

# Preface

The present thesis entitled “*A new class of special functions suggested by the generalized hypergeometric function and its  $q$ -analogue*” incorporates the investigation carried out by me under the guidance of Dr. B. I. Dave, Associate Professor, Department of Mathematics, Faculty of Science, The Maharaja Sayajirao University of Baroda, Vadodara.

The subject matter of this thesis is a generalization of the well-known hypergeometric function which as a particular case gives the extended exponential, trigonometric and hyperbolic functions. Their  $q$ -analogues are also derived. With the aid of these exponential functions, the Bessel functions are generated in context of this extended theory.

There are total seven chapters in this thesis. Chapter 1 introduces briefly, the generalized hypergeometric function, the confluent hypergeometric function, Sikkema’s function and the Bessel function. It also includes the basic hypergeometric series,  $q$ -Calculus and the  $q$ -Bessel function.

Chapter 2 deals with a generalization of hypergeometric function abbreviated as the  $\ell$ -Hypergeometric function which contains as main results the convergence of its series, differential equation it satisfies and eigen function property it possesses.

These results are published in the journal: *Annali dell’Universita’ Di Ferrara*, (2016), **62**(1):23-38.

In Chapter 3, the  $q$ -analogues of results derived in Chapter 2 are obtained. Another  $q$ -analogue of the  $\ell$ -Hypergeometric function is also obtained in this chapter.

The main results are published in the journal: *Bollettino dell’Unione Matematica Italiana*, (2016), **8**(4):239-256.

Chapter 4 contains the detailed study of the generalization of the  $\ell$ -Hypergeometric function which as particular cases gives rise to extended exponential ( $\ell$ -H exponential function), trigonometric and hyperbolic functions.

Certain results of these extensions are published in the journal: *Annali dell’Universita’ Di Ferrara*, (2016), **62**(1):23-38.

The  $\ell$ -analogues of Ramanujan's theorem and Kummer's first formula are also obtained by means of the particular case of the generalized  $\ell$ -Hypergeometric function.

Chapter 5 deals with the  $q$ -analogue of the generalized  $\ell$ -Hypergeometric function whose series convergence, difference equation, eigen function property and  $q$ -contiguous function relations are derived. The  $q$ -analogues of exponential function are extended as a special case of the  $q$ -generalized  $\ell$ -Hypergeometric function. The series transformation for this  $q$ -generalized  $\ell$ -Hypergeometric function is also obtained in this chapter. A part of this chapter is published in the journal: *Bollettino dell'Unione Matematica Italiana*, (2016), **8**(4):239-256.

Chapter 6 covers the derivation of several properties of the extended Bessel function generated with the aid of  $\ell$ -H exponential function considered in Chapter 4.

The last chapter, that is Chapter 7 incorporates the  $q$ -analogues of the results obtained in Chapter 6.

A shortened version of the contents of these Chapters may be found in the following published papers/communicated work.

1. A new generalization of  $q$ -hypergeometric function, *Boll. Unione. Mat. Ital.*, **8**(4), (2016), 239-256.  
DOI: <http://link.springer.com/article/10.1007/s40574-015-0041-1>
2. Some new class of special functions suggested by the confluent hypergeometric function, *Annali dell'Universita' Di Ferrara*, (2016), **62**(1):23-38.  
DOI: <http://link.springer.com/article/10.1007/s11565-015-0238-3>
3. A new class of functions suggested by the  $q$ -hypergeometric function. (Communicated).
4. A new class of functions suggested by the generalized hypergeometric function. (Communicated).
5. A new class of functions suggested by the generalized basic hypergeometric function. (Communicated).

6. A new extension of the Bessel function. (Communicated).
7. Certain properties of the extended  $q$ -Bessel function. (Communicated).

The following are the papers presented in national/international conferences/workshops:

1. Presented the paper entitled “*Extended Hypergeometric function and Its Properties*” in the National Conference (Sponsored by UGC) during March 14-15, 2015 at Department of Mathematics, Faculty of Science, The Maharaja Sayajirao University of Baroda, Vadodara.
2. Presented the paper entitled “*Some New Functions Suggested by Generalized Hypergeometric function*” at the International Conference of SSFA (Society for Special Functions and their Applications) organized by Department of Mathematics, Amity University, Noida (U.P.) during September 10-12, 2015.
3. Presented the paper entitled “*Another generalization of basic hypergeometric function*” in the National workshop (Sponsored by UGC, NBHM) during February 25-27, 2016 at Department of Mathematics, Mohanlal Sukhadia University, Udaipur, Rajasthan.