

Preface

The present thesis entitled “A UNIFICATION OF GENERALIZED MITTAG - LEFFLER FUNCTIONS, FAMILY OF BESSEL FUNCTION AND THEIR q -ANALOGUES” incorporates the investigations carried out by me under the able guidance of Dr. B. I. Dave, Associate Professor, Department of Mathematics, Faculty of Science, The Maharaja Sayajirao University of Baroda, Vadodara.

The subject matter of the thesis is to propose such a generalized structure of Mittag-Leffler function which besides unifying certain generalizations occurring in the literature, also yields certain inequality relations.

The particularizations of this proposed function include the Bessel function, Generalized Bessel-Maitland function, Lommel function, Struve function, also Dotsonko function, a particular form ($m = 2$) of extension of Mittag-Leffler function due to Saxena and Nishimoto, and the Elliptic function.

There are total seven chapters in the thesis. Chapter-1 introduces briefly, the Mittag-Leffler function, definitions and formulas used in the work, Hypergeometric series, Basic hypergeometric series, Integral transforms, q -Integral transforms, Fractional integral and derivatives, Fractional q -integral and q -derivatives.

Chapter-2 begins with the introduction of unification of certain generalizations of Mittag-Leffler function. The main results include the absolute convergence test of the series represented by the unified function. The subsequent properties include order and its type, asymptotic estimate, differential equation, Eigen function property, Mellin-Barnes contour integral representation. Certain mixed recurrence type relations are derived; and the results involving integral transforms namely, Euler-Beta transform, Mellin-Barnes transform, Laplace transform and Whittaker transform are obtained. The special cases such as the generalized hypergeometric function, generalized Laguerre polynomial, Fox H-function etc. are also illustrated.

In Chapter-3, two q -analogues of this unified function are defined and their properties are obtained. Chapter-4 and Chapter-5 contain the results pertaining to fractional differentiation, integration and their q -forms. In Chapter-6, the unified ML-function is exploited to yield a generalized Konhauser polynomial (GKP. Some interesting properties involving inequality are derived. Besides this, the generating function and inverse series relations are obtained. Chapter-7 deals with the “ q -treatment” of the results of Chapter-6.

A shortened version of the contents of the chapters may be found in the following published papers/communicated work.

1. Prajapati, J., Dave, B. and Nathwani, B., *On a unification of generalized Mittag-Leffler Function and family of Bessel Functions*, Advances in Pure Mathematics **3**(2013), 127–137.
2. Prajapati, J. and Nathwani, B., *Recurrence relation of a unified generalized Mittag-Leffler-Function*, Palestine Journal of Mathematics **3**(1)(2014), 94–98.
MR# 3109941
3. Prajapati, J. and Nathwani, B., *Fractional calculus of a unified Mittag-Leffler function*, Ukrainian Mathematical Journal **66**(2015), 1267–1280.
MR# 3334434
4. Nathwani, B. and Dave, B., *Generalized Mittag-Leffler function and its properties*, The Mathematics Student, (to appear).
5. Nathwani, B. and Dave, B., *An extended q -Mittag-Leffler function and its properties*, (communicated).
6. Nathwani, B. and Dave, B., *Fractional q -calculus of an extended q -Mittag-Leffler function*, (communicated)
7. Nathwani, B. and Dave, B., *Inequalities involving Mittag-Leffler type Konhauser polynomial*, (communicated).
8. Nathwani, B. and Dave, B., *Inequalities involving Mittag-Leffler type q -Konhauser polynomial*, (communicated).

The following are the papers presented in national/international conferences.

1. Presented a paper entitled "Mittag-Leffler type Generalized Konhauser Polynomial and its properties" in International Conference on "Special Functions and their applications (ICSFA 2015)(XIV Annual Conference of the Society for Special Functions and their applications) and Symposium on Fractional Calculus and their application in Special Functions" organized by Department of Mathematics, Amity Institute of Applied Sciences, Amity University, UP, Noida, India held on September 10-12, 2015.

2. Presented a paper entitled "Unification of Mittag-Leffler function and family of Bessel Function" in National Conference on "Current Developments in Analysis and its applications" organized by Department of Mathematics, Faculty of Science, The Maharaja Sayajirao University of Baroda, Vadodara on March 14-15, 2015.
3. Presented a paper entitled "Unification of q-ML function and family of q-Bessel Function" in International conference on "The works of Srinivas Ramanujan and their applications in Science and Engineering" and International Symposium on "New trends in applications of Mathematics in Science and Engineering specially in Fractional Calculus, Bio-informatics and Special Functions (ICSFA-2014)" at Department of Applied Sciences and Civil Engineering, RJIT, BSF Academy, Tekanpur, Gwalior(MP) on 22-23, December 2014.
4. Presented paper on "Certain Properties of Unified Mittag-Leffler Function" in "Regional Indian Science Congress Conference-2012" organized by The Maharaja Sayajirao University of Baroda, Vadodara held on September 16, 2012.
5. Presented a paper entitled "A q-Extension of Generalized Mittag-Leffler Function and Family of Bessel Function" in International Conference on "Special Functions and their Applications (ICSFA-2012) and Symposium on Life and Works of Ramanujan" organized by Department of Applied Mathematics and Humanities, Saradar Vallabhbhai National Institute of Technology, Surat during June 27-29, 2012 and received **M. I. Qureshi best paper presentation award**.
6. Presented a paper entitled "Unification of Generalized Mittag-Leffler Function and Certain Hypergeometric functions and its Properties" in International Conference on "Special Functions and their Applications (ICSFA-2011) and Symposium on Works of Ramanujan" organized by Department of Mathematics and Statistics, J. N. Vyas University, Jodhpur during July 28-30, 2011.

(B. V. Nathwani)

Vadodara,
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