CHAPTER - 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

Inventory means the reserved physical stock of items, maintained to accomplish the present and future demand. Here the term "items" is used in a broad sense. These items can be raw materials, semi-finished goods, finished goods, the workforce, or any resources. It is necessary for retailers, manufacturers, companies, states, countries, etc., to maintain the sufficient amount of inventory of required items. But, different costs are associated with the inventory, such as interest on capital investment, ordering cost, transportation cost, handling cost, storage cost, insurance cost, wastage cost, preservation technology cost. So, an excessive amount of inventory will lead to additional costs and wastage. On the other hand, an insufficient amount of inventory will lead to the loss of goodwill and opportunity. To avoid the above two situations and to maintain an adequate amount of inventory, it is necessary to order the correct amount (lot) at the right time at regular intervals. So, the problem of inventory arises, which requires a solution to the following questions.

(1) How much to order?

(2) When to order?

Researchers have developed several economic order quantity (EOQ) models and derived optimal order quantity and optimal length of the order cycle such that the total cost becomes minimum, or the total profit becomes maximum.

Mathematical modeling of inventory systems has to develop inherently based on certain assumptions, such as assumptions regarding budget, demand, lead time, inflation, shortages, backlogging, trade credit, price discounts, planning horizon, etc.

Mathematical modelling of deteriorating inventory systems is an interesting subset of inventory theory. Deterioration is one of the basic characteristics of almost all commodities. Deterioration means damage, decay, loss of utility, spoilage, vaporization, obsolescence, etc. Dairy products, pharmaceutical products, agricultural products, animal husbandry products, and many other products are deteriorating by nature. So, it is necessary to study the inventory systems for deteriorating items. On hand inventory for deteriorating items can be described by the following differential equation:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D \tag{1.1}$$

Where, $I(t)$ = inventory level at time t, $\theta(t)$ = deterioration rate at time t and D = demand function.

The above differential equation laid a foundation for the inventory modelling of deteriorating items, which is a first order linear differential equation and its solution can be obtained as follows.

$$I(t) = \frac{\int -D(t)\, e^{\int \theta(t)\, dt}\, dt + C}{e^{\int \theta(t)\, dt}} \tag{1.2}$$

Where, C is a constant and its value can be obtained by using the boundary conditions.

Demand and deterioration may be deterministic or probabilistic. Our work is based on the assumption of probabilistic deterioration, more specifically we assumed the lifetime of the items follows Weibull distribution.

Two-parameter Weibull distribution for lifetime (t) of items with scale parameter $\alpha\ (> 0)$ and shape parameter $\beta\ (\geq 0)$ is given by

$$f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}} \tag{1.3}$$

Two-parameter Weibull distributed deterioration rate function is given by

$$\theta(t) = \frac{f(t)}{1 - F(t)} = \frac{\alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}}{e^{-\alpha t^{\beta}}} = \alpha\beta t^{\beta-1} \qquad (1.4)$$

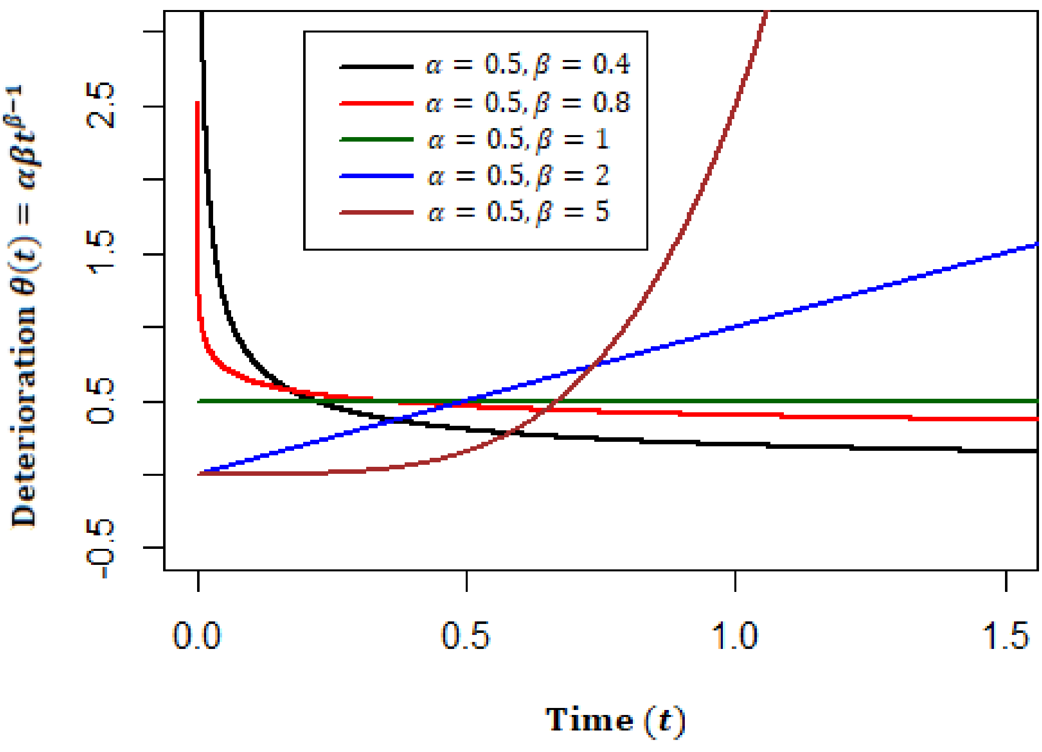
Special cases:

If $\beta = 1$ then $\theta(t) = \alpha$ (i.e. constant deterioration).

If $\beta = 2$ then $\theta(t) = 2\alpha t$ (i.e. linear deterioration).

This implies that the constant deterioration case and linear deterioration case both are particular cases of the Weibull deterioration.

Figure 1.1 Weibull distribution deterioration



From Figure 1.1, we can observe that when $\beta < 1$ the deterioration rate decreases with respect to time (t) and when $\beta > 1$ the deterioration rate increases with respect to time(t). When $\beta = 1$ the deterioration rate is constant. So, the Weibull distribution is suitable to study the increasing, decreasing, or constant deterioration phenomenon.

Researchers often assume that the deterioration process starts as soon as the commodities enter into the inventory system but, it is not true for all commodities. Fruits, vegetables, foodstuffs have a short span of maintaining fresh quality, in which there is almost no spoilage. This phenomenon is known as non-instantaneous deterioration. If T_d is the duration in which the item has no deterioration, then the three-parameter Weibull distribution with location parameter T_d is given by

$$f(t) = \alpha\beta(t - T_d)^{\beta-1}e^{-\alpha(t-T_d)^{\beta}} \tag{1.5}$$

Three-parameter Weibull distribution deterioration rate function is given by

$$\theta(t) = \alpha\beta(t - T_d)^{\beta-1} \tag{1.6}$$

Special case: If $T_d = 0$, then $\theta(t) = \alpha\beta t^{\beta-1}$, which is deterioration rate of two-parameter Weibull distribution.

This study deals with building some inventory models for deteriorating items. Particularly in this thesis, some inventory models are developed for Weibull distributed deterioration.

1.2 Literature Review

In the past few years, deteriorating inventory systems are studied extensively. In classical economic order quantity (EOQ) model the deteriorating fenomenon of the items was ignored. The research work in the field of deteriorating inventry modeling begun with Whitin (1957), who proposed an inventory model for fashion items deteriorating at the end of prescribed storage period. Ghare and Schrader (1963) first proposed exponential deteriorating inventory model with constant demand. Li et al.

(2010), Bakker et al. (2012), Goyal and Giri (2001), Nahmias (1982), Raafat (1991), and Janssen et al. (2016) from time to time provided the detailed review of the literature on deteriorating inventory systems.

In traditional EOQ models, it was assumed that the retailer must pay off as soon as the items are received. That would not be always true in today's competitive business environment. In practice the supplier may provide the retailer some credit period to settle the account in order to attract and motivate new customers and increase sales. Obviously, supplier charges interest at some rate on the remaining amount if retailer is not able to settle before given credit period. Haley and Higgins (1973) investigated the relationship between inventory policy and trade credit policy in the context of the basic lot-size model for the first time. Goyal (1985) introduced an EOQ model under permissible delay in payments. He ignored the difference of selling and purchase price. Later Dave (1985) corrected Goyal's model by assuming that the selling price should be higher than its purchase price. Shah et al. (1988) studied the same model allowing shortages. Shah (1993) extended an EOQ model in which delays in payment are permissible and items in inventory deteriorate at a constant rate. Aggarwal and Jaggi (1995) further extended the Goyal's model for deteriorating items. Jamal et al. (1997) developed an inventory model for deteriorating items allowing shortages under permissible delay in payments. Davis and Gaither (1985) developed EOQ models for firms offering a one-time opportunity to delay in payments by their suppliers for an order of a commodity. Khouja and Mehrez (1996) discussed the problem that the supplier offers a permissible delay in payments only when the order quantity is larger than a predetermined quantity. Shinn and Hwang (2003) developed a model for optimal pricing and ordering policies under order-size dependent delay in payments. Lokhandwala et al. (2005) derived optimal ordering policies under conditions of extended payment privileges. Goyal et al. (2007) introduced optimal ordering policies when the supplier provides a progressive interest scheme. Manna et al. (2008) derived optimal pricing and lot-size policies with weibull deterioration under trade credit. Ouyang et al. (2009) considered partially permissible delay in payments inked to order quantity. The review article by Molamohamadi et al. (2014) provided a comprehensive survey of published literature for the inventory models with permissible delay in payments or trade credit. Singh and Panda (2015) considered generalized weibull deterioration rate with price dependent demand under inflation and trade credit. Pervin at al. (2016) developed a deteriorating inventory model for declining demand market under trade credit. Sundara Rajan and Uthayakumar (2017) derived optimal pricing and ordering policies for deteriorating items under trade credit and inflation over a finite planning horizon. Kumar and Kumar (2016) provided a genetic algorithm for the solution of weibull deteriorating inventory model under inflation and permissible delay in payments. Shaikh (2017) developed an inventory model for with three parameter wibull deterioration under mixed type financial trade credit. He considered that the demand is dependent on selling price and frequency of advertisement.

Majority of the studies on deteriorating inventory systems assumed that the deterioration rate of the items is not controllable. But nowadays different preservation technologies are available to control the deterioration of products. Cooling, freezing, and drying are some methods to control the deterioration of vegetables, fruits, meat, dairy products, medicines, etc. Vacuuming, Irradiation, High pressure, Biopreservation, etc. are modern preservation technologies. Total profit can be maximized by opting preservation technology for items having significant deterioration. Also, in today's competitive environment, preservation technology investment has become necessary for producers, suppliers, and retailers to reduce the deterioration rate and economic losses due to deterioration. Sensitivity analysis of some studies (Yang et al. (2006), Tsao and Sheen (2008), Geetha and Uthayakumar (2010)) shows that the lower deterioration rate is beneficial from an economic point of view. Hsu et al. (2010) for the first time, developed a deteriorating inventory model with constant demand and constant deterioration when the retailer invests on the preservation technology to reduce the deterioration rate of the product. Dye and Hsieh (2012) extended the model of Hsu et al. (2010) with time-varying deterioration and partial backlogging by assuming the preservation technology cost as a function of the replenishment cycle. Lee and Dye (2012) developed a deteriorating inventory model with stock dependent demand and partial backlogging by allowing preservation technology cost. Chen and Dye (2013) and Dye and Hsieh (2013) obtained optimal solutions for deteriorating inventory models with fluctuating demand and preservation technology cost allowing trade credit. Dem and Singh (2013) formulated an integrated production inventory model with preservation technology. They considered single-manufacturer-single buyer supply chain problem with fluctuating demand and the concept of preservation technology on the buyer's side.

Researchers often assume that the deterioration process starts as soon as the commodities enter into the inventory system but, it is not true for all commodities. Fruits, vegetables, foodstuffs have a short span of maintaining fresh quality, in which there is almost no spoilage. This phenomenon is known as non-instantaneous-deterioration. Dye (2013) studied the effect of preservation technology investment on a non-instantaneous deteriorating inventory model, also established several structural properties on finding the optimal replenishment and preservation technology strategies. Mishra (2014) developed a deteriorating inventory model for non-instantaneous deteriorating items with linear demand and holding cost considering preservation technology cost when deterioration period starts. Tsao (2016) developed a non-instantaneous deteriorating inventory model for joint location, inventory, and preservation decisions under permissible delay in payments. Bardhan et al. (2017) developed a non-instantaneous deteriorating inventory model with stock dependent demand and preservation technology. Pal et al. (2018) derived optimal inventory policies for non-instantaneous deteriorating products with preservation technology and constant demand, assuming random deterioration start time.

Summary of the thesis:

Our work is divided into six chapters, Chapter 1-6. Chapter 1 includes an introduction and a detailed literature review. Our contribution to the inventory models for items having Weibull distribution deterioration is presented in Chapter 2-6.

In chapter 2, we derived an inventory model for Weibull deteriorating items with exponential demand and time-varying holding cost. Sensitivity analysis shows how the total profit is affected by different parameters. The solution of the given example is obtained by using optimize function in R programming. This is a simplest model without shortages, preservation technology investment, trade credit, and price and advertisement dependent demand. Paper based on this chapter is published in the journal Global and Stochastic Analysis (ISNN: 2248-9444) in 2016.

In chapter 3, an inventory model is developed for Weibull deteriorating items when the demand is exponential. We assumed the supplier provides some credit period to settle the account, and beyond the credit period, he will charge a certain rate of interest on the outstanding amount. Linear holding cost is considered, and the deterioration cost is taken as the difference between purchasing price and salvage value. A numerical example is provided, and sensitivity analysis is made to check the effect of various parameters on the inventory system. We found that as permissible delay period increase, the corresponding cost decrease. That is, as suppliers allow more days to settle the account, the retailer can earn more profit through sales revenue. Also, the supplier can attract and motivate new customers to increase his sales. Paper based on this chapter is published in the journal IAPQR Transactions (ISNN: 0970-0102) in 2017.

In chapter 4, we extended the developed model of chapter 3 allowing shortages and partial backlogging with price and advertisement frequency dependent demand. We considered the price, frequency of advertisement, length of order cycle and shortage period as decision variables and found optimal values simultaneously to maximize the profit. So, this model provides joint pricing, advertisement and inventory ordering policies to the retailer to maximize the total profit. An algorithm is provided to get the optimal solution. The solution is obtained by using the package DEoptimR in R programming. This package provides a global optimum solution using the differential evolution stochastic algorithm.

In chapter 5, we formulated an inventory model for items, having Weibull distribution deterioration by allowing preservation technology investment. We assumed the demand is constant, and it is possible to reduce the deterioration rate of the item by investing in preservation technology. Further, shortages are allowed and partially backlogged. Both instantaneous and non-instantaneous cases are considered. This model provides optimal preservation technology investment and optimal ordering policies to the retailer to maximize the total profit. Paper based on this chapter is published in the journal International Journal of Statistics and Reliability Engineering. ISSN: 2350-0174 (Print), 2456-2378(Online). http://ijsreg.com/index.php/ijsre/article/view/499

In chapter 6, we developed a profit maximization model for non-instantaneous deteriorating items with trade credit by optimizing the frequency of advertisement, selling price, preservation technology investment, and inventory policies simultaneously. This is an extended model of the developed model of chapter 4 allowing preservation technology investment for non-instantaneous deteriorating items. This model reveals that the non-instantaneous deteriorating items require less preservation technology investment than the instantaneous deteriorating items. The profit for the non-instantaneous deteriorating items is more than the profit for the instantaneous deteriorating items. When the cost per advertisement is less, the retailer can earn more profit through increasing advertisement frequency. Paper based on this chapter is published in the journal OPSEARCH (ISNN: 0030-3887) in 2019. https://doi.org/10.1007/s12597-019-00427-7

1.3 Terminology

Inventory: The physical stock that is stored to fulfill the present and future demand.

Demand: The required quantity of an item for consumption that consumers purchase at a particular time for a period. During our study we considered different demand functions. In chapter 2 and 3 we considered an exponential demand, in chapter 4 we considered constant demand while in chapter 5 and 6 we considered advertisement and price dependent demand.

(a) Constant demand function:

$$D(t) = D$$

Where, D is a constant.

(b) Exponential demand function:

$$D(t) = ke^{\gamma t}$$

Where, k is a scale parameter γ is a shape parameter.

(c) Demand function depend on frequency of advertisement and price:

$$D(A, P) = A^m a P^{-b}$$

Where, m is the shape parameter ($0 \leq m < 1$), A (> 0) is the frequency of advertisement, a(> 0) is the scaling factor, P is the selling price, and. b(≥ 1) is the index of price elasticity. Since $\frac{\partial D(A,P)}{\partial A} > 0$ and $\frac{\partial D(A,P)}{\partial P} < 0$, the demand function is a decreasing function of price (P) and increasing function of the advertisement frequency(A), this reflects a real situation.

Deterioration: Some items may lose their original freshness, quality, quantity, value, potential, efficiency, utility, or usability with time. The process of losing originality is known as deterioration.

Instantaneous deterioration: If the deterioration process begins as soon as the item enters into the inventory system, then the deterioration process is termed instantaneous deterioration.

Non-instantaneous deterioration: Some items do not start to deteriorate as soon they enter into the inventory system and maintain the originality for some duration, which is known as non-instantaneous deterioration.

Deterioration cost: When any item or commodity in the inventory system deteriorate (due to damage, decay, loss of utility, spoilage, obsolescence), there are two possibilities it may be completely useless or partially usable. The deterioration cost includes the disposal (or elimination or destroy) cost or the difference between purchase cost and salvage value.

Shortages: Shortages arise when the inventory becomes empty. Due to shortages the present and the future demand cannot be fulfilled, and hence there will be a loss of opportunity and goodwill.

In chapter 1 and 2, shortages are not allowed, while in chapter 3-6, shortages are allowed and partially backlogged.

The fraction of unsatisfied demand backlogged is given by

$$De^{-\delta(T-t)}$$

where backlogging parameter δ is a positive constant and $(T-t)$ is the waiting time.

Backlogging: When shortages occur, some customers may wait until the required item arrives in the inventory. That is, the unfulfilled demand which occurs due to the shortages will be fulfilled as soon as the new inventory arrives. The process of fulfilling the shortages later is known as backlogging.

Salvage value: The salvage value of any item is the difference between the purchase price and the depreciation in the price due to deterioration.

Lead time: Lead time is the length of the time duration between placing an order and receiving the order.

Permissible delay in payment (Trade credit): This is one of the common practices in businesses. In this practice, a retailer need to not pay the whole amount instantly to the supplier for his required order quantity, because the supplier allows some time duration for the retailer to settle the account. If the retailer can't settle the total amount within the permissible delay period, then the supplier will charge interest at some rate on the remaining amount.

Preservation technology: We considered the proportion of the reduced deterioration rate as given by Hsu et al. (2010)

$$m(\xi) = 1 - e^{-k\times\xi}$$

Where, $k(\geq 0)$ is the simulation coefficient representing the percentage increase in $m(\xi)$ per dollar increase in ξ.

When $\xi = 0$, the reduced deterioration rate $m(\xi) = 0$, and $\lim_{\xi \to \infty} m(\xi) = 1$. We can set constraint on PT investment $0 \leq \xi \leq \xi'$, where, ξ' is the maximum PT investment allowed.

In chapter 5 and 6 we considered preservation technology investment.

Ordering cost: Ordering cost is the cost of placing an order and receiving the order. It includes cost associate with finding a supplier, clerical process cost, inspection cost, handling cost, etc. Ordering cost incurs once per order.

Holding cost: Obviously, to store any physical item the storage space is required. The storage space or warehouse size depend on the quantity to be stored. Holding cost is the cost includes the required warehouse rent, insurance cost to avoid any unexpected losses due to accidents, wages paid to the workers to handle and maintain the record.