CHAPTER

8

STUDY OF HEAT AND MASS TRANSFER CHARACTERISTICS OF THREE DIMENSIONAL MHD NANOFLUID FLOW IN ROTATING SYSTEM

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Chapter 8

STUDY OF HEAT AND MASS TRANSFER CHARACTERISTICS OF THREE DIMENSIONAL MHD NANOFLUID FLOW IN ROTATING SYSTEM

In this chapter, three dimensional CuO – water nanofluid flow between two horizontal parallel plates through porous medium in a rotating system is scrutinized. Micro mixing in suspensions is taken into account while calculating viscosity. Previous studies anticipated that there aren't any slip velocities among nanoparticles. In natural convection of nanofluids, due to slip mechanisms such as Brownian motion and thermophoresis, the nanoparticles could not accompany fluid molecules, so volume fraction of nanofluids may not be uniform anymore and there would be a variable concentration of nanoparticles in a mixture.

8.1 Introduction

Vital role of heat transfer in various engineering procedures has given a new motivation for a further comprehensive study of enhancement techniques. Heat transfer arises in various systems such as automobiles, heat exchangers, buildings, electronic devices and refrigerators. Use of nanofluid is one such technique for heat transfer augmentation due to its higher thermal conductivity. Another technique for heat transfer improvement is the use of porous media, for example, the application of copper foams in heat exchangers. In recent times, the practice of both porous media and nanofluid has attracted extensive investigations. In porous media, the contact surface area between solid surface and liquid is more, and additionally, suspended nanoparticles in nanofluid enrich the effective thermal conductivity. Consequently, it appears that using porous media together with nanofluid can enhance the efficiency of thermal systems. In recent years, effect of magnetic field in different engineering applications such as the cooling of reactors and many metallurgical processes involved in the cooling of continuous tiles has been more considerable. Also, in several engineering processes, materials manufactured by extrusion processes and heat treated materials traveling between a feed roll and a wind up roll on convey belts possess the characteristics of a moving continuous surface.

Sheikholeslami et al. [83] and Kataria and Mittal [28] examined the MHD nanofluid flow and heat transfer amid two horizontal plates in a rotating system. Squeezing unsteady nanofluid flow and heat transfer has been studied by Sheikholeslami and Ganji [75]. Both studies assumed that there aren't any slip velocities between nanoparticles. Sheikholeslami and Ganji [73] discussed three dimensional nanofluid flow considering heat and mass transfer in a rotating system in the presence of magnetic field.

8.2 Novelty of the Problem

Effect of magnetic field on heat and mass transfer in presence of thermal radiation between horizontal parallel plates is analyzed in the present investigation. System considered is a rotating system. Novelty of the present work is the inclusion of often neglected effects such as Brownian motion, nanoparticle volume fraction and micro mixing in suspensions. Homotopy analysis method is employed to solve the resulting system of ordinary differential equations. The effects of relevant parameters on mass transfer are discussed in detail.

8.3 Mathematical Formulation of the Problem

It is assumed that CuO – water nanofluid flows between two horizontal parallel plates placed L units apart through a porous medium. A coordinate system (x, y, z) is such that origin is at the lower plate as shown in Figure 8.1. The lower plate is stretched by two equal forces in opposite directions. The plates along with the fluid rotate about y axis with angular velocity Ω. A uniform magnetic flux with density B is applied along y-axis.

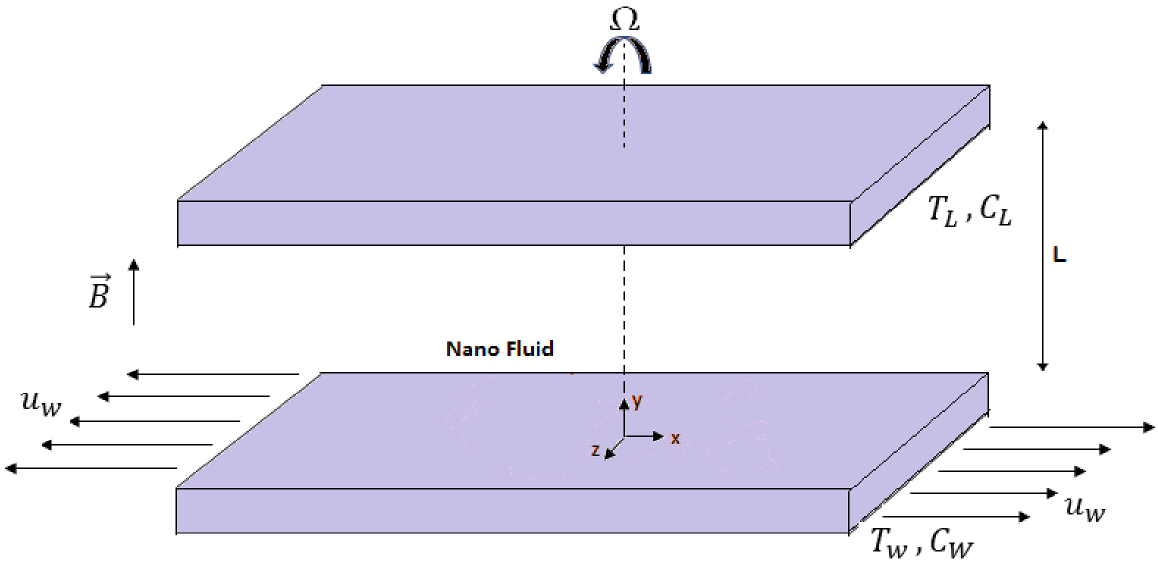

Figure 8.1: Physical Sketch of the Problem

Under these assumptions, governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{8.1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+2\Omega w\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma_{nf}B^2u-\frac{\mu_{nf}\varphi}{k_1}u, \tag{8.2}$$

$$\rho_{nf}\left(v\frac{\partial v}{\partial y}\right)=\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{8.3}$$

$$\rho_{nf}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-2\Omega w\right)=\mu_{nf}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\sigma_{nf}B^2w-\frac{\mu_{nf}\varphi}{k_1}w, \tag{8.4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{(\rho c_p)_s}{(\rho c_p)_f}\left(\frac{D_T}{T_w}\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right)+\right.$$

$$\left.D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right)\right)-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y}, \tag{8.5}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)+\frac{D_T}{T_w}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right), \tag{8.6}$$

where

$$\rho_{nf} = (1-\emptyset)\rho_f + \emptyset\rho_s, \tag{8.7}$$

$$\sigma_{nf} = \sigma_f[1 + \frac{3(\sigma-1)\emptyset}{(\sigma+2)-(\sigma-1)\emptyset}], \sigma = \frac{\sigma_s}{\sigma_f}, \tag{8.8}$$

$$\left(\rho c_p\right)_{nf} = (1-\emptyset)(\rho c_p)_f + \emptyset(\rho c_p)_s\,, \tag{8.9}$$

$$k_{nf} = k_f\left[1 - 3\frac{\emptyset\left(k_f-k_s\right)}{2k_f+k_s+\emptyset\left(k_f-k_s\right)}\right]. \tag{8.10}$$

Considering effect of micro mixing in suspensions on viscosity,

$$\mu_{nf} = \mu_{static} + \mu_{Brownian} = \frac{\mu_f}{(1-\emptyset)^{2.5}} + \frac{k_{Brownian}*\mu_f}{k_f*Pr_f}. \tag{8.11}$$

Considering temperature difference within the flow to be sufficiently small, using Taylor's series and neglecting higher terms, q_r [61] becomes

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} = -\frac{4\sigma^*}{3k^*}\frac{\partial\left(4{T_0}^3T-3{T_0}^4\right)}{\partial y}. \tag{8.12}$$

Thermo – physical properties of base fluid (water) and nanoparticles (CuO) are as in Table 1.1.

Boundary conditions are

$$u = ax;\ v = 0;\ w = 0;\ T = T_w\ at\ y = 0, \tag{8.13}$$

$$u = 0;\ v = 0;\ w = 0;\ T = T_L\ at\ y = L. \tag{8.14}$$

Introducing non dimensional variables

$$\eta = \frac{y}{L},\ u = axf'(\eta), v = -ahf(\eta),\ w = axg(\eta),\ \theta(\eta) = \frac{T-T_L}{T_w-T_L},\ C(\eta) = \frac{C-C_L}{C_w-C_L}. \tag{8.15}$$

Therefore, the governing momentum, energy and mass transfer equations for this problem are given in dimensionless form by:

$$a_1 f^{iv} - Re(f'f'' - ff''') - 2Krg' - \left(a_3 M^2 + \frac{a_1}{k}\right) f'' = 0, \tag{8.16}$$

$$a_1 g'' - Re(f'g - fg') + 2Krf' - \left(a_3 M^2 + \frac{a_1}{k}\right) g = 0, \tag{8.17}$$

$$\theta'' + PrRea_2 f\theta' + NbC'\theta' + Nt\theta'^2 = 0, \tag{8.18}$$

$$NbC'' + Nt\theta'' + Nb \cdot Re \cdot Sc \cdot f \cdot C' = 0, \tag{8.19}$$

subject to

$$f = 0, f' = 1, g = 0, \theta = 1,\ C = 1\ at\ \eta = 0, \tag{8.20}$$

$$f = 0, f' = 0, g = 0, \theta = 0,\ C = 0\ at\ \eta = 1, \tag{8.21}$$

where

$$Pr = \frac{\mu_f \left(\rho c_p\right)_f}{\rho_f k_f}, \tag{8.22}$$

$$Sc = \frac{\mu_f}{\rho_f D}, \tag{8.23}$$

$$M = \frac{\sigma_f {B_0}^2 L^2}{\rho_f v_f}, \tag{8.24}$$

$$\frac{1}{\kappa} = \frac{v\varphi^2}{k_1 v_f}, \tag{8.25}$$

$$Kr = \frac{\Omega L^2}{v_f}, \tag{8.26}$$

$$Re = \frac{aL^2}{v_f}, \tag{8.27}$$

$$Ec = \frac{(aL)^2}{(c_p)_f(\theta_0 - \theta_L)}, \tag{8.28}$$

$$\alpha = \frac{k_{nf}}{(\rho c_p)_{nf}}, \tag{8.29}$$

$$Nb = \frac{(\rho c_p)_s D_B C_L}{(\rho c_p)_f \alpha}, \tag{8.30}$$

$$Nt = \frac{(\rho c_p)_s D_B T_L}{(\rho c_p)_f \alpha T_w}, \tag{8.31}$$

$$b_0 = 1 - \emptyset, \tag{8.32}$$

$$b_1 = \left(b_0 + \emptyset \frac{\rho_s}{\rho_f}\right), \tag{8.33}$$

$$b_2 = \frac{1}{{b_0}^{2.5}}, \tag{8.34}$$

$$b_3 = \left(b_0 + \emptyset \frac{(\rho c_p)_s}{(\rho c_p)_f}\right), \tag{8.35}$$

$$b_4 = \frac{k_{nf}}{k_f}, \tag{8.36}$$

$$b_5 = \frac{\sigma_{nf}}{\sigma_f}, \tag{8.37}$$

$$a_1 = \frac{1}{{b_0}^{2.5} b_1}, \tag{8.38}$$

$$a_2 = \frac{b_3}{b_4 + Nr}, \tag{8.39}$$

$$a_3 = \frac{b_5}{b_1}, \tag{8.40}$$

$$a_4 = \frac{b_2}{b_4}, \tag{8.41}$$

$$Nr = \frac{16\sigma^* {T_0}^3}{3k^* k_f}. \tag{8.42}$$

8.4 Solution by Homotopy analysis Method

Equations (8.16) – (8.19) with the boundary conditions (8.20) – (8.21) are solved HAM [40].

Initial guess is given by:

$$f_0(\eta) = \frac{-2}{e^2 - 4e + 3} + \frac{e-1}{e-3}\eta + \frac{2-e}{e^2 - 4e + 3} e^{\eta} + \frac{e}{e^2 - 4e + 3} e^{-\eta}; \tag{8.43}$$

$$g_0(\eta) = 0; \theta_0(\eta) = 1 - \eta; \tag{8.44}$$

$$C_0(\eta) = 1 - \eta; \tag{8.45}$$

with auxiliary linear operators:

$$L_f = \frac{\partial^4 f}{\partial \eta^4} - \frac{\partial^2 f}{\partial \eta^2}, \; L_g = \frac{\partial^2 g}{\partial \eta^2} - \frac{\partial g}{\partial \eta}, \; L_\theta = \frac{\partial^2 \theta}{\partial \eta^2}, \; L_C = \frac{\partial^2 C}{\partial \eta^2}, \tag{8.46}$$

satisfying

$$L_f(C_1 + C_2\,\eta + C_3 e^{\eta} + C_4 e^{-\eta}) = 0, \tag{8.47}$$

$$L_g(C_5 + C_6 e^{\eta}) = 0, \tag{8.48}$$

$$L_\theta(\,C_7 + C_8\eta) = 0, \tag{8.49}$$

$$L_C(\,C_9 + C_{10}\eta) = 0, \tag{8.50}$$

where $c_1,\ \ c_2, \dots, c_{10}$ are the arbitrary constants.

The zeroth order deformation problems are constructed as follows:

$$(1-p)L_f\big[\hat{f}(\eta;\,p) - f_0(\eta)\big] = p\hbar_f N_f\big[\hat{f}(\eta;\,p), \hat{g}(\eta;\,p), \hat{\theta}(\eta;\,p), \hat{C}(\eta;\,p)\big], \tag{8.51}$$

$$(1-p)L_g[\hat{g}(\eta;\,p) - g_0(\eta)] = p\hbar_g N_g\big[\hat{f}(\eta;\,p), \hat{g}(\eta;\,p), \hat{\theta}(\eta;\,p), \hat{C}(\eta;\,p)\big], \tag{8.52}$$

$$(1-p)L_\theta\big[\hat{\theta}(\eta;\,p) - \theta_0(\eta)\big] = p\hbar_\theta N_\theta\big[\hat{f}(\eta;\,p), \hat{g}(\eta;\,p), \hat{\theta}(\eta;\,p), \hat{C}(\eta;\,p)\big], \tag{8.53}$$

$$(1-p)L_C\big[\hat{C}(\eta;\,p) - C_0(\eta)\big] = p\hbar_C N_C\big[\hat{f}(\eta;\,p), \hat{g}(\eta;\,p), \hat{\theta}(\eta;\,p), \hat{C}(\eta;\,p)\big], \tag{8.54}$$

subject to the boundary conditions:

$$\hat{f}(0;\,p) = 0, \quad \hat{f}'(0;\,p) = 1; \tag{8.55}$$

$$\hat{f}(1;\,p) = 0, \quad \hat{f}'(1;\,p) = 0; \tag{8.56}$$

$$\hat{g}(0;\,p) = 0, \quad \hat{g}(1;\,p) = 0; \tag{8.57}$$

$$\hat{\theta}(0;\,p) = 1, \quad \hat{\theta}(1;\,p) = 0; \tag{8.58}$$

$$\hat{C}(0;\,p) = 1, \quad \hat{C}(1;\,p) = 0. \tag{8.59}$$

Nonlinear operators are defined as

$$N_f\left[\hat{f}(\eta;\ p), \hat{g}(\eta;\ p), \hat{\theta}(\eta;\ p), \hat{C}(\eta;\ p)\right] = \ a_1 \frac{\partial^4 \hat{f}}{\partial \eta^4} - Re\left(\frac{\partial \hat{f}}{\partial \eta}\frac{\partial^2 \hat{f}}{\partial \eta^2} - \hat{f}\frac{\partial^3 \hat{f}}{\partial \eta^3}\right) - 2Kr\ \frac{\partial \hat{g}}{\partial \eta} - \left(a_3 M^2 + \frac{a_1}{k}\right)\frac{\partial^2 \hat{f}}{\partial \eta^2}, \tag{8.60}$$

$$N_g\left[\hat{f}(\eta;\ p), \hat{g}(\eta;\ p), \hat{\theta}(\eta;\ p), \hat{C}(\eta;\ p)\right] = a_1 \frac{\partial^2 \hat{g}}{\partial \eta^2} - Re\left(g\ \frac{\partial \hat{f}}{\partial \eta} - f\ \frac{\partial \hat{g}}{\partial \eta}\right) + 2Kr\ \frac{\partial \hat{f}}{\partial \eta} - \left(a_3 M^2 + \frac{a_1}{k}\right) g\ , \tag{8.61}$$

$$N_\theta\left[\hat{f}(\eta;\ p), \hat{g}(\eta;\ p), \hat{\theta}(\eta;\ p), \hat{C}(\eta;\ p)\right] = \ \frac{\partial^2 \hat{\theta}}{\partial \eta^2} + PrRea_2 \hat{f}\frac{\partial \hat{\theta}}{\partial \eta} + Nb\frac{\partial \hat{C}}{\partial \eta}\frac{\partial \hat{\theta}}{\partial \eta} + Nt\left(\frac{\partial \hat{\theta}}{\partial \eta}\right)^2, \tag{8.62}$$

$$N_C\left[\hat{f}(\eta;\ p), \hat{g}(\eta;\ p), \hat{\theta}(\eta;\ p), \hat{C}(\eta;\ p)\right] = \ Nb\frac{\partial^2 \hat{C}}{\partial \eta^2} + Nt\frac{\partial^2 \hat{\theta}}{\partial \eta^2} + NbReSc\hat{f}\,\frac{\partial \hat{C}}{\partial \eta}\ , \tag{8.63}$$

where $\hat{f}(\eta;\ p), \hat{g}(\eta;\ p), \widehat{\theta}(\eta;\ p)$ and $\hat{C}(\eta;\ p)$ are unknown functions with respect to η and p. $\hbar_f$, $\hbar_g, \hbar_\theta$ and $\hbar_C$ are the non-zero auxiliary parameters and N_f, N_g, N_θ and N_C are the nonlinear operators.

Also, $p \in (0,\ 1)$ is an embedding parameter. For $p = 0$ and $p = 1$,

$$\hat{f}(\eta; 0) = f_0(\eta), \hat{f}(\eta; 1) = f(\eta), \tag{8.64}$$

$$\hat{g}(\eta; 0) = g_0(\eta), \hat{g}(\eta; 1) = g(\eta), \tag{8.65}$$

$$\hat{\theta}(\eta; 0) = \ \theta_0(\eta), \hat{\theta}(\eta; 1) = \ \theta(\eta), \tag{8.66}$$

$$\hat{C}(\eta; 0) = \ C_0(\eta), \hat{C}(\eta; 1) = \ C(\eta). \tag{8.67}$$

In other words, when variation of p is taken from 0 to 1 then $\hat{f}(\eta;\ p)$, $\hat{g}(\eta;\ p), \hat{\theta}(\eta;\ p)$ and $\hat{C}(\eta;\ p)$ vary from $f_0(\eta)$, $g_0(\eta)$, $\theta_0(\eta)$ and $C_0(\eta)$ to $f(\eta), g(\eta), \theta(\eta)$ and $C(\eta)$ respectively.

Applying Taylor's series expansion:

$$\hat{f}(\eta;\ p) = f_0(\eta) + \textstyle\sum_{m=1}^{\infty} f_m(\eta) p^m, \tag{8.68}$$

$$\hat{g}(\eta;\ p) = g_0(\eta) + \textstyle\sum_{m=1}^{\infty} g_m(\eta) p^m, \tag{8.69}$$

$$\hat{\theta}(\eta;\ p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m, \tag{8.70}$$

$$\hat{C}(\eta;\ p) = C_0(\eta) + \sum_{m=1}^{\infty} C_m(\eta) p^m, \tag{8.71}$$

where

$$f_m(\eta) = \frac{1}{m!} \left[\frac{\partial^m f(\eta;\, p)}{\partial p^m} \right]_{p=0}, \tag{8.72}$$

$$g_m(\eta) = \frac{1}{m!} \left[\frac{\partial^m g(\eta;\, p)}{\partial p^m} \right]_{p=0}, \tag{8.73}$$

$$\theta_m(\eta) = \frac{1}{m!} \left[\frac{\partial^m \theta(\eta;\, p)}{\partial p^m} \right]_{p=0}, \tag{8.74}$$

$$C_m(\eta) = \frac{1}{m!} \left[\frac{\partial^m C(\eta;\, p)}{\partial p^m} \right]_{p=0}. \tag{8.75}$$

It should be noted that the convergence of above series strongly depend on $\hbar_f, \hbar_g, \hbar_\theta$ and $\hbar_C$. Assuming that these nonzero auxiliary parameters are chosen so that Equations (8.68) – (8.71) converge at $p = 1$. Hence, one can obtain the following:

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{8.76}$$

$$g(\eta) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta), \tag{8.77}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta), \tag{8.78}$$

$$C(\eta) = C_0(\eta) + \sum_{m=1}^{\infty} C_m(\eta). \tag{8.79}$$

Differentiating the zeroth order deformation (8.51) – (8.54) and (8.55) – (8.59) m times with respect to p and substituting $p = 0$, and finally dividing by $m!$; m^{th} order deformation ($m \geq 1$) is

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_f R_{f,m}(\eta), \tag{8.80}$$

$$L_g[g_m(\eta) - \chi_m g_{m-1}(\eta)] = \hbar_g R_{g,m}(\eta), \tag{8.81}$$

$$L_{\theta}[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)]=\hbar_{\theta}R_{\theta,m}(\eta), \tag{8.82}$$

$$L_C[C_m(\eta)-\chi_m C_{m-1}(\eta)]=\hbar_C R_{C,m}(\eta), \tag{8.83}$$

subject to the boundary conditions

$$f_m(0)=f_m'(0)=0, \tag{8.84}$$

$$f_m(1)=f_m'(1)=0, \tag{8.85}$$

$$g_m(0)=g_m(1)=0, \tag{8.86}$$

$$\theta_m(0)=\theta_m(1)=0, \tag{8.87}$$

$$C_m(0)=C_m(1)=0, \tag{8.88}$$

with

$$R_{f,m}(\eta)=a_1 f_{m-1}^{iv}-Re\left(\sum_{j=0}^{m-1} f_j' f_{m-1-j}''-\sum_{j=0}^{m-1} f_j f_{m-1-j}'''\right)-2Krg_{m-1}'-\left(a_3M^2+\frac{a_1}{k}\right)f_{m-1}'', \tag{8.89}$$

$$R_{g,m}(\eta)=a_1 g_{m-1}''-Re\left(\sum_{j=0}^{m-1} f_j' g_{m-1-j}-\sum_{j=0}^{m-1} f_j g_{m-1-j}'\right)+2Krf_{m-1}'-\left(a_3M^2+\frac{a_1}{k}\right)g_{m-1}, \tag{8.90}$$

$$R_{\theta,m}(\eta)=\theta_{m-1}''+PrRea_2\sum_{j=0}^{m-1} f_j\theta_{m-1-j}'+Nb\sum_{j=0}^{m-1} C_j'\theta_{m-1-j}'+Nt\sum_{j=0}^{m-1}\theta_j'\theta_{m-1-j}', \tag{8.91}$$

$$R_{C,m}(\eta)=NbC_{m-1}''+Nt\theta_{m-1}''+NbReSc\sum_{j=0}^{m-1} f_j C_{m-1-j}', \tag{8.92}$$

with

$$\chi_m=\begin{cases}0, & m\leq 1\\ 1, & m\geq 1\end{cases}. \tag{8.93}$$

Solving the corresponding m[th] order deformation equations,

$$f_m(\eta)=f_m^*(\eta)+C_1+C_2\,\eta+C_3e^{\eta}+C_4e^{-\eta}, \tag{8.94}$$

$$g_m(\eta) = g_m^*(\eta) + C_5 + C_6 e^{\eta}, \tag{8.95}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_7 + C_8\eta, \tag{8.96}$$

$$C_m(\eta) = C_m^*(\eta) + C_9 + C_{10}\eta. \tag{8.97}$$

Here f_m^* , g_m^*, θ_m^* and C_m^* are given by are particular solutions of the corresponding mth order equations and the constants $C_i(i = 1,2, \ldots ,10)$ are to be determined by boundary conditions.

8.4.1 Convergence of solution

Convergence of solutions and rate of approximations strongly depend on the values of auxiliary parameters $\hbar_f, \hbar_g, \hbar_\theta$ and $\hbar_C$. For this purpose the associated h-curves are plotted in Figure 8.2 – 8.5. These figures clearly suggest admissible ranges for the auxiliary parameters.

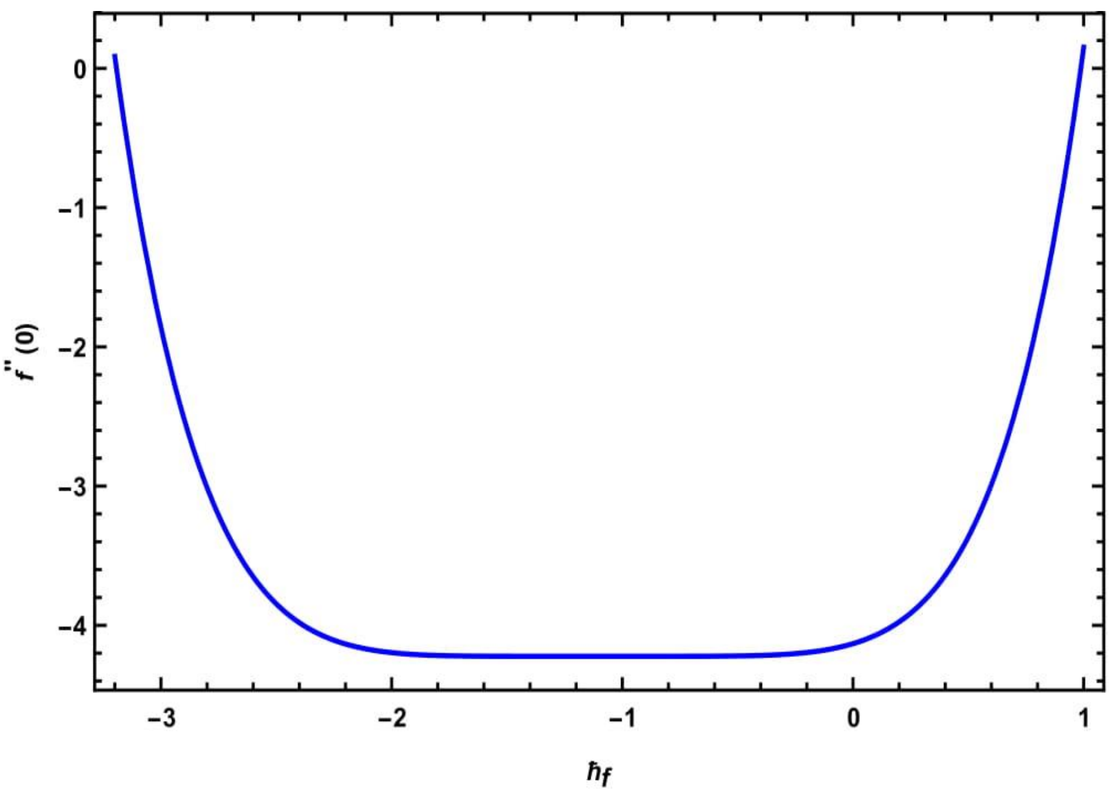

Figure 8.2: H-Curve of $f''(0)$ for $\hbar_f$ at $\kappa = 0.2,\ \phi = 0.04,\ M = 0.5,\ Kr = 0.5,$ $Pr = 6.2,\ Re = 0.5,\ Nt = 0.5,\ Nb = 0.5,\ Sc = 0.5\ and\ Nr = 2.0.$

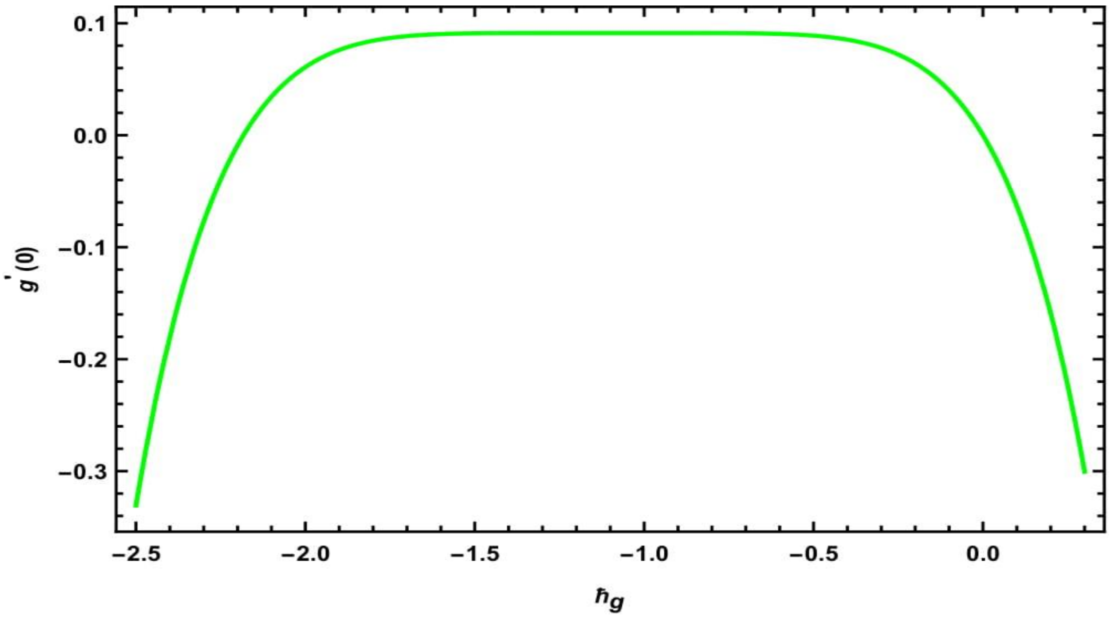

Figure 8.3: H-Curve of $g'(0)$ for $\hbar_g$ at $\kappa = 0.2, \phi = 0.04,\ M = 0.5,\ Kr = 0.5,$ $Pr = 6.2,\ Re = 0.5,\ Nt = 0.5,\ Nb = 0.5,\ Sc = 0.5\ and\ Nr = 2.0.$

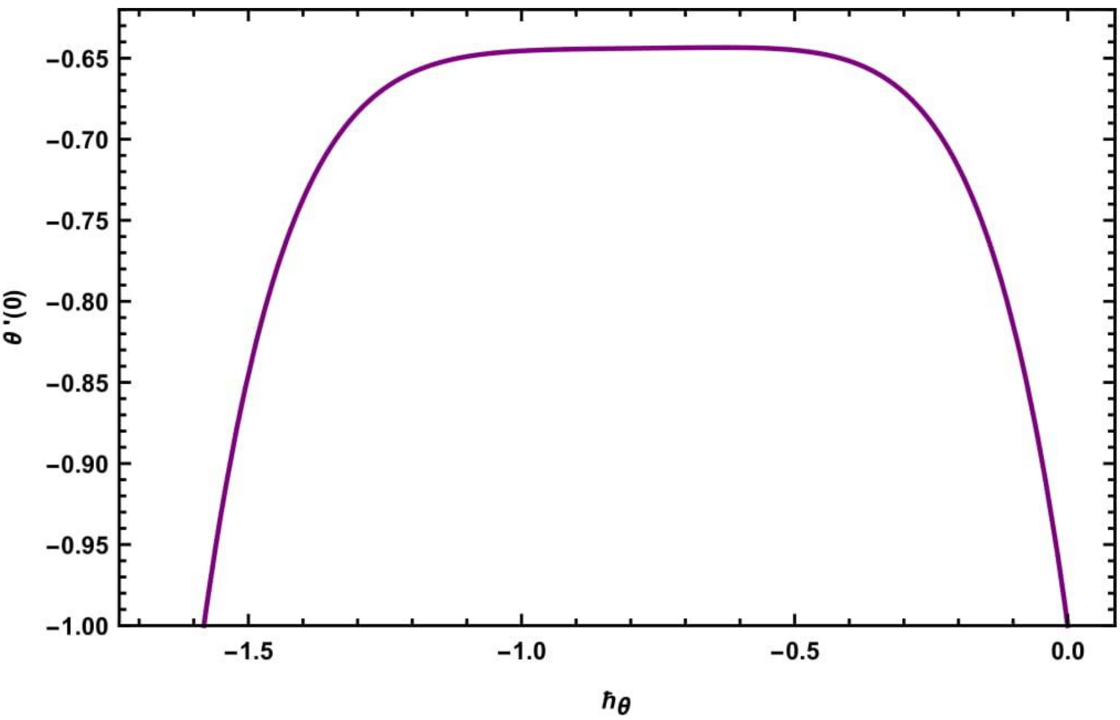

Figure 8.4: H-Curve of $\theta'(0)$ for $\hbar_\theta$ at $\kappa = 0.2,\ \phi = 0.04,\ M = 0.5,\ Kr = 0.5,$ $Pr = 6.2,\ Re = 0.5,\ Nt = 0.5,\ Nb = 0.5,\ Sc = 0.5\ and\ Nr = 2.0.$

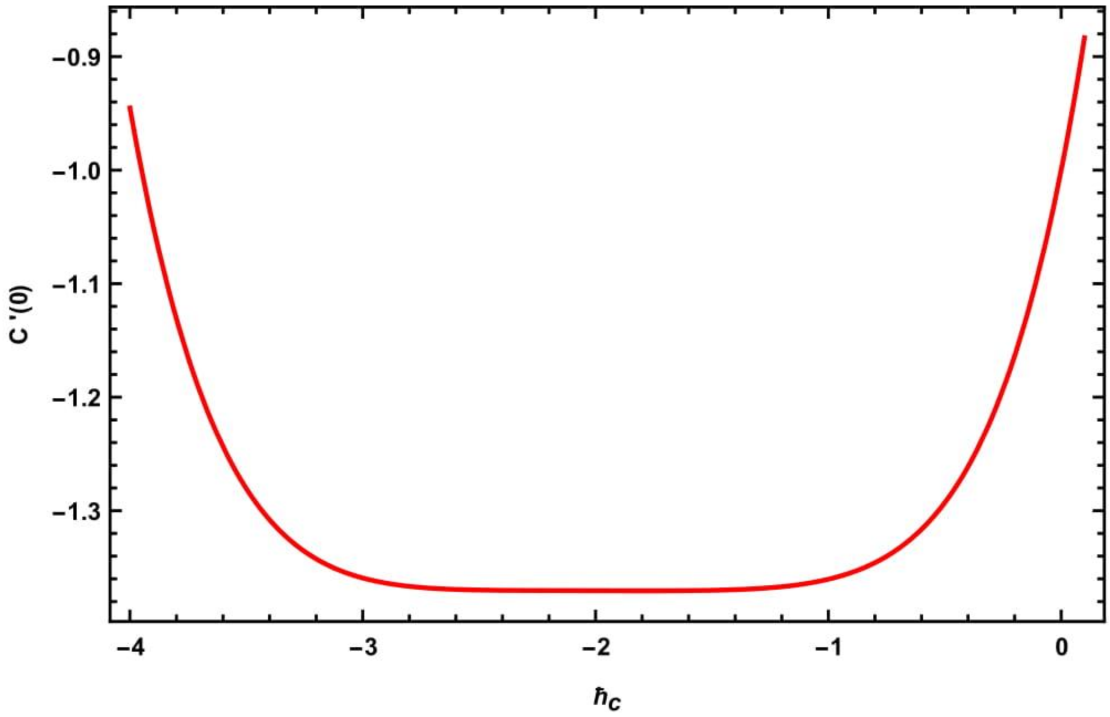

Figure 8.5: H-Curve of $C'(0)$ for $\hbar_C$ at $\kappa = 0.2,\ \phi = 0.04,\ M = 0.5,\ Kr = 0.5,$ $Pr = 6.2, Re = 0.5,\ Nt = 0.5,\ Nb = 0.5,\ Sc = 0.5\ and\ Nr = 2.0.$

8.5 Results and Discussion

This section is dedicated to the physics of the problem through graphical representation of concentration profile. Solutions are obtained using Mathematica. Effects of different parameters: Reynolds number Re, Radiation parameter Nr, Rotation parameter Kr, Permeability parameter κ, Prandtl number Pr, Thermophoretic parameter Nt, Brownian parameter Nb and Schmidt Number Sc on fluid flow is represented through Figures 8.6 – 8.17. Effect of Nb on concentration profile is evident from Figures 8.6. It is observed that concentration decreases with increase in Nb. From Figure 8.7, it is concluded that thermal radiation parameter Nr tends to reduce concentration throughout the flow field. Figures 8.8 shows that concentration can be decreased by increasing values of thermophoresis parameter Nt.

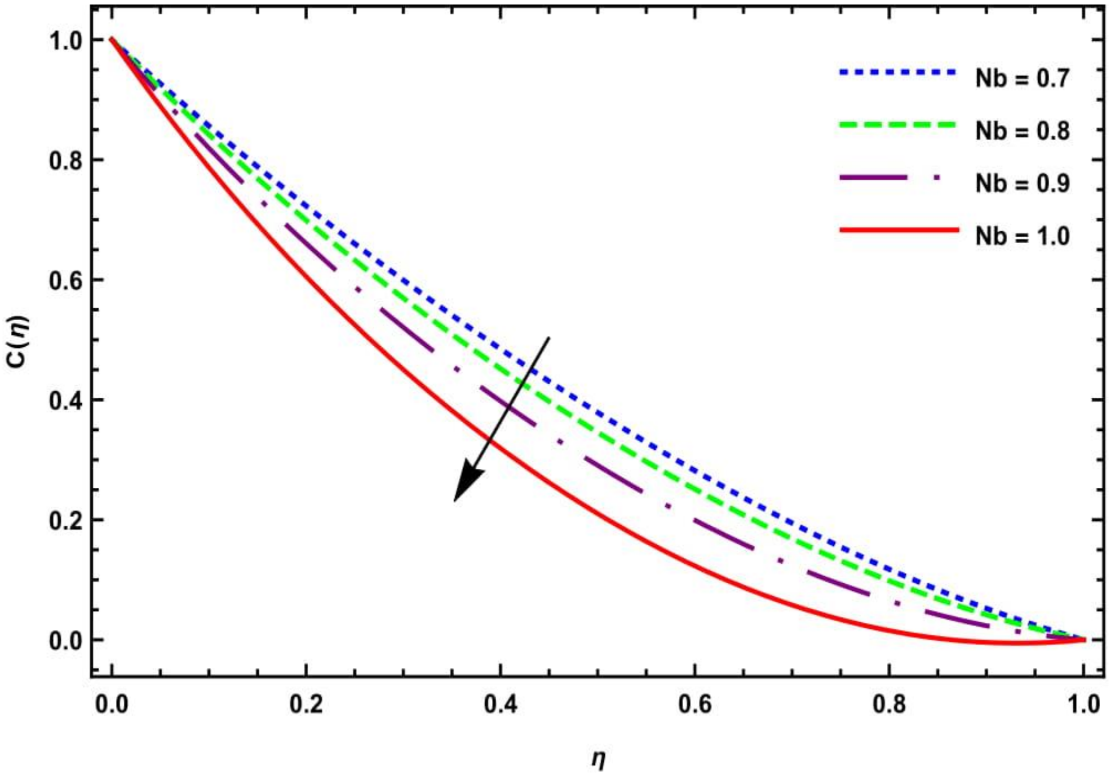

Figure 8.6: Concentration profile C for η and Nb at $\phi = 0.04,\ M = 0.5,\ Pr = 6.2,$ $\kappa = 0.2,\ Re = 0.5,\ Nt = 0.5,\ Kr = 0.5,\ Sc = 0.5\ and\ Nr = 2.0.$

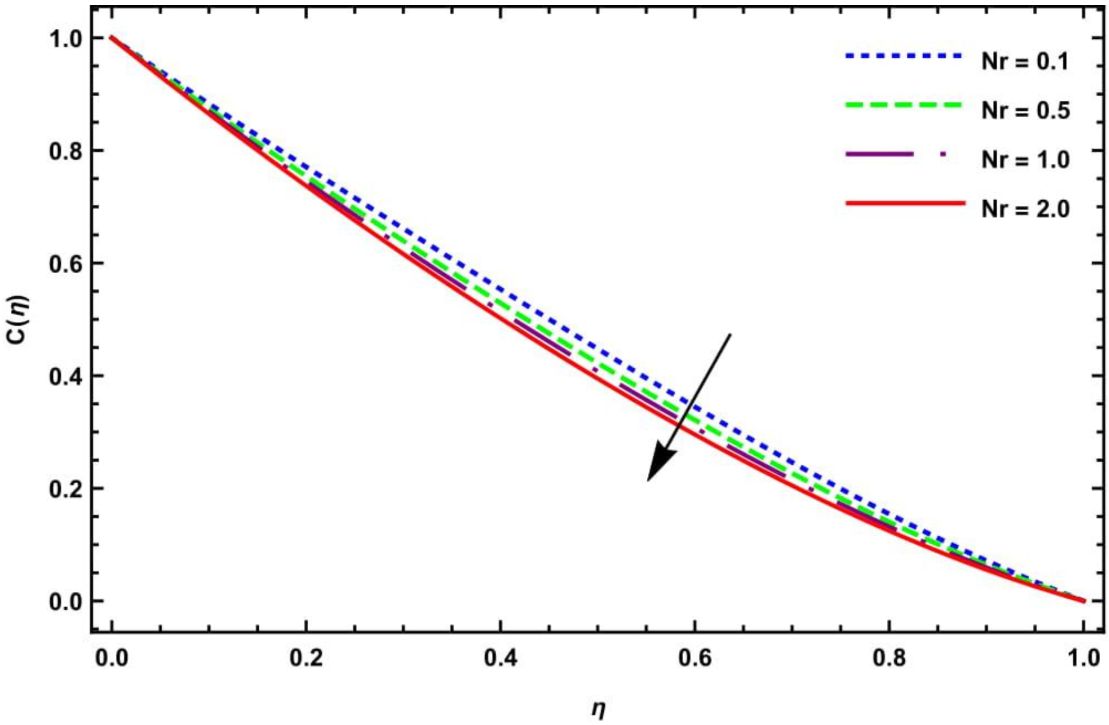

Figure 8.7: Concentration profile C for η and Nr at $\phi = 0.04,\ M = 0.5,\ Pr = 6.2,$ $\kappa = 0.2,\ Re = 0.5,\ Nb = 0.5, Kr = 0.5,\ Sc = 0.5\ and\ Nt = 0.5$

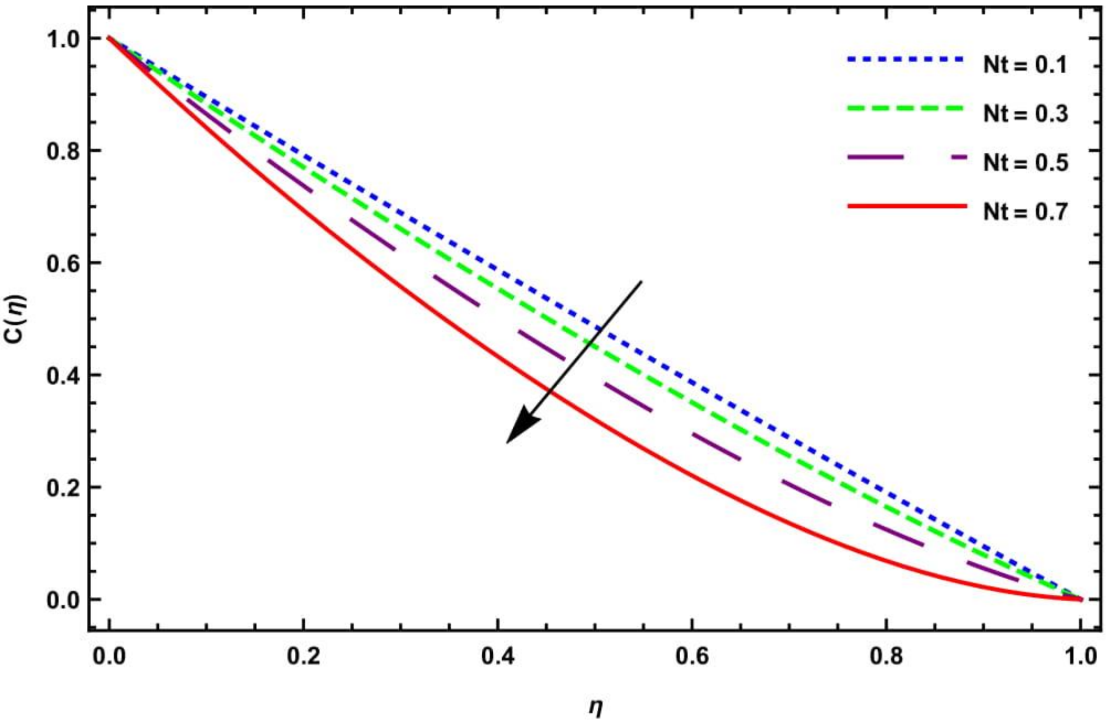

Figure 8.8: Concentration profile for η and Nt at $\phi = 0.04,\ M = 0.5,\ Pr = 10,$ $\kappa = 0.2,\ Re = 0.5,\ Nb = 0.5, Kr = 0.5,\ Sc = 0.5\ and\ Nr = 2.0.$

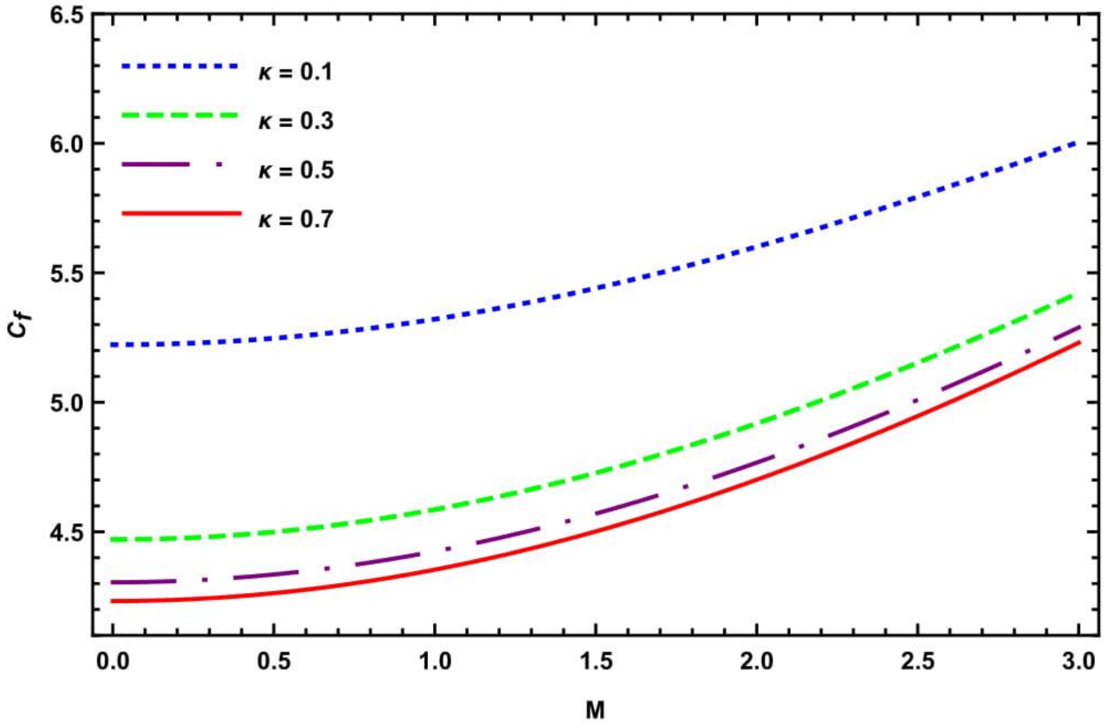

Figure 8.9: Effect of M and κ on C_f at $\phi = 0.04, Nt = 0.5,\ Pr = 6.2,\ Re = 0.5,$ $Nb = 0.5,\ Kr = 0.5,\ Sc = 0.5\ and\ Nr = 2.0.$

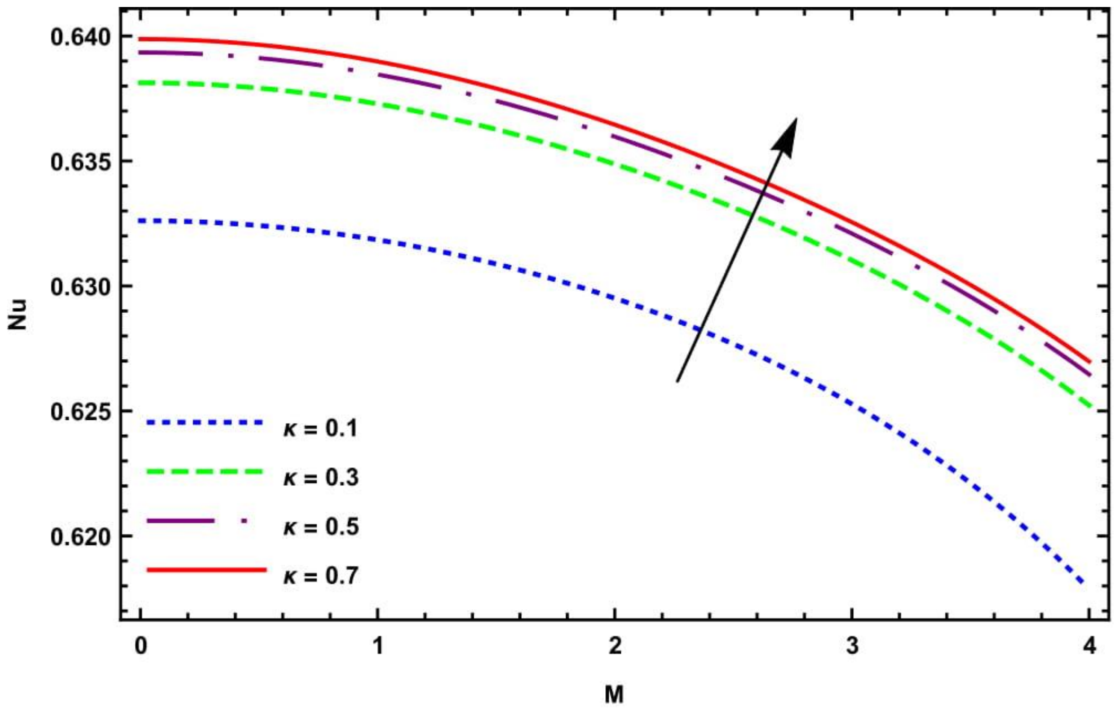

Figure 8.10: Effect of M and κ on Nu at $Kr = 0.5,\ Nt = 0.5,\ Pr = 6.2,\ \phi = 0.04,$ $Nb = 0.5,\ Re = 0.5,\ Sc = 0.5\ and\ Nr = 2.0.$

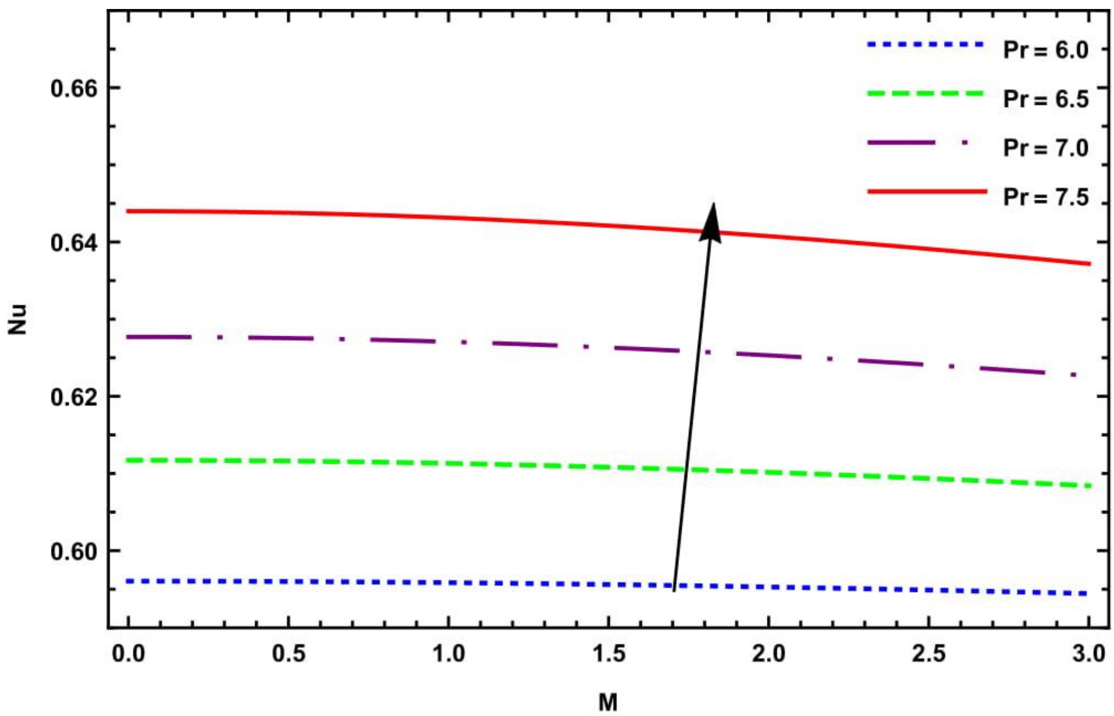

Figure 8.11: Effect of M and Pr on Nu at $\kappa = 0.2,\ Nr = 2.0,\ \phi = 0.04,\ Nt = 0.5,$ $Kr = 0.5,\ Re = 0.5,\ Sc = 0.5\ and\ Nb = 0.5.$

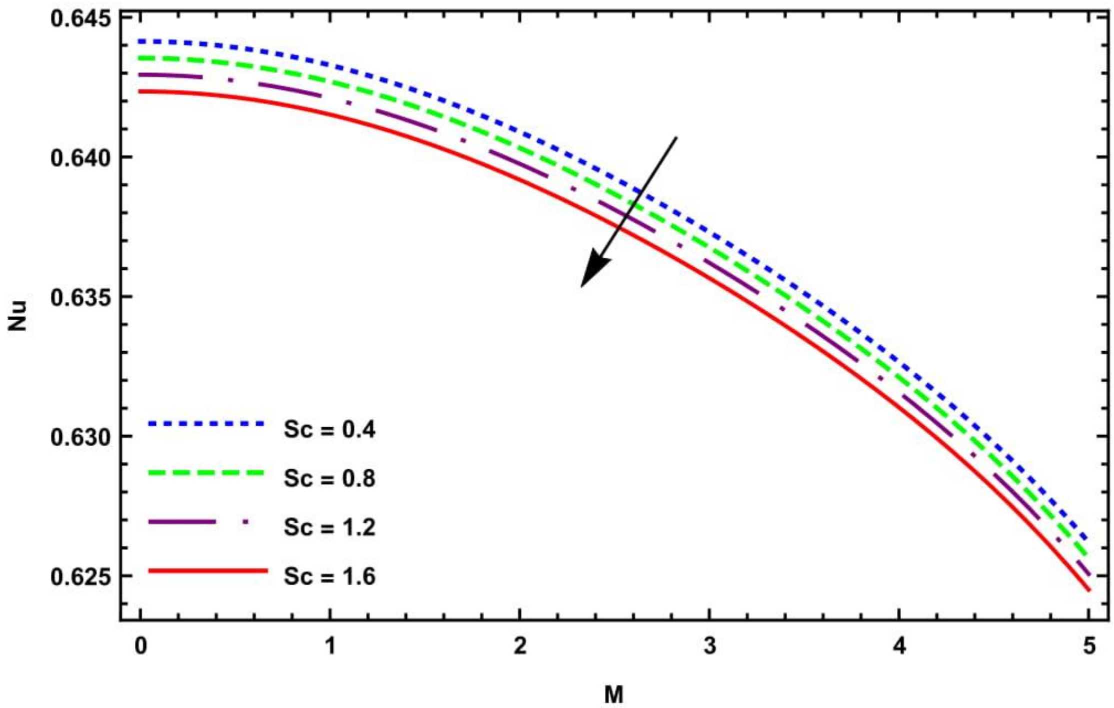

Figure 8.12: Effect of M and Sc on Nu at $\kappa = 0.2,\ Nr = 2.0,\ \phi = 0.04,\ Nt = 0.5,$ $Kr = 0.5,\ Pr = 6.2,\ Re = 0.5\ and\ Nb = 0.5.$

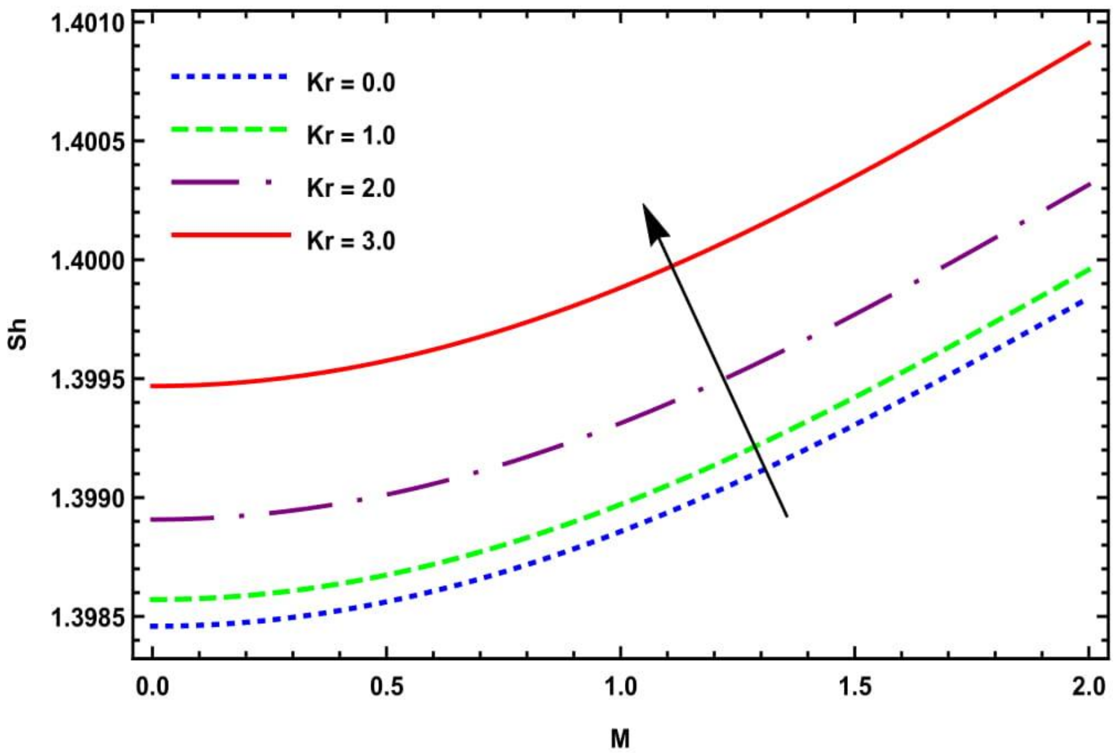

Figure 8.13: Effect of M and Kr on Sh at $\kappa = 0.2,\ Nr = 2.0,\ \phi = 0.04,\ Nt = 0.5,$ $Sc = 0.5,\ Pr = 6.2,\ Re = 0.5\ and\ Nb = 0.5.$

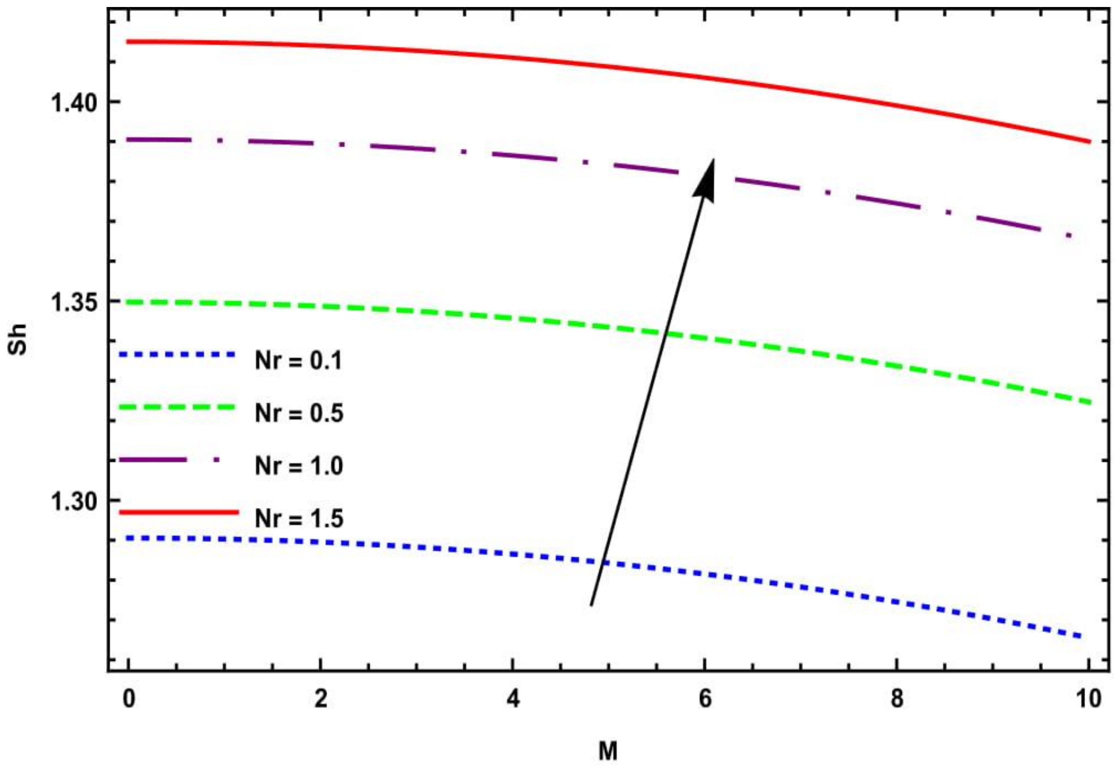

Figure 8.14: Effect of M and Nr on Sh at $\kappa = 0.2,\ Nb = 0.5,\ \phi = 0.04, Nt = 0.5,$ $Sc = 0.5,\ Pr = 6.2,\ Re = 0.5\ and\ Kr = 0.5.$

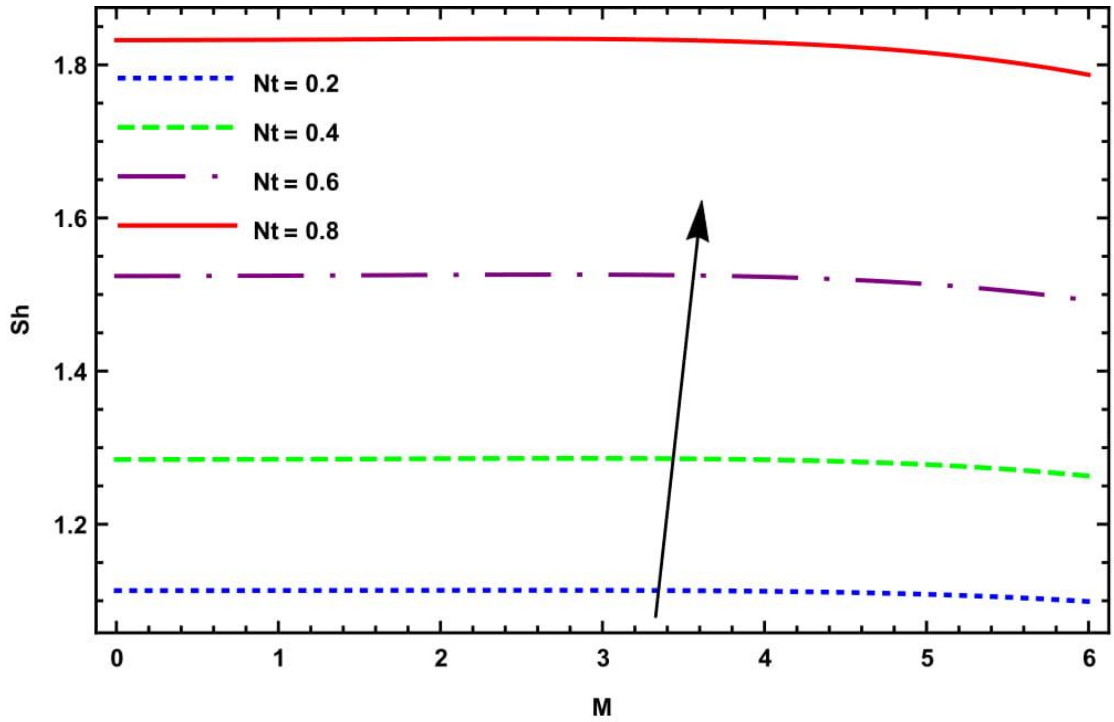

Figure 8.15: Effect of M and Nt on Sh at $\kappa = 0.2,\ Nr = 2.0,\ \phi = 0.04,\ Nb = 0.5,$ $Sc = 0.5,\ Pr = 6.2,\ Re = 0.5\ and\ Kr = 0.5.$

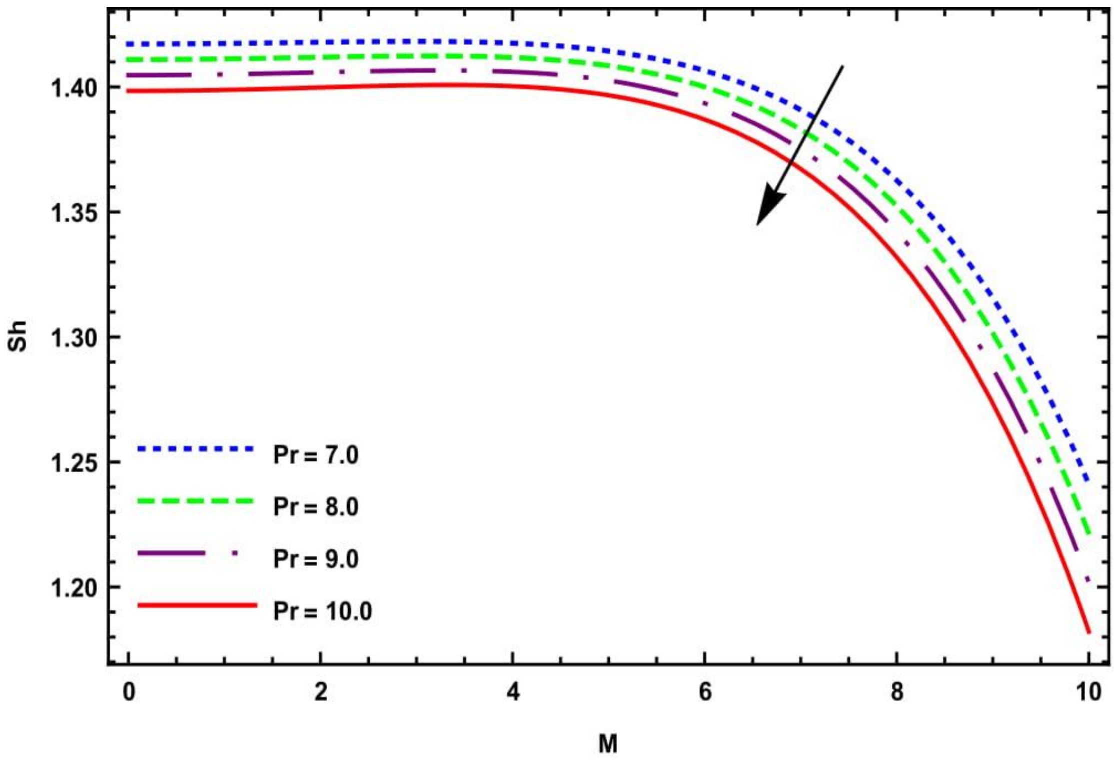

Figure 8.16: Effect of M and Pr on Sh at $\kappa = 0.2,\ Nr = 2.0,\ Nt = 0.5, Nb = 0.5,$ $Sc = 0.5,\ \phi = 0.04,\ Re = 0.5\ and\ Kr = 0.5.$

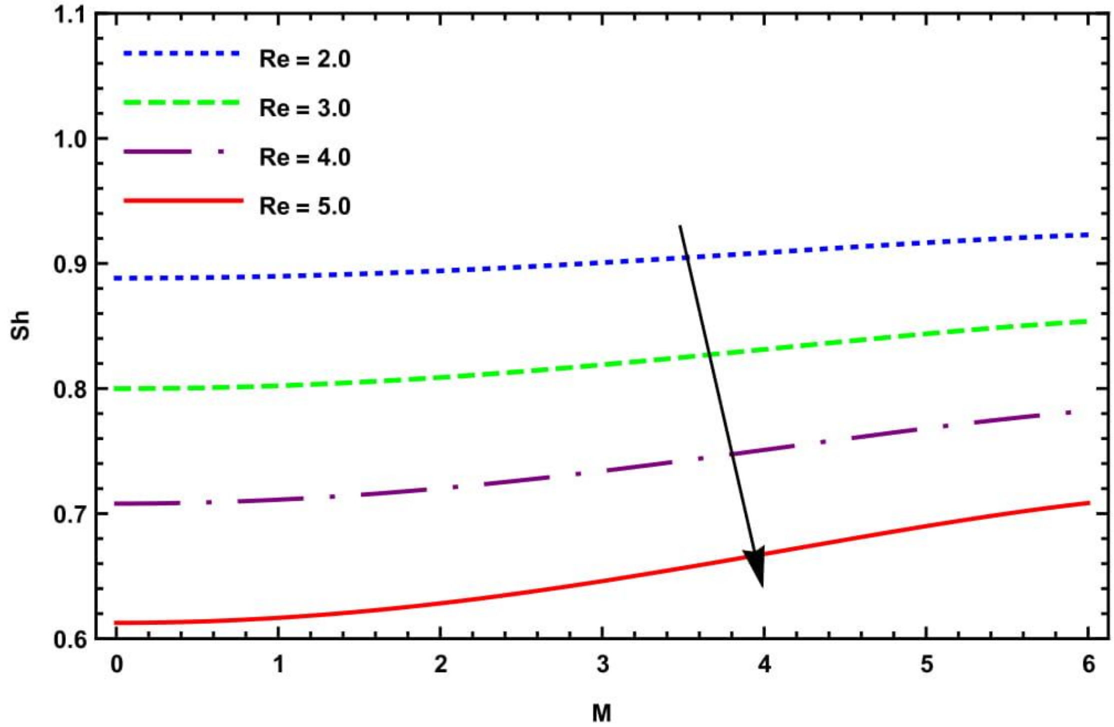

Figure 8.17: Effect of M and Re on Sh at $\kappa = 0.2,\ Nr = 2.0,\ Nt = 0.5,\ Nb = 0.5,$ $Sc = 0.5,\ \phi = 0.04,\ Pr = 6.2\ and\ Kr = 0.5.$

Figure 8.9 show skin friction for different values of κ. It is seen that, Skin friction decrease with increase in κ. Figures 8.10 – 8.12 illustrate effects of different physical parameter on Nusselt number. It is evident that κ and Pr tends to increase the values of Nusselt number while Sc have reverse effects on it. Figures 8.13 – 8.17 illustrate effects of different physical parameters on Sherwood Number. It is seen that Nr tends to increase the values of Sherwood number while Pr, Re, Kr and Nt have decreasing tendency.

8.6 Conclusion

The most important concluding remarks can be summarized as follows:

- Concentration declines with increasing value of Nt and Nb.
- Nusselt number increases with increase in Pr but decreases with increase in Sc.
- Nr has positive effect on Sherwood number.
- Kr, Pr, Re and Nt have decreasing tendency on Sherwood number.