Chapter IV

A FERTILITY DECISION MAKING MODEL FOR SEX PREFERENCES+

4.1 RATIONALE

The main purpose of the study is to look for a suitable model of fertility decision making with respect to sex preferences. Such a model would measure the effects of sex preferences on the birth rate and other current fertility indices, and would serve as a guide in the formulation of national population policies, in the light of the sex preferences that prevail in the country. A number of mathematical studies, as reviewed earlier (see Chapter II) have examined the effect of sex preferences on family size. However such studies are not particularly useful for the purpose of examining sex preference effects on current fertility. To this end, a fertility decision making model has been developed in the following section.

4.2 THE MODEL

The model to study the effects of allowing couples to satisfy their sex preferences on fertility, can be looked upon as a controlled experiment. Mainly two sets of current fertility rates would be obtained, one assuming usual reproductive behaviour (where reproduction is, by and large, at the observed

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+ Some of the results of this chapter have been published in a paper by the present author, entitled "A Model to Study Changes in Current Fertility Under Different Sex Preferences", Demography India, 16(2), 1987.

level i.e. unaffected by any specific planning - control set) and the other with specific rules for stopping after achieving certain specified family size composition (experimental set). All the input parameters except those of stopping rules will be identical for deriving those two sets of fertility rates. The difference in the fertility rates of the two sets (control and experimental) is a measure of the impact of stopping rules adopted by the couples with regard to the sex composition of their children, on fertility. To obtain fertility rate, the analysis would be carried out in two segments. The first segment derives estimates of birth probabilities for a given age and age at marriage of a woman through probability model. The second segment involves estimation of various fertility rates from the age at marriage and age specific birth probability matrices derived in the first segment. This is done through a simple projection technique (Venkatacharya, 1972). In the following section the derivation of birth probabilities are given and it is followed by the derivation of various current fertility rates from them.

4.2.1 Derivation of Birth Probabilities

To derive age specific and age at marriage specific birth probabilities the following assumptions are made :

1. A woman is not pregnant at the time of consummation of her marriage and continues to be in marital union until she attains 45 years of age.
2. There is a one to one correspondence between a conception and a live birth i.e. each conception leads to a live birth.
3. The length of infecundable exposure associated with a conception is constant for all the ages of the women and does not vary from woman to woman.
4. Couples can conceive or take decision regarding further fertility when the age of the previous child born is α years.
5. The probability that $(i+1)^{th}$ conception occurs during the time interval $(t, t+dt)$, given that the i^{th} conception occurred prior to time point $t-\theta$ is $\pi dt + O(dt)$ where $t \geq i\theta$, $\pi > 0$ and zero otherwise. Here $\theta = \max(G+M, G+\alpha)$, where G is the period of gestation and M is the period of postpartum amenorrhea. This assumption is equivalent to the assumption that waiting time for a conception after a woman becomes fecund following a live birth or after her marriage in case of first conception follows exponential distribution with mean time for conception $(1/\pi)$.
6. The probability of a woman giving birth to two children in the same year is zero.

7. Sex of each child is independent of the sexes of the other children born to a mother.

8. Infant and child mortality occurs in the first α years of life (say in the first one or two years) and later no death occurs among the children until the couples complete their reproduction.

9. The probability that a child born will be male is p. The probability of twins or multiple births is zero. The probability that a child will be a male is assumed to be constant, and same for all parents.

10. β_x is the probability that a woman is not sterile at age x (and before).

11. Birth control is complete. Couples stop reproduction as soon as the desired sex composition and/or size (in terms of b surviving sons and g surviving daughters and/or a total of s surviving children) are satisfied.

Some of the assumptions are no doubt strong but they may be considered as first approximations to the real process. Since the present study needs to develop a differencing model to study the changes in fertility under different sex preferences, fecundability (π) over age is assumed to be constant for simplification of the model. However, β_x is taken as a variable which to some extent takes care that risk of conception varies over age x. Assumption (8) is not far fetched; the mortality after the first few years is comparatively low.

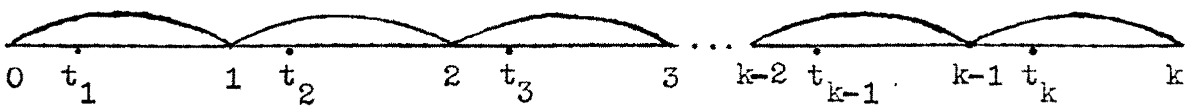
Moreover, it is kept open, α can be fixed depending on the mortality pattern of a country. The assumption (11) is meant for fixing parameters in the experimental group where it is assumed that a couple has a preference for a fixed minimum number of boys and a fixed minimum number of girls. The fixed - minima model also implies that sex preference can only increase, not decrease, fertility (McClelland, 1983). The accuracy of the results depends on the validity of the assumption in a population under study. Nevertheless, the present model is useful for estimating the maximum possible impact of sex preference on fertility.

Birth Probabilities for Control Group

As mentioned earlier, to measure the impact of sex preference on fertility it is necessary to derive two sets of birth probabilities, one assuming stopping rule and the other without stopping rule. To obtain age and age at marriage specific birth probabilities in the absence of stopping rules, that is for the control group, the following procedure is adopted :

For the convenience of computation, the probability of a woman giving birth to a child at a specific age is taken as the probability of a woman giving birth to a child in the corresponding year of marriage. (This is the case if the date of marriage and woman's birth date are almost close). It is further assumed that there is one to one correspondence between conception and live birth and the gestation period associated with it is constant and equal to 9 months or 3/4 year. The probability of occurrence of a birth in a given year depends on the timing of conception.

Let us consider time sequence of occurrences of births to a woman during the reproductive period since her marriage. Let '0' coincide with the date of marriage, the points 1, 2, ... k denote completion time points of the 1st, 2nd, ... k^{th} year of marriage and $t_k = k-1+\frac{1}{4} = k-\frac{3}{4}$.



(Each arc denotes the duration of one year)

It may be noted that the interval (t_{k-1}, t_k) denotes the conception time interval for a birth in the k^{th} year of marriage $(t_0 = 0)$.

Let W_k be the waiting time for the k^{th} conception and θ be the rest period $(\theta = \max(G+M, G+\alpha))$ associated with a conception. The total waiting times for the 1st, 2nd, ... k^{th} conception from the date of marriage are W_1, $W_1+W_2+\theta$, ... $W_1+W_2+\dots+W_k+(k-1)\theta$ respectively.

Let C_1, C_2, ... C_k denote respectively the events that a birth takes place in the 1st, 2nd, ... k^{th} year of marriage in the control group where a couple does not follow any stopping rule regarding preference for sex composition and/or size of the family. A birth takes place to a woman in k^{th} year under the condition that she is not sterile in the relevant period. For convenience of notation, the conditioning event is ignored but to show that event is conditional, its probability is indicated by an asterik. Having obtained conditional probabilities, unconditional probabilities can be obtained by multiplying them by the relevant probabilities of non sterility.

Then,

C_k = The event that a birth takes place in the k^{th} year of marriage

= (A birth of order one takes place in the k^{th} year of marriage) U

(A birth of order two takes place in the k^{th} year of marriage) U

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- -

(A birth of order k takes place in the k^{th} year of marriage)

= (A conception, leading to 1st order birth, takes place in the interval (t_{k-1}, t_k)) U

(A conception, leading to 2nd order birth, takes place in the interval (t_{k-1}, t_k)) U

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(A conception, leading to k^{th} order birth, takes place in the interval (t_{k-1}, t_k))

$$= \left[t_{k-1} < W_1 < t_k \right] \cup \left[t_{k-1} < W_1 + W_2 + \theta < t_k \right] \cup \ldots\ldots$$

$$\ldots\ldots\ldots \left[t_{k-1} < W_1 + W_2 + \ldots + W_{k-1} + (k-1)\theta + W_k < t_k \right]$$

$$= \bigcup_{m=1}^{k} \left[t_{k-1} < W_1 + W_2 + \ldots + W_{m-1} + (m-1)\theta + W_m < t_k \right]$$

Where,

$$W_0 \equiv 0 .$$

In other words, the event C_k can be written as

$$C_k = \bigcup_{m=1}^{k} C_{k,m}$$

where $C_{k,m} = \left[t_{k-1} < W_1 + W_2 + \ldots + W_{m-1} + (m-1)\theta + W_m < t_k \right]$

and $C_{k,1} \ldots C_{k,k}$ are mutually exclusive events in view of assumption (6).

Since $W_1, W_2, \ldots W_k$ are independent identically distributed exponential random variables with parameter π, $W_1 + W_2 + \ldots + W_k$ follows gamma distribution, given by its probability density

$$f(x;k,\pi) = \begin{cases} \dfrac{\pi^k}{\Gamma k} e^{-\pi x} x^{k-1} & x > 0 \\ 0 & \text{Otherwise} \end{cases}$$

where the parameters k and π are both positive.

Thus,

$$P^*(C_k) = \sum_{m=1}^{k} Pr.(C_{k,m})^{\ddagger}$$

$$= \sum_{m=1}^{k} Pr.\left[t_{k-1} < W_1 + W_2 + \cdots + W_{m-1} + (m-1)\theta + W_m < t_k \right]$$

$$= \sum_{m=1}^{k} Pr.\left[t_{k-1} - (m-1)\theta < z_m < t_k - (m-1)\theta \right]$$

where, $Z_m = W_1 + W_2 + \cdots + W_m$.

$$P^*(C_k) = Pr.\left[Z_m > t_{k-1} - (m-1)\theta \right] - Pr.\left[Z_m > t_k - (m-1)\theta \right]$$

$$= Pr.\left[Z_m > r \right] - Pr.\left[Z_m > r+1 \right],$$

where, $r = t_{k-1} - (m-1)\theta$.

For $r > 0$ the term $Pr.\left[Z_m > r \right]$ is evaluated as follows :

$$Pr.\left[Z_m > r \right] = \int_r^{\infty} f(x;m,\pi)dx$$

$$= \int_r^{\infty} \frac{\pi^m}{\Gamma m} x^{m-1} e^{-\pi x} dx$$

$$= \frac{\pi^m}{\Gamma m} \left[-\frac{1}{\pi} x^{m-1} e^{-\pi x} \right]_r^{\infty} -$$

$$\frac{\pi^m}{\Gamma m} \int_r^{\infty} \left[-\frac{1}{\pi} e^{-\pi x} \right] (m-1)x^{m-2} dx$$

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‡ To avoid confusion between P* and P in the equation, Pr. is used instead of P

$$= \frac{\pi^{m-1}}{(m-1)!} r^{m-1} e^{-\pi r} + \frac{\pi^{m-1}}{\Gamma m-1} \int_r^{\infty} x^{(m-1)-1} e^{-\pi x} dx$$

$$= \frac{e^{-\pi r} (\pi r)^{m-1}}{(m-1)!} + Pr.\left[Z_{m-1} > r \right]$$

$$= \sum_{n=0}^{m-1} \frac{e^{-\pi r} (\pi r)^n}{n!} ,$$ the cumulative probability upto (m-1) of the Poisson distribution with mean πr.

Similarly,

$$Pr.\left[Z_m > r+1 \right] = \sum_{n=0}^{m-1} \frac{e^{-\pi(r+1)}}{n!} \left\{ \pi(r+1) \right\}^n .$$

Therefore, the probability of a woman giving birth in the k^{th} year of marriage in the control group is given by

$$P^*(C_k) = \sum_{m=1}^{k} Pr.\left[r < Z_m < r+1 \right]$$

where,

$$Pr.\left[r<Z_m<r+1 \right] = \begin{cases} \sum_{n=0}^{m-1} \frac{e^{-\pi r} (\pi r)^n}{n!} - \sum_{n=0}^{m-1} \frac{e^{-\pi(r+1)} \left\{\pi(r+1)\right\}^n}{n!} & \text{if } r>0 \\ 1 - \sum_{n=0}^{m-1} \frac{e^{-\pi(r+1)}\left\{\pi(r+1)\right\}^n}{n!} & \text{if } r\leq 0<r+1 \\ 0 & \text{if } r+1\leq 0 . \end{cases}$$

In the above expression

$$r = t_{k-1} - (m-1)\theta , \ \theta = \max(G+\alpha , G+M) \text{ and } t_k = k - \frac{3}{4} .$$

It should be noted that k does not exceed 45-y, where y is the age at marriage of a woman. In fact, for $k > 45-y$, $P^*(C_k) = 0$. It may be recalled that probability of a woman giving birth to two children in the same year is assumed to be zero. Therefore, the maximum number of children by the k^{th} year of marriage is k. This assumption however can be relaxed easily. In that case, the maximum number of children by the k^{th} year of marriage is k/G.

The birth probabilities derived above are in fact conditional probabilities with the condition that a woman is not sterile. To get the unconditional birth probability of a woman in the k^{th} year of her marriage, namely $P(C_k)$ it is necessary to multiply $P^*(C_k)$ by the relevant probability for non-sterility of the woman. Thus, since β_x denotes the probability that a woman is not sterile at age x the unconditional birth probability, $f^c_{x,y}$ for a woman of age x who married at age y is given by

$$P(C_k) = f^c_{x,y} = P^*(C_{x-y+1})\beta_x$$

where $x = 15, 16, \ldots\ldots.\ 44$

$y = 15, 16, \ldots\ldots.\ 35$ (assuming that all women get married by 35 years of age)

and $x \geq y$.

The maximum age at marriage is assumed to be 35 years, considering the marriage pattern in India. The assumption can be relaxed and y can be allowed to go beyond 35 (years).

Birth Probabilities for Experimental Group

To obtain the corresponding age and age at marriage specific birth probabilities in the experimental group, the following procedure is adopted.

Let E_k (S(b,g,s)) be the event that a birth takes place to a woman in the k^{th} year of marriage in the experimental group where a couple is assumed to adopt scheme S(b,g,s) of preference for sex and/or size. As per scheme S(b,g,s), a couple will stop reproduction as soon as they have b surviving sons and g surviving daughters or a total of s surviving children ($s \geq b+g \geq 1$, $b,g \geq 0$). If a couple will stop reproduction as soon as they have b surviving sons and g surviving daughters (no limit on total family size 's') the scheme is denoted as S(b,g,.), and if a couple will stop reproduction as soon as they have a total of s living children (no sex preference), the scheme is denoted as S(.,.,s).

In order to derive the probability of giving birth in the k^{th} year, $P(E_k(S(b,g,s))) = P(E_k)$, it is necessary to understand first the chances of not achieving desired sex composition and/or size in the $(k-1)^{th}$ year or before.

Probability that at parity m there are u boys and v girls who survived at least first α years of their life, and the rest (m-u-v) are non-surviving children, is given by

$$P_m(u,v,p_\alpha,q_\alpha) = \frac{m}{u!v!(m-u-v)!} \; p_\alpha^u \, q_\alpha^v \, (1-p_\alpha - q_\alpha)^{m-u-v}$$

where p_α is the probability that child born is a boy and he survives the first α years and q_α is the corresponding probability for a female child. They are given by

$$p_\alpha = (1-\delta_\alpha)p \quad \text{and} \quad q_\alpha = (1-\epsilon_\alpha)q,$$

where p is the probability that child born is a boy,

q is the probability that child born is a girl,

δ_α is the probability that the child does not survive first α years, given that it is a male child

and ϵ_α is the probability that the child does not survive first α years, given that it is a female child.

Let D_m be the event that desired level in terms of b surviving sons and g surviving daughters or in terms of s total living children is not achieved by the couple at the m^{th} parity or before. In other words, for $m \geq \min(s, b+g)$

D_m = [among the m children born the number of boys surviving is less than b, the number of girls surviving is greater than or equal to g and the total number of children surviving is less than s] U

[Among the m children born the number of girls surviving is less than g, the number of boys surviving is greater than or equal to b and the total number of children surviving is less than s] U

[Among the m children born the number of boys surviving is less than b, the number of girls surviving is less than g and the total number of children surviving is less than s].

Mathematically, the event D_m can be written in terms of X_m, the number of surviving sons and Y_m, the number of surviving daughters at the m^{th} parity, as

$$\begin{aligned} D_m &= \left[X_m < b,\ Y_m \geq g,\ X_m + Y_m < s \right] \cup \\ &\quad \left[X_m \geq b,\ Y_m < g,\ X_m + Y_m < s \right] \cup \\ &\quad \left[X_m < b,\ Y_m < g,\ X_m + Y_m < s \right] \\ &= \left[X_m < b,\ g \leq Y_m < s - X_m \right] \cup \\ &\quad \left[Y_m < g,\ b \leq X_m < s - Y_m \right] \cup \\ &\quad \left[X_m < b,\ Y_m < g,\ X_m + Y_m < s \right]. \end{aligned}$$

It may be noted that usually $s \geq b+g$ and in this case

$$\left[X_m < b,\ Y_m < g,\ X_m + Y_m < s \right] = \left[X_m < b,\ Y_m < g \right].$$

Therefore, the probability, $P(D_m)$, that desired level in terms of b and g or s is not achieved at the m^{th} parity or before, denoted by $Q_m(b,g,s)$, is given by

$$Q_m(b,g,s) = P(D_m) = \begin{cases} 1 & \text{if } m < \min(s,b+g) \\ F_1+F_2+F_3 & \text{if } m \geq \min(s,b+g) \end{cases}$$

where,

$$F_1 = \begin{cases} \sum_{u=0}^{b-1} \sum_{v=g}^{s-u-1} P_m(u,v,p_\alpha, q_\alpha) & \text{if } b \geq 1 \\ 0 & \text{if } b = 0 \end{cases}$$

$$F_2 = \begin{cases} \sum_{v=0}^{g-1} \sum_{u=b}^{s-v-1} P_m(u,v,p_\alpha,q_\alpha) & \text{if } g \geq 1 \\ 0 & \text{if } g = 0 \end{cases}$$

$$F_3 = \begin{cases} \sum_{u=0}^{b-1} \sum_{v=0}^{\min(g-1,s-u-1)} P_m(u,v,p_\alpha,q_\alpha) & \text{if } b\geq 1 \text{ and } g\geq 1 \\ 0 & \text{if } b=0 \text{ or } g=0 \end{cases}$$

In the experimental group, if a couple is assumed to continue reproduction until it achieves the desired level in terms of b surviving sons and g surviving daughters (without any limit on the total living children) the event D_m obviously needs to be modified to derive the formulae for $Q_m(b,g,.)$. Similarly, if a couple is assumed to stop reproduction as soon as it achieves the desired level in terms of 's' total living children (irrespective of the sex of the living children) the event D_m needs to be modified to obtain the formulae for $Q_m(.,.,s)$.

Let H_m be the event that the desired level of b surviving sons and g surviving daughters is not achieved by the couple at the m^{th} parity or before.

In other words, for $m \geq b+g$

$$H_m = \left[X_m<b,\ Y_m \geq g\right] \cup \left[X_m \geq b,\ Y_m<g\right] \cup \left[X_m<b,\ Y_m<g\right].$$

Therefore, the probability, $P(H_m)$, denoted by $Q_m(b,g,.)$ is given by

$$Q_m(b,g,.) = P(H_m) = \begin{cases} 1 & \text{if } m < b+g \\ F_1'+F_2'+F_3' & \text{if } m \geq b+g \end{cases}$$

where,

$$F_1' = \begin{cases} \sum_{u=0}^{b-1} \sum_{v=g}^{m-u} P_m(u,v,p_\alpha,q_\alpha) & \text{if } b \geq 1 \\ 0 & \text{if } b = 0 \end{cases}$$

$$F_2' = \begin{cases} \sum_{v=0}^{g-1} \sum_{u=b}^{m-v} P_m(u,v,p_\alpha,q_\alpha) & \text{if } g \geq 1 \\ 0 & \text{if } g = 0 \end{cases}$$

$$F_3' = \begin{cases} \sum_{u=0}^{b-1} \sum_{v=0}^{g-1} P_m(u,v,p_\alpha,q_\alpha) & \text{if } b \geq 1 \text{ and } g \geq 1 \\ 0 & \text{if } b=0 \text{ or } g=0 . \end{cases}$$

Let T_m be the event that the desired level of 's' living children is not achieved by the couple at the m^{th} parity or before.

In other words, for $m \geq s$

$$T_m = \left[X_m + Y_m < s \right] .$$

Therefore, the probability, $P(T_m)$, denoted by $Q_m(.,.,s)$ is given by

$$Q_m(.,.,s) = P(T_m) = \begin{cases} 1 & \text{if } m < s \\ \sum_{u+v=0}^{s-1} P_m(u,v,p_\alpha,q_\alpha) & \text{if } m \geq s \end{cases}$$

$$= \begin{cases} 1 & \text{if } m < s \\ \sum_{u=0}^{s-1} \sum_{v=0}^{s-u-1} P_m(u,v,p_\alpha,q_\alpha) & \text{if } m \geq s \,. \end{cases}$$

(Note: s=0 is meaningless)

Now the derivation of $P^*(E_k)$(and $P(E_k)$), the conditional (and unconditional) probability that a birth takes place in the k^{th} year of marriage of a woman in the experimental group, is similar to that of $P^*(C_k)$ (and $P(C_k)$) in the control group.

In order to find $P^*(E_k(S(b,g,s)))$ it is noted that

$E_1 = C_1 = \left[t_0 < W_1 < t_1 \right]$, where $t_0 = 0$

and for $k \geq 2$

$$E_k = \left[t_{k-1} < W_1 < t_k \right] \cup \left[\bigcup_{m=1}^{k-1} \left\{ t_{k-1} < W_1 + W_2 + \ldots + W_{m+1} + m\theta < t_k \right\} \cap R_m \right].$$

The event R_m is D_m, H_m or T_m depending on the scheme of preferences $S(b,g,s)$, $S(b,g,.)$ or $S(.,.,s)$ adopted by the couple. These events and probabilities of their occurrence have already been discussed in the preceding section. It is now possible to obtain $P^*(E_1)$ and $P^*(E_k)$ on the same lines as $P^*(C_1)$ and $P^*(C_k)$ are derived. The final expressions for $P^*(E_1)$ and $P^*(E_k)$ are shown in the following.

Under the condition that all fecund women are exposed to the risk of conception and the couples adhere to stopping rule $S(b,g,s)$, the probability of a woman giving birth in the 1st and the k^{th} year of marriage are given by

$$P^*(E_1) = \int_0^{t_1} \pi e^{-\pi x}\, dx = 1-e^{-\pi t_1}$$

and for $k \geq 2$

$$P^*(E_k) = Pr.\left[t_{k-1} < W_1 < t_k \right] + \sum_{m=1}^{k-1} Pr.\left[t_{k-1} < W_1 + W_2 + \ \cdots\ + W_{m+1} + m\theta < t_k \right] P(R_m)$$

$$= \left\{ e^{-\pi t_{(k-1)}} \right\}(1-e^{-\pi}) + \sum_{m=1}^{k-1} Pr.\left[r' < Z_{m+1} < r'+1 \right] Q_m .$$

In the above expression

$r' = t_{k-1} - m\theta$ and $Z_{m+1} = W_1 + W_2 + \ \cdots\ + W_{m+1}$,

where W_1, W_2, ... W_{m+1} are independent identically distributed exponential random variables.

It is further noted that Q_m is $Q_m(b,g,s)$ or $Q_m(b,g,.)$ or $Q_m(.,.,s)$ depending on the scheme of preference adopted by the couple. They are known from the presentation made in the preceding section. The formulae for Pr.$[r' < Z_{m+1} < r'+1]$ in the above equation can be derived on the same lines as Pr.$[r < Z_m < r+1]$ is derived for the control group. Therefore,

$$\Pr.\left[r' < Z_{m+1} < r'+1\right] = \begin{cases} \sum_{n=0}^{m} \dfrac{e^{-\pi r'}(\pi r')^n}{n!} - \sum_{n=0}^{m} \dfrac{e^{-\pi(r'+1)}\left[\pi(r'+1)\right]^n}{n!} & \text{if } r'>0 \\ 1 - \sum_{n=0}^{m} \dfrac{e^{-\pi(r'+1)}}{n!}\left[\pi(r'+1)\right]^n & \text{if } r' \leq 0 < r'+1 \\ 0 & \text{if } r'+1 \leq 0 \end{cases}$$

Where, $r' = t_{k-1} - m\theta$ and $\theta = \max(G+M, G+\alpha)$.

Substituting $t_k = k - 3/4$ in the above expression for $P^*(E_k)$, the probability of a woman giving birth in the k^{th} year of marriage in the experimental group can be obtained.

Further, $f^e_{x,y}$ the unconditional birth probability that a birth takes place to a woman of age x who married at age y in the experimental group is given by

$$f^e_{x,y} = P^*(E_{x-y+1})\beta_x$$

where, $x = 15, 16, \ldots, 44$

$y = 15, 16, \ldots, 35$

and $x \geq y$.

4.2.2 Derivation of Current Fertility Rates

Having obtained the estimates of age at marriage and age specific birth probabilities under control and various sex preference assumptions, the corresponding fertility rates during 1981 and their trends in the next fifteen years are derived. The current fertility indices mainly considered are Age Specific Marital Fertility Rate (ASMFR), Total Marital Fertility Rate (TMFR), General Marital Fertility Rate (GMFR) and Crude Birth Rate (CBR). For this purpose, it is necessary to derive the currently married women by their current age at each future year. This is done by projecting the single year currently married women in 1981 into future years, by making use of appropriate joint survival ratio and taking into account new entrants through marriage at each year. The details of obtaining currently married women by current age in a given year are described below.

To project the currently married women (aged 15-44) the following assumptions are made :

1. Marriage is assumed to be universal and no remarriage is considered. The age specific marriage probabilities (m_y) are assumed to correspond to the pattern shown in Table 5.2 (Chapter V).
2. The level of mortality for males and females is assumed to correspond to the e_0^o shown in Table 5.3 (Chapter V).

3. To obtain the joint survival ratio of a woman in married state, it is assumed that age differential between husband and wife is 5 years. Making use of e_o^o for the concerned period (see assumption 2), the single year survival ratios for males and females (S_x^m and S_x^f) are first obtained from the results of a paper by Sinha (1972) which provides complete life tables based on Coale and Demeny's Model (West) Life Tables. Appropriate joint survival ratio (S_x^{mf}) is obtained as $S_x^{mf} = S_x^f \, S_{x+5}^m$ (x = 15, 16, ..., 44).
4. The ratio of newly married women in a year to the currently married women of the age group 15-44 in the preceding year, is assumed to be constant throughout the projection period. This ratio is estimated to be around 5.5 percent from the census data of 1971 and 1981 (see Chapter V for details).
5. The proportion m_y of women marrying at a particular age y in a year to all women that got married during the same year, is assumed to be constant throughout the projection period. Under this assumption, in the year J

$$m_y^J = P \text{ (a newly wed woman in the year J is of age y)}$$
$$= m_y .$$

The 1981 census has revealed that the population of India is 685.185 million as on 1st March 1981. Further, the total number of currently married females in the age group 15-44 is reported to be about 115.776 million. The quinquennial age distribution of the currently married females based on five percent sample data (Govt. of India, 1983) is used to derive single year age distribution (see Table 5.1, Chapter V). To project the 1981 single year currently married females (in the age group 15-44) at each future year, the following procedure is adopted.

Let W_x^J = the number of currently married women aged x in the J^{th} year (J = 1981, 1982, ... 1996)

W^J = the total number of currently married women in the age group 15-44 in the J^{th} year

$$(W^J = \sum_{x=15}^{44} W_x^J).$$

and E_x^J = the number of new entrants through marriage into the currently married group at age x in the J^{th} year.

In view of assumptions 4 and 5 mentioned earlier, we have

$$E_x^J = (0.055)\ W^{J-1}\ m_x$$

Therefore, the number of currently married women age x in the J^{th} year is given by

$$W_x^J = W_{x-1}^{J-1}\ S_{x-1}^{mf} + E_x^J\ .$$

This procedure is repeated from 1981 onward to obtain age specific currently married women for each of the years during 1981-96.

Computation of Various Fertility Indices

Given the constancy of birth probabilities $f^e_{x,y}$ or $f^c_{x,y}$ over the time as well as of the corresponding probabilities (m_y) of marriage (under assumption 5), the single year age specific marital fertility rate (F_x) is not supposed to show variation over the years and is therefore independent of J. So is the total marital fertility rate (T). However, the ASMFRs by five year age groups, the GMFR (G^J) or crude birth rate (derived later in the section) may be expected to vary, depending on the extent of changes in the population age-composition over time. The single year ASMFRs (F_x) as well as TMFR (T), derived from F_x, are independent of J and can be obtained by

$$F_x = P(\text{A married woman of age x gives birth to a child})$$

$$= \sum_{y=15}^{35} P(\text{A woman of age x gives birth to a child and the woman was married at age y})$$

$$= \sum_{y=15}^{\min(x,35)} f_{x,y}\, m_y \qquad (x = 15, 16, \ldots, 44)$$

and $T = \sum_{x=15}^{44} F_x$.

Where m_y = Probability of a single woman marrying at age y

$f_{x,y}$ = Probability of occurrence of a birth to a woman given that she is aged x and is married at age y.

Note that $f_{x,y}$ is either $f^c_{x,y}$ or $f^e_{x,y}$ depending on the group (control or experimental) for which the fertility rates are to be obtained.

It is now possible to derive various other measures of current fertility. The absolute number of births to the currently married women aged x in the J^{th} year can be obtained as

$$b^J_x = W^J_x \, F_x \qquad (J = 1981, 1982, \ldots\ldots 1996)$$

and the absolute number of births to the women in the age group 15-44 in the J^{th} year can be obtained as

$$b^J = \sum_{x=15}^{44} b^J_x = \sum_{x=15}^{44} W^J_x \, F_x$$

Where,

W^J_x = Number of currently married women aged x in the year J.

Relating these births to currently married women, general marital fertility rate (G^J) as well as ASMFRs in the conventional five year age groups (${}_5F^J_x$) for the control and experimental set in a given year J can be obtained.

$$G^J = \frac{b^J}{W^J} = \sum_{x=15}^{44} b_x^J \Big/ \sum_{x=15}^{44} W_x^J , \qquad (J=1981, 1982 .... 1996)$$

$$_5F_x^J = {}_5b_x^J \,/\, {}_5W_x^J , \qquad (x = 15,20,25,30,35,40)$$

Where,

$$_5b_x^J = \sum_{i=0}^{4} b_{x+i}^J$$

and $$_5W_x^J = \sum_{i=0}^{4} W_{x+i}^J .$$

Remarks: The above results can also be used to estimate Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR). GRR is a slight modification of the total fertility rate. The only distinction is that the numerator of the GRR is based on female births instead of total births. The NRR also uses the age specific female birth rates; however, it is based on the survivors of a cohort rather than on a cohort without mortality (The cohort is taken directly from a Life Table).

It is also possible to estimate the birth rate of the population under the different sex preference and stopping rule assumptions for the J^{th} year. To do this the mid-year population for the J^{th} year is required as the absolute number of births in the J^{th} year is already known. In other words, the mid-year population for each of the year 1981, 1982, ... 1996 is required. For this the derivation of total population at each future year during 1981-1997 is necessary. This is done by simply projecting the 1981 census population by age and sex into future years, by making use of appropriate survival ratios which are again selected with the help of the assumed level of male and female e_o^o of the respective period (see assumption 2). Further, in future years, the births which take place during a year under a particular fertility assumption, are to be added to survivors of the previous year's population. For this, the use is made of the absolute number of births in each calender year obtained through the present birth probability matrices, and they are distributed by sex assuming the sex ratio at birth to be 105 males per 100 females. The enumerated population of 1981 census is taken to be the population at the beginning of 1981. Therefore, after knowing the population at the beginning of each year during 1981-1997, the mid-year population during each of the years 1981-96 can be obtained and hence the birth rate. Denoting by P^T and $\bar{P}^T$ the population at the beginning and the middle of the calender year T, we have

$$\bar{P}^T = (P^T + P^{T+1})/2 \qquad (T = 1981, 1982 \ldots 1996).$$

Then the birth rate can be obtained as

$$B^T = b^T / \bar{P}^T$$

Where b^T is the number of births in the calender year T.

4.3 EXTENSION OF THE MODEL

4.3.1 Independent Effect of Sex Preference

The decision making model presented in the preceding section (4.2) measures the combined effect of sex preference and child mortality on the level of current fertility, rather than just the effect of sex preference. This is because model takes into account mortality among children. This implies that couples continue reproduction until the minimum desired number of surviving children by sex is achieved. The consideration of mortality is vital in the study of sex preference, since desired family size composition conceived by couples is with reference to living children and not live birth. Therefore, assumptions regarding infant and child mortality make the model more realistic and such a model would be more useful for policy makers to understand the implications of allowing couples to attain a specified number of living children of each sex on the level of current fertility, although the model, as such, may not reveal the independent effect of sex preference.

The basic approach to obtain just the effect of sex preference, is the same as that used in the section 4.2. In this regard, the derivation of age and age at marriage specific birth probabilities in the control and experimental group is similar to that of $P(C_k)$ and $P(E_k)$ respectively in section 4.2.1. The only difference is that while framing the stopping rules regarding sex preference in the experimental group, the section 4.2.1 considered mortality among the children born. This is not considered here. However, the corresponding formulae of $P(C_k)$ and $P(E_k)$ for the present one can also be derived from the results presented in section 4.2.1. The later procedure is followed here.

It may be recalled that while computing the probability $Q_m(b,g,\alpha)$ of not achieving a desired family size composition at the attained parity m or before in section 4.2.1, mortality among children born was considered. If mortality among children is ignored, the value of the parameter α will reduce to zero, that is, $\alpha = 0$, $\delta_\alpha = 0$ and $\epsilon_\alpha = 0$.

Then in the expression

$$P_m(u,v,p_\alpha,q_\alpha) = \frac{m!}{u!v!(m-u-v)} \quad p_\alpha^u \, q_\alpha^v \, (1-p_\alpha-q_\alpha)^{m-u-v} ,$$

$p_\alpha = p$, $q_\alpha = q$ and $m = u+v$ so that

$$P_m \,(u,\, m-u,\, p,\, q) = \frac{m}{u!(m-u)!} \quad p^u \, q^{m-u} ,$$

$$u=0,\ 1,\ 2,\ \ldots,\ m .$$

Then for $m \geq b+g$, the event

$$H_m = \left[X_m<b,\ Y_m\geq g\right] \cup \left[X_m\geq b,\ Y_m<g\right] \cup \left[X_m<b,\ Y_m<g\right]$$

is infact $\left[X_m<b,\ Y_m\geq g\right] \cup \left[X_m\geq b,\ Y_m<g\right]$.

The event H_m can also be written as

$$H_m = [X_m < b] \cup [X_m < g] \text{ because}$$

$$[X_m < b] \subset [Y_m \geq g] \text{ for } m \geq b+g$$

and $$[Y_m < g] \subset [X_m \geq b] .$$

Therefore, the probability, that a couple will not achieve the sex composition of b sons and g daughters at parity m or before, denoted by $Q_m^{'}(b,g)$, is given by

$$Q_m^{'}(b,g) = \begin{cases} 1 & \text{if } m < b+g \\ F(m,p,b) + F(m,q,g) & \text{if } m \geq b+g \end{cases}$$

where

$$F(m,p,b) = \begin{cases} 0 & \text{if } b = 0 \\ \sum_{u=0}^{b-1} \frac{m}{u!(m-u)!} p^u q^{m-u} & \text{if } b \geq 1 \end{cases}$$

and $$F(m,q,g) = \begin{cases} 0 & \text{if } g = 0 \\ \sum_{v=0}^{g-1} \frac{m!}{v!(m-v)!} q^v p^{m-v} & \text{if } g \geq 1 . \end{cases}$$

Note that s, the limit on the total number of children, is ignored. This will be taken care of later.

Since $\alpha = 0$, $\theta = \max(G, G+M) = G+M$. Replacing the values of $Q_m(b,g,.)$ by $Q_m'(b,g)$ and θ by $h = G+M$ in the formulae for $P^*(E_k)$ in section 4.2.1, the probabilities of a woman giving birth in the 1st and the k^{th} year of marriage (for $k \geq 2$) in the experimental group are derived (under the assumption that the couples adhere to stopping rules as per new scheme $S'(b,g,s)$ of sex preference).

$$P^*(E_1') = 1 - e^{-\pi t_1}$$

and for $k \geq 2$

$$P^*(E_k') = \left\{e^{-\pi t(k-1)}\right\}(1-e^{-\pi}) + \sum_{m=1}^{\min\{k-1,s-1\}} \Pr.\left[r_1 < Z_{m+1} < r_1+1\right] Q_m'(b,g)$$

where

$$\Pr.\left[r_1 < Z_{m+1} < r_1+1\right] = \begin{cases} \sum_{n=0}^{m} \dfrac{e^{-\pi r_1}(\pi r_1)^n}{n!} - \sum_{n=0}^{m} \dfrac{e^{-\pi(r_1+1)}\left[\pi(r_1+1)\right]^n}{n!} & \text{if } r_1 > 0 \\ 1 - \sum_{n=0}^{m} \dfrac{e^{-\pi(r_1+1)}\left[\pi(r_1+1)\right]^n}{n!} & \text{if } r_1 \leq 0 < r_1+1 \\ 0 & \text{if } r_1+1 \leq 0 . \end{cases}$$

In the above expression

$r_1 = t_{k-1} - mh$, $h = G+M$, $t_k = k - 3/4$ and $Z_{m+1} = W_1+W_2+ \ldots\ldots +W_{m+1}$, where $W_1, W_2, \ldots, W_{m+1}$ are independent identifically distributed exponential random variables.

Remarks :

(1) It may be noted that 's' is introduced under the summation to put an upper limit on the total number of children (live births).

(2) If a couple has a particular sex preference say b sons and g daughters but no upper limit on the total number of children, the above formulae can still be used by taking s large (say 50).

(3) When a couple has no particular sex preference and wishes to stop reproduction with s children, the above formulae can be used with the modification that

$$Q_m'(b,g) \doteq 1 \text{ for all } m.$$

The birth probabilities derived above are infact conditional probabilities with the condition that a woman is not sterile. The unconditional birth probability, $f_{x,y}^{e'}$ for a woman of age x who married at age y is given by

$$P(E_k') = f_{x,y}^{e'} = P^*(E_{x-y+1}')\beta_x$$

where, β_x is the probability that a woman is not sterile at age x ;

$x = 15, 16, \ldots\ldots 44;$

$y = 15, 16, \ldots\ldots 35;$

and $x \geq y$.

Let $C_k^{'}$ be the event that a birth takes place to a woman in the k^{th} year of marriage in the control group where a couple is assumed to follow no stopping rules regarding adoption of any particular sex composition and/or size of the family.

The corresponding derivation of $P^*(C_k^{'})$ (and $P(C_k^{'})$) in the control group is similar to that of $P^*(E_k^{'})$ (and $P(E_k^{'})$) in the experimental group. The only difference is that $Q_m^{'}(b,g)$ is not required to be considered in the control group. However, the formula for $P^*(C_k^{'})$ can be derived from the results of experimental group where b and g can be taken very large so that $Q_m^{'}(b,g) = 1$ for all m, and s is also taken large enough to remove its effect on the summation in the formulae for $P^*(E_k^{'})$. This is because it is assumed that there is no limit on b, g or s in the control group. Thus one can obtain the formulae for $P^*(C_k^{'})$ and $P(C_k^{'})$ from $P^*(E_k^{'})$ and $P(E_k^{'})$ by taking $Q_m^{'}(b,g) = 1$ for all m, with s assumed large. These probabilities can be rewritten as

$$P^*(C_k') = \sum_{m=1}^{k} \Pr.\left[r_1' < Z_m < r_1' + 1\right]$$

where,

$$\Pr.\left[r_1' < Z_m < r_1'+1\right] = \begin{cases} \sum_{n=0}^{m-1} \frac{e^{-\pi r_1'}(\pi r_1')^n}{n!} - \sum_{n=0}^{m-1} \frac{e^{-\pi(r_1'+1)}\left[\pi(r_1'+1)\right]^n}{n!} & \text{if } r_1' > 0 \\ 1 - \sum_{n=0}^{m-1} \frac{e^{-\pi(r_1'+1)}\left[\pi(r_1'+1)\right]^n}{n!} & \text{if } r_1' \leq 0 < r_1'+1 \\ 0 & \text{if } r_1'+1 \leq 0 . \end{cases}$$

In the above expression

$$Z_m = W_1 + W_2 + \ldots\ldots + W_m; \quad r_1' = t_{k-1} - (m-1)h;$$

$$h = G+M; \text{ and } t_k = k - 3/4 .$$

Further, the unconditional birth probability, $f_{x,y}^{c'}$ for a woman of age x who married at age y in the control group is given by

$$P(C_k') = f_{x,y}^{c'} = P^*(C_{x-y+1}')\beta_x$$

where $x = 15, 16, \ldots\ldots 44;$

$y = 15, 16, \ldots\ldots 35;$

and $x \geq y$.

Having obtained the estimates of $f^{c'}_{x,y}$ and $f^{e'}_{x,y}$ under control and various sex preference assumptions, the corresponding fertility rates can be obtained by replacing $f^{c}_{x,y}$ and $f^{e}_{x,y}$ by $f^{c'}_{x,y}$ and $f^{e'}_{x,y}$ in the various indices of fertility (such as F_x, T, G^J and B^T) in section 4.2.2. Difference between the fertility rates of control and experimental sets is a measure of the impact of sex preference on current fertility.

4.3.2 Heterogeneity of Sex Preferences

The decision making models presented in the sections 4.2 and 4.3.1 assume that the sex preferences within the population are homogeneous. In other words, if all couples wish to have b sons and g daughters and are allowed to achieve the desired minimum, its effect on current fertility can be examined from the results presented in the previous sections. If individual preferences vary and are allowed to achieve their respective desired minimum of each sex, its over all effect on current fertility can be measured by the same decision making model with the procedure suggested below.

Suppose schemes S_1, S_2, ... and S_N of sex preference are followed by $P(S_1)$, $P(S_2)$, ... and $P(S_N)$ proportion of couples respectively, so that

$$P(S_1) + P(S_2) + \dots\dots + P(S_N) = 1$$

Using the notations adopted earlier, the probability of occurrence of a birth to a woman in the experimental group, given that she is aged x and is married at age y ($y \leq x$), is given by

$$f^e_{x,y}(S) = P(E^S_k) = P(E_k(S_1))\ P(S_1) + P(E_k(S_2)\ P(S_2) + \dots\dots\dots + P(E_k(S_N))\ P(S_N)$$

where $k = x-y+1$, and $P(E_k(S_1))$, $P(E_k(S_2))$, ... and $P(E_k(S_N)$ are the conditional probabilities of a woman giving birth in the experimental group.

Derivation of $P(E_k(S_1))$, $P(E_k(S_2))$, ... and $P(E_k(S_N))$ in the above equation are similar to that of $P(E_k)$ in section 4.2.1 when couples adhere to scheme S_1, S_2 ..... and S_N respectively. Therefore,

$$f^e_{x,y}(S) = f^e_{x,y}(S_1)P(S_1) + f^e_{x,y}(S_2)P(S_2) + \dots\dots\dots + f^e_{x,y}(S_N)P(S_N)\ .$$

Replacing $f^e_{x,y}$ by $f^e_{x,y}(S)$ in section 4.2.2, the corresponding fertility rates for the experimental group can be obtained. These fertility rates are now compared with the rates derived earlier for the control group. The difference is the measure of the overall effect of sex preferences on fertility in a population where individual sex preferences vary.

4.4 REMARKS

The entire work has been programmed for computer analysis. As mentioned earlier, data from India have been utilized to illustrate the various models developed in this chapter, the results of which are presented in Chapters VI and VII. However the results of the model illustrating the overall effect of sex preferences on fertility (for a given set of values of the parameters) are not presented in either of these two chapters, but are deferred to Chapter VIII in order to avoid duplication of the same. Similarly, the discussion on the index NRR is also presented only in Chapter VIII. The various input parameters used in the models are discussed in the chapter that follows.