

CHAPTER - 4

**JOINT PRICING ADVERTISEMENT AND
INVENTORY POLICIES FOR DETERIORATING
ITEMS UNDER TRADE CREDIT WITH SHORTAGES
AND PARTIAL BACKLOGGING**

4.1. Introduction

The sales of any item majorly depend on its price and promotional advertisement (publicity). In this chapter we extended the model developed in chapter-3 for inventory systems where the demand depends on price and advertisement, allowing shortages and partial backlogging. In a competitive business environment, to increase market share it is necessary to decide the optimal selling price and optimal cost for promotional advertisement. So, instead of considering the price and advertisement cost as constant values, we consider them as decision variables and then optimize them such that the total profit function becomes maximum.

4.2 Assumptions

- Demand is a function of selling price and frequency of advertisement.

We assume the demand function as (described by Kotler (1972)).

$$D(A, P) = A^m a P^{-b}$$

- Where, m is the shape parameter ($0 \leq m < 1$), $A (> 0)$ is the frequency of advertisement, $a (> 0)$ is the scaling factor, P is the selling price, and. $b (\geq 1)$ is the index of price elasticity. Since $\frac{\partial D(A,P)}{\partial A} > 0$ and $\frac{\partial D(A,P)}{\partial P} < 0$, the demand function is a decreasing function of price (P) and increasing function of the advertisement frequency(A), this reflects a real situation.
- The deterioration rate $\theta(t)$ of an item in the inventory system follows the two parameter Weibull distribution deterioration rate, given by

$$\theta(t) = \alpha\beta t^{\beta-1}; \text{ where } 0 \leq \alpha \ll 1, \beta > 0$$

- The supplier provides some credit period.
- Deteriorated items have no resale value.
- Instant and infinite replenishment rate.
- The inventory system involves only one item.
- Shortages are permitted with partially backlogging. The portion of the unsatisfied demand that backlogged is $D(A, P)e^{-\delta(T-t)}$ where backlogging parameter δ is a positive constant and $(T - t)$ is the waiting time.

4.3 Model Development

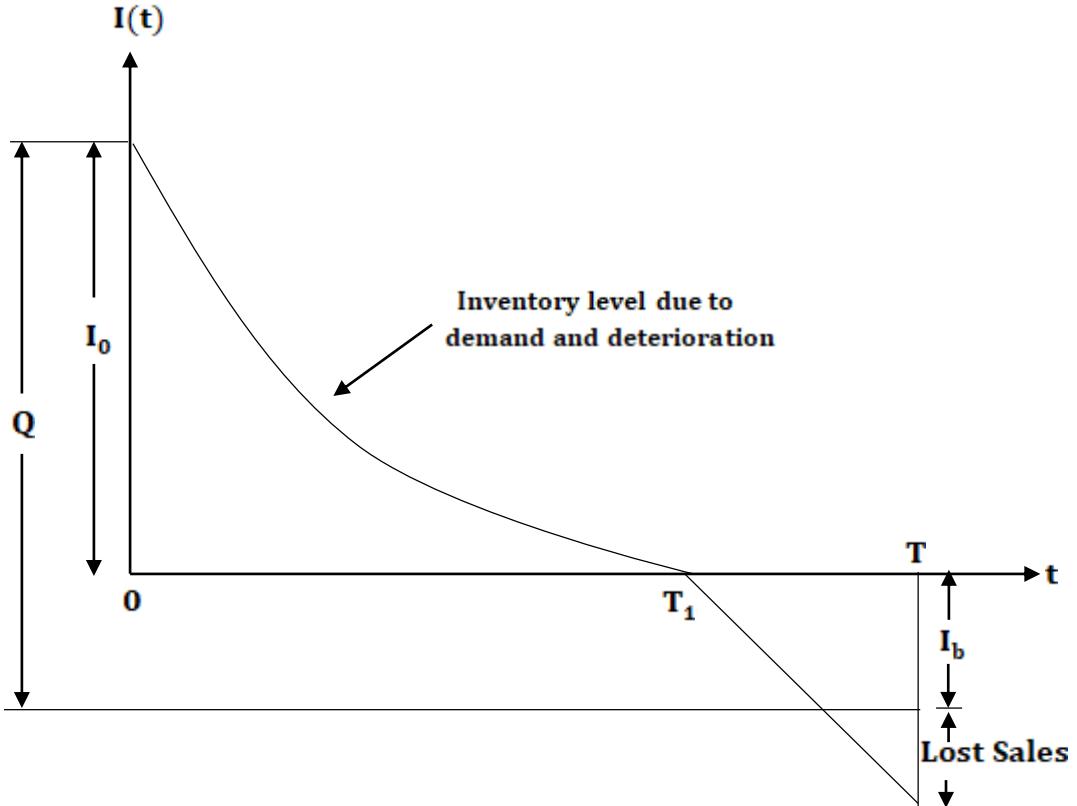
As shown in figure 4.3.1, initially the inventory system has I_0 units and due to the demand and deterioration, the inventory level will continuously decrease with time during the time interval $[0, T_1]$. At time T_1 the inventory reaches zero and the demand will be partially backlogged during $[T_1, T]$.

The rate of change in inventory level is given by the differential equations 4.3.1 and 4.3.2.

$$\frac{dI_1(t)}{dt} + \alpha\beta t^{\beta-1} I_1(t) = -D(A, P), \quad 0 \leq t \leq T_1 \quad (4.3.1)$$

$$\frac{dI_2(t)}{dt} = -D(A, P)e^{-\delta(T-t)}, \quad T_1 \leq t \leq T \quad (4.4.2)$$

Figure 4.3.1 Graphical depiction of the inventory level.



Solution of the equation (4.3.1) using the boundary condition $I_1(T_1) = 0$ is

$$I_1(t) = D \left[(T_1 - t) + \frac{\alpha}{\beta + 1} (T_1^{\beta+1} - t^{\beta+1}) - \alpha (T_1 t^\beta - t^{\beta+1}) - \frac{\alpha^2}{(\beta + 1)} (T_1^{\beta+1} t^\beta - t^{2\beta+1}) \right] \quad (4.3.3)$$

Solution of the equation (4.3.2) is

$$I_2(t) = \frac{-D(A, P)}{\delta} [e^{-\delta(T-t)} - e^{-\delta(T-T_1)}] \quad (4.3.4)$$

The maximum backlogged inventory is

$$I_B = \frac{D(A, P)}{\delta} [1 - e^{-\delta(T-T_1)}] \quad (4.3.5)$$

From equation (4.3.3), the initial quantity at $t = 0$ is

$$I_0 = D(A, P) \left[T_1 + \frac{\alpha}{(\beta+1)} T_1^{(\beta+1)} \right] \quad (4.3.6)$$

Ordering cost per order is

$$OC = C_0 \quad (4.3.7)$$

Number of deteriorated units during $[0, T_1]$ is

$$I_0 - \int_0^{T_1} D(A, P) dt = D(A, P) \frac{\alpha}{(\beta+1)} T_1^{(\beta+1)} \quad (4.3.8)$$

Deterioration cost is

$$DC = \frac{C_d D(A, P) \alpha}{(\beta+1)} T_1^{(\beta+1)} \quad (4.3.9)$$

Lost sale cost is

$$LSC = C_s D(A, P) \left[T - T_1 - \frac{1}{\delta} + \frac{e^{-(T-T_1)}}{\delta} \right] \quad (4.3.10)$$

Sales Revenue

$$\begin{aligned} SR &= P \left[\int_0^{T_1} D(A, P) dt + \int_{T_1}^T D(A, P) e^{-\delta(T-t)} dt \right] \\ &= PD(A, P) \left[T_1 + \frac{1}{\delta} - \frac{e^{-\delta(T-T_1)}}{\delta} \right] \end{aligned} \quad (4.3.11)$$

Holding cost

$$\begin{aligned}
 HC &= C_h \left[\int_0^{T_1} I_1(t) dt \right] \\
 &= C_h D(A, P) \left[\frac{T_1^2}{2} - \frac{\alpha}{(\beta+1)} T_1^{(\beta+2)} - \frac{\alpha^2}{2(\beta+1)^2} T_1^{2(\beta+1)} \right]
 \end{aligned} \tag{4.3.12}$$

Advertisement cost

$$AC = C_a A \tag{4.3.13}$$

If the supplier allows credit period M units of the time to settle the account, then the possible cases are,

$$(1) 0 \leq M \leq T_1$$

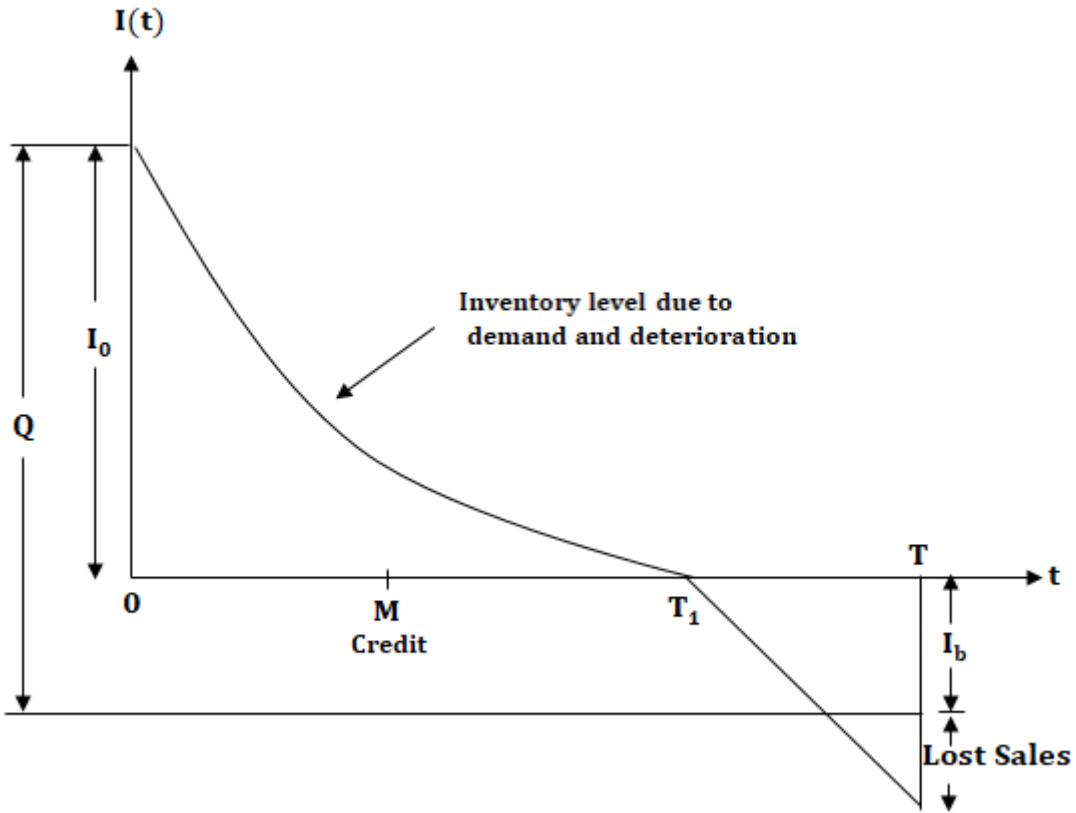
$$(2) T_1 < M \leq T.$$

4.3.1 Case 1: $0 \leq M \leq T_1$

Interest Charged:

$$\begin{aligned}
 IC_1 &= CI_c \left[\int_M^{T_1} I_1(t) dt \right] \\
 &= CI_c D(A, P) \left[\frac{(T_1-M)^2}{2} - \frac{\alpha T_1^{\beta+1}}{\beta+1} (T_1 - M) - \frac{\alpha}{\beta+1} (T_1^{\beta+2} - M^{\beta+2}) \right. \\
 &\quad \left. + \frac{\alpha^2}{2(\beta+1)^2} (T_1^{\beta+1} - M^{\beta+1})^2 \right]
 \end{aligned} \tag{4.3.14}$$

Figure 4.3.1.1 Inventory level when $0 \leq M \leq T_1$.



Interes earned

$$\begin{aligned}
 IE_1 &= PI_e \int_0^M D(A, P)t \, dt \\
 &= \frac{PI_e D(A, P) M^2}{2} \tag{4.3.15}
 \end{aligned}$$

Total profit per unit time:

$$\begin{aligned}
 TP_1(A, T_1, T, P, \xi) &= \frac{1}{T} [SR - PC - DC - LSC - HC - OC - AC - IC_1 + IE_1] \\
 &= \frac{1}{T} \left[PD(A, P) \left[T_1 + \frac{1}{\delta} - \frac{e^{-\delta(T-T_1)}}{\delta} \right] \right]
 \end{aligned}$$

$$\begin{aligned}
& -CD(A, P) \left[T_1 + \frac{\alpha}{(\beta+1)} T_1^{(\beta+1)} + \frac{1}{\delta} [1 - e^{-\delta(T-T_1)}] \right] \\
& - \frac{C_d D(A, P) \alpha}{(\beta+1)} T_1^{(\beta+1)} - C_s D(A, P) \left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta} \right] \\
& - C_h D(A, P) \left[\frac{T_1^2}{2} - \frac{\alpha}{(\beta+1)} T_1^{(\beta+2)} - \frac{\alpha^2}{2(\beta+1)^2} T_1^{2(\beta+1)} \right] \\
& - C_l D(A, P) \left[\frac{(T_1 - M)^2}{2} - \frac{\alpha T_1^{\beta+1}}{\beta+1} (T_1 - M) \right. \\
& \quad \left. - \frac{\alpha}{\beta+1} (T_1^{\beta+2} - M^{\beta+2}) + \frac{\alpha^2}{2(\beta+1)^2} (T_1^{\beta+1} - M^{\beta+1})^2 \right] \\
& + \frac{PI_e D(A, P) M^2}{2} \tag{4.3.16}
\end{aligned}$$

4.3.2 Case 2: $T_1 \leq M \leq T$

Interest charged:

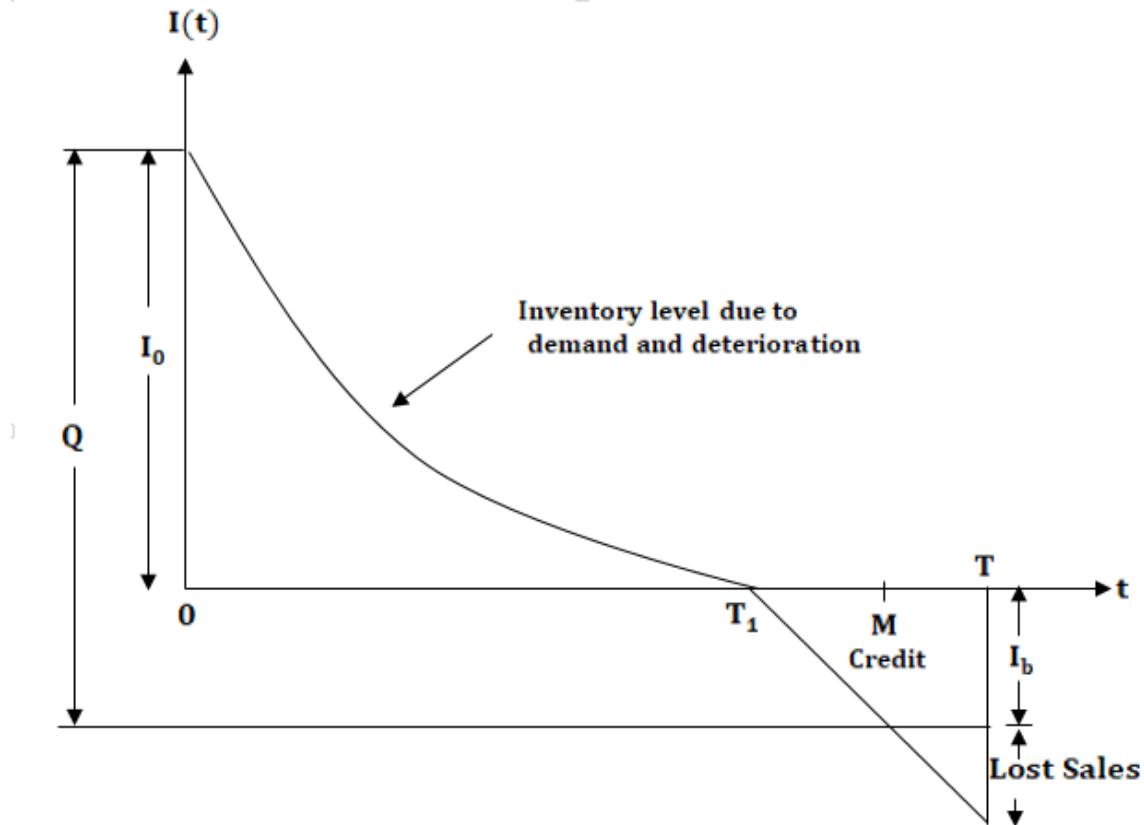
In this case, the interest charged will be zero. *i.e.* $IC_2 = 0$

Interest earned:

$$IE_2 = PI_e \int_0^{T_1} D(A, P) t \, dt + (M - T_1) \int_0^M D(A, P) t \, dt$$

$$= PI_e D(A, P) T_1 \left(M - \frac{T_1^2}{2} \right) \quad (4.3.17)$$

Figure 4.3.2.1 Inventory level when $T_1 \leq M \leq T$



Total profit per unit time:

$$\begin{aligned} TP_2(A, T_1, T, P, \xi) &= \frac{1}{T} [SR - PC - DC - LSC - HC - OC - AC - IC_1 + IE_1] \\ &= \frac{1}{T} \left[PD(A, P) \left[T_1 + \frac{1}{\delta} - \frac{e^{-\delta(T-T_1)}}{\delta} \right] \right. \\ &\quad \left. - CD(A, P) \left[T_1 + \frac{\alpha}{(\beta+1)} T_1^{(\beta+1)} + \frac{1}{\delta} [1 - e^{-\delta(T-T_1)}] \right] \right] \end{aligned}$$

$$\begin{aligned}
& - \frac{C_a D(A, P) \alpha}{(\beta + 1)} T_1^{(\beta+1)} - C_s D(A, P) \left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta} \right] \\
& - C_h D(A, P) \left[\frac{T_1^2}{2} - \frac{\alpha}{(\beta + 1)} T_1^{(\beta+2)} - \frac{\alpha^2}{2(\beta + 1)^2} T_1^{2(\beta+1)} \right] \\
& - C_o - C_a A + PI_e D(A, P) \mathbf{T}_1 (\mathbf{M} - \frac{T_1^2}{2}) \quad (4.3.18)
\end{aligned}$$

The optimal order quantity corresponding to the optimal solution (A^*, T_1^*, T^*, P^*) is

$$Q^* = D(A, P) \left[T_1 + \frac{\alpha}{(\beta + 1)} T_1^{(\beta+1)} + \frac{1}{\delta} [1 - e^{-\delta(T-T_1)}] \right] \quad (4.3.19)$$

Procedure for finding the optimal order policy:

Algorithm:

Step 1: Set numerical values to all the parameters in appropriate units.

Step 2: Set $A = 1$ and $i = 1$.

Step 3: Find the optimal solution of $TP_i(T_1, T, P | A)$. Then obtain the corresponding total profit $TP_i(A, T_1^*, T^*, P^*)$ and go to next step.

Step 4: Set $A' = A + 1$ and repeat step 3 to get $TP_1(A', T_1^*, T^*, P^*)$ and go to next step.

Step 5: If $TP_i(A', T_1^*, T^*, P^*) \geq TP_i(A, T_1^*, T^*, P^*)$ then set $A = A'$ and go to step 3.

Otherwise go to next step.

Step 6: Set the solution $(A^*, T_1^*, T^*, P^*)^i = (A, T_1^*, T^*, P^*)^i$ and the profit $TP_i^* = TP_i((A^*, T_1^*, T^*, P^*)^i)$.

Step 7: Set $A = 1, i = 2$ and repeat the step 3 to step 6, then go to next step.

Step 8: If $M < T_1^{*1}$ and $M \geq T_1^{*2}$ then the optimal solution is $(A^*, T_1^*, T^*, P^*)^1 = (A, T_1^*, T^*, P^*)^1$ and the maximum profit is $TP_1^* = TP_1((A^*, T_1^*, T^*, P^*)^1)$.

Step 9: If $M < T_1^{*1}$ and $M > T_1^{*2}$ then the optimal solution is $(A^*, T_1^*, T^*, P^*)^2 = (A, T_1^*, T^*, P^*)^2$ and the maximum profit is $TP_2^* = TP_2((A^*, T_1^*, T^*, P^*)^2)$.

Step 10: If $M < T_1^{*1}$ and $M > T_1^{*2}$ then the optimal solution is $(A^*, T_1^*, T^*, P^*)^i = (A, T_1^*, T^*, P^*)^i$ ($i = 1$ or 2) for which the profit $TP_i^* = TP_i((A^*, T_1^*, T^*, P^*)^i)$ is maximum.

Step 11: If $M < T_1^{*1}$ and $M \geq T_1^{*2}$ then the optimal solution is $(A^*, T_1^*, T^*, P^*)^1 = (A, T_1^*, T^*, P^*)^2$ and the maximum profit is $TP_1^* = TP_2^*$.

Step 12: Compute the corresponding optimal order quantity Q^* from Eq. (4.3.19). Stop.

[Note: $(A^*, T_1^*, T^*, P^*)^i$ is the solution of case- i and $TP_i((A^*, T_1^*, T^*, P^*)^i)$ is the total profit of case- i]

During the execution of the above algorithm, for a fixed value of A, we can obtain the optimal solution which maximizes the total profit function with the constraints using software like MATHEMATICA, MATLAB, PYTHON, R, MATHCAD, etc.

For a fixed value of the variable A , the necessary and sufficient conditions to maximize the total profit function $TP_i(T_1, T, P|A)$ are $\frac{\partial TP_2}{\partial T_1} = 0$,

$\frac{\partial TP_2}{\partial T} = 0$, $\frac{\partial TP_2}{\partial P} = 0$, provided that the Hessian matrix $H = \begin{bmatrix} \frac{\partial^2 TP_i}{\partial T_1^2} & \frac{\partial^2 TP_i}{\partial T_1 T} & \frac{\partial^2 TP_i}{\partial T_1 P} \\ \frac{\partial^2 TP_i}{\partial T T_1} & \frac{\partial^2 TP_i}{\partial T^2} & \frac{\partial^2 TP_i}{\partial T P} \\ \frac{\partial^2 TP_i}{\partial P T_1} & \frac{\partial^2 TP_i}{\partial P T} & \frac{\partial^2 TP_i}{\partial P^2} \end{bmatrix}$ is

a negative definite.

4.4 Examples

Table 4.4.1 Numerical examples

Example	M	α	β	a	b	m	δ	C	C_0	C_a	C_h	C_d	C_s	I_c	I_e
1	0.0822 (30 days)	0.4	2	500000	2	0.04	0.5	10	300	80	1.5	1	4	0.12	0.09
2	0.411 (150 days)	0.4	2	500000	2	0.04	0.5	10	300	80	1.5	1	4	0.12	0.09

Table 4.4.2 Computational results of Example-1.

A	T₁	T	P	TP₁
1	0.32598	0.49277	20.76018	11113.46
2	0.35008	0.53354	20.83791	11290.83
3	0.37286	0.57248	20.91344	11344.38
4	0.39427	0.60925	20.98466	11350.87
5	0.41434	0.64392	21.05316	11333.82
6	0.43330	0.67685	21.11821	11303.40

Table 4.4.3 Computational results of Example-2.

A	T₁	T	P	TP₂
1	0.34892	0.43650	20.21610	11753.53
2	0.36895	0.47447	20.28628	11931.80
3	0.38751	0.51028	20.3511	11979.70
4	0.40453	0.54371	20.41178	11978.41

5 0.42023 0.57504 20.46869 11952.72

4.5 Concavity

Figure 4.5.1 Concavity of TP_1 w.r.t P .

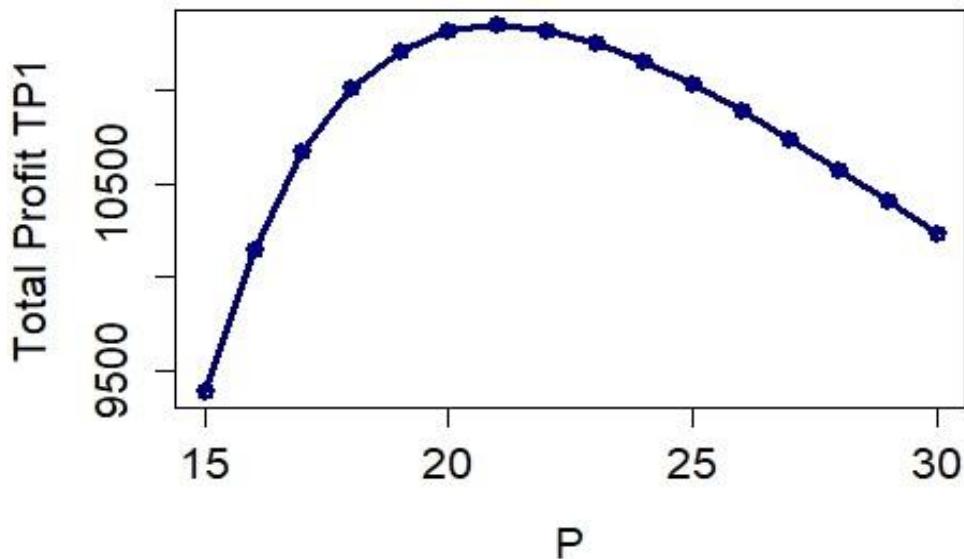


Figure 4.5.2 Concavity of TP_1 w.r.t T_1

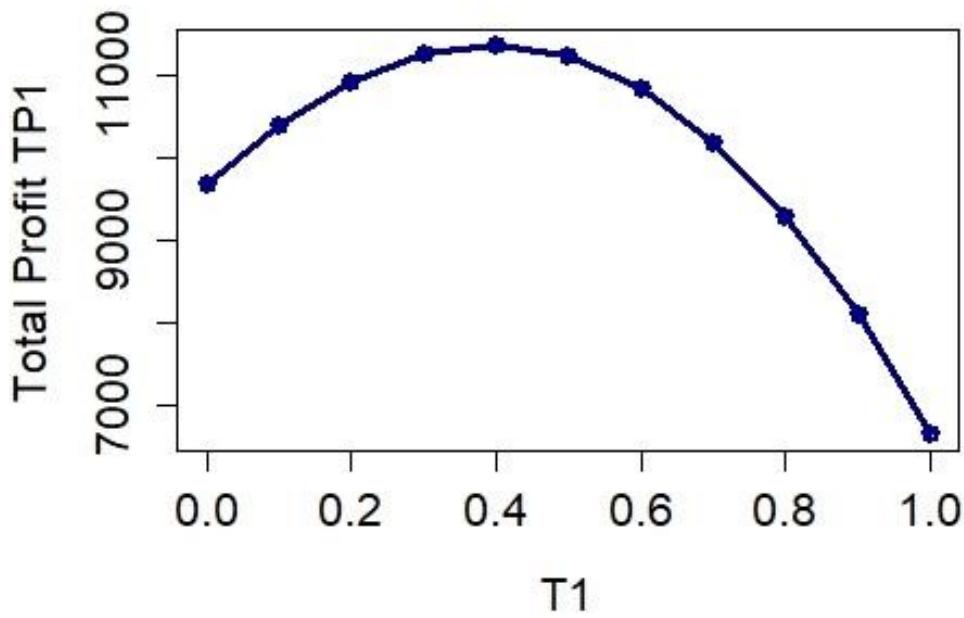


Figure 4.5.3 Concavity of TP_1 w.r.t T .

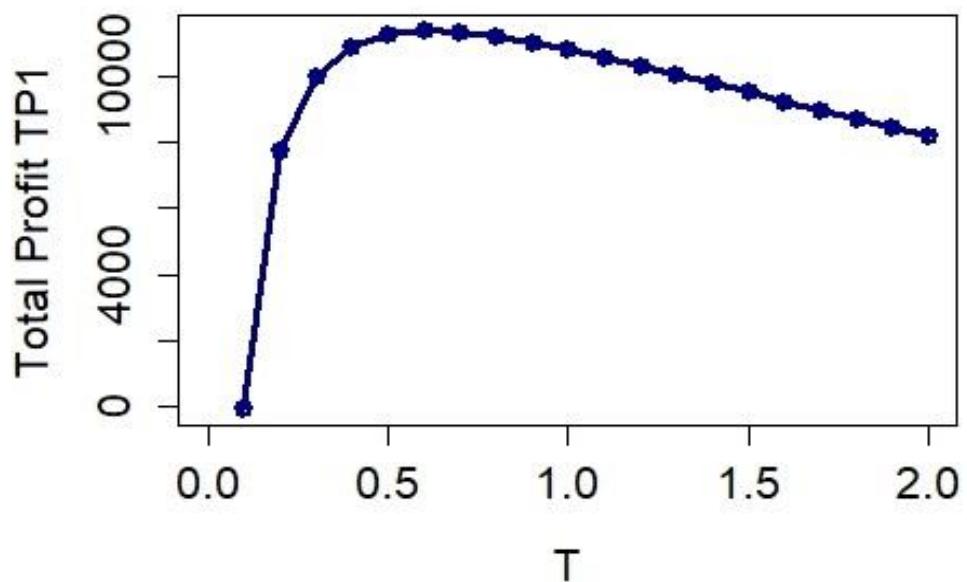
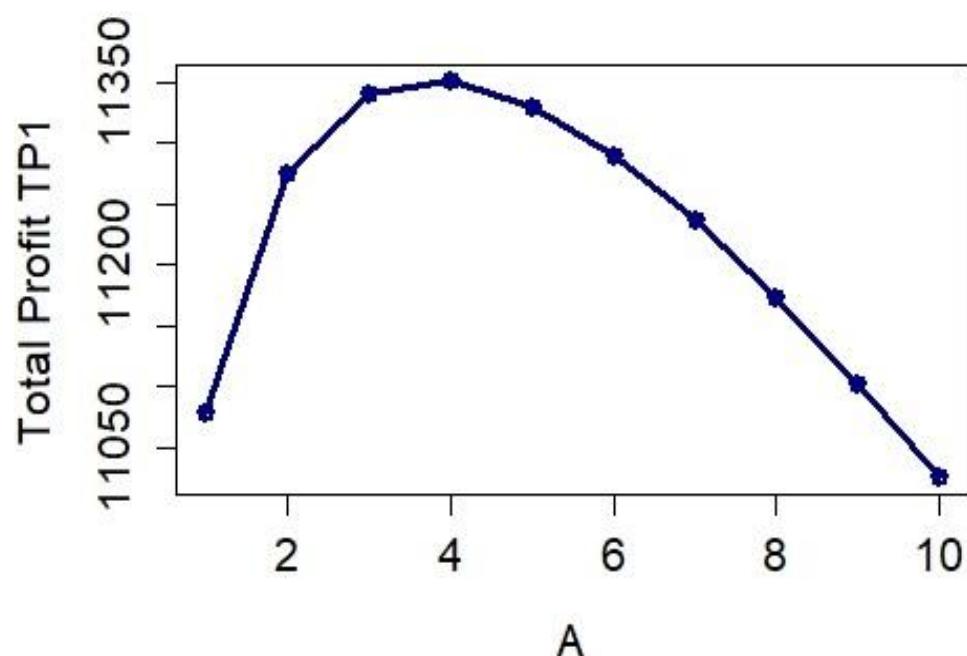


Figure 4.5.4 Concavity of TP_1 w.r.t A .



The figures 4.5.1-4.5.6 clearly indicates that the total profit function TP_1 is a concave function with respect to different variables.

Figure 4.5.5 Concavity of $TP_1(A, T_1, T, P)$ w.r.t P and T

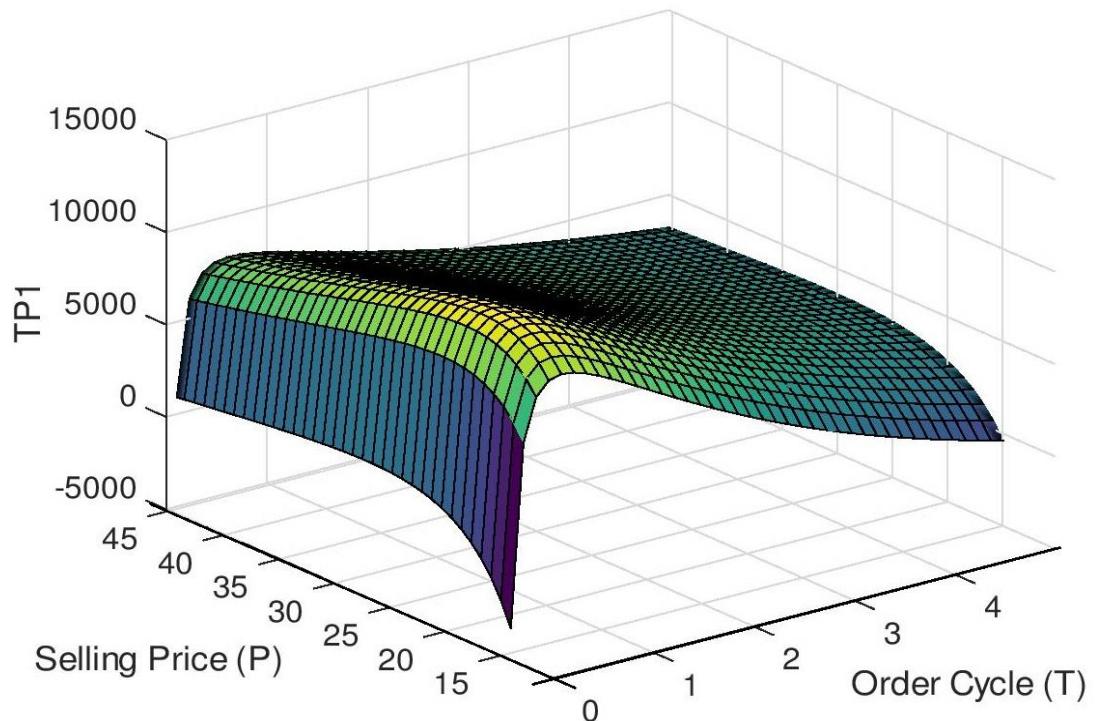
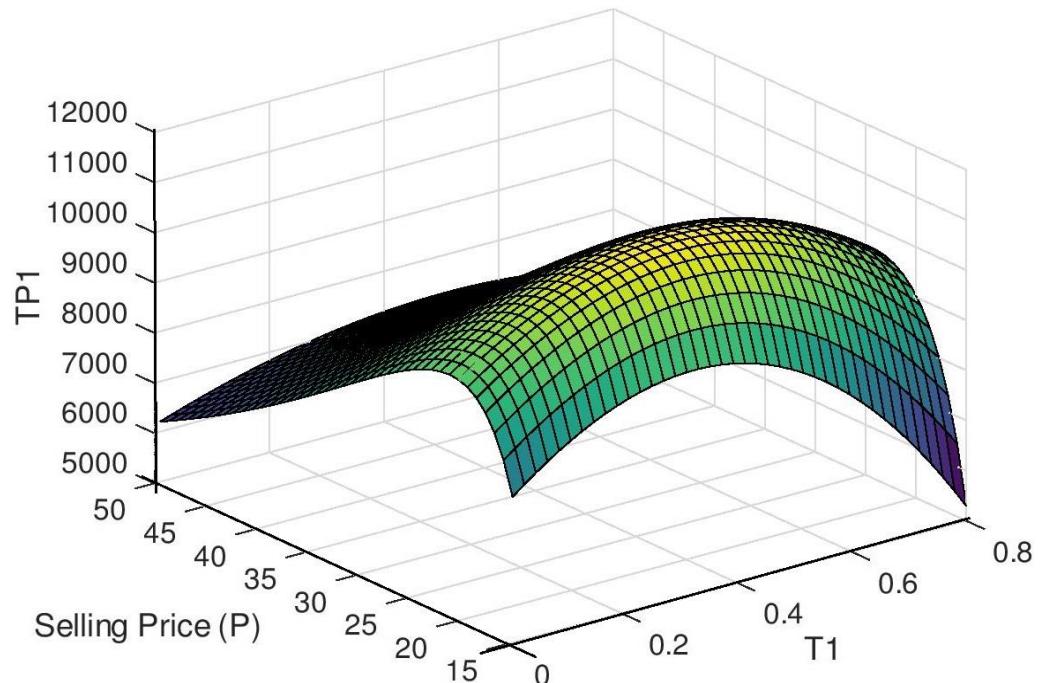


Figure 4.5.6 Concavity of $TP_1(A, T_1, T, P)$ w.r.t P and T_1 .

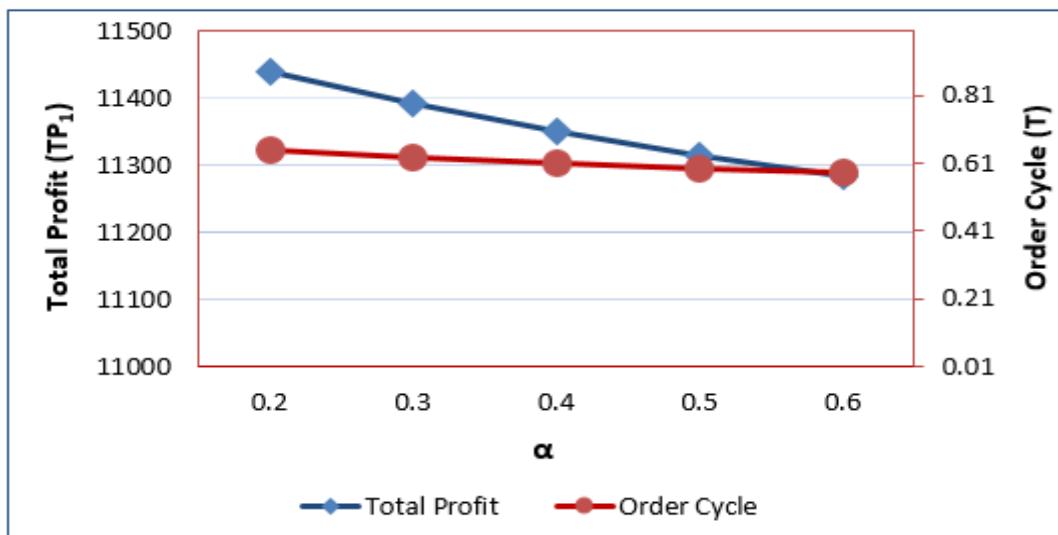


4.6 Sensitivity Analysis

4.6.1 Sensitivity of the parameter α .

Parameter		A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)
α	0.2	4	0.44632	0.65035	21.00397	774.153	11439.04
	0.3	4	0.41733	0.62724	20.99314	748.153	11391.84
	0.4	4	0.39427	0.60925	20.98466	727.638	11350.87
	0.5	4	0.37524	0.59471	20.97814	710.835	11314.66
	0.6	4	0.35904	0.58253	20.97203	696.649	11282.23

Figure 4.6.1 Effect of α on T and TP_1



Observations from the table 4.6.1 and figure 4.6.1: As the value of α increase,

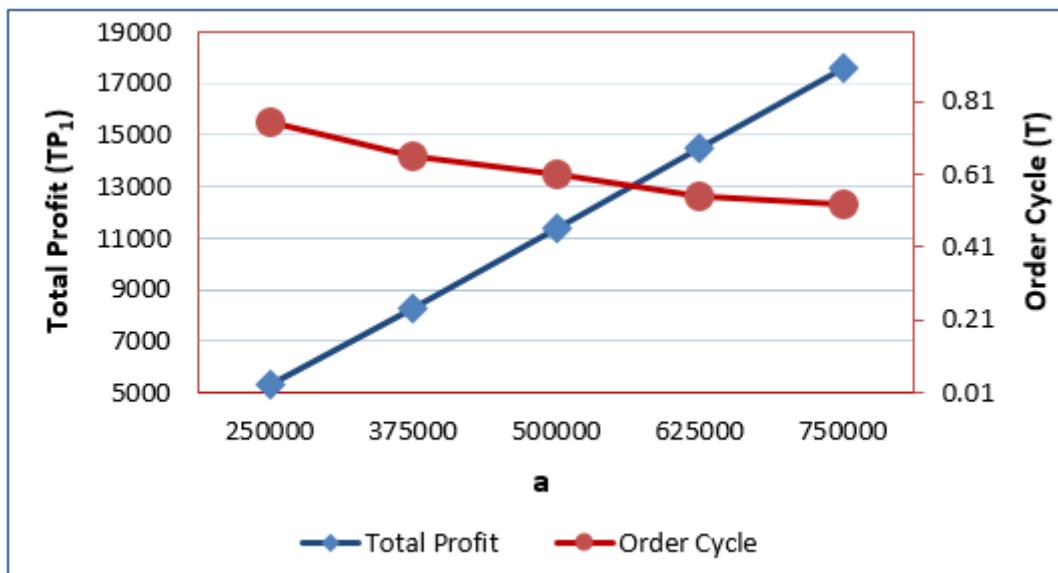
- (a) The total profit (TP_1) decreases.
- (b) The length of the order cycle (T) decreases.
- (c) The order Quantity (Q) decreases.
- (d) The selling price (P) decreases.

When the scale parameter α increase, the deterioration rate increase and which result in decrement in total profit.

4.6.2 Sensitivity of the parameter a .

Parameter	A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)	
a	250000	2	0.47574	0.75104	21.26672	424.687	5287.18
	375000	3	0.42332	0.65953	21.08353	578.475	8289.13
	500000	4	0.39427	0.60925	20.98466	727.638	11350.87
	625000	4	0.35745	0.54612	20.86206	825.115	14456.90
	750000	5	0.34668	0.52778	20.82734	968.779	17598.19

Figure 4.6.2 Effect of a on T and TP_1



Observations from the table 4.6.2 and figure 4.6.2: As the value of a increase,

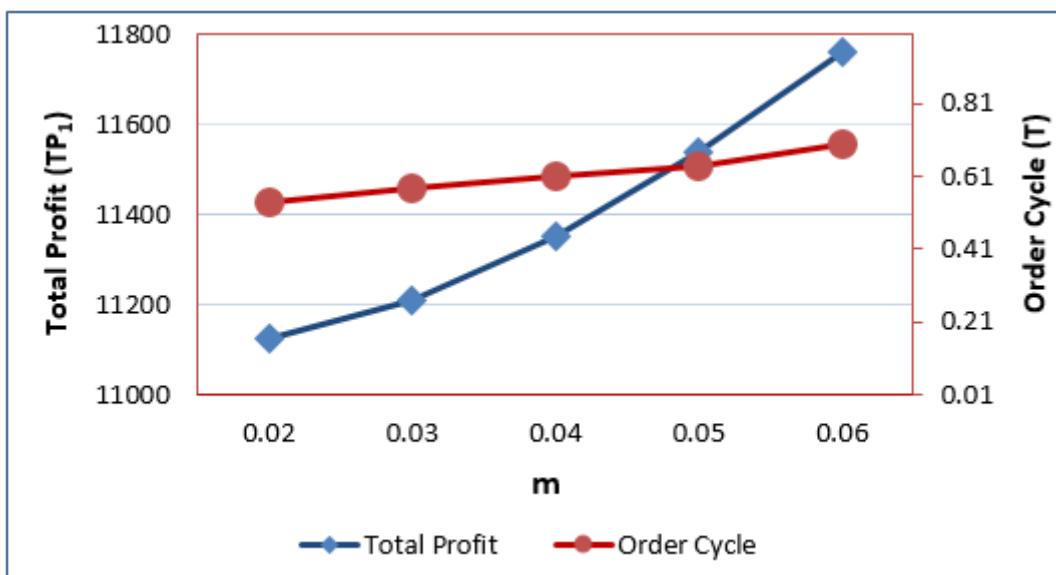
- (a) The total profit (TP_1) increases.
- (b) The length of the order cycle (T) decreases.
- (c) The order Quantity (Q) increases.
- (d) The selling price (P) decreases.

The total profit is very sensitive with respect to the parameter a .

4.6.3 Sensitivity of the parameter m.

Parameter	A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)	
m	0.02	2	0.35218	0.53714	20.84433	623.878	11123.55
	0.03	3	0.37472	0.57558	20.91884	676.458	11210.16
	0.04	4	0.39427	0.60925	20.98466	727.638	11350.87
	0.05	5	0.41140	0.63886	21.04283	777.952	11535.42
	0.06	7	0.44358	0.69475	21.15406	868.015	11760.46

Figure 4.6.3 Effect of m on T and TP_1



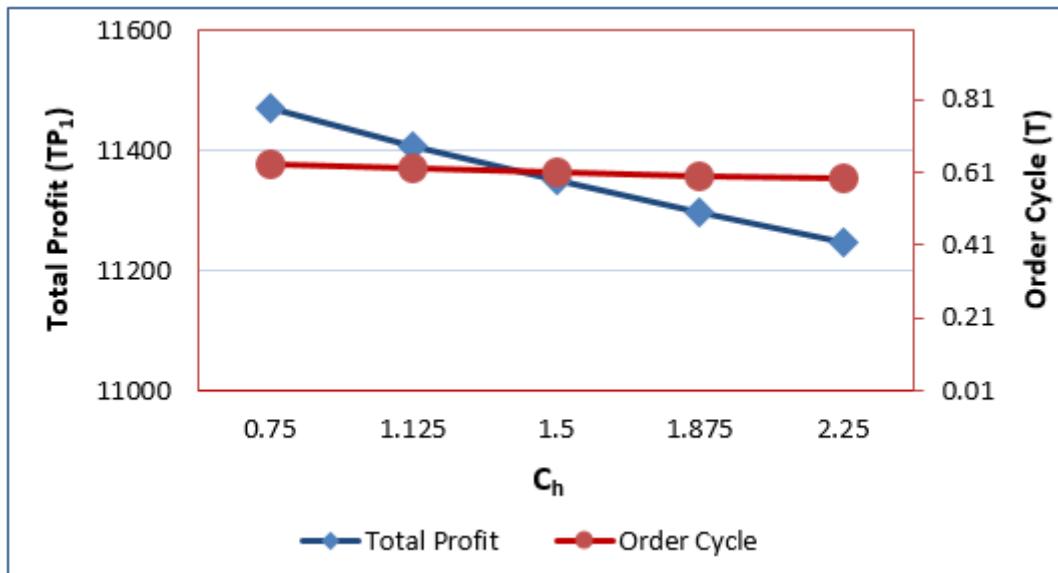
Observations from the table 4.6.3 and figure 4.6.3: As the value of m increase,

- (a) The total profit (TP_1) increases.
- (b) The length of the order cycle (T) increases.
- (c) The order Quantity (Q) increases.
- (d) The selling price (P) increases.

4.6.4 Sensitivity of the parameter C_h .

Parameter	A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)	
C_h	0.75	4	0.42910	0.62959	20.90178	762.613	11468.71
	1.125	4	0.41121	0.61913	20.94657	744.365	11407.78
	1.5	4	0.39427	0.60925	20.98466	727.638	11350.87
	1.875	4	0.37821	0.59977	21.01765	712.053	11297.66
	2.25	4	0.36307	0.59081	21.04622	697.654	11247.89

Figure 4.6.4 Effect of C_h on T and TP_1



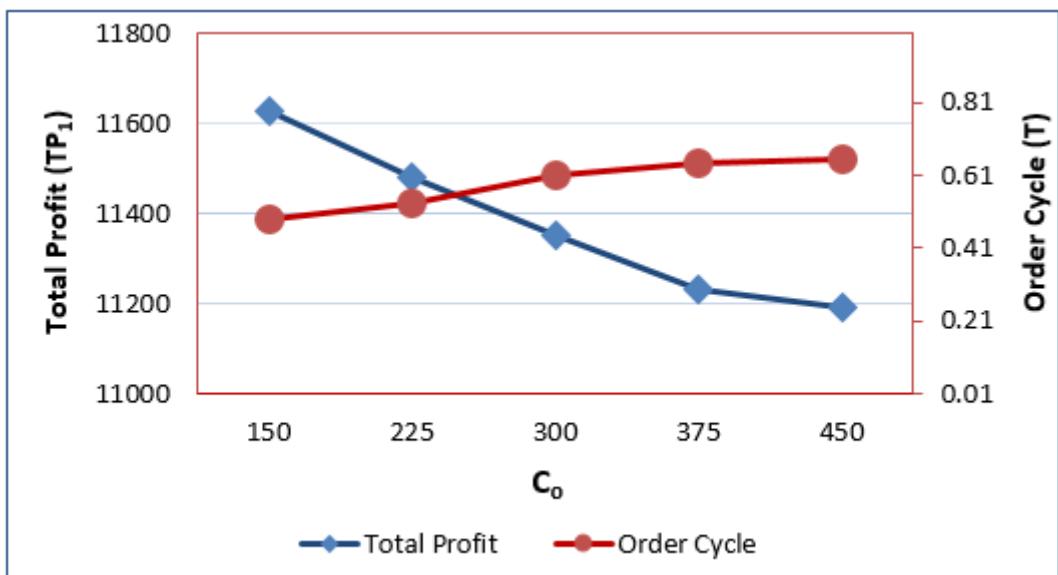
Observations from the table 4.6.4 and figure 4.6.4: As the value of C_h increase,

- (a) The total profit (TP_1) decreases.
- (b) The length of the order cycle (T) decreases.
- (c) The order Quantity (Q) decreases.
- (d) The selling price (P) increases.

4.6.5 Sensitivity of the parameter C_o .

Parameter	A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)	
C_o	150	3	0.32339	0.48839	20.75117	590.001	11627.14
	225	3	0.34924	0.53215	20.83490	637.550	11480.17
	300	4	0.39427	0.60925	20.98466	727.638	11350.87
	375	4	0.41460	0.64439	21.05409	764.472	11231.22
	450	4	0.42119	0.65584	21.07630	776.398	11192.76

Figure 4.6.5 Effect of C_o on T and TP_1



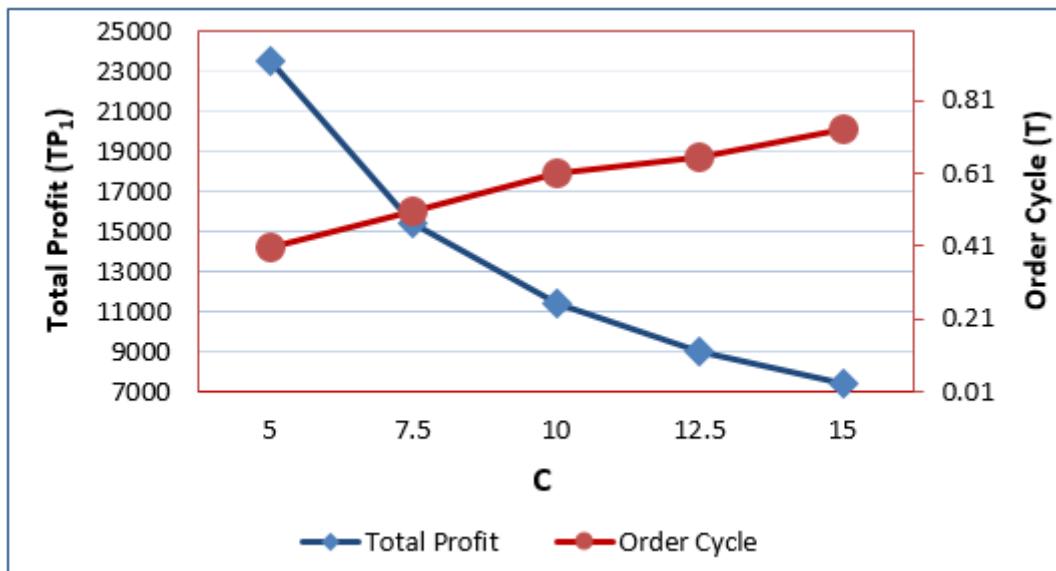
Observations from the table 4.6.5 and figure 4.6.5: As the value of C_o increase,

- (a) The total profit (TP_1) decreases.
- (b) The length of the order cycle (T) increases.
- (c) The order Quantity (Q) increases.
- (d) The selling price (P) increases.

4.6.6 Sensitivity of the parameter C .

Parameter	A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)	
C	5	5	0.26381	0.40531	10.46524	1961.52	23462.06
	7.5	4	0.33048	0.50696	15.69489	1081.80	15372.97
	10	4	0.39427	0.60925	20.98466	727.638	11350.87
	12.5	3	0.42357	0.65567	26.20820	494.579	8951.367
	15	3	0.46721	0.72939	31.52982	381.823	7361.575

Figure 4.6.6 Effect of C on T and TP_1



Observations from the table 4.6.6 and figure 4.6.6: As the value of C increase,

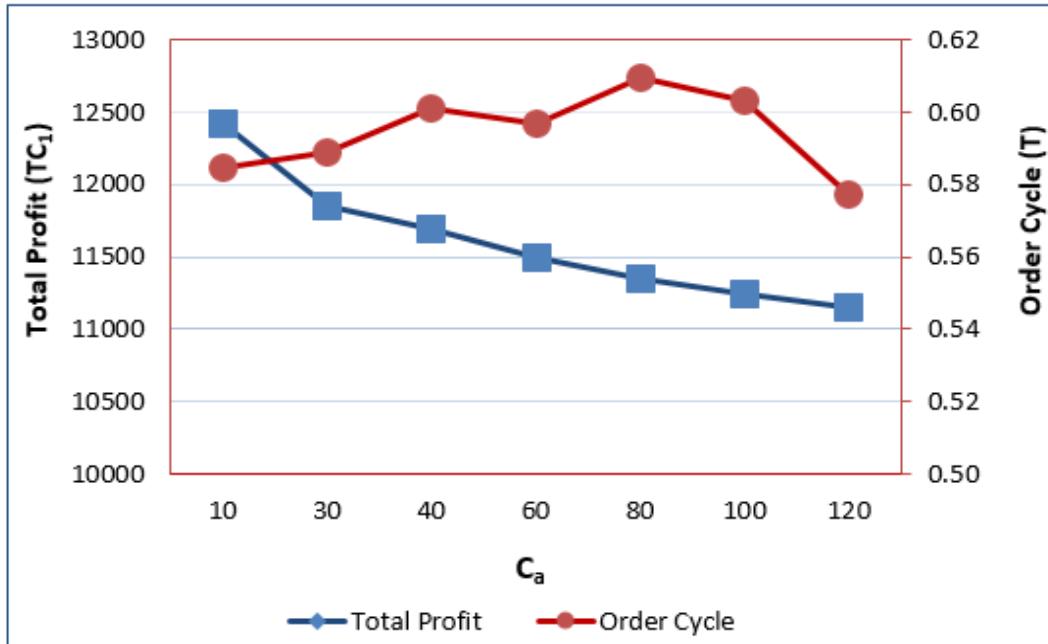
- (a) The total profit (TP_1) decreases.
- (b) The length of the order cycle (T) increases.
- (c) The order Quantity (Q) decreases.
- (d) The selling price (P) increases.

We can observe that the selling price and the total profit is more sensitive with respect to the purchase cost.

4.6.7 Sensitivity of the parameter C_a .

Parameter	A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)	
C_a	10	32	0.38010	0.58486	20.93703	762.615	12425.45
	30	10	0.38236	0.58875	20.94456	732.25	11846.64
	40	8	0.38943	0.60098	20.96826	739.106	11698.79
	60	5	0.38704	0.59683	20.96060	720.872	11495.02
	80	4	0.39427	0.60925	20.98466	727.638	11350.87
	100	3	0.39062	0.60290	20.97204	712.693	11242.29
	120	2	0.37559	0.57713	20.92229	674.507	11146.77

Figure 4.6.7 Effect of C_a on T and TP_1



Observations from the table 4.6.7 and figure 4.6.7: As the value of C_a increase,

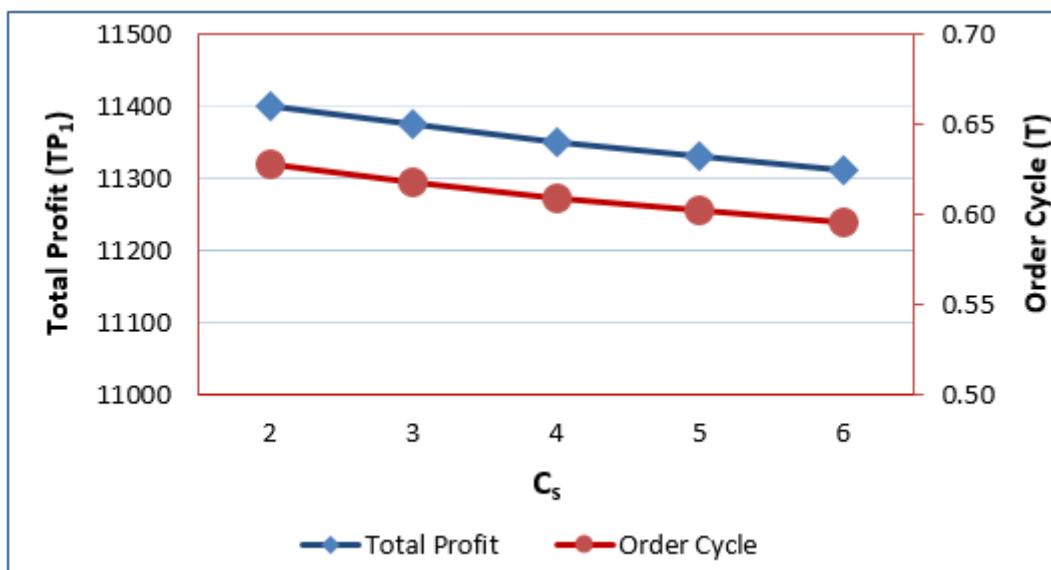
- (a) The total profit (TP_1) decreases.
- (b) The length of the order cycle (T) is not continuously increasing or decreasing.
- (c) The order quantity (Q) not continuously increasing or decreasing.
- (d) The selling price (P) increases.

Also, in table 4.6.7 we can observe that when the cost per advertisement is low, the frequency of advertisement is low and as the cost per advertisement increase the frequency of advertisement decreases.

4.6.8 Sensitivity of the parameter C_s .

Parameter	A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)	
C_s	2	4	0.38398	0.62748	20.8576	754.168	11400.05
	3	4	0.38945	0.61769	20.92566	739.900	11374.06
	4	4	0.39427	0.60925	20.98466	727.638	11350.87
	5	4	0.39854	0.60177	21.03498	716.940	11330.00
	6	4	0.40242	0.59523	21.07929	707.615	11311.12

Figure 4.6.8 Effect of C_s on T and TP_1



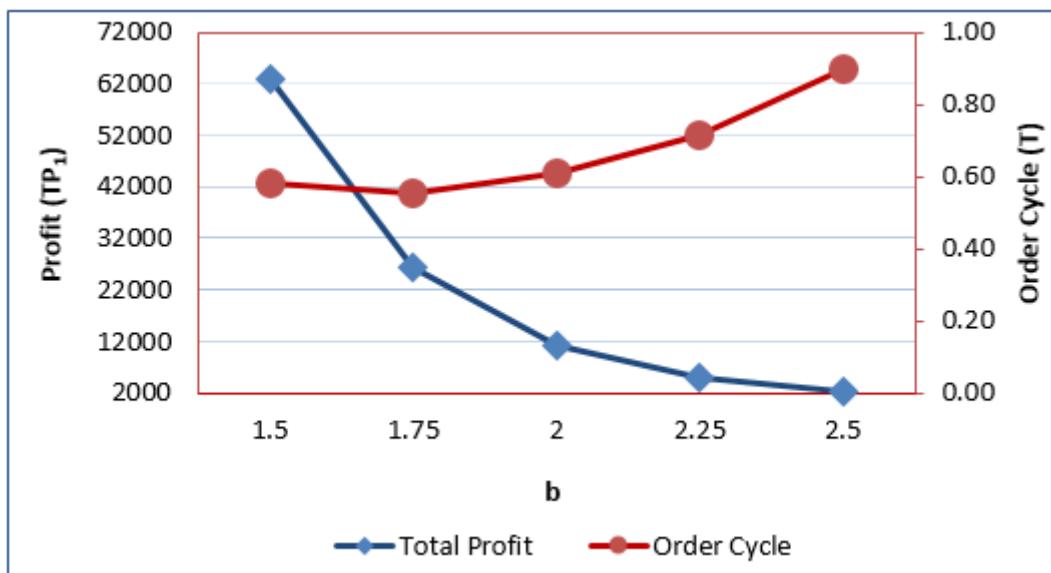
Observations from the table 4.6.8 and figure 4.6.8: As the value of C_s increase,

- (a) The total profit (TP_1) decreases.
- (b) The length of the order cycle (T) decreases.
- (c) The order Quantity (Q) decreases.
- (d) The selling price (P) increases.

4.6.9 Sensitivity of the parameter C_s .

Parameter	A^*	T_1^*	T^*	P^*	Q^*	Profit (TP_1^*)
b	1.5	19	0.43865	0.58021	31.78575	1841.159
	1.75	8	0.38715	0.55540	24.46770	26196.70
	2	4	0.39427	0.60925	20.98466	727.638
	2.25	2	0.43129	0.71563	19.00102	482.177
	2.5	1	0.50239	0.89600	17.81327	2129.22

Figure 4.6.9 Effect of b on T and TP_1



Observations from the table 4.6.9 and figure 4.6.9: As the value of b increase,

- (a) The total profit (TP_1) decreases.
- (b) The length of the order cycle (T) is not linearly related with b .
- (c) The order Quantity (Q) decreases.
- (d) The selling price (P) decreases.

The total profit is very sensitive with respect to the parameter b .

4.7 Conclusion

In this chapter, we attempted to obtain the optimal values of the selling price, frequency of advertisement, and inventory ordering policies simultaneously. Also, permissible delay in payment and partial backlogging of shortages are allowed. The model developed in this chapter reflects the more practical scenario of the real-life inventory system. Using the suggested algorithm we can obtain the frequency of advertisement for which the total profit is maximum. The total profit is very sensitive with respect to the demand parameters a , m and b . So, to apply this model in industry these parameters must be known and reliable.