CHAPTER - 3

AN INVENTORY MODEL FOR WEIBULL DETERIORATING ITEMS WITH EXPONENTIAL DEMAND UNDER PERMISSIBLE DELAY IN PAYMENTS

3.1 Introduction

In this chapter, we extended the model developed in chapter 1 by allowing a permissible delay in payments. Permissible delay in payments (or trade credit) is one of the best practices in the competitive business environment. In this practice, a retailer need to not pay the whole amount instantly to the supplier for his required order quantity, because the supplier allows some time duration for the retailer to settle the account. If the retailer fails to settle the account within the credit period, then the supplier will charge him some rate of interest for the period beyond the credit period.

3.2 Assumptions

- The demand D is a function of time, given by

  D (t) = $ke^{\gamma t}$; $|\gamma| \ll 1$

- The holding cost C_h is a linear function of time, given by

  $C_h(t) = x + yt$; where, x and y are constants.

- The deterioration rate $\theta(t)$ of an item in the inventory system follows the two parameter Weibull distribution deterioration rate, given by

  $\theta(t) = \alpha\beta t^{\beta-1}$; where $0 \leq \alpha \ll 1,\ \beta > 0$

- The supplier provides some credit period.
- Deteriorated items have no resale value.
- Lead time is negligible.
- Instant and infinite replenishment rate.
- The inventory system involves only one item.

3.3 Model Development

As shown in chapter-2 figure 2.3.1, at $t = 0$ the initial inventory in the system is I_0. Due to the demand and deterioration, the inventory level will continuously decrease with time and become zero at time $T = 0$. The rate of change in inventory is given by the differential equation (3.3.1).

$$\frac{dI(t)}{dt} = -\alpha\beta t^{\beta-1} I(t) - ke^{\gamma t}, \qquad 0 \leq t \leq T \tag{3.3.1}$$

Solving the equation (3.3.1) using the boundary condition $I(T) = 0$ we get

$$I(t) = k\left[(T-t) + \frac{\gamma(T^2 - t^2)}{2} + \frac{\alpha\left(T^{\beta+1} - t^{\beta+1}\right)}{\beta+1} + \frac{\alpha\gamma\left(T^{\beta+2} - t^{\beta+2}\right)}{\beta+2} + \alpha t^{\beta+1} - \alpha T t^{\beta}\right] \tag{3.3.2}$$

The initial order quantity at $t = 0$ is

$$I_0 = k\left[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}\right] \tag{3.3.3}$$

The total variable inventory cost includes ordering cost, deterioration cost, holding cost, and interest charged minus interest earned.

The ordering cost per unit time is

$$OC = \frac{C_O}{T} \tag{3.3.4}$$

The total demand during the cycle period $[0, T]$ is

$$\int_0^T D(t)\, dt = \int_0^T k\, e^{\gamma t}\, dt = \frac{k}{\gamma}[e^{\gamma T} - 1] \tag{3.3.5}$$

The number of deteriorated units during the period $[0, T]$ is

$$I_0 - \int_0^T D(t)\,dt$$

$$= k\left[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}\right] - \frac{k}{\gamma}[e^{\gamma T} - 1] \tag{3.3.6}$$

Cost due to deterioration per unit time is

$$DC = \frac{kC_d}{T}\left[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}\right] - \frac{kC_d}{\gamma T}[e^{\gamma T} - 1] \tag{3.3.7}$$

Inventory holding cost per unit time is

$$HC = \frac{1}{T}\int_0^T (x + yt)I(t)\,dt$$

$$= \frac{xk}{T}\left[\frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\gamma T^{\beta+3}}{\beta+3}\right]$$

$$+ \frac{yk}{T}\left[\frac{T^3}{6} + \frac{\gamma T^4}{8} + \frac{\alpha\beta T^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha\gamma T^{\beta+4}}{2(\beta+4)}\right] \tag{3.3.8}$$

3.3.1 Case – 1: $(\boldsymbol{M \le T})$

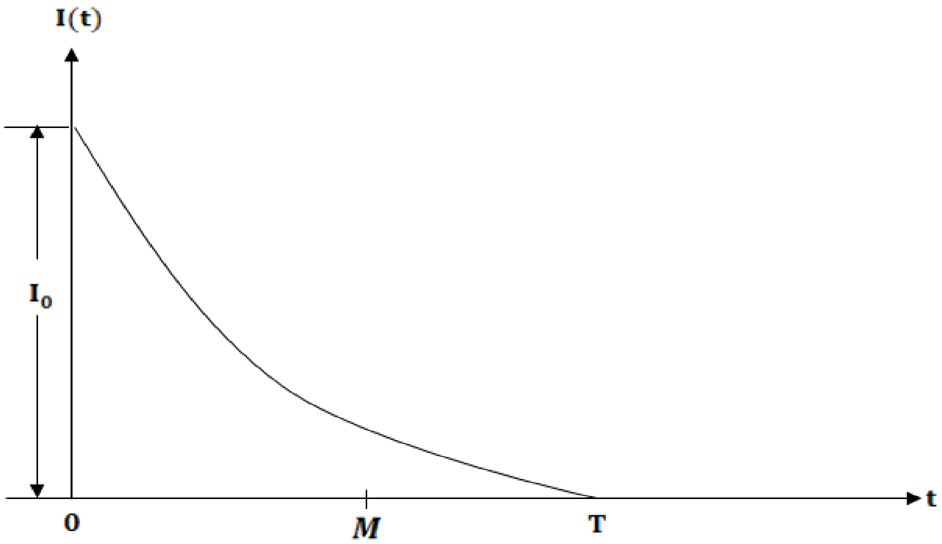
Interest charged per unit time for the inventory not being sold after the due time M in a cycle is.

$$IC_1 = \frac{Ci_c}{T}\int_M^T I\,(t)\,dt$$

$$= \frac{kCi_c}{T}\left[Tt - \frac{t^2}{2} + \frac{\gamma}{2}\left(T^2 t - \frac{t^3}{3}\right) + \frac{\alpha}{\beta+1}\left(T^{\beta+1}t - \frac{t^{\beta+2}}{\beta+2}\right) + \frac{\alpha\gamma}{\beta+2}\left(T^{\beta+2}t - \frac{t^{\beta+3}}{\beta+3}\right) + \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{\alpha T t^{\beta+1}}{\beta+1}\right]_M^T \tag{3.3.9}$$

$$= \frac{kCi_c}{T}\left[\left[\frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha T^{\beta+2}}{\beta+2} + \frac{\alpha\gamma T^{\beta+3}}{\beta+3} - \frac{\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right] - \left[M - \frac{M^2}{2} + \frac{\gamma}{2}\left(T^2 M - \frac{M^3}{3}\right) + \frac{\alpha}{\beta+1}\left(T^{\beta+1}M - \frac{M^{\beta+2}}{\beta+2}\right) + \frac{\alpha\gamma}{\beta+2}\left(T^{\beta+2}M - \frac{M^{\beta+3}}{\beta+3}\right) + \frac{\alpha M^{\beta+2}}{\beta+2} - \frac{\alpha T M^{\beta+1}}{\beta+1}\right]\right] \tag{3.3.10}$$

Figure 3.3.1 Graphical depiction of the inventory level when $\boldsymbol{M \leq T}$.



Interest earned per unit time by the sales revenue in $(0, M)$ in a cycle.

$$IE_1 = \frac{Pi_e}{T}\int_0^M ke^{\gamma t} t\, dt$$

$$= \frac{kPi_e}{T}\int_0^M (1+\gamma t)t\, dt$$

$$= \frac{kPi_e}{T}\left[\frac{M^2}{2} + \frac{\gamma M^3}{3}\right] \qquad (3.3.11)$$

Total cost per unit time is

$$TC_1(T) = OC + DC + HC + IC_1 - IE_1$$

$$= \frac{C_O}{T} + \frac{kC_d}{T}\left[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}\right] - \frac{kC_d}{\gamma T}\left[e^{\gamma T} - 1\right]$$

$$+ \frac{xk}{T}\left[\frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\gamma T^{\beta+3}}{\beta+3}\right]$$

$$+ \frac{yk}{T}\left[\frac{T^3}{6} + \frac{\gamma T^4}{8} + \frac{\alpha\beta T^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha\gamma T^{\beta+4}}{2(\beta+4)}\right]$$

$$+ \frac{kCi_c}{T}\left[\frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha T^{\beta+2}}{\beta+2} + \frac{\alpha\gamma T^{\beta+3}}{\beta+3} - \frac{\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$- \frac{kCi_c}{T}\left[\begin{array}{c} TM - \frac{M^2}{2} + \frac{\gamma}{2}\left(T^2M - \frac{M^3}{3}\right) + \frac{\alpha}{\beta+1}\left(T^{\beta+1}M - \frac{M^{\beta+2}}{\beta+2}\right) \\ + \frac{\alpha\gamma}{\beta+2}\left(T^{\beta+2}M - \frac{M^{\beta+3}}{\beta+3}\right) + \frac{\alpha M^{\beta+2}}{\beta+2} - \frac{\alpha T M^{\beta+1}}{\beta+1} \end{array}\right]$$

$$- \frac{kPi_e}{T}\left[\frac{M^2}{2} + \frac{\gamma M^3}{3}\right] \qquad (3.3.12)$$

$$\frac{\partial TC_1(T)}{\partial T} = -\frac{C_O}{T^2} + kC_d\left[\frac{\gamma}{2} + \frac{\alpha\beta T^{\beta-1}}{\beta+1} + \frac{\alpha\gamma(\beta+1)T^{\beta}}{\beta+2}\right]$$

$$-\frac{kC_d}{\gamma T^2}\left[\gamma T e^{\gamma T} - e^{\gamma T} + 1\right]$$

$$+xk\left[\frac{1}{2} + \frac{2\gamma T}{3} + \frac{\alpha\beta T^{\beta}}{\beta+2} + \frac{\alpha\gamma(\beta+2)T^{\beta+1}}{\beta+3}\right]$$

$$+yk\left[\frac{T}{3} + \frac{3\gamma T^2}{8} + \frac{\alpha\beta T^{\beta+1}}{2(\beta+3)} + \frac{\alpha\gamma(\beta+3)T^{\beta+2}}{2(\beta+4)}\right]$$

$$+kCi_c\left[\frac{1}{2} + \frac{2\gamma T}{3} + \frac{\alpha(\beta+1)T^{\beta}}{\beta+2} + \frac{\alpha\gamma(\beta+2)T^{\beta+1}}{\beta+3} - \frac{\alpha T^{\beta}}{(\beta+2)}\right]$$

$$-kCi_c\left[\begin{array}{l}\frac{M^2}{2T^2} + \frac{\gamma}{2}\left(M + \frac{M^3}{3T^2}\right) + \frac{\alpha}{\beta+1}\left(\beta T^{\beta-1}M + \frac{M^{\beta+2}}{(\beta+2)T^2}\right) \\ \quad + \frac{\alpha\gamma}{\beta+2}\left((\beta+1)T^{\beta}M + \frac{M^{\beta+3}}{(\beta+3)T^2}\right) - \frac{\alpha M^{\beta+2}}{(\beta+2)T^2}\end{array}\right]$$

$$+\frac{kPi_e}{T^2}\left[\frac{M^2}{2} + \frac{\gamma M^3}{3}\right] \qquad (3.3.13)$$

$$\frac{\partial^2 TC_1(T)}{\partial T^2} = \frac{2C_O}{T^3} + kC_d\left[\frac{\alpha\beta(\beta-1)T^{\beta-2}}{\beta+1} + \frac{\alpha\gamma\beta(\beta+1)T^{\beta-1}}{\beta+2}\right]$$

$$-\frac{kC_d}{\gamma}\left[\left(\frac{T\gamma^2 e^{\gamma T} - \gamma e^{\gamma T}}{T^2}\right) - \left(\frac{T\gamma e^{\gamma T} - 2Te^{\gamma T}}{T^4}\right) - \frac{2}{T^3}\right]$$

$$+xk\left[\frac{2\gamma}{3} + \frac{\alpha\beta^2 T^{\beta-1}}{\beta+2} + \frac{\alpha\gamma(\beta+1)(\beta+2)T^{\beta}}{\beta+3}\right]$$

$$+yk\left[\frac{1}{3} + \frac{3\gamma T}{4} + \frac{\alpha\beta(\beta+1)T^{\beta}}{2(\beta+3)} + \frac{\alpha\gamma(\beta+2)(\beta+3)T^{\beta+1}}{2(\beta+4)}\right]$$

$$+kCi_c\left[\frac{2\gamma}{3}+\frac{\alpha\beta(\beta+1)T^{\beta-1}}{\beta+2}+\frac{\alpha\gamma(\beta+1)(\beta+2)T^{\beta}}{\beta+3}-\frac{\alpha\beta T^{\beta-1}}{(\beta+2)}\right]$$

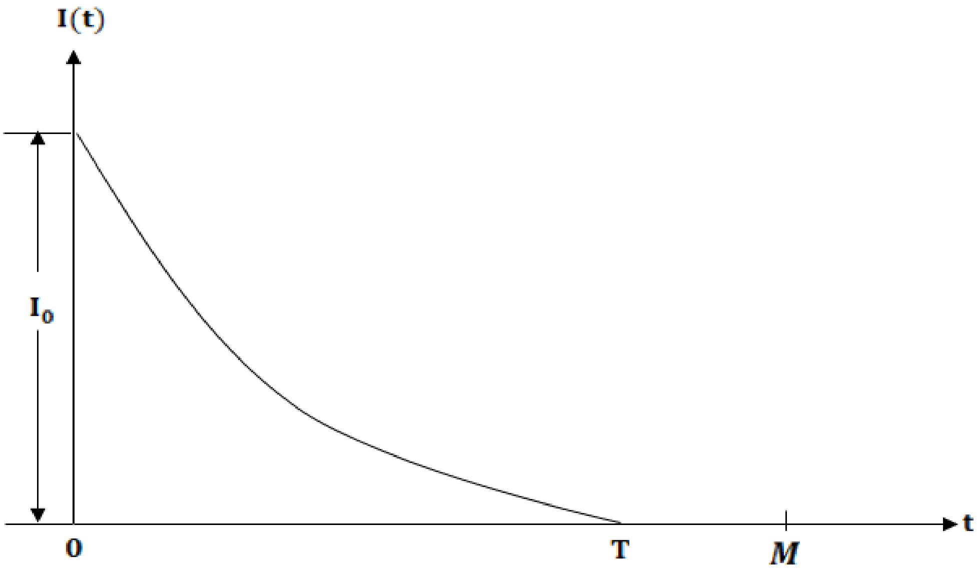
$$-kCi_c\left[\begin{array}{l}-\frac{M^2}{T^3}-\frac{\gamma M^3}{3T^3}+\frac{\alpha}{\beta+1}\left(\beta(\beta-1)MT^{\beta-2}-\frac{2M^{\beta+2}}{(\beta+2)T^3}\right)\\+\frac{\alpha\gamma}{\beta+2}\left(\beta(\beta+1)MT^{\beta-1}-\frac{2M^{\beta+3}}{(\beta+3)T^3}\right)+\frac{2\alpha M^{\beta+2}}{(\beta+2)T^3}\end{array}\right]$$

$$-\frac{2kPi_e}{T^3}\left[\frac{M^2}{2}+\frac{\gamma M^3}{3}\right] \tag{3.3.14}$$

Solving the equation $\frac{\partial TC_1(T)}{\partial T}=0,$ we get the optimal value of the order cycle length $T=T_1^*$ provided that $\frac{\partial^2 TC_1(T)}{\partial T^2}>0$ at $T=T_1^*$.

3.3.2 Case – 2: $(\boldsymbol{M}\geq\boldsymbol{T})$

Figure 3.3.2 Graphical depiction of the inventory level when $\boldsymbol{M}\geq\boldsymbol{T}$.



In this case, since the permissible delay period M is greater than the cycle length, the interest payable is zero. The interest earned per unit time is

$$IE_2 = \frac{Pi_e}{T}\left[\int_0^T ke^{\gamma t} t\, dt + (M - T)\int_0^T ke^{\gamma t}\, dt\right]$$

$$= \frac{kPi_e}{T}\left[MT + \frac{\gamma MT^2}{2} - \frac{T^2}{2} - \frac{\gamma T^3}{6}\right] \tag{3.3.15}$$

In this case, the total cost per unit time is given by

$$TC_2(T) = OC + DC + HC - IE_1$$

$$TC_2(T) = \frac{C_O}{T}$$

$$+\frac{kC_d}{T}\left[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}\right] - \frac{kC_d}{\gamma T}\left[e^{\gamma T} - 1\right]$$

$$+\frac{xk}{T}\left[\frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\gamma T^{\beta+3}}{\beta+3}\right]$$

$$+\frac{yk}{T}\left[\frac{T^3}{6} + \frac{\gamma T^4}{8} + \frac{\alpha\beta T^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha\gamma T^{\beta+4}}{2(\beta+4)}\right]$$

$$-\frac{kPi_e}{T}\left[MT + \frac{\gamma MT^2}{2} - \frac{T^2}{2} - \frac{\gamma T^3}{6}\right] \tag{3.3.16}$$

$$\frac{\partial TC_2(T)}{\partial T} = -\frac{C_O}{T^2} + kC_d\left[\frac{\gamma}{2} + \frac{\alpha\beta T^{\beta-1}}{\beta+1} + \frac{\alpha\gamma(\beta+1)T^{\beta}}{\beta+2}\right]$$

$$-\frac{kC_d}{\gamma T^2}\left[\gamma T e^{\gamma T} - e^{\gamma T} + 1\right]$$

$$+xk\left[\frac{1}{2} + \frac{2\gamma T}{3} + \frac{\alpha\beta T^{\beta}}{\beta+2} + \frac{\alpha\gamma(\beta+2)T^{\beta+1}}{\beta+3}\right]$$

$$+yk\left[\frac{T}{3}+\frac{3\gamma T^2}{8}+\frac{\alpha\beta T^{\beta+1}}{2(\beta+3)}+\frac{\alpha\gamma(\beta+3)T^{\beta+2}}{2(\beta+4)}\right]$$

$$-kPi_e\left[\frac{\gamma M}{2}-\frac{1}{2}-\frac{\gamma T}{3}\right] \tag{3.3.17}$$

$$\frac{\partial^2 TC_2(T)}{\partial T^2}=\frac{2C_O}{T^3}$$

$$+kC_d\left[\frac{\alpha\beta(\beta-1)T^{\beta-2}}{\beta+1}+\frac{\alpha\gamma\beta(\beta+1)T^{\beta-1}}{\beta+2}\right]$$

$$-\frac{kC_d}{\gamma}\left[\left(\frac{T\gamma^2e^{\gamma T}-\gamma e^{\gamma T}}{T^2}\right)-\left(\frac{T\gamma e^{\gamma T}-2Te^{\gamma T}}{T^4}\right)-\frac{2}{T^3}\right]$$

$$+xk\left[\frac{2\gamma}{3}+\frac{\alpha\beta^2T^{\beta-1}}{\beta+2}+\frac{\alpha\gamma(\beta+1)(\beta+2)T^{\beta}}{\beta+3}\right]$$

$$+yk\left[\frac{1}{3}+\frac{3\gamma T}{4}+\frac{\alpha\beta(\beta+1)T^{\beta}}{2(\beta+3)}\right.$$

$$\left.+\frac{\alpha\gamma(\beta+2)(\beta+3)T^{\beta+1}}{2(\beta+4)}\right]$$

$$+\frac{kPi_e\gamma}{3} \tag{3.3.18}$$

Solving the equation $\frac{\partial TC_2(T)}{\partial T}=0,$ we get the optimal value of the order cycle length $T=T_2^*$ provided that $\frac{\partial^2 TC_2(\mathrm{T})}{\partial T^2}>0$ at $T=T_2^*$.

The optimal order quantity is

$$Q^*=k\left[T^*+\frac{\gamma {T^*}^2}{2}+\frac{\alpha {T^*}^{\beta+1}}{\beta+1}+\frac{\alpha\gamma {T^*}^{\beta+2}}{\beta+2}\right] \tag{3.3.19}$$

Procedure for finding the optimal order policy:

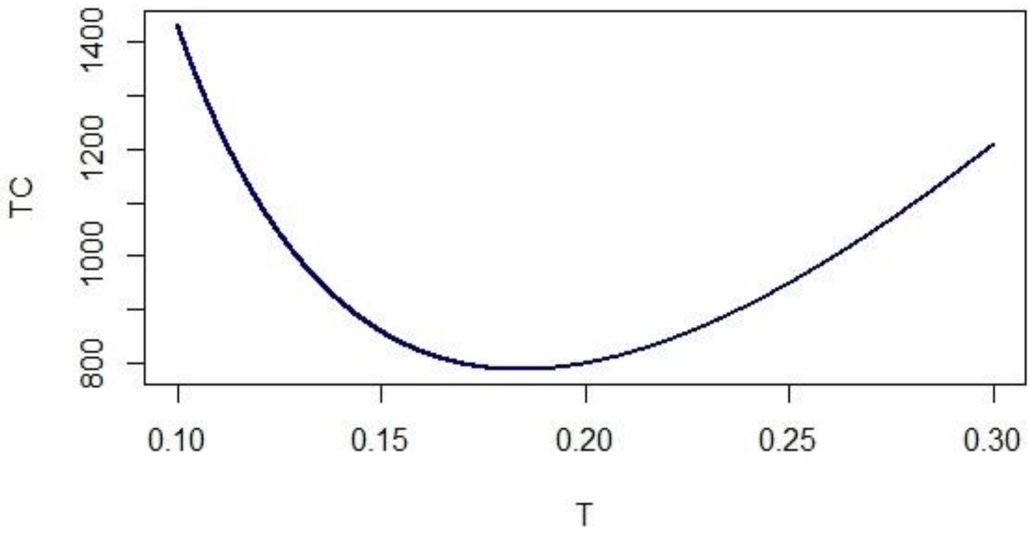
- If $M < T_1^*$ and $M \ngtr T_2^*$, then the optimal order cycle will be $T^* = T_1^*$. Obtain Q^* from equation (3.3.19) and the corresponding TC* from equation (3.3.12).
- If $M \nless T_1^*$ and $M > T_2^*$, then the optimal order cycle will be $T^* = T_2^*$. Obtain Q^* from equation (3.3.19) and the corresponding TC* from equation (3.3.16).
- If $M < T_1^*$ and $M > T_2^*$, then compare $TC_1(T_1^*)$ and $TC_2(T_2^*)$. The optimal order cycle will be $T^* = T_i^*$ ($i = 1$ or 2) for which $TC_i(T_i^*)$ is minimum.
- If $M \nless T_1^*$ and $M \ngtr T_2^*$, then the optimal order cycle will be $T^* = M$. Obtain Q^* from equation (3.3.19) and the corresponding TC* from equation (3.3.12) or (3.3.16). [In this case $TC_1(M) = TC_2(M)$]

3.4 Examples

Example 1: Taking $C_O = 250,\ C = 200,\ C_d = 180,\ P = 245,\ \alpha = 0.04,\ \beta = 2,$ $k = 500,\ \gamma = 0.02,\ x = 4,\ y = 0.05,\ i_c = 0.15,\ i_e = 0.09,\ M = 0.1644$ (60 days). Solving in R programming we get the solution as follows.

$T^* = T_1^* = 0.184063$ (67.18 days), $Q^* = 92.242583$ and $TC^* = 788.1295352$.

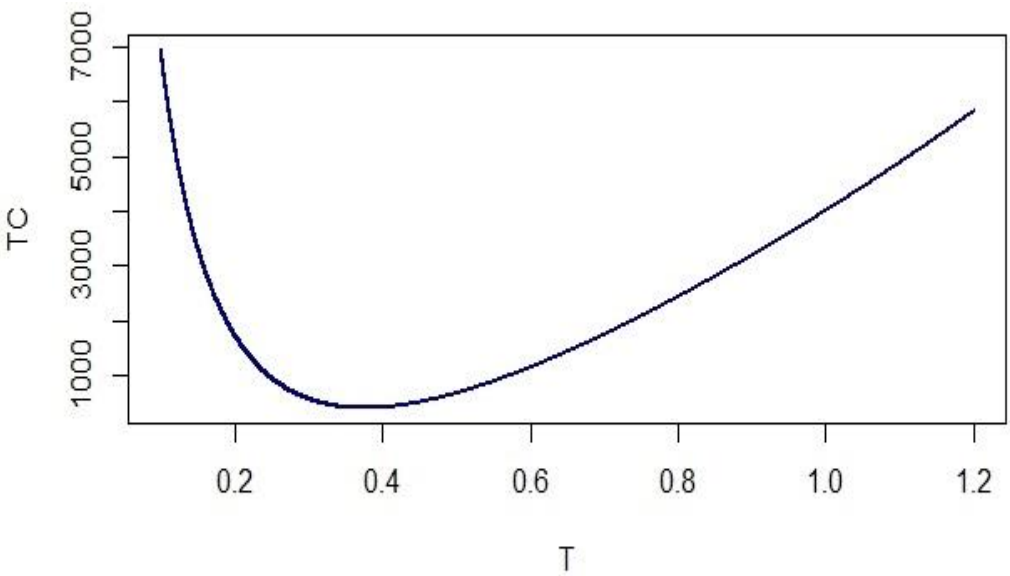
Figure 3.4.1 Convexity of the total cost function TC.



Example 2: Taking $C_O = 500,\ C = 200,\ C_d = 6,\ P = 245,\ \alpha = 0.06,\ \beta = 4,\ k = 500,\ \gamma = 0.06,\ x = 4,\ y = 0.5,\ i_c = 0.15,\ i_e = 0.05,\ M = 0.4$ (146 days).
Solving in R programming we get the solution as follows.

$$T^* = T_2^* = 0.349394 \text{ (92.48 days)},\ Q^* = 176.5599 \text{ and } TC^* = 392.72$$

Figure 3.4.2 Convexity of the total cost function TC.



Figures 3.4.1 and 3.4.2 reveals that the total cost function is convex.

3.5 Sensitivity Analysis

Table 3.5.1 Effect of % change in different parameters on T*, Q* and TC*.

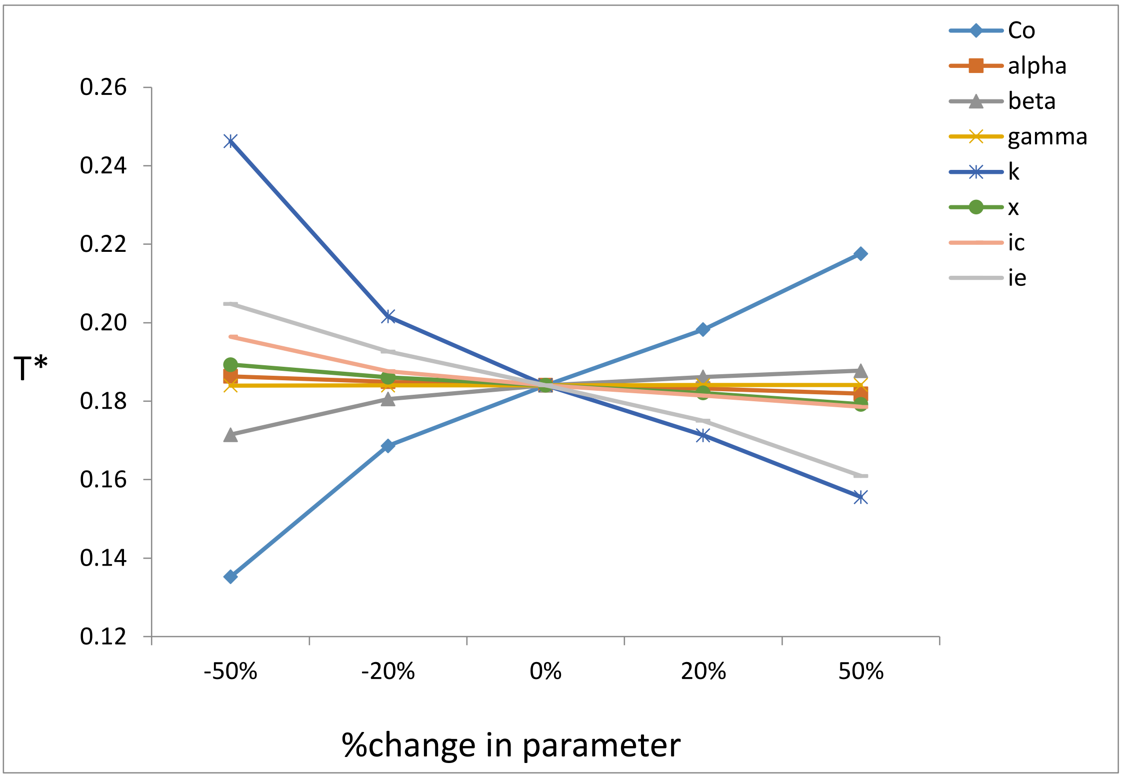
Parameter		%Change	T*	Q*	TC*	Remark
C_O	375	+50%	0.2176001	109.10571	1410.411	M < T*
	300	+20%	0.1982220	99.35954	1049.704	M < T*
	200	-20%	0.1686146	84.48149	504.5993	M < T*
	125	-50%	0.1352463	67.73113	12.9800	M > T*
α	0.06	+50%	0.1819031	91.17735	808.314	M < T*
	0.048	+20%	0.1831843	91.80924	796.261	M < T*
	0.032	-20%	0.1849645	92.68715	779.919	M < T*
	0.02	-50%	0.1863574	93.37398	767.4495	M < T*
β	3	+50%	0.1877492	94.05708	752.2759	M < T*
	2.4	+20%	0.1861662	93.27582	765.3838	M < T*
	1.6	-20%	0.1805404	90.52318	839.3633	M < T*
	1	-50%	0.1714679	86.17564	1070.936	M < T*
γ	0.03	+50%	0.1839816	92.37617	787.2979	M < T*
	0.024	+20%	0.1840295	92.29509	787.8092	M < T*
	0.016	-20%	0.1840994	92.19211	788.4335	M < T*
	0.01	-50%	0.1841569	92.11620	788.8585	M < T*
k	750	+50%	0.1555858	116.73017	449.2162	M > T*
	600	+20%	0.1712940	102.81421	664.3585	M < T*
	400	-20%	0.2015934	80.67969	889.786	M < T*

	250	-50%	0.2462420	61.60977	974.658	M < T*
x	6	+50%	0.1792281	89.81310	879.1967	M < T*
	4.8	+20%	0.1820839	91.24813	824.849	M < T*
	3.2	-20%	0.1861078	93.27027	751.0052	M < T*
	2	-50%	0.1893028	94.87603	694.53	M < T*
y	0.075	+50%	0.1840557	92.23907	788.2003	M < T*
	0.06	+20%	0.1840603	92.24108	788.1579	M < T*
	0.04	-20%	0.1840668	92.24459	788.1012	M < T*
	0.025	-50%	0.1840711	92.24660	788.0587	M < T*
i_c	0.225	+50%	0.1785766	89.48581	793.9161	M < T*
	0.18	+20%	0.1814277	90.91835	790.8878	M < T*
	0.12	-20%	0.1876641	94.05232	784.4239	M < T*
	0.075	-50%	0.1964639	98.47564	775.6501	M < T*
i_e	0.135	+50%	0.1609414	80.62807	354.8847	M > T*
	0.108	+20%	0.1750138	87.69588	621.8006	M < T*
	0.072	-20%	0.1926557	96.56124	946.6695	M < T*
	0.045	-50%	0.2048141	102.67425	1172.061	M < T*

In Figures 3.5.1 and 3.5.2, as the ordering cost (A) increases, T*, Q* and TC* increases. The total inventory cost (TC*) is very sensitive with respect to the ordering cost. 50% decrement in ordering cost results in almost nearer to zero total cost.

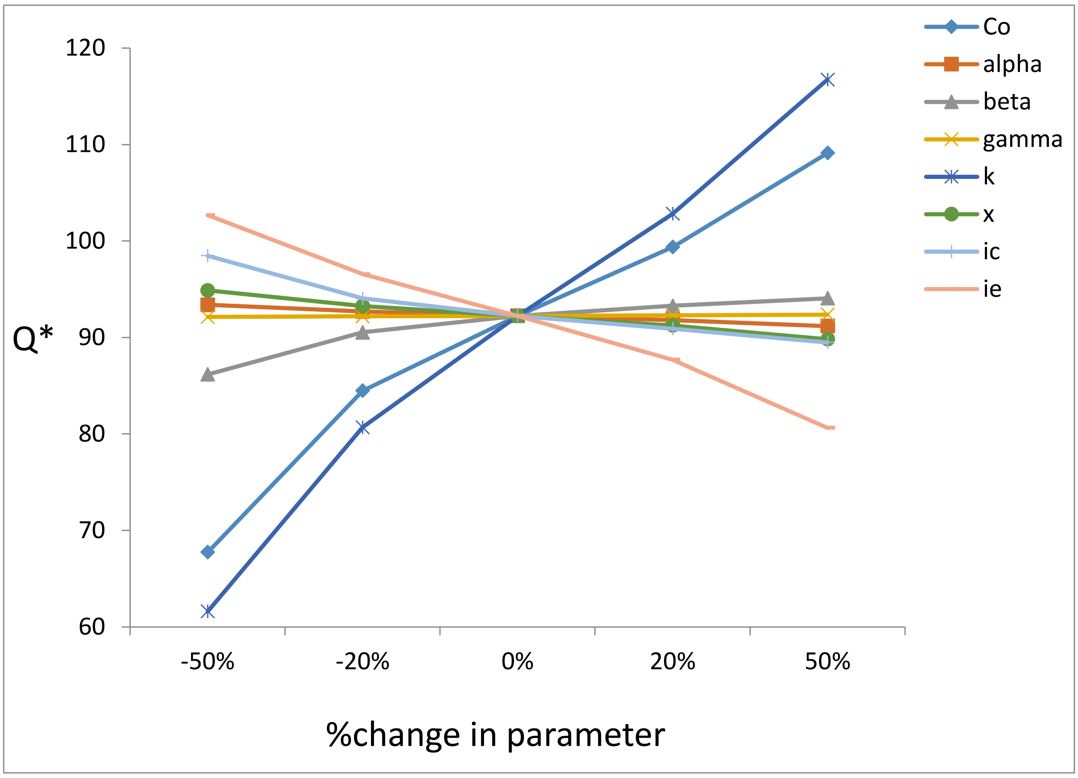
As the scale parameter α increases, T* and Q* decreases but TC* increases. Obviously as the value of α increases, deterioration will increase and hence the corresponding cost also increase.

Figure 3.5.1 Change in T*.



As shape parameter β increases, T* and Q* increases but TC* decreases. That is, even if the order cycle and order quantity increases the total cost decreases. This happens due to the characteristic of β and interest earned. For $\beta = 1$ the deterioration rate is constant (equal to α) and as β increases (i.e. for larger value of β) the deterioration rate in the beginning is low and increases with the time, but retailer earns interest from sales revenue right from the beginning.

Figure 3.5.2 Change in Q*.



As demand parameters k and γ increases, T* decreases, Q* increases and TC* decreases. Generally, as demand increases the order quantity increases and the corresponding cost also increases, but in this case TC* decreases due to the interest earned from sales revenue during the permissible delay period. In table 3.5.1 as rate of interest charged i_c increases, T* and Q* decreases but TC* increases.

The solution provided in this model is such that the total cost TC* does not change drastically even if the rate of interest charged is high. As the rate of interest earned increases, T*, Q* and TC* decreases. In figure 3.5.3 we can observe that TC* is very sensitive about interest earned.

Figure 3.5.3 Change in TC*.

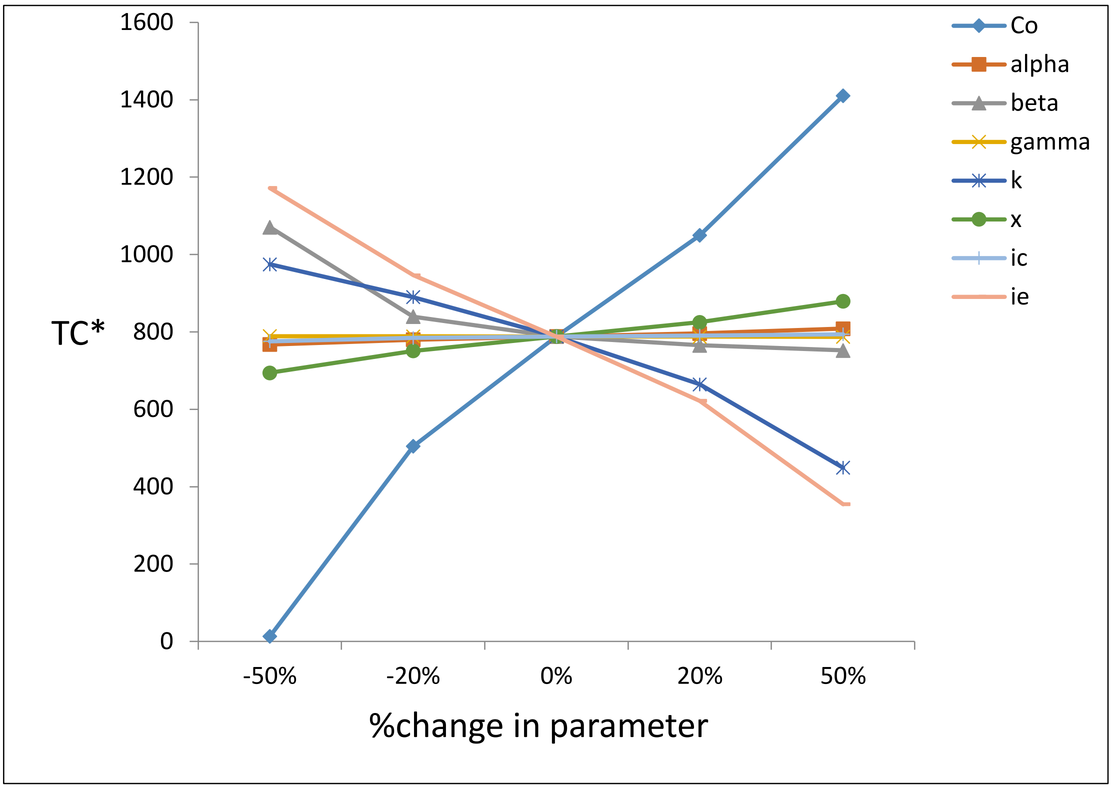


Table 3.5.2 Effect of Permissible Delay Period.

M	T*	Q*	TC*
M=0.041096 (15 days)	0.168308	84.327503	2356.005
M=0.082192 (30 days)	0.171614	85.988039	1797.533
M=0.123288 (45 days)	0.176928	88.657539	1276.053
M=0.164384 (60 days)	0.184061	92.241578	788.3133
M=0.205479 (75 days)	0.189441	94.945393	328.7198

In table 3.5.2 as the permissible delay period increases the total cost decreases. So, when the supplier allows a long credit period, the retailer can take advantage of it.

When the supplier does not allow any permissible delay period, for a retailer there will be no interest earned and no interest charged. In this case $M = 0, i_e = 0$ and $i_c = 0$. Putting $M = 0, i_e = 0$ and $i_c = 0$ in equations (3.3.12) and (3.3.16) the total cost functions TC_1 and TC_2 reduce to the total cost function given in chapter 2. That means the model developed in chapter 2 is a particular case of the model developed in this chapter.

3.6 Conclusion

In this chapter we investigated the effect of permissible delay in payments. It is found that when supplier provide permissible delay in payment the total cost decreases significantly. The practice of permissible delay is beneficial for both supplier and retailer. By allowing permissible delay in payments supplier can attract and motivate new customers to increase his sales, but at the same time supplier has to take care of the default risk associated with the length of permissible delay period.