

APPENDIX A

A.1 Sigma-system of Vertical Coordinate

For an atmosphere which is represented by a system of nine number of levels in the vertical, a parameter  $Q_k$  where  $k$  represents any level, may be defined as

$$Q_k = \frac{2k-1}{17} \quad , \quad k = \frac{1}{2}, 1, 1\frac{1}{2}, 2, \dots, 9 \quad (\text{A.1.1})$$

From above,

$$Q = 0, \text{ for } k = \frac{1}{2} \quad (\text{top of the atmosphere})$$

and

$$Q = 1, \text{ for } k = 9 \quad (\text{earth's surface}).$$

The sigma coordinate which is defined as the ratio of pressure at any level to the pressure at the surface may be introduced as a function of  $Q$ .

Thus,

$$\sigma_k = \frac{p_k}{p_*} = F(Q_k) \quad (\text{A.1.2})$$

where  $p_*$  is the surface pressure.  $\sigma$  is zero at the top of the atmosphere ( $p=0$ ) and is unity at the earth's surface ( $p = p_*$ ). By choosing a proper form of the function  $F$ , the locations of levels between the top and the bottom of the atmosphere can be adjusted as required. In the present study, the

distribution of levels in the vertical is such that there is greater resolution (i.e. smaller pressure intervals) between successive levels at the extremities of the atmosphere. This has been achieved by applying the condition

$$\frac{d\sigma}{dQ} = 0 \quad (A.1.3)$$

at the top and the bottom of the atmosphere.

Suppose

$$\sigma = F(Q) = AQ^3 + BQ^2 + CQ + D$$

$$\frac{d\sigma}{dQ} = 3AQ^2 + 2BQ + C = 0 \quad ; \quad \text{therefore } C = 0$$

at the top,

$$\sigma = Q = 0 \quad ; \quad \text{therefore } D = 0$$

at the bottom,

$$\begin{aligned} \sigma = Q = 1 \quad ; \quad \text{therefore } A + B &= 1 \\ \text{and } 3A + 2B &= 0 \end{aligned}$$

This yields,

$$A = -2 \quad \text{and} \quad B = 3$$

The equation (A.2) for  $\sigma$  which would give maximum resolution at the extremities of the atmosphere takes the form (Manabe et al, 1961)

$$\sigma_k = Q_k^2 (3 - 2Q_k) \quad (A.1.4)$$

where  $Q_k$  is defined as in equation (A.1.1).

In table A.1 are shown the values of  $Q, \sigma$ , pressure, height, and temperature for various of  $k$ . Height, and temperature values are obtained from the U.S. Standard atmosphere, 1962. The surface pressure is 1013.25 mb.

Table A.1 : The Sigma system in the vertical with associated values of Pressure (P), Height (H), and Temperature (T)

K	Q	$\sigma$	P (MB)	H (KM)	T (C)
0.5	.000	.000	000.	$\infty$	----
1.0	.059	.010	10.10	31.14	- 45.51
1.5	.118	.038	38.77	22.28	- 54.30
2.0	.176	.082	83.52	17.37	- 56.50
2.5	.235	.140	141.89	13.99	- 56.50
3.0	.294	.209	211.40	11.45	- 56.50
3.5	.353	.286	289.57	9.41	- 46.10
4.0	.412	.369	373.91	7.69	- 34.78
4.5	.471	.456	461.97	6.16	- 24.99
5.0	.529	.544	551.26	4.85	- 16.55
5.5	.588	.631	639.32	3.72	- 9.17
6.0	.647	.714	723.69	2.75	- 2.88
6.5	.706	.791	801.86	1.93	2.45
7.0	.785	.860	871.36	1.25	6.85
7.5	.824	.918	929.72	0.72	10.32
8.0	.882	.962	974.47	0.33	12.87
8.5	.941	.990	1003.15	0.09	14.45
9.0	1.000	1.000	1013.25	0.00	15.00