examine study of heat transfer on the peristaltic transport of a fractional second grade fluid in a vertical tube.

In nature, some Non-Newtonian fluids behave like elastic solid that is, no flow occurs with small shear stress. Casson fluid is one of such fluids. This fluid has distinct features and is quite famous recently. Casson [5] fluid model was introduced in 1959 for the prediction of the flow behavior of pigment-oil suspensions.

The word magnetohydrodynamics (MHD) is derived from magneto meaning magnetic field, hydro meaning fluid, and dynamics meaning movement. Today MHD has developed into a vast field of applied and fundamental research in astrophysics, engineering, communication, confinement of plasma for controlled fusion and physical science. Research works in the magnetohydrodynamics have been advanced significantly during the last few decades after the pioneer work of Hartmann [6] in liquid metal duct flows under external magnetic field. Singh and Singh [7] consider MHD flow past a semi-infinite vertical permeable wall. Kataria et al. [8] deals with MHD micropolar fluid between two vertical walls, whereas Kataria and Mittal [9-10] deals with MHD nanofluid flow in the presence of thermal radiation through porous medium. Many researchers like, Hayat et al. [11-12] discuss MHD flow of second grade fluid and Samiulhaq et al. [13] obtain the solution of MHD flow of a second grade fluid in a Porous Medium with ramped wall temperature. Hussnan et al. [14] deals with natural convection flow past an oscillating plate with Newtonian heating whereas, Rashidi et al. [15] consider heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation. Seth et al. [16-17] consider Hall and heat absorbing effects on MHD flow in the presence of thermal radiation and chemical reaction with ramped wall temperature. Recently, Kataria and Patel [18-19] consider thermal radiation and chemical reaction effects on MHD Casson fluid flow in embedded porous medium with ramped wall temperature whereas, Kataria and Patel [20] deals with soret and heat generation effects on MHD second grade fluid flow with ramped wall temperature and ramped surface concentration.

In this thesis, effect of magnetic field on one-, two- and three-dimensional unsteady free convective non-Newtonian fluid flow with Heat and Mass transfer in porous medium is discussed. This thesis consists of seven chapters.

Chapter 1 is taken to build up a stronger structure in logical manner to provide knowledge of fundamentals of MHD flow, basic concepts of non-Newtonian fluid, heat and mass transfer effects, radiation effects, heat generation effects and Soret effects. A brief history of the development of the subject is also given. Relevant literature has been surveyed. Further, Laplace transform technique for solving system of linear partial differential equations and Homotopy analysis method for solving system of non-linear equations are discussed.

Study of MHD flow of Non-Newtonian fluid is instrumental, as there are many engineering applications. The flow pattern of such fluids has been studied for industrial oils, slurry flows, dilute polymer solutions etc. In chapter 2, effects of magnetic field and Soret effects are studied on radiating and chemically reactive non-Newtonian fluid flow with ramped boundary conditions.

Axes are chosen as follows. $x'-$axis which is drawn vertically is the wall and y' – axis is drawn horizontally. Magnetic field of strength B_0 is in opposite direction to fluid flow. When time $t' \leq 0$, plate and fluid are stationary having surface concentration C'_∞ and constant temperature of fluid and the plate is assumed to be T'_∞. During time between $0 < t' \leq t_0$ velocity, temperature and concentration are $u'_\infty + (u'_w - u'_\infty)\ {t'}/{t_0}$, $T'_\infty + (T'_w - T'_\infty)\ {t'}/{t_0}$ and $C'_\infty + (C'_w - C'_\infty)\ {t'}/{t_0}$ respectively, whereas for time $t' > t_0$, they remain constant u'_w, T'_w and C'_w respectively. Effect of viscous dissipation induced by magnetic and electrical field are neglected. Here flow is considered as one-dimensional laminar flow, and the fluid is incompressible Non-Newtonian fluid. The equations which are governed for all these assumptions, are derived using Boussinesq's approximation. They are as follows.

$$\frac{\partial u'}{\partial t'} = \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{\mu \phi}{\rho k_1{}'} u' + g\beta'_T (T' - T'_\infty) + g\beta'_C (C' - C'_\infty) \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r{}'}{\partial y'} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + D_T \frac{\partial^2 T'}{\partial y'^2} - k_2' \, (C' - C'_\infty) \quad (3)$$

With following initial and boundary conditions:

$$u' = 0, \; T' = T'_\infty, \; C' = C'_\infty; \; as \;\; y' \geq 0 \; and \;\; t' \leq 0,$$

$$u' = \begin{cases} u'_\infty + (u'_w - u'_\infty)\, {t'}/{t_0} \; if \;\; 0 < t' < t_0 \\ \quad u'_w \qquad\qquad\qquad\quad if \quad t' \geq t_0 \end{cases}; \;\; as \;\; t' > 0 \; and \; y' = 0 \;\; ,$$

$$T' = \begin{cases} T'_\infty + (T'_w - T'_\infty)\, {t'}/{t_0} \; if \;\; 0 < t' < t_0 \\ \quad T'_w \qquad\qquad\qquad\quad if \quad t' \geq t_0 \end{cases}; \;\; as \;\; t' > 0 \; and \; y' = 0 \;\; ,$$

$$C' = \begin{cases} C'_\infty + (C'_w - C'_\infty)\, {t'}/{t_0} \; if \;\; 0 < t' < t_0 \\ \quad C'_w \qquad\qquad\qquad\quad if \quad t' \geq t_0 \end{cases}; \;\; as \;\; t' > 0 \; and \; y' = 0 \;\; ,$$

$$u' \to 0, T' \to T'_\infty, \;\; C' \to C'_\infty; \; as \;\; y' \to \infty \; and \; t' \geq 0 \quad (4)$$

The local radiant for the case of an optically thin gray gas is expressed by Rosseland approximation [21]

$$\frac{\partial q_r{}'}{\partial y'} = -4a^* \sigma^* (T'^4_\infty - T'^4) \quad (5)$$

The equations are transformed using transformation variables to obtain the system of linear ordinary differential equations. Laplace transform technique is used to solve the governing dimensionless system of linear ordinary differential equations. Analytic solutions for governing equations of MHD flow of Non-Newtonian fluid through porous medium with constant fluid velocity and ramped velocity are obtained. Effects of chemical reaction, thermal radiation and thermo-diffusion on velocity, temperature and concentration profiles are perceived through various graphs.

Results of Chapter 2 is published in **International Journal of Advanced Science and Technology (Scopus) (Ref. [22])** and **Journal of Emerging Technologies Engineering and Innovative Research {Earlier in UGC list) (Ref. [23])**.

Many flow problems with various geometries and different mechanical and thermal boundary conditions have also been studied. Chapter 3 deals with the investigation of exact solution of heat generation/absorption effect on unsteady natural convective MHD Second grade fluid flow. Such study may find application in fire dynamics in insulations and geothermal energy systems etc.

Governing equations of Boussinesq's approximation under above assumptions are as follows.

$$\frac{\partial u'}{\partial t'} = \left(\nu + \frac{\alpha_1}{\rho}\frac{\partial}{\partial t'}\right)\frac{\partial^2 u'}{\partial y'^2} + g\beta'_T(T' - T'_\infty) - \frac{\sigma B_0{}^2}{\rho}u' - \frac{\emptyset}{k_1{}'}\left(\nu + \frac{\alpha_1}{\rho}\frac{\partial}{\partial t'}\right)u' + g\beta'_C(C' - C'_\infty) \tag{6}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r{}'}{\partial y'} + \frac{Q_0}{\rho c_p}(T' - T'_\infty) \tag{7}$$

$$\frac{\partial C'}{\partial t'} = D_M\frac{\partial^2 C'}{\partial y'^2} - k_2'\,(C' - C'_\infty) \tag{8}$$

With following initial and boundary conditions:

$$u' = 0,\quad T' = T'_\infty,\ \ C' = \ C'_\infty;\ as\ \ y' \geq 0\ and\ \ t' \leq 0$$

$$u' = \frac{U_0\cos\omega' t'}{U_0\sin\omega' t'\ as}\ t' > 0\ and\ y' = 0\ ,$$

$$T' = \begin{cases} T'_\infty + (T'_w - T'_\infty)\,{}^{t'}\!/_{t_0}\ if\ \ 0 < t' < t_0 \\ \qquad T'_w \qquad\qquad\qquad if\quad t' \geq t_0 \end{cases},$$

$$C' = \begin{cases} C'_\infty + (C'_w - C'_\infty)\,{}^{t'}\!/_{t_0}\ if\ \ 0 < t' < t_0 \\ \qquad C'_w \qquad\qquad\qquad if\quad t' \geq t_0 \end{cases};\ y' = 0$$

$$u' \to 0, T' \to T'_\infty,\ \ C' \to C'_\infty;\ as\ \ y' \to \infty\ and\ t' \geq 0 \tag{9}$$

Exact solution for fluid velocity; Temperature and Concentration is obtained using the Laplace transform technique. For both thermal plates, analytic expressions of Nusselt Number and Sherwood Number are derived and presented in tabular form. The effects of Magnetic parameter M, second grade fluid α, Heat generation/absorption parameter H, thermal radiation parameter Nr, chemical reaction parameter Kr in time variable t on velocity, temperature and concentration profiles are discussed through several graphs.

This result is published in **International Journal of Scientific Research in Mathematical and Statistical Sciences. (Earlier in UGC list) (Ref. [24])**.

There is another type of non-Newtonian fluid known as Casson fluid. Casson fluid exhibits yield stress. It is well known that Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear, i.e., if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move. The examples of Casson fluid are of the type are as follows: jelly, tomato sauce, honey, soup, concentrated fruit juices, etc. Sandeep et al. [25] discussed modified kinematic viscosity model for 3D-Casson fluid flow. The main objective of chapter 4 is to develop mathematical modeling of Soret and Dufour effects on MHD Casson fluid flow in the presence of thermal radiation.

Boundary layer equations in MHD flow under consideration are.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{10}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+g\beta_T(T-T_\infty)+g\beta_\emptyset(\emptyset-\emptyset_\infty) \tag{11}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{k_4}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q_0}{\rho c_p}(T-T_\infty)+\frac{D_e k_T}{C_s C_p}\frac{\partial^2 \emptyset}{\partial y^2} \tag{12}$$

$$\left(u\frac{\partial \emptyset}{\partial x}+v\frac{\partial \emptyset}{\partial y}\right)=D_e\frac{\partial^2 \emptyset}{\partial y^2}-k_1(\emptyset-\emptyset_\infty)+\frac{D_e k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{13}$$

Subject to

$$u = ax;\ v = 0;\ T = T_w;\ \emptyset = \emptyset_w\ at\ y = 0 \tag{14}$$

$$u\to 0, T\to T_\infty,\ \emptyset\to\emptyset_\infty;\ as\ y\to\infty\ and\ t\geq 0 \tag{15}$$

$$\eta = y\sqrt{\frac{a}{\nu}},\ u=axf'(\eta), v=-\sqrt{a\nu}f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \emptyset(\eta)=\frac{\emptyset-\emptyset_\infty}{\emptyset_w-\emptyset_\infty} \tag{16}$$

The simplified systems of ordinary differential equations are solved using the Homotopy analysis method introduced by Liao [25]. The effects of the pertinent parameters are discussed through graphs. Skin friction, Nusselt number and Sherwood number are presented in tabular form. This result included in Chapter 4, is published in **Materials Today: Proceedings (Elsevier) (Ref. [27])**. Viscoelastic fluid behavior is a type of non-Newtonian fluid formed by a viscous component and an elastic one. Examples of viscoelastic fluids are paints, some biological fluids, and DNA suspensions, etc. Several features make the viscoelastic fluids very interesting and of industrial importance. A proper understanding of viscoelasticity is key for industrial applications. Viscoelastic fluids are common in very important applications [28-30]. Recently, many researchers done works on MHD flow of viscoelastic fluid [31-32]. Two dimensional MHD flow problems are of more importance and realistic compared to one dimensional problems. Due to this reason, Chapter 5 is dedicated to the study of the steady two-dimensional MHD flow of viscoelastic fluid over stretching/shrinking sheet considering the effects of Brownian motion, thermal radiation, and chemical reaction.

Two-dimensional incompressible steady viscoelastic fluid over a stretching/shrinking surface is considered. The origin is taken as stagnation point. Plate is assumed to be along x-axis and is subject to forces of magnitude bx applied in opposite directions keeping origin fixed. Flow is along positive y direction. Components of velocity along x and y axis are assumed to be u and v respectively. Here we assign a magnetic field perpendicular to the stretching sheet. The governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{17}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=U_\infty\frac{dU_\infty}{dx}+v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}\left(u+k_0 v\frac{\partial u}{\partial y}\right)-\ k_0\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right) \tag{18}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2 \tag{19}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}\right)-R(C-C_\infty)+D_B\left(\frac{\partial^2 C}{\partial y^2}\right) \tag{20}$$

Where $U_\infty = ax,\quad a>0$ is the straining velocity of the flow is the straining constant, where u and v are the velocity component in the x and y directions. T_∞ and C_∞ are denoted for the ambient values of T and C, when y tends towards infinity.

The boundary conditions for the above defined model are:

$$u=U_w=bx,\ \ v=0,\ \ -k\frac{\partial T}{\partial y}=h(T_w-T),\ \ C=Cw \text{ at } y=0.$$

$$u\to U_\infty=ax, T\to T_\infty,\ \ C\to C_\infty, as\ y\to\infty. \tag{21}$$

Where the temperature has linear relationship with temperature gradient, T_w and C_w are the temperature of the fluid and concentration at the wall.

The prevailing partial differential equations are transformed into ordinary differential equations. Homotopy analysis method is used to solve the nonlinear systems. Obtained solutions are graphed to show the effects of different parameters on velocity, temperature, and concentration profiles. The effects of different prevailing parameters on velocity, temperature, and concentration profiles are shown graphically which is obtained from the Mathematica code.

In recent time, many researchers have been involved in doing research on 3 dimensional MHD flow of various Non-Newtonian fluids due to their fascinating and noteworthy engineering focus with respect to utility and applications. It is established on the principle that particles of fluid would be structurally continuous. Porous media flow has many practical applications removals of heat from nuclear fuel, underground disposal of radioactive waste material, food storage, production of papers, oil exploration etc. Purpose of chapter 6 is to investigate semi analytic solution of Dufour and Soret effects on unsteady MHD Casson fluid flow past over vertical plate embedded in porous medium in a rotating system. This study may find applications in fire dynamics.

It is assumed that Casson fluid flows between two horizontal parallel plates placed L units apart through a porous medium. A coordinate system (x, y, z) is such that origin is at the lower plate. The lower plate is stretched by two equal forces in opposite directions. The plates along with the fluid rotate about y axis with angular velocity Ω. A uniform magnetic flux with density B is applied along y-axis. Under these assumptions, governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{22}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + 2\Omega w\right) = \mu\left(1 + \frac{1}{\gamma}\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma B^2 u - \frac{\mu\varphi}{k_1{}'}u \tag{23}$$

$$\rho\left(v\frac{\partial v}{\partial y}\right) = \mu\left(1 + \frac{1}{\gamma}\right)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{24}$$

$$\rho\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} - 2\Omega w\right) = \mu\left(1 + \frac{1}{\gamma}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \sigma B^2 w - \frac{\mu\varphi}{k_1{}'}w \tag{25}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{(\rho c_p)_s}{(\rho c_p)_f}\left(\frac{D_T}{T_w}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right)\right.$$

$$\left. + D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right)\right) - \frac{\partial q_r{}'}{\partial y} + \frac{D_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{26}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) + \frac{D_T}{T_w}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{27}$$

Subject to boundary conditions

$$u = ax;\ v = 0;\ w = 0;\ T = T_w\ at\ y = 0$$

$$u = 0;\ v = 0;\ w = 0;\ T = T_L\ at\ y = L \tag{28}$$

Some variables are transformed and converted to dimensionless. The system is then transformed to ordinary differential equations with corresponding initial conditions and boundary conditions. This system of equations with initial conditions and boundary conditions are solved using method of Homotopy Analysis. Expressions for fluid velocity (in all three directions), temperature and concentration profiles are obtained. The features of the velocity, temperature and concentration are analyzed by plotting graphs and the physical aspects are studied for different parameters. Major findings are published in **Journal of Applied Sciences and Engineering (Scopus) (Ref. [33])**.

Chapter 7 deals with effect of magnetic field on heat and mass transfer features of three-dimensional water based composite nanofluid flow between two horizontal parallel plates. System under consideration is rotating. Aim of the study is to develop the mathematical modeling for

Brownian motion effects on MHD flow water based composite nanofluid in porous medium with thermal radiation. The governing equations are.

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0 \tag{29}$$

$$\rho_{hnf}\left(u'\frac{\partial u'}{\partial x}+v'\frac{\partial u'}{\partial y}+2\Omega w'\right)=\mu_{hnf}\left(\frac{\partial^2 u'}{\partial x^2}+\frac{\partial^2 u'}{\partial y^2}\right)-\sigma_{hnf}{B_0}^2u'-\frac{\mu_{hnf}\phi}{{k_1}'}u' \tag{30}$$

$$\rho_{hnf}\left(v'\frac{\partial v'}{\partial y}\right)=\mu_{hnf}\left(\frac{\partial^2 v'}{\partial x^2}+\frac{\partial^2 v'}{\partial y^2}\right) \tag{31}$$

$$\rho_{hnf}\left(u'\frac{\partial w'}{\partial x}+v'\frac{\partial w'}{\partial y}-2\Omega w'\right)=\mu_{hnf}\left(\frac{\partial^2 w'}{\partial x^2}+\frac{\partial^2 w'}{\partial y^2}\right)-\sigma_{hnf}{B_0}^2w'-\frac{\mu_{hnf}\phi}{{k_1}'}w' \tag{32}$$

$$\left(\rho c_p\right)_{hnf}\left(u'\frac{\partial T'}{\partial x}+v'\frac{\partial T'}{\partial y}+w'\frac{\partial T'}{\partial z}\right)=k_{nf}\left(\frac{\partial^2 T'}{\partial x^2}+\frac{\partial^2 T'}{\partial y^2}+\frac{\partial^2 T'}{\partial z^2}\right)+\mu_{hnf}\left(\frac{D_T}{T_w}\left(\left(\frac{\partial T'}{\partial x}\right)^2+\left(\frac{\partial T'}{\partial y}\right)^2+\left(\frac{\partial T'}{\partial z}\right)^2\right)+D_B\left(\frac{\partial C}{\partial x}\frac{\partial T'}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T'}{\partial y}+\frac{\partial C}{\partial z}\frac{\partial T'}{\partial z}\right)\right)-\frac{\partial q_r'}{\partial y} \tag{33}$$

$$u'\frac{\partial C}{\partial x}+v'\frac{\partial C}{\partial y}+w'\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)+\frac{D_T}{T_w}\left(\frac{\partial^2 T'}{\partial x^2}+\frac{\partial^2 T'}{\partial y^2}+\frac{\partial^2 T'}{\partial z^2}\right) \tag{34}$$

From Ref. [34], we write.

$$\rho_{hnf}=\left\{(1-\emptyset_2)\left[(1-\emptyset_1)\rho_{bf}+\emptyset_1\rho_{s1}\right]\right\}+\emptyset_2\rho_{s2} \tag{35}$$

$$\frac{\sigma_{hnf}}{\sigma_{bf}}=\frac{\sigma_{S_2}+2\sigma_{bf}-2\emptyset_2(\sigma_{bf}-\sigma_{S_2})}{\sigma_{S_2}+2\sigma_{bf}+\emptyset_2(\sigma_{bf}-\sigma_{S_2})}, \tag{36}$$

$$\frac{\sigma_{bf}}{\sigma_f}=\frac{\sigma_{S_1}+2\sigma_f-2\emptyset_1(\sigma_f-\sigma_{S_1})}{\sigma_{S_1}+2\sigma_f+\emptyset_1(\sigma_f-\sigma_{S_1})} \tag{37}$$

$$\left(\rho c_p\right)_{hnf}=\left\{(1-\emptyset_2)\left[(1-\emptyset_1)\left(\rho c_p\right)_f+\emptyset_1\left(\rho c_p\right)_{s1}\right]\right\}+\emptyset_2\left(\rho c_p\right)_{s2} \tag{38}$$

$$\frac{k_{hnf}}{k_{bf}}=\frac{k_{S_2}+(n-1)k_{bf}-(n-1)\emptyset_2(k_{bf}-k_{S_2})}{k_{S_2}+(n-1)k_{bf}+\emptyset_2(k_{bf}-k_{S_2})}, \tag{39}$$

Where

$$\frac{k_{bf}}{k_f}=\frac{k_{S_1}+(n-1)k_f-(n-1)\emptyset_1(k_f-k_{S_1})}{k_{S_1}+(n-1)k_f+\emptyset_1(k_f-k_{S_1})}, \tag{40}$$

$$\mu_{hnf}=\frac{\mu_{bf}}{(1-\emptyset_1)^{2.5}(1-\emptyset_2)^{2.5}} \tag{41}$$

A semi analytic Homotopy analysis method is implemented to solve the dimensionless system of ordinary differential equations. Computations were performed graphically to analyze the behaviors of several variables on the flow, heat and mass transfer is scrutinized. Numerical values of skin friction coefficient, local Nusselt number and Sherwood number are obtained and presented in graphical form. It is seen that velocity increases with decrease in magnetic parameter. It is also seen that thermal radiation, magnetic field and thermophoresis tends to improve heat transfer process whereas thermophoresis parameter, Schmidt number and Brownian motion tends to reduce mass transfer process.

Results of this chapter are published in **International Journal of Applied and Computational Mathematics (Springer) (Ref. [35])**.

Bibliography:

[1] W. Tan and T. Masuoka, Stokes' first problem for a second grade fluid in a porous half-space with heated boundary, Int J Non-Linear, 40 (2005) 515–522.

[2] M. M. Rashidi, S. A. Majid, A. Mostafa, Application of homotopy analysis method to the unsteady squeezing flow of a second-grade fluid between circular plates, Math Probl Eng., 18 (2010) 706840.

[3] T. Hayat, M. Qasim, S.A. Shehzad, A. Alsaedi, Unsteady stagnation point flow of second grade fluid with variable free stream, Alexandria Engineering Journal, 53 (2014) 455–461.

[4] M. Hameed, A. Khan, R. Ellahi, M. Raza, Study of magnetic and heat transfer on the peristaltic transport of a fractional second grade fluid in a vertical tube, Engineering Science and Technology, an International Journal, 18 (2015) 496-502.

[5] N. Casson, A flow equation for the pigment oil suspensions of the printing ink type, C.C. Mills (Eds.), Rheology of Disperse Systems, Newyork Oxford: Pergamon, (1959) 84-104.

[6] J. Hartmann, Hg-dynamics I theory of the laminar flow of an electrically conductive liquid in a homogenous magnetic field, Det Kongelige Danske Videnskabernes Selskab Mathematiskfysiske Meddeleser XV (1937) 1–27.

[7] R. K. Singh and A. K. Singh, MHD Free Convective Flow Past a Semi-infinite Vertical Permeable wall, Applied Mathematics and Mechanics, 33 (2012) 1207-1222.

[8] H. R. Kataria, H. R. Patel, R. K. Singh, Effect of magnetic field on unsteady natural convective flow of a micropolar fluid between two vertical walls, Ain Shams Engineering Journal, 8 (2015) 87-102

[9] H. R. Kataria and A. S. Mittal, Mathematical model for velocity and temperature of gravity-driven convective optically thick nanofluid flow past an oscillating vertical plate in presence of magnetic field and radiation, Journal of the Nigerian Mathematical Society, 34 (2015) 303–317.

[10] H. Kataria, A. Mittal, Velocity, mass and temperature analysis of gravity-driven convection nanofluid flow past an oscillating vertical plate in the presence of magnetic field in a porous medium, Applied Thermal Engineering, 110 (2017) 864–874.

[11] T. Hayat, N. Ahmed, M. Sajid, S. Asghar, On the MHD flow of a second grade fluid in a porous channel, Computers & Mathematics with Applications, 54 (2007) 407–414.

[12] T. Hayat, M. Nawaz, M. Sajid, S. Asghar, The effect of thermal radiation parameter on the flow of a second grade fluid, Computers and Mathematics with Applications, 58 (2009) 369-379.

[13] Samiulhaq, S. Ahmad, D. Vieru, I. Khan, S. Shafie, Unsteady Magnetohydrodynamic Free Convection Flow of a Second Grade Fluid in a Porous Medium with Ramped Wall Temperature PLOS ONE 9(5) (2015): e88766. doi: 10.1371/journal.pone.0088766.

[14] A. Hussnan, Z. Ismail, I. Khan, S. Shafie, unsteady mhd free convection flow in a porous medium with constant mass diffusion and Newtonian heating, The European Physical Journal - Plus, (2014) 129: 46. doi:10.1140/epjp/i2014-14046-x

[15] M. M. Rashidi, B. Rostami, N. Freidoonimehr, S. Abbasbandy, Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects, Ain Shams Engineering Journal, 5 (2014) 901–912.

[16] G. S. Seth, R. Sharma, S. Sarkar, Hydromagnetic Natural Convection Heat and Mass Transfer Flow with Hall Current of a Heat Absorbing and Radiating Fluid Past an Accelerated Moving Vertical Plate with Ramped Temperature in a Rotating Medium, J. Appl. Fluid Mech, 8 (2015) 7-20.

[17] G. S. Seth, A K. Singha, R Sharma, MHD natural convection flow with hall effects, radiation and Heat absorption over an exponentially accelerated vertical Plate with ramped temperature, Ind. J. Sci. Res., and Tech, 5 (2015) 10-22.

[18] H. R. Kataria and H. R. Patel, Radiation and chemical reaction effects on MHD Casson fluid flow past an oscillating vertical plate embedded in porous medium, Alexandria Engineering Journal 55 (2016) 583-595.

[19] H. R. Kataria and H. R. Patel, soret and heat generation effects on MHD Casson fluid flow past an oscillating vertical plate embedded through porous medium, Alexandria Engineering Journal, 55 (2016) 2125-2137.

[20] H. R. Kataria, H. R. Patel, Effect of thermo-diffusion and parabolic motion on MHD Second grade fluid flow with ramped wall temperature and ramped surface concentration, Alexandria Engineering Journal, 57 (1) (2018) 73-85.

[21] Rosseland, S.: Astrophysik und atom-theoretischeGrundlagen. Springer, Berlin (1931).

[22] R. P. Mehta, H. R. Kataria, Study of Soret effect of MHD flow of Non-Newtonian fluid with radiation and chemical reaction, International Journal of Advanced Science and Technology, 29 (04) (2020) 7588 – 7602.

[23] R. P. Mehta, H. R. Kataria, Mathematical analysis of Soret effect on non-Newtonian fluid flow through porous medium in presence of magnetic field, Journal of Emerging Technologies Engineering and Innovative Research, volume 5 | Issue 7| page No 279 – 286

[24] R. P. Mehta, H. R. Kataria, Magnetic Field a Heat Generation Effects on Second Grade Fluid Flow past an Oscillating Vertical Plate in Porous Medium, International Journal of Scientific Research in Mathematical and Statistical Sciences, 7 (2) (2020) 01-08.

[25] N. Sandeep, O. K. Koriko, I. L. Animasaun, modified kinematic viscosity model for 3D-Casson fluid flow within boundary layer formed on a surface at absolute zero, Journal of Molecular Liquids, 221 (2016) 1197–1206.

[26] Liao, S.J.: Beyond Perturbation: Introduction to Homotopy Analysis Method. Chapman and Hall/C.R.C. Press, Boca Raton (2003).

[27] R. P. Mehta, H. R. Kataria, R. R. Darji, 2D model of Casson fluid flow considering Soret and dufour effects in presence of radiation, Materials Today: Proceedings, https://doi.org/10.1016/j.matpr.2020.11.061.

[28] S. H. Sadek, F. T. Pinho, M. A. Alves, Electro-elastic flow instabilities of viscoelastic fluids in contraction/expansion micro-geometries, Journal of Non-Newtonian Fluid Mechanics, 283 (2020) 104293.

[29] Y. Wang, H. Ma, W. Cai, H. Zhang, X. Zheng, A POD-Galerkin reduced-order model for two-dimensional Rayleigh-Bénard convection with viscoelastic fluid, International Communications in Heat and Mass Transfer, 117 (2020) 104747.

[30] S. Nadeem, A. Amin, N. Abbas, On the stagnation point flow of nanomaterial with base viscoelastic micropolar fluid over a stretching surface, Alexandria Engineering Journal, 59 (3) (2020) 1751-1760.

[31] X. Chen, W. Yang, X. Zhang, F. Liu, Unsteady boundary layer flow of viscoelastic MHD fluid with a double fractional Maxwell model, Applied Mathematics Letters, 95 (2019) 143-149.

[32] R. K. Lodhi, K. Ramesh, Comparative study on electroosmosis modulated flow of MHD viscoelastic fluid in the presence of modified Darcy's law, Chinese Journal of Physics, https://doi.org/10.1016/j.cjph.2020.09.005

[33] R. P. Mehta, H. R. Kataria, Cross diffusion effects on motion of three dimensional Casson fluid flow past between two horizontal plates in a porous medium, Journal of Applied Science and Engineering, Vol. 23, No 2, Page 319-331

[34] Mittal, A.S., Kataria, H.R.: Three dimensional CuO–water nanofluid flow considering Brownian motion in the presence of radiation. Karbala Inter. J. Mod. Sci. 4(3), 275–286 (2018)

[35] R. P. Mehta, H. R. Kataria, Influence of Magnetic Field, Thermal Radiation and Brownian motion on Water-Based Composite Nanofluid Flow Passing Through a Porous Medium, Int. J. Appl. Comput. Math 7, 7 (2021). https://doi.org/10.1007/s40819-020-00938-8

PUBLISHED / ACCEPTED RESEARCH ARTICLES

1. R. P. Mehta, H. R. Kataria, magnetic field and heat generation effects on second grade fluid flow past an oscillating vertical plate in porous medium International Journal of Scientific Research in Mathematical and Statistical Sciences, Volume-7, Issue-1, pp.01-05, February (2020), E-ISSN: 2348-4519 **(Earlier in UGC list)**
2. R. P. Mehta, H. R. Kataria, Study of Soret effect of MHD flow of Non-Newtonian fluid with radiation and chemical reaction. International Journal of Advanced Science and Technology,29(04), 7588 - 7602. http://sersc.org/journals/index.php/IJAST/article/view/28175 **(Scopus)**
3. R. P. Mehta, H. R. Kataria, Radiative Effect on Parabolic Motion of Casson Fluid Flow Past Over Vertical Plate Embedded in a Porous Medium, Mathematics Today Vol.34(A) (April 2018 – Special Issue) 100-114 ISSN 0976-3228, e-ISSN 2455-9601 **(Earlier in UGC list)**
4. R. P. Mehta, H. R. Kataria, Mathematical analysis of Soret effect on non-Newtonian fluid flow through porous medium in presence of magnetic field, Journal of Emerging Technologies Engineering and Innovative Research **(Earlier in UGC list)**
5. R. P. Mehta, H. R. Kataria, Influence of Magnetic Field, Thermal Radiation and Brownian Motion on Water-Based Composite Nanofluid Flow Passing Through a Porous Medium, International Journal of Applied and Computational Mathematics 7 (1) (2020), 1-24. **(Springer)**
6. R. P. Mehta, H. R. Kataria, Cross diffusion effects on motion of three dimensional cassion fluid flow past between two horizontal plates in a porous medium. Journal of Applied Science and Engineering 23 (2), (2020) 319-331. **(Scopus)**

7. R. P. Mehta, H. R. Kataria, R. R. Darji, 2D Model of Casson Fluid Flow considering Soret and Dufour effects in presence of radiation, Materials Today **(Elsevier)**

COMMUNICATED RESEARCH WORK

1. R. P. Mehta, H. R. Kataria, Brownian motion and thermophoresis effects on MHD flow of viscoelastic fluid over stretching/shrinking sheet in the presence of thermal radiation and chemical reaction

PRESENTED RESEARCH WORK IN CONFERENCE

1. R. P. Mehta, H. R. Kataria, Heat generation / absorption effects on fluid past an oscillating vertical plate through porous medium, International conference on recent advances in theoretical and computational partial differential equations with applications, Punjab University, Chandigarh, December 05 – 09, 2016.
2. R. P. Mehta, H. R. Kataria, H. R. Patel, Effect of radiation on casson fluid flow past over an oscillating vertical plate embedded in porous medium, International Conference on Futuristic Trends in Engineering, Science, Pharmacy and Management., A. D. Publication, Vadodara, December 23, 2016
3. Mathematical Analysis of three dimensional nanofluid flow in a rotating system considering thermal interfacial resistance and micro mixing in suspensions through porous medium, International Conference on Advances in Pure and Applied Mathematics, Ganpat University, Mehsana, December 22 – 24, 2017.
4. R. P. Mehta, H. R. Kataria, Mathematical analysis of Soret effect on non-Newtonian fluid flow through porous medium in presence of magnetic field, International Conference on Advances in Engineering and Technology (AIET – 2018), Silver Oak Group of Institutes, July 9 – 10, 2018.
5. R. P. Mehta, H. R. Kataria, R. R. Darji, 2D Model of Casson Fluid Flow considering Soret and Dufour effects in presence of radiation, 21st International Conference on Science, Engineering and Technology (ICSET-2020) Vellore Institute of Technology, Vellore from 30th November 2020 to 1st December 2020