CHAPTER - 6

JOINT PRICING, ADVERTISEMENT, PRESERVATION TECHNOLOGY INVESTMENT AND INVENTORY POLICIES FOR NON-INSTANTANEOUS DETERIORATING ITEMS UNDER TRADE CREDIT

6.1 Introduction

In this chapter we extend the model developed in chapter 4 allowing preservation technology investment. We assume the deterioration rate can be reduced through investing in preservation technology, and the demand depends on the price and frequency of advertisement. We simultaneously optimize the preservation technology investment, selling price, frequency of advertisement, and ordering policies for non-instantaneous deteriorating items.

To obtain the optimal solution an iterative algorithm is provided, then the proposed model is illustrated through numerical examples. The concavity of the profit function w.r.t decision variables shown graphically. The sensitivity analysis investigates the impact of each parameter on decision policies. Preservation technology investment and credit period are beneficial for the retailer, and can also earn more profit through advertisement. Value-added food products, such as soft drinks, bottled fruit juice, packed fruits, cake, bread, processed meat, etc., needs preservation technology also their demand depends on the price as well as marketing. Profit maximization of such items can be studied with the help of the model developed in this chapter.

6.2 Assumptions

- The inventory system involves a single non-instantaneous deteriorating item.
- Demand is a function of selling price and advertisement frequency. We assume the demand function as (described by Kotler (1972).) follows:

$$D(A, P) = A^m a P^{-b}$$

Where, m is the shape parameter ($0 \le m < 1$), A (> 0) is the frequency of advertisement, $a(> 0)$ is the scaling factor, P is the selling price, and. $b(\ge 1)$ is the index of price elasticity. Since $\frac{\partial D(A,P)}{\partial A} > 0$ and $\frac{\partial D(A,P)}{\partial P} < 0$, the demand function is a decreasing function of price (P) and increasing function of the advertisement frequency(A), this reflects a real situation.

- The lifetime (t) of the product follows three-parameter Weibull distribution $f(t) = \alpha\beta(t - T_d)^{\beta-1}e^{-\alpha(t-T_d)^{\beta}}$, where α (> 0) is the scale parameter, β (> 0) is the shape parameter and T_d (≥ 0) (deterioration free life) is the location parameter. The cumulative distribution function is $F(t) = \int_{T_d}^{t} f(t)dt = 1 - e^{-\alpha(t-T_d)^{\beta}}$, hence the deterioration rate is $\frac{f(t)}{1-F(t)} = \alpha\beta(t - T_d)^{\beta-1}$.
- The deterioration rate can be reduced through investing in preservation technology. The proportion of reduced deterioration rate is $m(\xi) = 1 - e^{-\eta\times\xi}$, where, $\eta(\ge 0)$ is the simulation coefficient representing the percentage increase in $m(\xi)$ per dollar increase in ξ. When $\xi = 0$, the reduced deterioration rate $m(\xi) = 0$, and for $\xi \to \infty$, $\lim_{\xi\to\infty} m(\xi) = 1$. But we set constraint $0 \le \xi \le \xi'$, where, ξ' is the maximum PT investment allowed.
- Instantaneous replenishment and infinite replenishment rate.
- Shortages are allowed and partially backlogged. The fraction of unsatisfied demand backlogged is $D(A, P)e^{-\delta(T-t)}$ for $t \in [T_1, T]$, where backlogging parameter δ is a positive constant and $(T - t)$ is the waiting time.
- The supplier provides some credit period.
- There is no salvage value or resale for the deteriorated items.

6.3 Model Development

As shown in figure 1, initially the inventory system has I_0 units. During the time interval $[0, T_d]$ there will be no deterioration and hence the inventory level decrease in this period due to demand only. During the interval $[T_d, T_1]$ the inventory level decrease due to demand and as well as deterioration, but in this period the deterioration rate will be reduced by investing in preservation technology. At time T_1 the inventory reaches zero and the demand will be partially backlogged during $[T_1, T]$. If the supplier allows credit period M units of the time to settle the account then the following three cases are possible.

(1) $0 \le M \le T_d$

(2) $T_d \le M \le T_1$

(3) $T_1 \le M \le T$

According to the above description, the differential equations representing the inventory status within different time intervals given by the equations (6.3.1 - 3).

$$\frac{dI_1(t)}{dt} = -D(A,P), \qquad 0 \le t \le T_d \tag{6.3.1}$$

$$\frac{dI_2(t)}{dt} = -\alpha\beta(t-T_d)^{\beta-1}\big(1-m(\xi)\big)\, I_2(t) - D(A,P), \qquad T_d \le t \le T_1 \tag{6.3.2}$$

$$\frac{dI_3(t)}{dt} = -D(A,P)e^{-\delta(T-t)}, \qquad T_1 \le t \le T \tag{6.3.3}$$

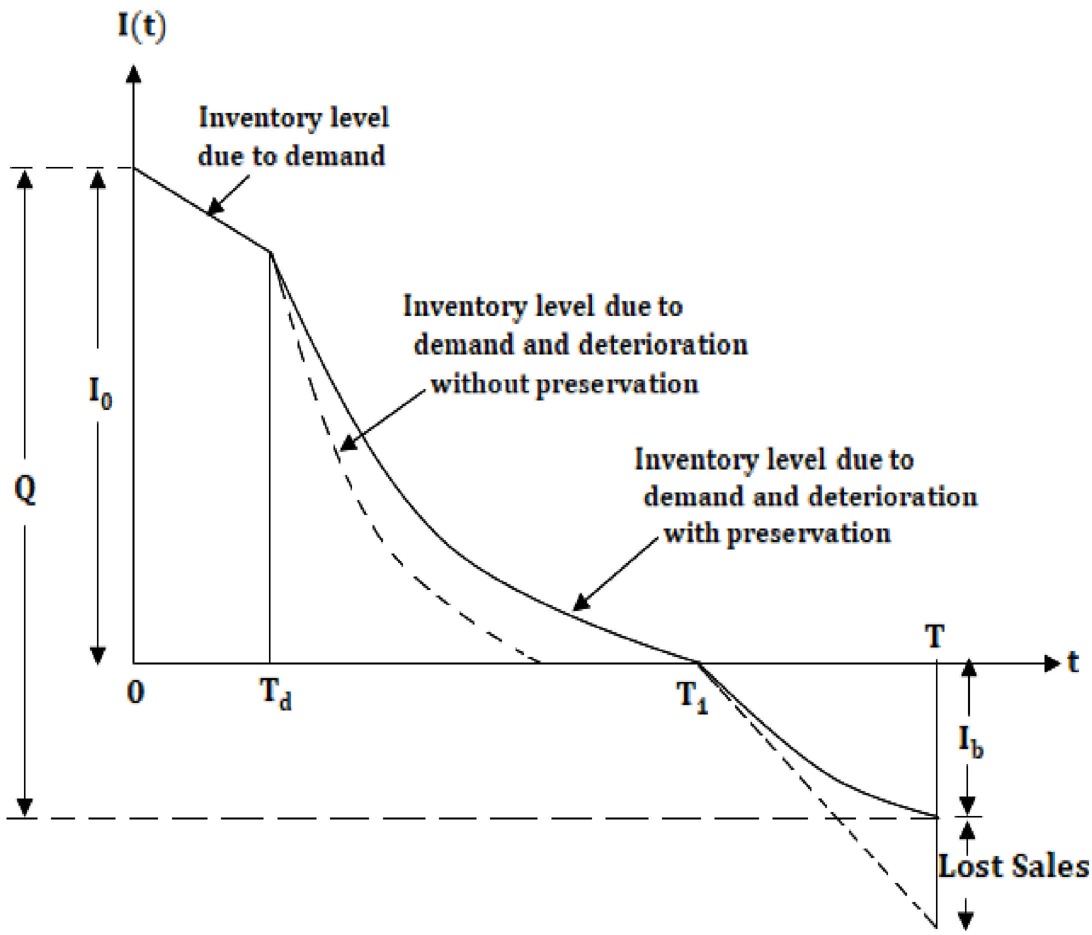
Using the boundary conditions $I_1(0) = I_0$, $I_2(T_1) = 0$ and $I_3(T_1) = 0$, we get the solution of equation (6.3.1), (6.3.2) and (6.3.3) respectively as follows.

$$I_1(t) = -D(A,P)t + I_0 \tag{6.3.4}$$

$$I_2(t) = D(A,P)\left[(T_1 - t) + \frac{\alpha\big(1-m(\xi)\big)}{(\beta+1)}\left\{(T_1 - T_d)^{(\beta+1)} - (T - T_d)^{(\beta+1)}\right\}\right] \times \left[1 - \alpha\big(1-m(\xi)\big)(t - T_d)\right] \quad (6.3.5)$$

$$\mathrm{I}_3(\mathrm{t}) = \frac{-D(A,P)}{\delta}\left[\mathrm{e}^{-\delta(\mathrm{T}-\mathrm{t})} - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}\right] \quad (6.3.6)$$

Figure 6.3.1 Graphical representation of the inventory system



Using the condition $I_1(T_d) = I_2(T_d)$

$$I_0 = D(A,P)\left[T_1 + \frac{\alpha\big(1-m(\xi)\big)}{(\beta+1)}(T_1 - T_d)^{(\beta+1)}\right] \tag{6.3.7}$$

The maximum amount of demand backlogged per cycle is obtained by putting $t = T$ in equation (6.3.6) and considering positive quantity.

$$I_B = \frac{D(A,P)}{\delta}\left[1 - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}\right] \tag{6.3.8}$$

Order quantity per cycle:

$$Q = I_0 + I_B$$

$$= D(A,P)\left[T_1 + \frac{\alpha\big(1-m(\xi)\big)}{\beta+1}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+1} + \frac{1}{\delta}\left[1 - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}\right]\right] \tag{6.3.9}$$

Purchase cost:

$$PC = C \cdot Q \tag{6.3.10}$$

Lost sale cost:

$$LSC = C_s \int_{T_1}^{\mathrm{T}} \left[D(A,P) - D(A,P)\mathrm{e}^{-\delta(\mathrm{T}-\mathrm{t})}\right]\,\mathrm{dt}$$

$$= C_s D(A,P)\left[T - T_1 - \frac{1}{\delta} + \frac{\mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}}{\delta}\right] \tag{6.3.11}$$

Deterioration cost:

$$DC = C_d\left[I_2(T_d) - \int_{T_d}^{T_1} D(A,P)\,dt\right]$$

$$= \frac{C_d D(A,P)\alpha\big(1-\mathrm{m}(\xi)\big)}{\beta+1}(T_1 - T_d)^{\beta+1} \tag{6.3.12}$$

Holding cost:

$$HC = C_h\left[\int_0^{T_d} \mathrm{I}_1(\mathrm{t})\ \mathrm{dt} + \int_{T_d}^{T_1} \mathrm{I}_2(\mathrm{t})\ \mathrm{dt}\right]$$

$$= C_h D(A,P)\left[\frac{T_1^2}{2} + \frac{\alpha\big(1-\mathrm{m}(\xi)\big)}{\beta+1}\mathrm{T_d}(\mathrm{T_1}-\mathrm{T_d})^{\beta+1}\right.$$

$$\left. + \frac{\alpha\beta\big(1-\mathrm{m}(\xi)\big)}{(\beta+1)(\beta+2)}(\mathrm{T_1}-\mathrm{T_d})^{\beta+2} - \frac{\alpha^2\big(1-\mathrm{m}(\xi)\big)^2}{2(\beta+1)^2}(\mathrm{T_1}-\mathrm{T_d})^{2(\beta+1)}\right] \tag{6.3.13}$$

Total sales revenue:

$$SR = P\left[\int_0^{T_1} D(A,P)\,dt + \int_{T_1}^{T} D(A,P)e^{-\delta(T-t)}\ dt\right]$$

$$= PD(A,P)\left[T_1 + \frac{1}{\delta}\big(1 - e^{-\delta(T-T_1)}\big)\right] \tag{6.3.14}$$

Preservation technology investment:

$$PTI = (T_1 - T_d)\xi \tag{6.3.15}$$

Advertisement cost:

$$AC = C_{\mathrm{a}}A \tag{6.3.16}$$

6.3.1 Case 1: $\mathbf{0 \leq M \leq T_d}$

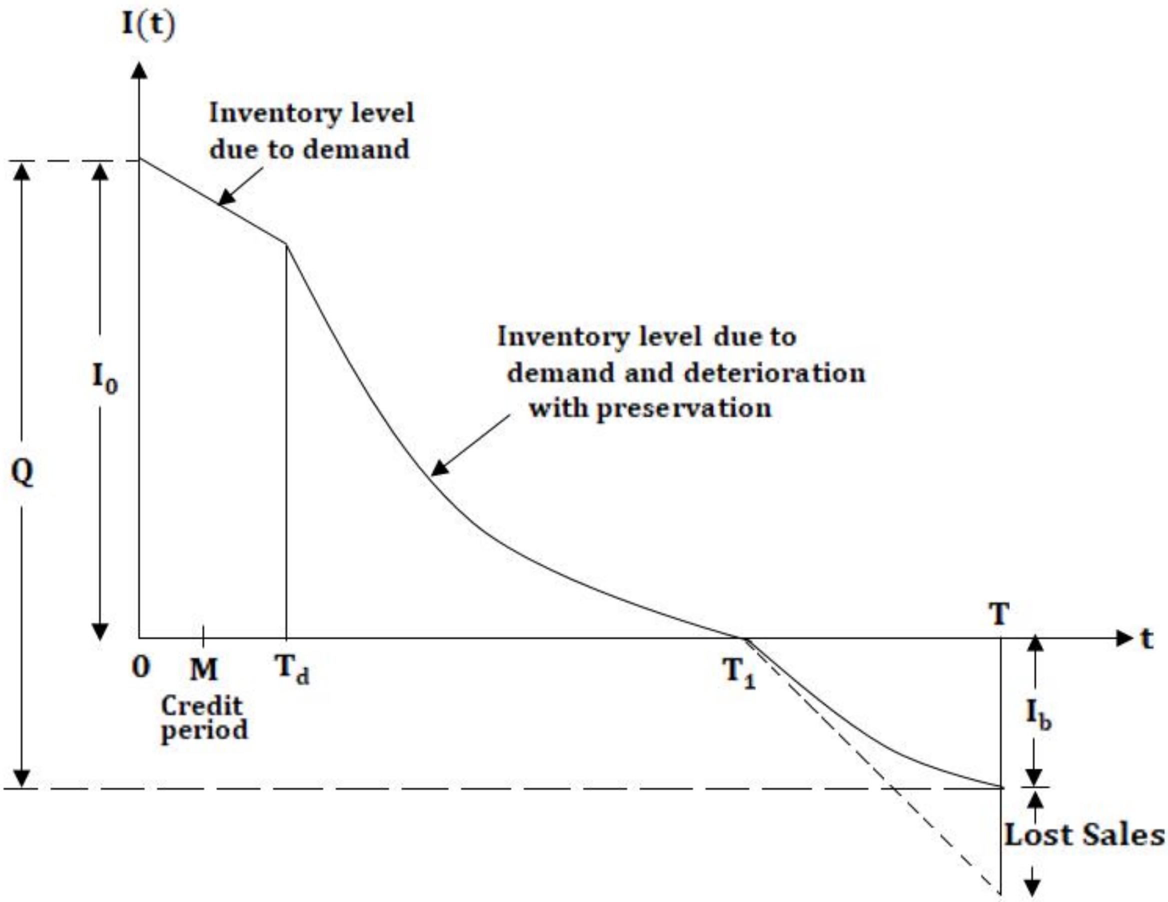
Interest Charged:

$$IC_1 = CI_c\left[\int_M^{T_d} I_1(t)\ dt + \int_{T_d}^{T_1} I_2(t)\ dt\right]$$

$$= CI_c D(A,P)\left[\frac{(T_1 - M)^2}{2} + \frac{\alpha\left(1 - m(\xi)\right)}{\beta + 1}(T_d - M)(T_1 - T_d)^{\beta+1}\right.$$

$$\left. + \frac{\alpha\beta\left(1 - m(\xi)\right)}{(\beta + 1)(\beta + 2)}(T_1 - T_d)^{\beta+2} - \frac{\alpha^2\left(1 - m(\xi)\right)^2}{2(\beta + 1)^2}(T_1 - T_d)^{2(\beta+1)}\right] \quad (6.3.17)$$

Figure 6.3.2 Inventory level when $0 \leq M \leq T_d$



Interest earned:

$$IE_1 = PI_e \int_0^M D(A,P)t \ dt$$

$$= \frac{PI_e D(A,P) M^2}{2} \qquad (6.3.18)$$

Total profit per unit time:

$$TP_1(A, T_1, T, P, \xi) = \frac{1}{T}[SR - PC - DC - LSC \ - HC - OC - PTI - AC - IC_1 + IE_1]$$

$$TP_1(A, T_1, T, P, \xi) = \frac{1}{T}\Big[PD(A,P)\left[T_1 + \frac{1}{\delta}\left(1 - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}\right)\right]$$

$$-CD(A,P)\left[T_1 + \frac{\alpha\big(1 - m(\xi)\big)}{\beta + 1}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+1} + \frac{1}{\delta}\left[1 - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}\right]\right]$$

$$-\frac{C_d D(A,P)\alpha\big(1 - \mathrm{m}(\xi)\big)}{\beta + 1}(T_1 - T_d)^{\beta+1}$$

$$-C_s D(A,P)\left[T - T_1 - \frac{1}{\delta} + \frac{\mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}}{\delta}\right]$$

$$-C_h D(A,P)\Bigg[\frac{T_1^2}{2} + \frac{\alpha\big(1 - \mathrm{m}(\xi)\big)}{\beta + 1}\mathrm{T}_\mathrm{d}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+1}$$

$$+\frac{\alpha\beta\big(1 - \mathrm{m}(\xi)\big)}{(\beta + 1)(\beta + 2)}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+2} - \frac{\alpha^2\big(1 - \mathrm{m}(\xi)\big)^2}{2(\beta + 1)^2}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{2(\beta+1)}\Bigg]$$

$$-C_O - (T_1 - T_d)\xi - C_\mathrm{a}A$$

$$-CI_c D(A,P)\Bigg[\frac{(T_1 - M)^2}{2} + \frac{\alpha\big(1 - m(\xi)\big)}{\beta + 1}(\mathrm{T}_\mathrm{d} - \mathrm{M})(\mathrm{T}_1 - T_d)^{\beta+1}$$

$$+\frac{\alpha\beta\big(1-m(\xi)\big)}{(\beta+1)(\beta+2)}(T_1-T_d)^{\beta+2}-\frac{\alpha^2\big(1-m(\xi)\big)^2}{2(\beta+1)^2}(T_1-T_d)^{2(\beta+1)}\Bigg]$$

$$+\frac{PI_eD(A,P)M^2}{2}\Bigg] \tag{6.3.19}$$

So, in this case, the objective is to maximize $Z_1 = TP_1(A,T_1,T,P,\xi)$.

$$\text{Subject to} \qquad \left.\begin{aligned} T_d &\le T_1 \\ T_1 &\le T \\ C &\le P \\ \xi &\le \xi' \end{aligned}\right\} \tag{6.3.20}$$

and $T_1 \ge 0, T \ge 0, P \ge 0, \xi \ge 0$, A is a positive integer $(A > 0)$.

When $M \ge T_d$ there are two possibilities either $T_d \le M \le T_1$ or $T_1 \le M \le T$.

6.3.2 Case 2: $\boldsymbol{T_d \le M \le T_1}$

Interest Charged:

$$IC_2 = CI_c\int_M^{T_1} I_2(t)\ dt$$

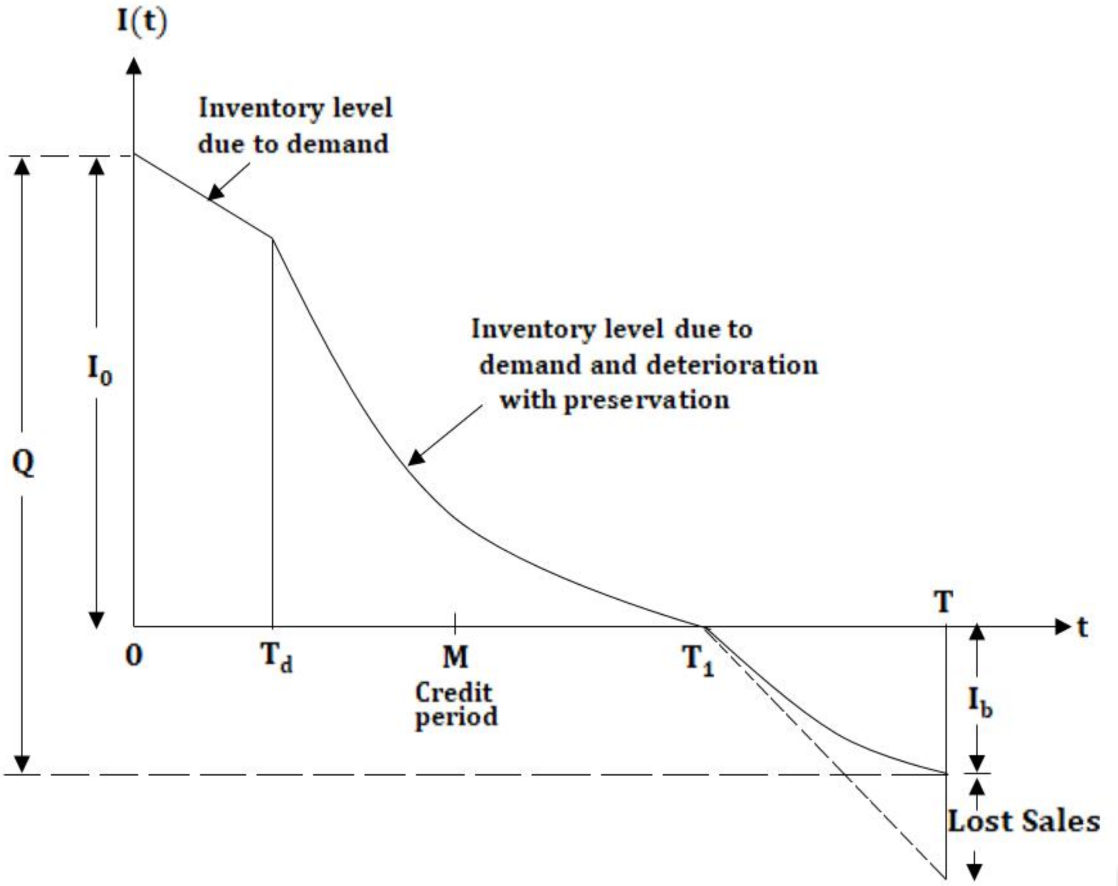
$$= CI_cD(A,P)\Bigg[\frac{(T_1-M)^2}{2}$$

$$-\frac{\alpha\big(1-m(\xi)\big)}{\beta+1}(T_1-M)\big\{(T_1-T_d)^{\beta+1}+(M-T_d)^{\beta+1}\big\}$$

$$-\frac{2\alpha\big(1-m(\xi)\big)}{(\beta+1)(\beta+2)}\big\{(T_1-T_d)^{\beta+2}-(M-T_d)^{\beta+2}\big\}$$

$$-\frac{\alpha^2\left(1-m(\xi)\right)^2}{2(\beta+1)^2}\left\{(T_1-T_d)^{\beta+1}-\ (M-T_d)^{\beta+1}\right\}^2\Bigg] \tag{6.3.21}$$

Figure 6.3.3 Inventory level when $T_d \leq M \leq T_1$



Interest earned:

$$IE_2 = PI_e \int_0^M D(A,P)t\ dt$$

$$= \frac{PI_e D(A,P)M^2}{2} \tag{6.3.22}$$

Total profit per unit time:

$$TP_2(A,T_1,T,P,\xi) = \frac{1}{T}[SR - PC - DC - LSC\ - HC - OC - PTI - AC - IC_2 + IE_2]$$

$$TP_2(A,T_1,T,P,\xi) = \frac{1}{T}\left[PD(A,P)\left[T_1 + \frac{1}{\delta}\left(1 - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}\right)\right]\right.$$

$$-CD(A,P)\left[T_1 + \frac{\alpha\left(1-m(\xi)\right)}{\beta+1}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+1} + \frac{1}{\delta}\left[1 - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}\right]\right]$$

$$-\frac{C_d D(A,P)\alpha\left(1-\mathrm{m}(\xi)\right)}{\beta+1}(T_1 - T_d)^{\beta+1}$$

$$-C_s D(A,P)\left[T - T_1 - \frac{1}{\delta} + \frac{\mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}}{\delta}\right]$$

$$-C_h D(A,P)\left[\frac{T_1^2}{2} + \frac{\alpha\left(1-\mathrm{m}(\xi)\right)}{\beta+1}\mathrm{T}_\mathrm{d}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+1}\right.$$

$$+\frac{\alpha\beta\left(1-\mathrm{m}(\xi)\right)}{(\beta+1)(\beta+2)}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+2}$$

$$\left.-\frac{\alpha^2\left(1-\mathrm{m}(\xi)\right)^2}{2(\beta+1)^2}(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{2(\beta+1)}\right]$$

$$-C_O - (T_1 - T_d)\xi - C_\mathrm{a}A$$

$$-CI_c D(A,P)\left[\frac{(T_1 - M)^2}{2}\right.$$

$$-\frac{\alpha\left(1-m(\xi)\right)}{\beta+1}(T_1 - \mathrm{M})\left\{(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+1} + (\mathrm{M} - \mathrm{T}_\mathrm{d})^{\beta+1}\right\}$$

$$-\frac{2\alpha\left(1-\mathrm{m}(\xi)\right)}{(\beta+1)(\beta+2)}\left\{(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+2} - (\mathrm{M} - \mathrm{T}_\mathrm{d})^{\beta+2}\right\}$$

$$\left.-\frac{\alpha^2\left(1-\mathrm{m}(\xi)\right)^2}{2(\beta+1)^2}\left\{(\mathrm{T}_1 - \mathrm{T}_\mathrm{d})^{\beta+1} - (\mathrm{M} - \mathrm{T}_\mathrm{d})^{\beta+1}\right\}^2\right]$$

$$\left.+\frac{PI_e D(A,P)M^2}{2}\right] \tag{6.3.23}$$

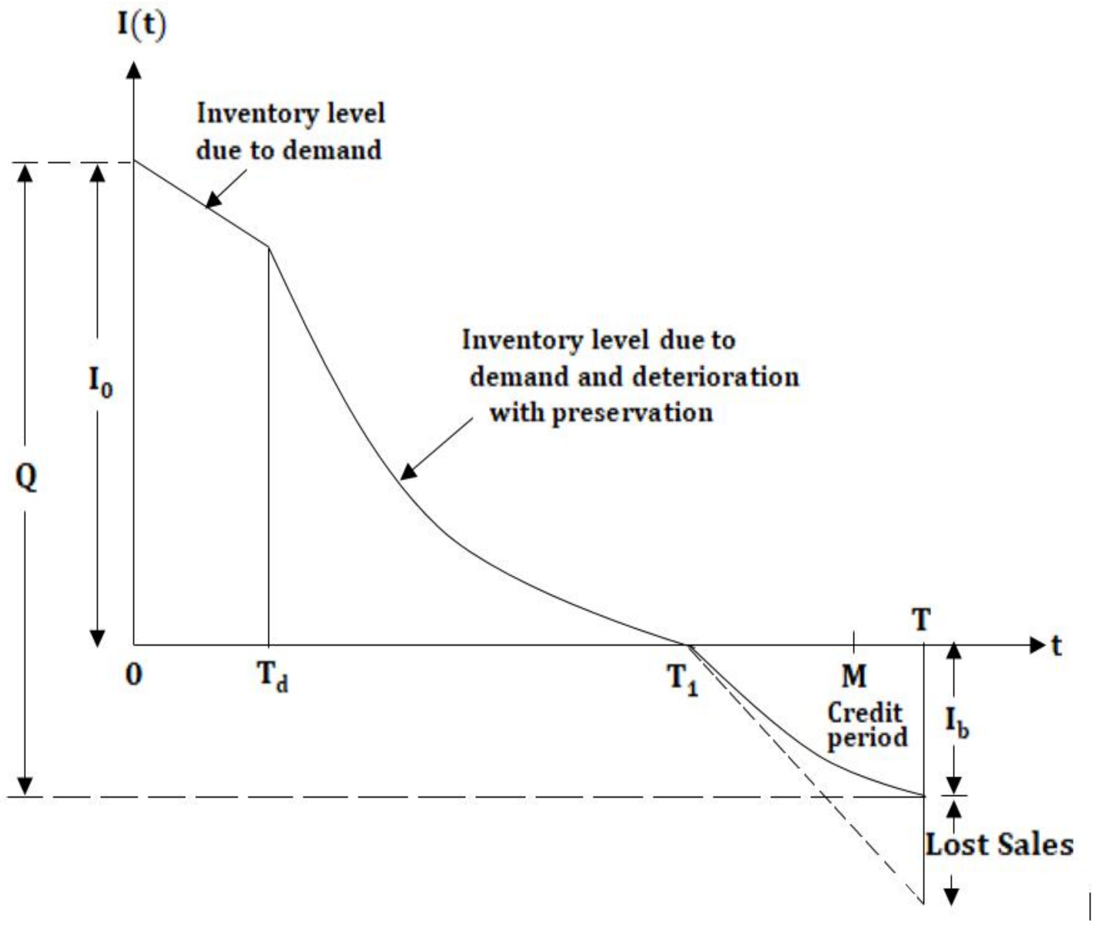
So, in this case, the objective is to maximize $Z_2 = TP_2(A, T_1, T, P, \xi)$.

$$\text{Subject to} \quad \left.\begin{array}{l} M \le T_1 \\ T_1 \le T \\ C \le P \\ \xi \le \xi' \end{array}\right\} \tag{6.3.24}$$

and $T_1 \ge 0, T \ge 0, P \ge 0, \xi \ge 0$, A is a positive integer ($A > 0$).

6.3.3 Case 3: $T_1 \le M \le T$

Figure 6.3.4 Inventory level when $T_1 \le M \le T$



Interest charged: In this case there is no interest charged

$$IC_3 = 0 \tag{6.3.25}$$

Interest earned:

$$IE_3 = PI_e\left[\int_0^{T_1} D(A,P)t\ dt + (M - T_1)\int_0^{T_1} D(A,P)\ dt\right]$$

$$= PI_e D(A,P)T_1(M - \frac{T_1}{2}) \tag{6.3.26}$$

Total profit per unit time:

$$TP_3(A,T_1,T,P,\xi) = \frac{1}{T}[SR - PC - DC - LSC\ - HC - OC - PTI - AC - IC_3 + IE_3]$$

$$TP_3(A,T_1,T,P,\xi) = \frac{1}{T}\Bigg[PD(A,P)\left[T_1 + \frac{1}{\delta}\left(1 - e^{-\delta(T-T_1)}\right)\right]$$

$$-CD(A,P)\left[T_1 + \frac{\alpha\big(1 - m(\xi)\big)}{\beta + 1}(T_1 - T_d)^{\beta+1} + \frac{1}{\delta}\left[1 - e^{-\delta(T-T_1)}\right]\right]$$

$$-\frac{C_d D(A,P)\alpha\big(1 - m(\xi)\big)}{\beta + 1}(T_1 - T_d)^{\beta+1}$$

$$-C_s D(A,P)\left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta}\right]$$

$$-C_h D(A,P)\Bigg[\frac{T_1^2}{2} + \frac{\alpha\big(1 - m(\xi)\big)}{\beta + 1}T_d(T_1 - T_d)^{\beta+1}$$

$$+\frac{\alpha\beta\big(1 - m(\xi)\big)}{(\beta + 1)(\beta + 2)}(T_1 - T_d)^{\beta+2}$$

$$-\frac{\alpha^2(1-m(\xi))^2}{2(\beta+1)^2}(T_1-T_d)^{2(\beta+1)}\Big]$$

$$-C_O-(T_1-T_d)\xi-C_aA$$

$$-0+PI_eD(A,P)T_1(M-\frac{T_1}{2})\Big] \tag{6.3.27}$$

So, in this case, the objective is to maximize $Z_3 = TP_3(A, T_1, T, P, \xi)$.

Subject to

$$\left.\begin{array}{l} T_1 \leq M \\ M \leq T \\ C \leq P \\ \xi \leq \xi' \end{array}\right\} \tag{6.3.28}$$

and $T_1 \geq 0, T \geq 0, P \geq 0, \xi \geq 0$, A is a positive integer $(A > 0)$.

The optimal order quantity corresponding to the optimal solution $(A^*, T_1^*, T^*, P^*, \xi^*)$ is

$$Q^* = D(A^*,\ P^*)\left[T_1^* + \frac{\alpha(1-m(\xi^*))}{\beta+1}(T_1^*-T_d)^{\beta+1} + \frac{1}{\delta}\left[1-e^{-\delta(T^*-T_1^*)}\right]\right] \tag{6.3.29}$$

6.4 Solution Methodology

For fixed T_1, T, P, and ξ the second order partial derivative of $TP_i(A, T_1, T, P, \xi)$ with respect to A gives,

$$\frac{\partial^2 TP_i}{\partial A^2} = \frac{m(m-1)D(A,P)}{TA^2}\left[P\left[T_1+\frac{1}{\delta}\left(1-e^{-\delta(T-T_1)}\right)\right]\right.$$

$$-\frac{C_d\alpha\big(1-\mathrm{m}(\xi)\big)}{\beta+1}(T_1-T_d)^{\beta+1}$$

$$-C_s\left[T-T_1-\frac{1}{\delta}+\frac{\mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}}{\delta}\right]$$

$$-C_h\left[\frac{T_1^2}{2}+\frac{\alpha\big(1-\mathrm{m}(\xi)\big)}{\beta+1}\mathrm{T_d}(\mathrm{T_1}-\mathrm{T_d})^{\beta+1}\right.$$

$$+\frac{\alpha\beta\big(1-\mathrm{m}(\xi)\big)}{(\beta+1)(\beta+2)}(\mathrm{T_1}-\mathrm{T_d})^{\beta+2}$$

$$\left.-\frac{\alpha^2\big(1-\mathrm{m}(\xi)\big)^2}{2(\beta+1)^2}(\mathrm{T_1}-\mathrm{T_d})^{2(\beta+1)}\right]$$

$$-X_i] \qquad (i=1,2,3) \tag{6.3.30}$$

where, $X_1=-CI_c\left[\frac{(T_1-M)^2}{2}+\frac{\alpha(1-m(\xi))}{\beta+1}(\mathrm{T_d}-\mathrm{M})(\mathrm{T}_{\mathbf{1}}-T_d)^{\beta+1}\right.$

$$\left.+\frac{\alpha\beta(1-\mathrm{m}(\xi))}{(\beta+1)(\beta+2)}(\mathrm{T_1}-\mathrm{T_d})^{\beta+2}-\frac{\alpha^2(1-\mathrm{m}(\xi))^2}{2(\beta+1)^2}(\mathrm{T_1}-\mathrm{T_d})^{2(\beta+1)}\right]+\frac{PI_eM^2}{2}$$

$$X_2=-CI_cD(A,P)\left[\frac{(T_1-M)^2}{2}-\frac{\alpha(1-m(\xi))}{\beta+1}(T_1-\mathrm{M})\{(\mathrm{T_1}-\mathrm{T_d})^{\beta+1}+(\mathrm{M}-\mathrm{T_d})^{\beta+1}\}\right.$$

$$-\frac{2\alpha(1-\mathrm{m}(\xi))}{(\beta+1)(\beta+2)}\{(\mathrm{T_1}-\mathrm{T_d})^{\beta+2}-(\mathrm{M}-\mathrm{T_d})^{\beta+2}\}$$

$$\left.-\frac{\alpha^2(1-\mathrm{m}(\xi))^2}{2(\beta+1)^2}\{(\mathrm{T_1}-\mathrm{T_d})^{\beta+1}-(\mathrm{M}-\mathrm{T_d})^{\beta+1}\}^2\right]+\frac{PI_eM^2}{2}$$

and $X_3=PI_eT_1(M-\frac{T_1}{2})$

Because of $0\le m<1$, $\frac{\partial^2 TP_i}{\partial A^2}<0$. Therefore, $TP_i(A,T_1,T,P,\xi)$ is concave with respect to A. So, the problem of finding the global optimal solution for the frequency of advertisement (A^*), reduces to find the local optimum solution.

Concavity of the total profit function, with respect to other decision variables, has been shown graphically by means of numerical examples in concavity section. The variable A is a positive integer, we suggest the following algorithm to find the optimal solution of the proposed inventory system.

Algorithm:

Step 1: Assign numerical values to all the parameters in appropriate units.

Step 2: Set $A = 1$.

Step 3: Compare M and T_d. If $M \leq T_d$ then go to step 4. Otherwise go to step 8.

Step 4: Find the optimal solution of $TP_1(T_1, T, P, \xi | A)$ subject to the constraints in Eq. (6.3.20).

Then obtain the corresponding total profit $TP_1(A, T_1^*, T^*, P^*, \xi^*)$ and go to next step.

Step 5: Set $A' = A + 1$ and repeat step 4 to get $TP_1(A', T_1^*, T^*, P^*, \xi^*)$ and go to next step.

Step 6: If $TP_1(A', T_1^*, T^*, P^*, \xi^*) \geq TP_1(A, T_1^*, T^*, P^*, \xi^*)$ then set $A = A'$ and go to step 4.

Otherwise go to next step.

Step 7: Set the optimal solution $(A^*, T_1^*, T^*, P^*, \xi^*) = (A, T_1^*, T^*, P^*, \xi^*)$. Go to step 18.

Step 8: Find the optimal solution of $TP_2(T_1, T, P, \xi | A)$ subject to the constraints in Eq. (6.3.24).

Then obtain the corresponding total profit $TP_2(A, T_1^*, T^*, P^*, \xi^*)$ and go to next step.

Step 9: Set $A' = A + 1$ and repeat step 8 to get $TP_2(A', T_1^*, T^*, P^*, \xi^*)$ and goto next step.

Step 10: If $TP_2(A', T_1^*, T^*, P^*, \xi^*) \geq TP_2(A, T_1^*, T^*, P^*, \xi^*)$ then set $A = A'$ and go to step 8.

Otherwise go to next step.

Step 11: Set the optimal solution $(A^*, T_1^*, T^*, P^*, \xi^*) = (A, T_1^*, T^*, P^*, \xi^*)$ and go to next step.

Step 12: Set $A = 1$.

Step 13: Find the optimal solution of $TP_3(T_1, T, P, \xi | A)$ subject to the constraints in Eq. (6.3.28).

Then obtain the corresponding total profit $TP_3(A, T_1^*, T^*, P^*, \xi^*)$ and go to next step.

Step 14: Set $A' = A + 1$ and repeat step 13 to get $TP_3(A', T_1^*, T^*, P^*, \xi^*)$ and go to next step.

Step 15: If $TP_3(A', T_1^*, T^*, P^*, \xi^*) \geq TP_3(A, T_1^*, T^*, P^*, \xi^*)$ then set $A = A'$ and go to step 13.

Otherwise goto next step.

Step 16: Set the optimal solution $(A^*, T_1^*, T^*, P^*, \xi^*) = (A, T_1^*, T^*, P^*, \xi^*)$ and go to next step.

Step 17: If $\text{Max}\{TP_2^*, TP_3^*\} = TP_2^*$ then the solution obtained in step 11 is the optimal.

If $\text{Max}\{TP_2^*, TP_3^*\} = TP_3^*$ then the solution obtained in step 16 is the optimal.

Step 18: Compute the corresponding optimal order quantity Q^* from Eq. (6.3.29). Stop.

While executing the above algorithm, for fixed A, we can obtain the optimal solution which maximizes the total profit function with constraints using software like MATLAB, MATHEMATICA, R, MATHCAD, etc.

For fixed value of the variable A the necessary and sufficient conditions to maximize the total profit function $TP_i(T_1, T, P, \xi|A)$ are as follows:

$\frac{\partial TP_2}{\partial T_1} = 0,\ \frac{\partial TP_2}{\partial T} = 0,\ \frac{\partial TP_2}{\partial P} = 0,\ \frac{\partial TP_2}{\partial T_\xi} = 0;$

Provided that the Hessian matrix $H = \begin{bmatrix} \frac{\partial^2 TP_i}{\partial {T_1}^2} & \frac{\partial^2 TP_i}{\partial T_1 T} & \frac{\partial^2 TP_i}{\partial T_1 P} & \frac{\partial^2 TP_i}{\partial T_1 \xi} \\ \frac{\partial^2 TP_i}{\partial T T_1} & \frac{\partial^2 TP_i}{\partial T^2} & \frac{\partial^2 TP_i}{\partial T P} & \frac{\partial^2 TP_i}{\partial T \xi} \\ \frac{\partial^2 TP_i}{\partial P T_1} & \frac{\partial^2 TP_i}{\partial P T} & \frac{\partial^2 TP_i}{\partial P^2} & \frac{\partial^2 TP_i}{\partial P \xi} \\ \frac{\partial^2 TP_i}{\partial \xi T_1} & \frac{\partial^2 TP_i}{\partial \xi T} & \frac{\partial^2 TP_i}{\partial \xi P} & \frac{\partial^2 TP_i}{\partial \xi^2} \end{bmatrix}$ is a negative definite.

6.5 Examples

Example 1 (case 1): Consider the following parameter values in apropriate unites.

$T_d = 0.15,\ M = 0.0822$ (30 days), $\alpha = 0.4,\ \beta = 2,\ a = 500000,\ b = 2,\ m = 0.04,$ $\delta = 0.5,\ C = \$10,\ C_{\text{O}} = \$300,\ C_{\text{a}} = \$80,\ C_{\text{h}} = \$1.5,\ C_{\text{d}} = \$0.5,\ C_{\text{s}} = \$8,\ I_{\text{c}} =$ $0.12,\ I_{\text{e}} = 0.09,\ \xi' = 500,\ m(\xi) = 1 - e^{-\eta \times \xi}$ where $\eta = 0.03$.

Since $M < T_d$, this is an example of case 1. For different values of A, maximizing $TP_1(A, T_1, T, P, \xi)$, subject to $T_1 \le T,\ C \le P,\ \xi \le \xi'$ (using R programming) the solutions are given in Table 6.5.1.

Table 6.5.1 Optimal solutions of TP_1 for fixed $\boldsymbol{A}$

$\boldsymbol{A}$	$\boldsymbol{T_1}$	$\boldsymbol{T}$	$\boldsymbol{P}$	$\boldsymbol{\xi}$	$\boldsymbol{TP_1}$
1	0.408688	0.531986	20.845830	41.652852	11181.30
2	0.447590	0.582849	20.938407	51.763384	11371.93
3	0.485023	0.631732	21.028218	60.037093	11438.98
4	**0.520403**	**0.677869**	**21.113219**	**66.972334**	**11459.13**
5	0.554201	0.721827	21.194088	72.935439	11455.83
6	0.586336	0.763542	21.271640	78.148412	11439.17

From table 6.5.1 the optimal solution for which the total profit function is maximum is

$A^* = 4,\ T_1^* = 0.520403,\ T^* = 0.677869,\ P^* = 21.113219,\ \xi^* = 66.972334.$

The corresponding optimal profit is $TP_1^* = 11459.13$ and order quantity is $Q^* = 797.6083$.

Example 2: Consider $M = 0.274$ (100 days) and other parameter values same as in example-1.

Since $M > T_d$, it may be of Case 2 or case 3. For different values of A, maximizing $TP_2(A, T_1, T, P, \xi)$ and $TP_3(A, T_1, T, P, \xi)$ using R programming the solutions are given in table 2 and table 3, respectively.

Table 6.5.2 Optimal solutions of TP_2 for fixed $\boldsymbol{A}$

$\boldsymbol{A}$	$\boldsymbol{T_1}$	$\boldsymbol{T}$	$\boldsymbol{P}$	$\boldsymbol{\xi}$	$\boldsymbol{TP_2}$
1	0.404514	0.502004	20.497432	39.441632	11445.25
2	0.444320	0.554132	20.589762	49.634536	11636.39
3	0.482101	0.603645	20.677882	57.831019	11701.90
4	**0.517957**	**0.650476**	**20.762055**	**64.642551**	**11720.11**
5	0.551772	0.694642	20.841378	70.495671	11714.84
6	0.584082	0.736711	20.917501	75.515276	11696.28

Table 6.5.3 Optimal solutions of TP_3 for fixed $\boldsymbol{A}$

$\boldsymbol{A}$	$\boldsymbol{T_1}$	$\boldsymbol{T}$	$\boldsymbol{P}$	$\boldsymbol{\xi}$	$\boldsymbol{TP_3}$
1	0.27400	0.37837	20.24564	0.00001	11366.53
2	0.27400	0.39504	20.27316	0.00013	11506.83
3	**0.27400**	**0.41169**	**20.30375**	**0.00037**	**11515.28**
4	0.27400	0.42797	20.33603	0.00059	11472.89
5	0.27400	0.44387	20.37007	0.00005	11404.96

From table 6.5.2 and table 6.5.3, Max$\{TP_2^*, TP_3^*\} = TP_2^* = 11720.11$. Hence this is an example of case 2, and the optimal solution is:

$A^* = 4,\ T_1^* = 0.517951,\ T^* = 0.650476,\ P^* = 20.762055,\ \xi^* = 64.642551.$

The corresponding optimal order quantity is $Q^* = 793.4279$.

Example 3: Consider $M = 0.5754$ (210 days) and other parameter values same as in example-1.

Since $M > T_d$, it may be of Case 2 or case 3. For different values of A, maximizing $TP_2(A, T_1, T, P, \xi)$ and $TP_3(A, T_1, T, P, \xi)$ using R programming the solutions are given in table 6.5.4 and table 6.5.5, respectively.

Table 6.5.4 Optimal solutions of TP_2 for fixed $\boldsymbol{A}$

$\boldsymbol{A}$	$\boldsymbol{T_1}$	$\boldsymbol{T}$	$\boldsymbol{P}$	$\boldsymbol{\xi}$	$\boldsymbol{TP_2}$
1	0.57540	0.63009	20.36286	75.52380	11898.79
2	0.57540	0.63984	20.36447	76.45455	12125.58
3	0.57540	0.64982	20.36628	76.98144	12212.27
4	**0.57540**	**0.65980**	**20.37008**	**77.34903**	**12241.56**
5	0.57540	0.66967	20.37460	77.64469	12239.78
6	0.57540	0.67948	20.38069	77.85076	12218.73

Table 6.5.5 Optimal solutions of TP_3 for fixed $\boldsymbol{A}$

$\boldsymbol{A}$	$\boldsymbol{T_1}$	$\boldsymbol{T}$	$\boldsymbol{P}$	$\boldsymbol{\xi}$	$\boldsymbol{TP_3}$
1	0.49403	0.57540	20.20258	61.74271	11956.51
2	0.49425	0.57540	20.20106	62.67526	12173.23
3	0.49443	0.57540	20.20139	63.26522	12246.93
4	**0.49816**	**0.58050**	**20.20966**	**64.33542**	**12261.01**
5	0.52703	0.62042	20.27567	69.78885	12247.32
6	0.55429	0.65813	20.33728	74.51923	12220.14

From table 6.5.4 and table 6.5.5, $\text{Max}\{TP_2^*, TP_3^*\} = TP_3^* = 12261.01$. Hence this is an example of case 3, and the optimal solution is:

$A^* = 4,\ T_1^* = 0.49816,\ T^* = 0.58050,\ P^* = 20.20966,\ \xi^* = 64.33542.$

The corresponding optimal order quantity is $Q^* = 750.0605$.

6.6 Concavity and Optimality

For example 2 of the above section, the total profit is plotted against each variable fixing other variables in Figure 6.6.1 to 6.6.5. From these figures, it is obvious that the total profit function TP_2 is concave with respect to each variable. Figures. 6.6.6-6.6.11 also reveals that the total profit functions TP_1, TP_2 and TP_3 are concave functions.

Fixing $A = 4$ in example 2, for the solution

$(T_1^*, T^*, P^*, \xi^*) = (0.517957, 0.650476, 20.762055, 64.642551)$ the gradient is $(-0.275, 0.032, 0.0005, 0.0002)$, which is close to zero.

Hessian matrix is

$$H = \begin{bmatrix} -22442.947579 & 16548.77 & 111.29610614 & 3.0198145 \\ 16548.774027 & -16548.45 & 28.37785091 & -0.00033836 \\ 109.5509 & 28.37785 & -58.9697889 & -0.054512286 \\ 3.019814 & -0.0003384 & -0.05451229 & -0.01697277 \end{bmatrix};$$

The eigenvalues of H are $-0.01542, -55.54260, -268.98330, -36304.99$. Therefore, the Hessian matrix is negative definite, and hence the solution is global maximum.

Figure 6.6.1 Concavity of the total profit function $TP_2(A, T_1, T, P, \xi)$ (example 2) with respect to A when other variables are fixed.

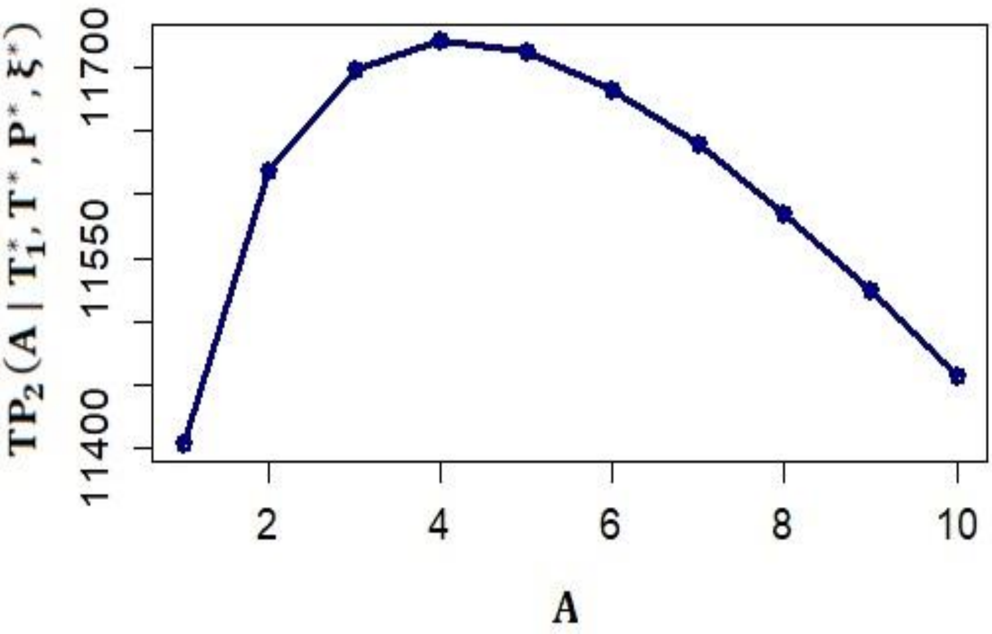


Figure 6.6. 2 Concavity of the total profit function $TP_2(A, T_1, T, P, \xi)$ (example 2) with respect to T_1 when other variables are fixed.

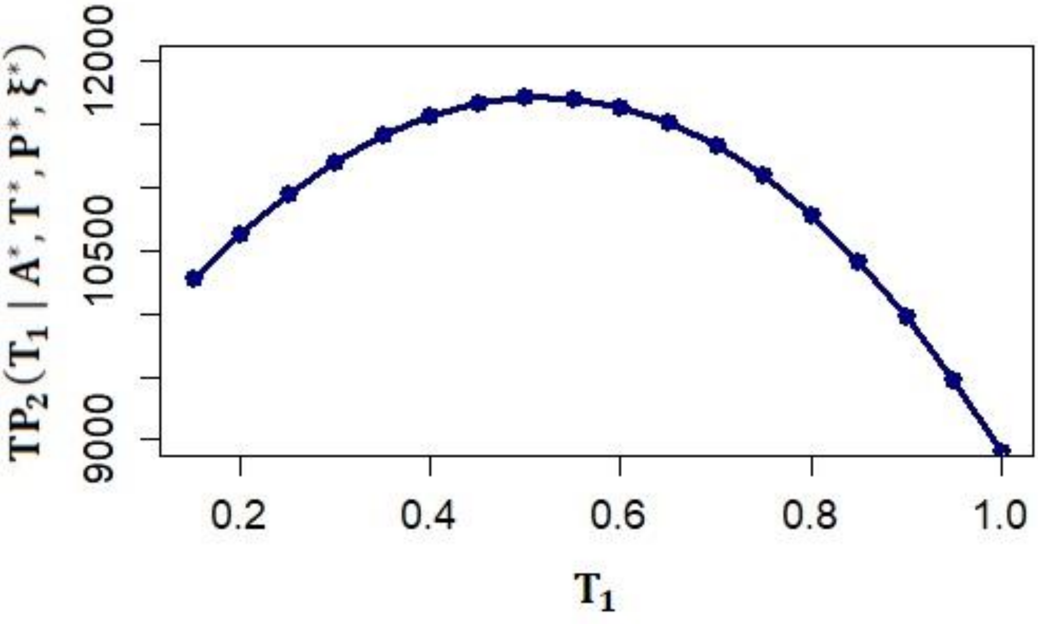


Figure 6.6.3 Concavity of the total profit function $TP_2(A, T_1, T, P, \xi)$ (example 2) with respect to T when other variables are fixed.

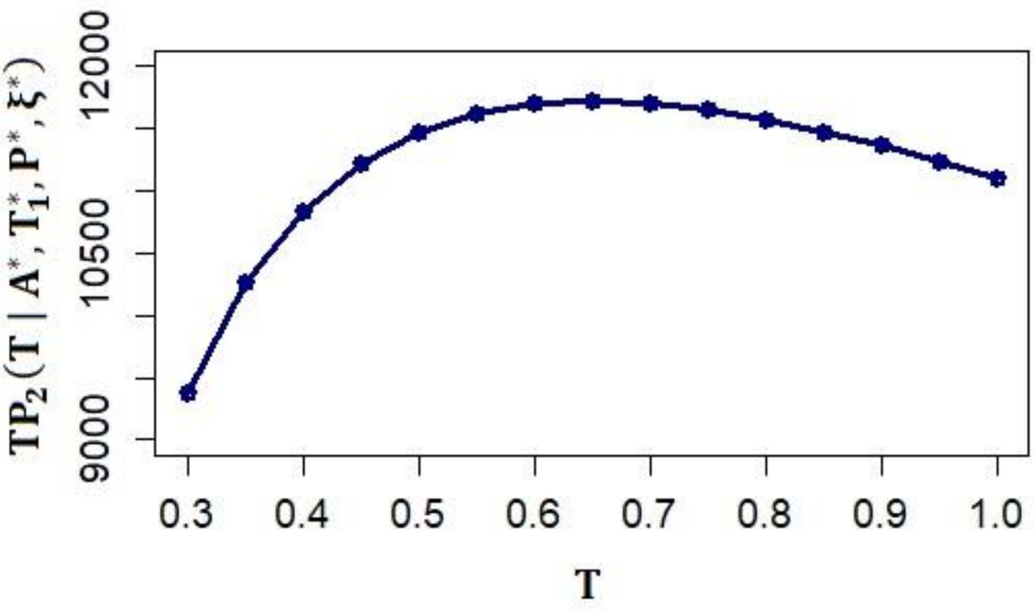


Figure 6.6.4 Concavity of the total profit function $TP_2(A, T_1, T, P, \xi)$ (example 2) with respect to P when other variables are fixed.

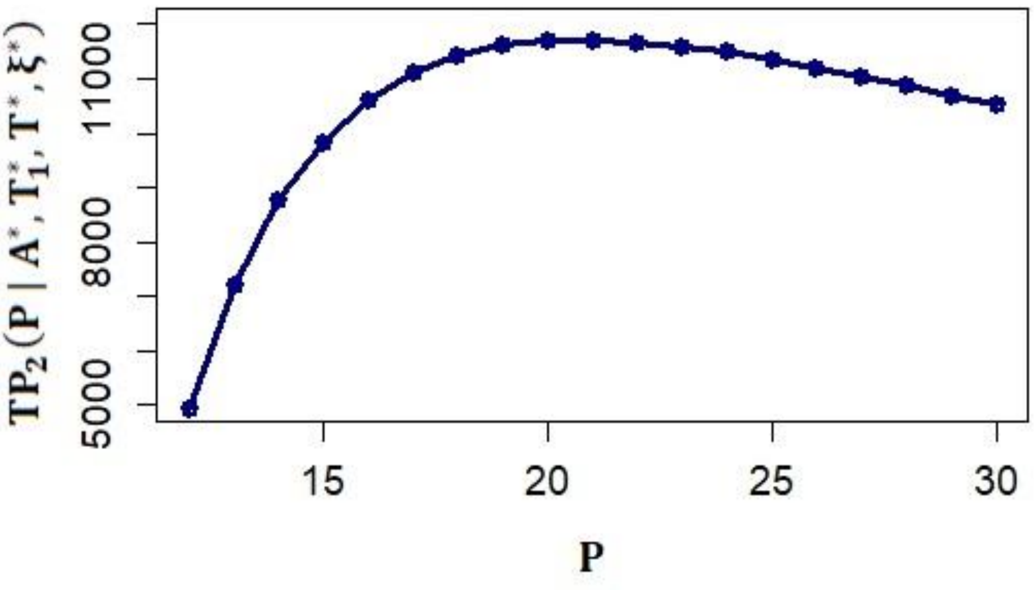


Figure 6.6.5 Concavity of the total profit function $TP_2(A, T_1, T, P, \xi)$ (example 2) with respect to ξ when other variables are fixed.

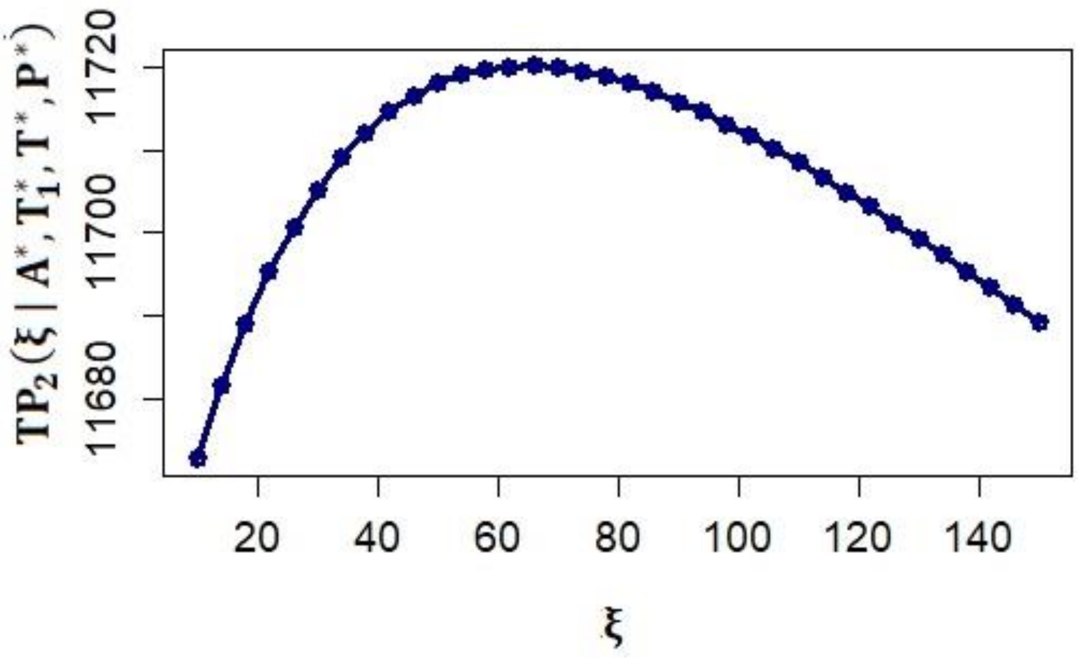


Figure 6.6.6 Concavity of TP_1 (Example-1) w.r.t T and P.

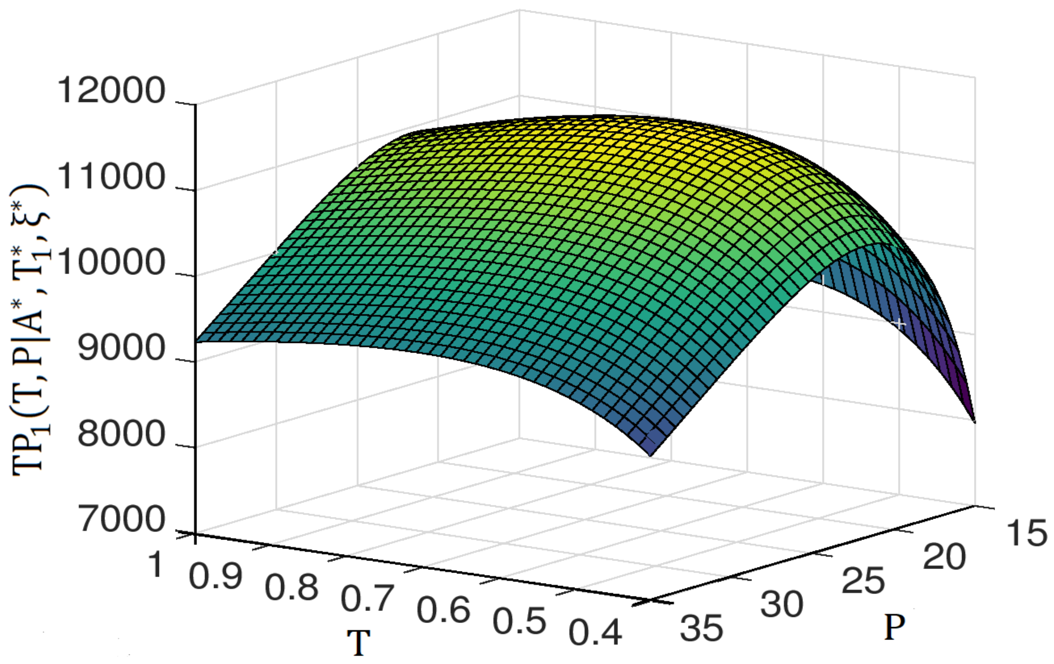


Figure 6.6.7 Concavity of TP_1 (Example-1) w.r.t T and ξ.

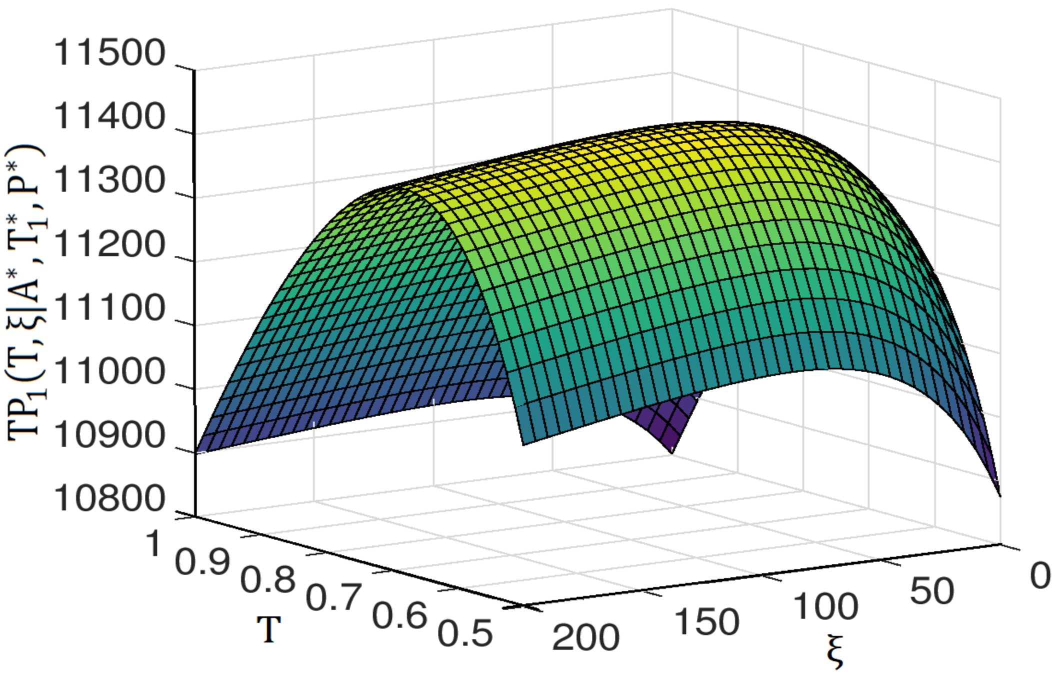


Figure 6.6.8 Concavity of TP_2 (Example-2) w.r.t T and P.

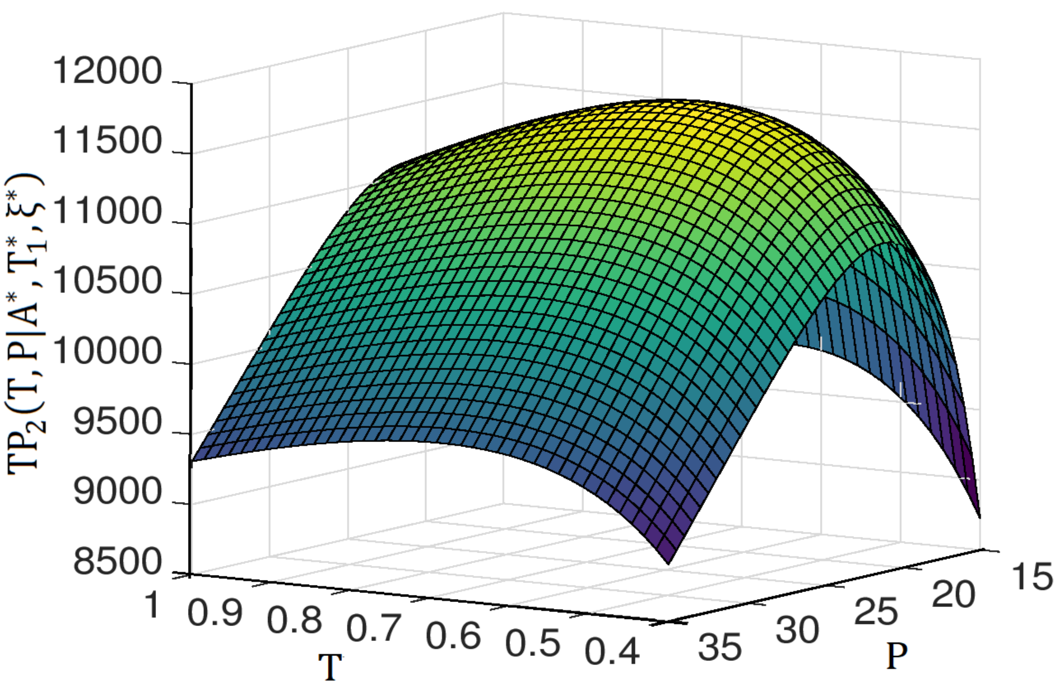


Figure 6.6.9 Concavity of TP_2 (Example-2) w.r.t T and ξ.

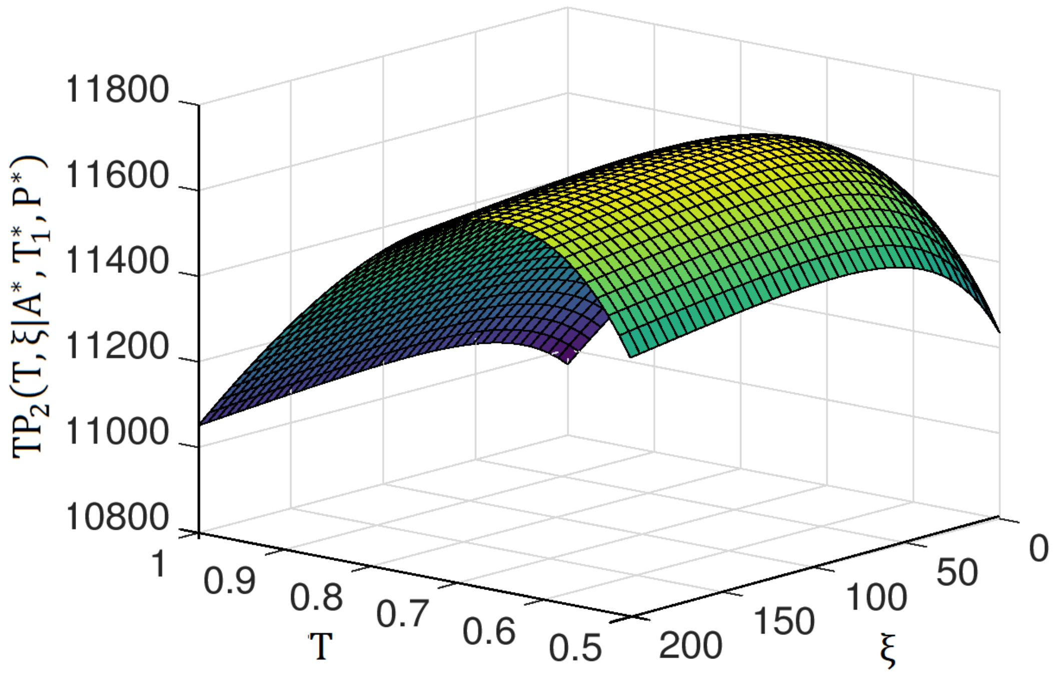


Figure 6.6.10 Concavity of TP_3 (Example-3) w.r.t T and P.

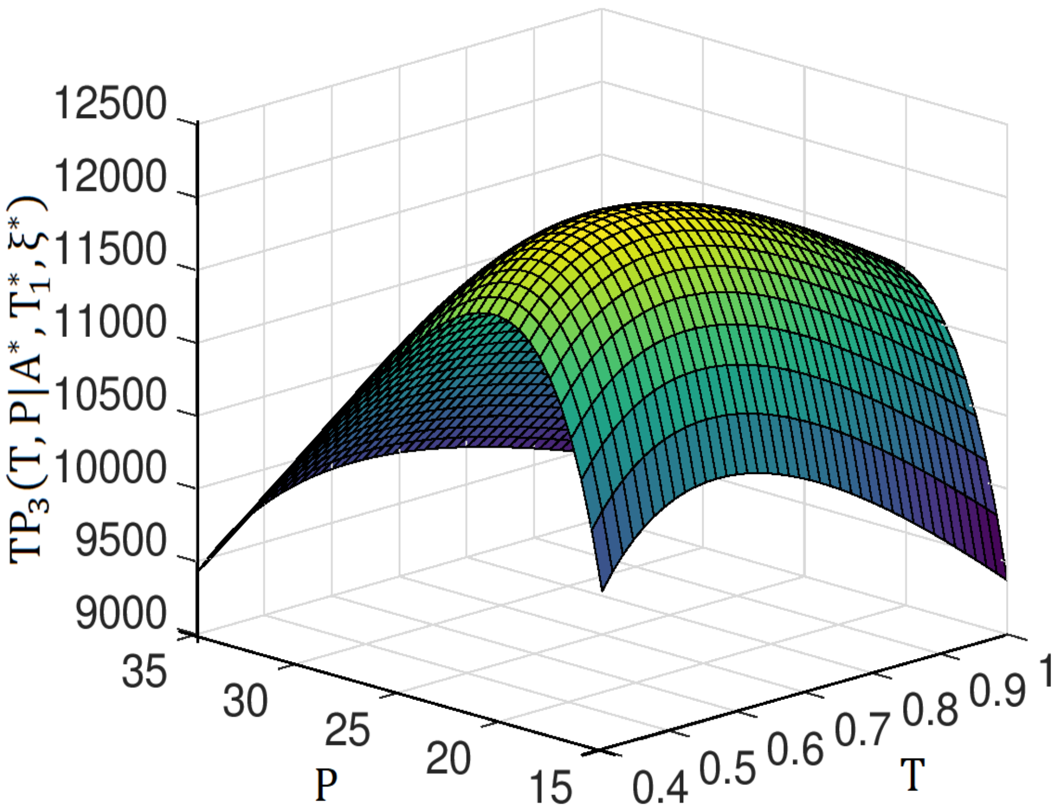
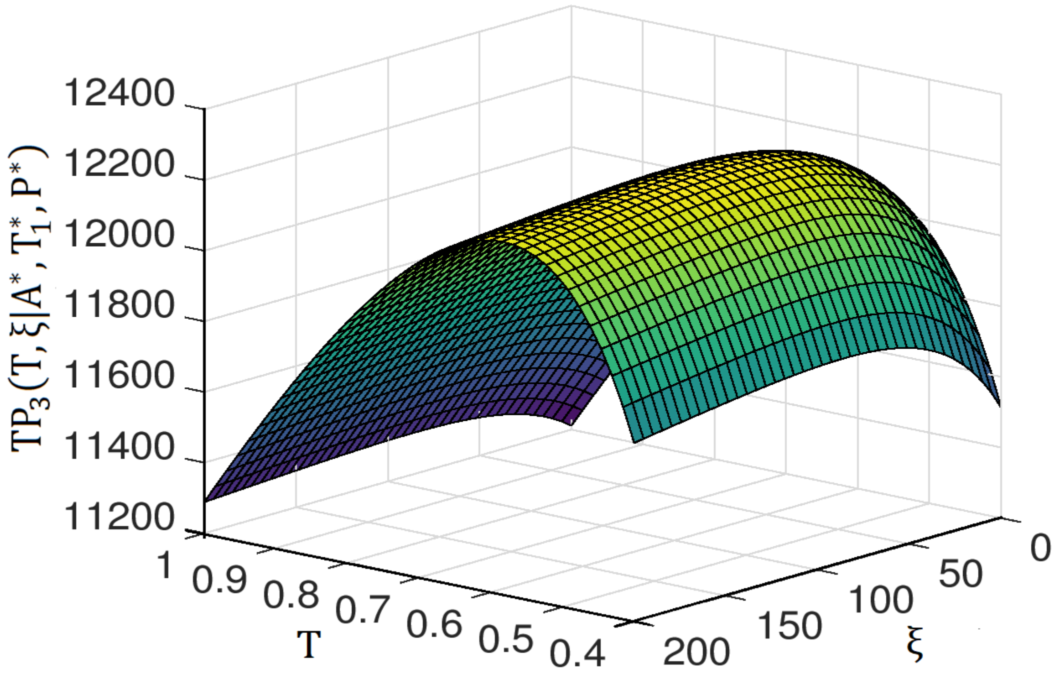


Figure 6.6.11 Concavity of TP_3 (Example-3) w.r.t T and ξ.



6.7 Sensitivity Analysis

Table 6.7.1 Computational results for different values of T_d and M.

T_d	M	A^*	T_1^*	T^*	P^*	ξ^*	Profit	Remark
0	0.0822	4	0.52732	0.68838	21.14265	85.99221	11423.10	Case 2
	0.2740	4	0.52403	0.66028	20.78973	87.41877	11680.12	Case 2
	0.5754	4	0.50343	0.58978	20.23549	88.52549	12215.83	Case 3
0.15	0.0822	4	0.52047	0.67788	21.11303	66.94617	11459.13	Case 1
	0.2740	4	0.51792	0.65044	20.76162	64.67911	11720.11	Case 2
	0.5754	4	0.49821	0.58057	20.20924	64.35627	12261.01	Case 3
0.25	0.0822	4	0.51990	0.67496	21.10296	47.52051	11481.69	Case 1
	0.2740	4	0.51667	0.64698	20.74995	43.67079	11743.74	Case 2
	0.5754	4	0.49709	0.57704	20.19649	41.74733	12287.45	Case 3

Table 6.7.1 reveals that when the supplier allows more credit period (M), the retailer earns more profit. The model assumes non-instantaneous deterioration, but it is also applicable for instantaneous deterioration case by taking $T_d = 0$. That means, the instantaneous deterioration case (i.e. $T_d = 0$) is a particular case of non-instantaneous deterioration case (i.e. $T_d > 0$). Table 6.7.1 shows that instantaneous deteriorating items need more PT investment.

Table 6.7.2 shows the computational results obtained by increasing each parameter of example 2 by -50%, -25%, +25% and +50%.

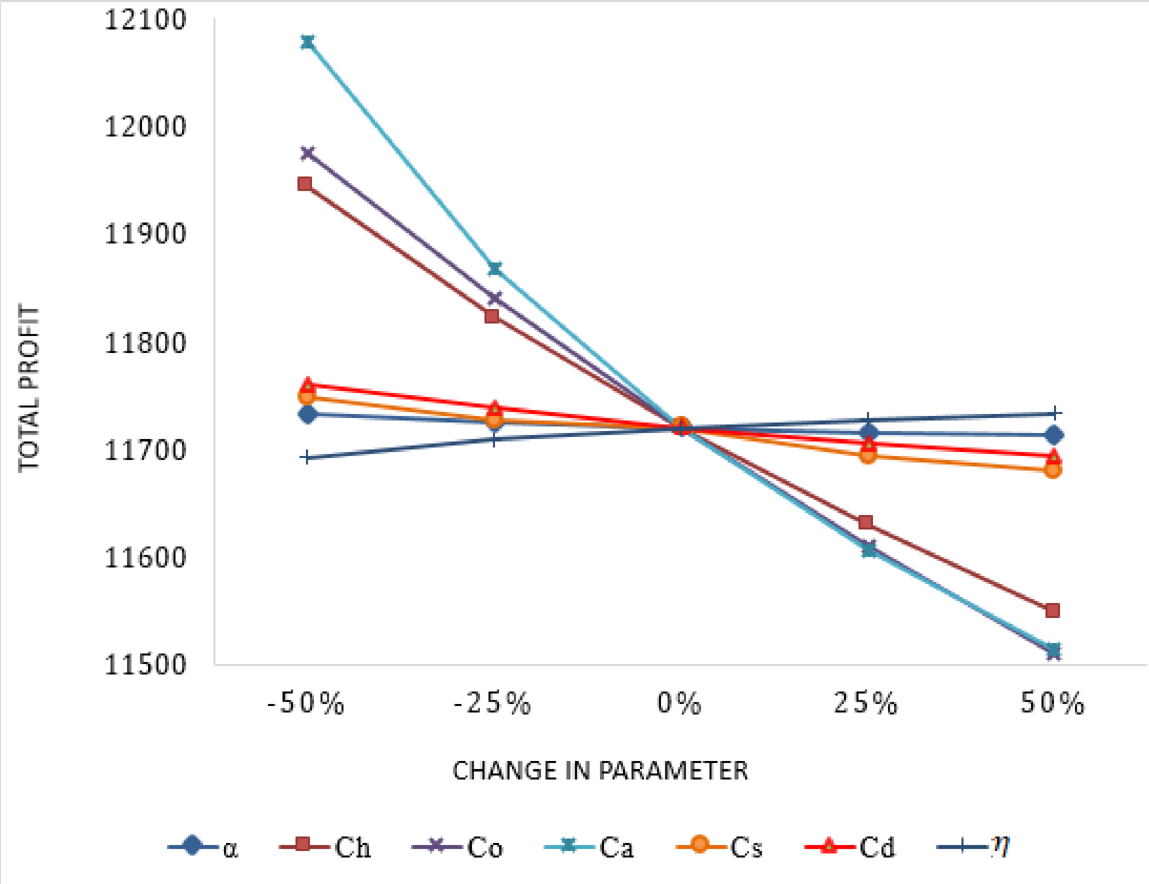
Table 6.7.2 Effect of different parameters on optimal results

Parameter		A^*	T_1^*	T^*	P^*	ξ^*	Q^*	**Profit** (TP_2^*)
α	0.2	4	0.52097	0.65230	20.76806	42.07581	795.307	11733.21
	0.3	4	0.51916	0.65120	20.76440	55.29689	794.177	11725.54
	0.5	4	0.51690	0.64980	20.75966	71.88940	792.749	11715.91
	0.6	4	0.51612	0.64935	20.75798	77.88195	792.295	11712.48
a	250000	3	0.68210	0.87806	21.19168	63.84241	507.077	5468.14
	375000	3	0.55720	0.70601	20.86815	60.95053	631.712	8561.82
	625000	5	0.49250	0.61469	20.69463	67.94421	952.102	14920.75
	750000	6	0.47446	0.58948	20.64731	70.85396	1109.01	18153.84
m	0.02	2	0.44738	0.55834	20.59708	49.79629	664.563	11464.43
	0.03	3	0.48480	0.60731	20.68446	57.96275	730.131	11563.96
	0.05	6	0.57883	0.72965	20.90467	75.35319	907.505	11927.78
	0.06	8	0.63112	0.79731	21.02671	83.63606	1014.445	12183.52
	0.75	5	0.65764	0.77909	20.66534	85.27541	939.266	11943.92

C_h	1.125	5	0.59874	0.73152	20.76080	77.47423	901.162	11822.16
	1.875	4	0.481863	0.62289	20.82987	58.06043	753.859	11630.08
	2.25	4	0.45156	0.60032	20.88911	51.91613	721.497	11548.88
C_O	150	3	0.40166	0.49776	20.48856	40.21037	617.479	11974.47
	225	4	0.48183	0.60314	20.67717	58.19567	742.163	11839.78
	375	5	0.58422	0.73697	20.91817	75.28870	892.573	11610.07
	450	5	0.61523	0.77744	20.99191	79.55606	934.532	11511.02
C	5	5	0.31135	0.39026	10.35501	36.54526	1934.094	24060.98
	7.5	5	0.43904	0.55153	15.58236	59.68624	1205.604	15818.49
	12.5	4	0.61271	0.77161	26.02568	71.32180	598.484	9270.85
	15	4	0.69879	0.88247	31.31551	75.56844	472.4117	7642.91
C_a	40	9	0.52758	0.66271	20.78385	67.36174	833.112	12077.96
	60	6	0.53193	0.66859	20.79472	67.54914	826.066	11867.07
	100	3	0.51149	0.64218	20.74734	63.14861	775.526	11605.57
	120	3	0.53949	0.67883	20.81299	67.84697	814.262	11514.73
C_s	4	4	0.50438	0.66958	20.67533	62.47933	820.765	11759.94
	6	4	0.51194	0.65895	20.72516	63.67690	805.455	11737.97
	10	4	0.52284	0.64350	20.79060	65.41710	783.630	11705.29
	12	4	0.52706	0.63785	20.81335	66.06682	775.718	11692.79
$\boldsymbol{\eta}$	0.015	4	0.49302	0.62805	20.73864	74.02351	768.468	11692.58
	0.0225	4	0.50904	0.64254	20.75396	71.29754	784.582	11709.08
	0.0375	4	0.52357	0.65539	20.76800	58.45208	798.831	11727.91

	0.045	4	0.52747	0.65875	20.77182	53.20539	8	11733.73

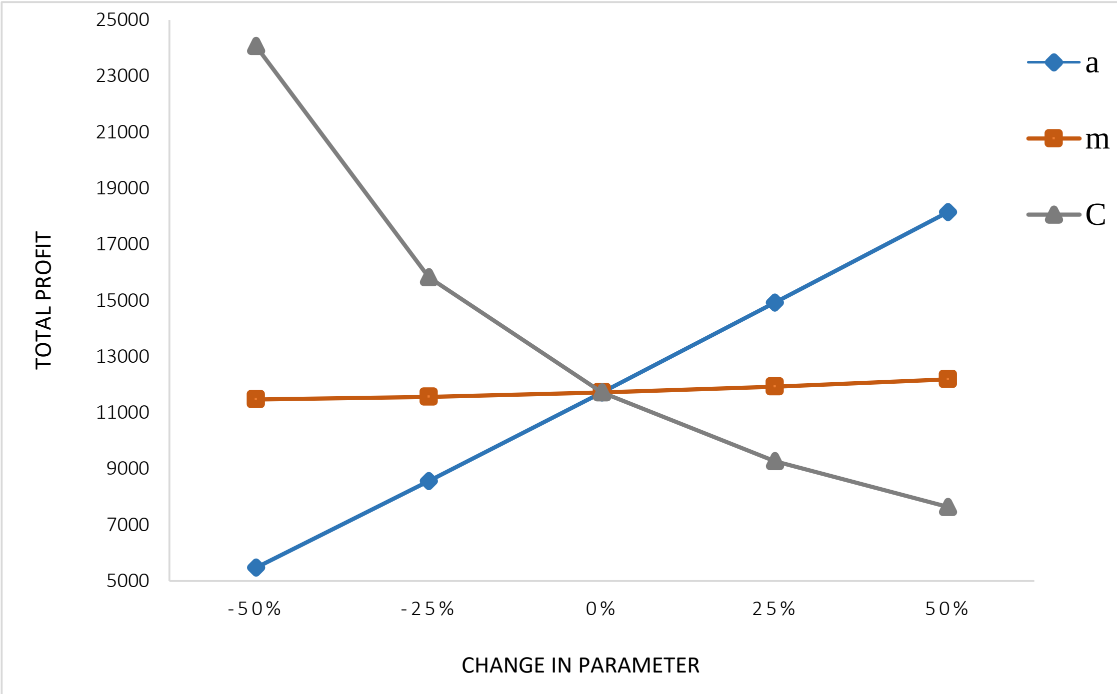
Figure 6.7.1 Effect of $\alpha, C_h, C_o, C_a, C_s, C_d$ and η on total profit.



Observations and managerial insights:

The total profit is less sensitive with the change in parameters α, C_d, and C_s. An increment in α increase the deterioration rate and increment in C_d increase the total deterioration cost but, preservation technology investment reduce the deterioration rate (number of deteriorating units) significantly, and hence profit is ineffective with the change in α and C_d. Hence, retailers are suggested to invest in preservation technology to reduce losses incurring due to deterioration. As the shortage cost C_s increases, our model decreases the shortage period $(T^* - T_1^*)$ (see table 6.7.2), which reduce lost sales and hence, profit is less effective with the change in C_s.

Figure 6.7.2 Effect of $a, m,$ and C on total profit



Increment in different cost parameters $C_h, C_o,$ and C_a results in a decrement in total profit. In table 6.7.2, increment in holding cost (C_h) decreases the optimal order cycle T^* while increment in ordering cost (C_o) increases the optimal order cycle (T^*). Hence, when the holding cost raises, the retailer is suggested to decrease the order cycle, and when the ordering cost rises, the retailer is suggested to increase the order cycle. As the advertisement cost (C_a) increase, the frequency of advertisement and total profit decreases. To increase the total profit, the retailer is suggested to increase the frequency of advertisement (A) when the advertisement cost (C_a) is less.

As the value of η increase, the total profit increases. Since the reduced deterioration rate is $(1 - m(\xi)) = e^{-\eta \times \xi}$, an increment in η will reduce the deterioration rate greatly, which results in a less preservation technology investment and more profit. The retailer need to invest more in preservation technology for smaller value of η.

The total profit is very sensitive with the change in parameters a and C. Increased value of the scale parameter (a) of the demand function will increase the demand, and hence increase the total profit. As purchase cost (C) increase, the optimal value of selling price (P^*) drastically increases. But, increased selling price (P^*) decrease the demand, and hence the total profit is decreasing drastically as C increases. As the shape parameter of demand (m) increase the total profit increases. In figure 6.7.2, it seems that the profit is less sensitive with the change in the parameter (m) this is due to assigning a smaller value to m ($m = 0.04$). The profit will drastically increase for the assignment of higher value to m.

6.8 Conclusion

In a competitive market environment, to get maximum revenue every business organization has to optimize all the possible strategies. In this chapter, our proposed model maximizes the total profit by optimizing the pricing, marketing, preservation, and inventory ordering policies. The preservation technology investment reduces faster deteriorations, which is beneficial to businesses based on agricultural products, bakery products, dairy products, and meat and fish products. The retailer can earn additional profit by taking advantage of credit period. More preservation technology investment is required for instantaneous deterioration case. So, the profit of non-instantaneous deterioration case will be more than the profit of instantaneous deterioration case. When the cost of advertisement is low, the retailer can earn more profit through increasing the frequency of advertisement.