PERISHABLE PRODUCTS STOCHASTIC INVENTORY MODELS UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENT

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By

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CERTIFICATE

This is to certify that the work incorporated in this thesis entitled **"Perishable Products Stochastic Inventory Models under Inflation and Permissible Delay in Payment"** submitted by Mrs. Khimya S Tinani for the award of the degree of Doctor of Philosophy in Statistics, Department of Statistics, Faculty of Science, The Maharaja Sayajirao University of Baroda, Vadodara, comprises the results of independent and original investigation carried out by her under my supervision. The material that has been obtained and used from other sources has been duly acknowledged in the thesis.

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DECLARATION

I hereby declare that the entire work embodied in this thesis has been carried out by me under the supervision and guidance of **Dr.(Mrs.) Deepa H. Kandpal**, Department of Statistics, Faculty of Science, The Maharaja Sayajirao University of Baroda, Vadodara. I also declare, to the best of my knowledge and belief, in this thesis no material is previously published or written by any other person and has not been submitted to any other university.

(Khimya S Tinani)

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(Khimya S Tinani)

CONTENT

CHAPTER 1

INTRODUCTION

CHAPTER 1

1.1. LITERATURE REVIEW:

Inventory can be considered as an accumulation of physical commodity that can be used to satisfy some future demand for that commodity. The main and foremost reason for maintaining inventory level is to shorten the gap between demand and supply for the commodity under consideration. Any inventory system consists of an input process and output process. The input process refers to supply either by means of production or purchase while the output process refers to demand due to which depletion of inventory occurs. Thus, supply is a replenishment process, whereas demand is a depletion process.

Though the inventories are essential and provide an alternative to production or purchase in future, they also mean lock up capital of an enterprise. Maintenance of inventories also costs money by way of expenses on stores, equipment, personnel, insurance etc. Thus excess of inventories are undesirable. This calls for controlling the inventories in the most profitable way. Hence inventory theory deals with the determination of the optimal level of such ideal resources. An excellent survey on research in inventory management in a single product, single location inventory environment is provided by Lee and Nahmias (1993).

The conventional inventory models can be placed in two categories: deterministic and stochastic. In deterministic models, all input data are assumed deterministic and are given and based upon the known data, a mathematical model is applied to minimize the total inventory costs. In stochastic models, a probabilistic distribution of the input data is specified, and a mathematical model is used to minimize the total expected inventory costs. Important contributions in the development of mathematical theory of inventories include the landmark articles by Wilson (1934), Arrow, Harris and Marshak (1951), Dvoretsky, Kiefer and Wolfowitz (1952a, 1952b), Bellman, Glicksberg and Gross (1955), Arrow, Karlin and Scarf (1958) and Wagner and Whitin (1958).

The original economic order inventory model (EOQ) assumes that inventory items are unaffected by time and replenishment is done instantaneously. However, this ideal case is not always applicable. Inventories are often replenished periodically at certain production rate, which is seldom infinite. Even for purchase items, when supply varies at the warehouse, it may take days for receiving department to completely transfer the supply into storage room.

Perishable inventory forms a large portion of total inventory and include virtually all foodstuffs, pharmaceuticals, fashion goods, electronic items, periodicals (magazines/ Newspapers), digital goods (computer software, video games, DVD) and many more as they lose value with time due to deterioration and/ or obsolescence. Perishable goods can be broadly classified into two main categories based on: (i) Deterioration (ii) Obsolescence .Deterioration refers to damage, spoilage, vaporization, depletion, decay (e.g. radioactive substances), degradation (e.g. electronic components) and loss of potency (e.g. chemicals and pharmaceuticals) of goods. Obsolescence is loss of value of a product, due arrival of new and better product. Perishable goods have continuous or discrete loss of utility and therefore can have either fixed life or random life. Fixed life perishable products have a deterministic, known and definite shelf life and examples of such goods are pharmaceuticals, consumer packed goods and photographic films. On the other hand, random life perishable products have a shelf life that is not known in advance and vary depending on variety factors including storage atmosphere. Items are discarded when they spoil and the time to spoilage is uncertain. For example, fruits, vegetables, dairy products, bakery products etc., have random life.

A large number of researchers developed the models in the area of deteriorating inventories. At first Whitin (1957) considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Various types of inventory models for items deteriorating at a constant rate were discussed by Roy, Chowdhury and Choudhuri (1983). A complete survey of the published literature in mathematical modeling of deteriorating inventory systems is given by Raafat (1991). Goyal and Giri (2001) developed recent trends of inventory models for deteriorating items. Teng and

Chang (2005) determined economic production quantity in an inventory model for deteriorating items.

In most inventory models it is implicitly assumed that the product to be ordered is always available (i.e. continuous supply availability), that is when an order is placed it is either received immediately or after a deterministic or perhaps random lead time. However if the product is purchased from another company (as in the JIT deliveries of parts and components), the supply of the product may sometimes be interrupted due to the supplier's equipment breakdowns, labor strikes or other unpredictable circumstances. Silver (1981) appears to be first author to discuss the need for models that deal with supplier uncertainty. Supply uncertainty can have a drastic impact on firms who fail to protect against it. Supply uncertainty has become a major topic in the field of inventory management in recent years. Notable events such as the 9/11terrorist attacks or major natural disasters have generated significant interest in supply chain disruption studies. Not only have modern events demonstrated the relevance and impact of uncertainty in supply, but analytical studies have also demonstrated the harm that can come from ignoring it. Supply disruptions can be caused by factors other than major catastrophes. More common incidents such as snow storms, customs delays, fires, strikes, slow shipments, etc. can halt production and/or transportation capability, causing lead time delays that disrupt material flow. Improved transportation and communication capabilities allow companies to expand globally, which also puts them at higher risk of supply variability.

There appears to be very few articles dealing with the issue of supply uncertainty phenomenon. One of the earliest articles to analyze the problem with supply interruptions was by Meyer, Rothkopf and Smith (1979). They investigated a system consisting of production facility subject to random failure and repair processes and developed expressions for the average inventory level and the fraction of time demand is met. Chao (1987) has used dynamic programming to find optimal policies for electric utility companies that may face market disruptions. Groenevelt, Pintelon and Seidmann (1992) modeled a deterministic demand production lot sizing problem where the effects of machine breakdowns and corrective maintenance on the economic lot sizing decisions are analyzed. Articles by Parlar and Berkin (1991) consider the supply uncertainty problem, for a class of EOQ model with a single supplier where the availability and unavailability periods constitute an alternating Poisson process. Parlar and Berkin (1991) assume that at any time the decision maker is aware of the availability status of the product although he does not know when the ON (available) and OFF (unavailable) periods will start and end. When the inventory level reaches the reorder point of zero and the status is ON, the order is received; otherwise the decision maker must wait until the product becomes available. Parlar and Perry (1996) generalized the formulation of Parlar and Berkin (1991) by first assuming that the reorder point r is a non negative decision variable instead of being equal to zero. In stochastic inventory models fixing the reorder point at zero usually results in sub optimal solutions. Hence, making r as another decision variable and solving the more general problem eliminates the above mentioned sub optimality. Kandpal and Gujarathi (2003,2006) has extended the model of Parlar & Perry (1996) by considering demand rate greater than one and for deteriorating items for single supplier.

The supplier’s market may not always be monopolistic as competitive spirit in the business has increased especially after induction of multinational companies. The duopolistic case can be explained as follows where the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 whereas state 3 denotes the non-availability of either of them. Status of both the suppliers is explained as below.

State	Status of supplier 1	Status of supplier 2
0	ON	ON
1	ON	OFF
2	OFF	ON
3	OFF	OFF

Here it is assumed that one may place order to either one of the two suppliers or partly to both when both suppliers are available (i.e. state 0 of the system).

In the classical, EOQ it is assumed that the payments for the goods received should be made immediately after the receipt of the order. Furthermore, while developing a mathematical model in inventory control, it is assumed that the payment will be made to the suppliers for the goods immediately after receiving the consignment. However, in day-to-day dealing it is found that a supplier allows a certain fixed period to settle the account. During this fixed period the supplier charges no interest, but beyond this period interest is charged by the supplier under the terms and conditions agreed upon, since inventories are usually financed through debt or equity. In case of debt financing, it is often a short term financing. Thus, interest paid here is nothing but the cost of capital or opportunity cost. Also, short-term loans can be thought of as having been taken from the suppliers on the expiry of the credit period. However, before the account has been settled, the customer can sell goods and continues to accumulate revenue and earn interest instead of paying the overdraft that is necessary if the supplier requires settlement of the account after replenishment. Interest earned can be thought of as a return on investment since the money generated through revenue can be ploughed back into the business. Therefore, it makes economic sense of the customer to delay the settlement of the replenishment account up to the last day of the credit period allowed by the supplier. If the credit period is less than the cycle length, the customer continues to accumulate revenue and earn interest on it for the rest of the period in the cycle, from the stock remaining beyond the credit period. The primary benefit of taking trade credit is that one can have savings in purchase cost and opportunity cost, which becomes quite relevant for deteriorating items. In such cases one has to procure more units than required in the given cycle to account for the deteriorating effects. In particular, when the unit purchase cost is high and decay is continuous, the saving due to delayed payment appears to be more significant than when the decay is continuous without delayed payments.

Goyal (1985) has studied an EOQ system with deterministic demands and delay in payments is permissible which was reinvestigated by Chand and Ward (1987). Mandal and Phaujdar (1989) extended Goyal's (1985) model to the case of shortages. Shah (1991) studied the same problem with uncertainty in the quantity received, resulting in a random duration between successive orders. Using the first two moments of the distribution of random quantities received, Shah (1991) arrived at modified results of Goyal (1985) by using probabilistic demand. The aspect of admissible delay in payments has been extended to the case of two level of storage by Shah and Shah (1992). Shah (1993) developed model for exponentially deteriorating items when delay in payments is permissible by assuming deterministic demand. Thereafter, under the same situation, Shah (1993) developed model for probabilistic demand. Chung (1998) studied the same model as Goyal (1985) and presented an alternative approach to find a theorem to determine the EOQ under condition of permissible delay in payments. In Goyal's (1985) model, it is assumed that no deterioration is allowed to occur and the capacity of the warehouse is unlimited. Aggarwal and Jaggi (1995) extended Goyal's model to the case of deterioration under permissible delay in payment. Jamal *et al.* (1997) developed EOQ model with constant deterioration rate and permissible delay in payments. An EOQ model for inventory control in the presence of trade credit is presented by Chung and Huang (2005). The optimal replenishment policy for EOQ models under permissible delay in payments is also discussed by Chung et al. (2002) and Chung and Huang (2003). An EOQ model under conditionally permissible delay in payments was developed by Huang (2007) and obtained the retailer's optimal replenishment policy under permissible delay in payments. Jaggi et al. (2007) developed an inventory model under two levels of trade credit policy by assuming the demand is a function of credit period offered by the retailer to the customers using discounted cash-flow (DCF) approach. Jaggi et al.(2008) developed a model retailer's optimal replenishment decisions with credit- linked demand under permissible delay in payments. Hou and Lin (2009) developed a cash flow oriented EOQ model with deteriorating items under permissible delay in payments and the minimum total present value of the costs is obtained. Tripathi and Misra (2010) developed EOQ model credit financing in economic ordering policies of non- deteriorating items with time- dependent demand rate in the presence of trade credit using a discounted cash-flow (DCF) approach.

From a financial standpoint, an inventory represents a capital investment and must compete with other assets for a firm's limited capital funds. The effects of inflation are not usually considered when an inventory system is analyzed because most people think that the inflation would not influence the inventory policy to any significant degree. Due to high inflation, the financial situation has changed in many developing countries, especially in politically turmoil countries such as united Germany, Russia and Iraq; so it is necessary to consider the effects of inflation on the inventory system. Following Buzacott (1975) and Misra (1979), Bierman and Thomas (1977) investigated the inventory decisions under an inflationary condition in a standard EOQ model. Misra (1979) developed a discount cost model in which the effects of both inflation and time value of money are considered. Chandra and Bahner (1985) developed models to investigate the effects of inflation and time value of money on optimal order policies. An inventory model with deteriorating items under inflation when a delay in payment is permissible is analyzed by Liao et al. (2000). Bhahmbhatt (1982) developed an EOQ model under a variable inflation rate and marked-up price. Ray and Chaudhuri (1997) presented an EOQ model under inflation and time discounting allowing shortages. B.R. Sarker, A.M.M. Jamal, Shajunwang (2000) developed Supply Chain Models for Perishable products under inflation and permissible delay in payment. Agrawal et al. (2009) developed a model on integrated inventory system with the effect of inflation and credit period. In this model the demand rate is assumed to be a function of inflation. This EOQ model is applicable when the inventory contains trade credit that supplier give to the retailer. Tripathi et al. (2010) developed an inventory model for non-deteriorating items and time-dependent demand under inflation when delay in payment is permissible.

Inventory model for non-deteriorating and deteriorating items with future supply uncertainty considering demand rate as d for single supplier was developed by Gujarathi and Kandpal (2003). We have developed stochastic inventory models for perishable products where the effect of inflation and permissible delay in payment was considered.

1.2. SUMMARY OF THESIS:

Chapter 1 gives a detailed introduction of stochastic inventory models for perishable items under inflation and permissible delay in payment and its need. An exhaustive literature survey on various models is discussed. Various real life examples and their application areas are discussed in this chapter.

Our contribution is divided into 7 chapters from chapter 2 to 8. Its brief account of the work done is as follows:

The thesis “Perishable Products Stochastic Inventory Models under Inflation and Permissible delay in payment” is divided into two parts.

Part I

Single supplier stochastic inventory models

Part II

Two suppliers stochastic inventory models

Part I deals with inventory models of future supply uncertainty with single supplier. Our contribution is divided into 2 chapters: **chapter 2 and chapter 3.** Every chapter is followed by an illustrative example clarifying the results obtained and their practical utility. Also sensitivity analysis is done with respect to some important parameters. This is in support of verification of the usual behaviour of economics variables and hence to exemplify the scope of their real life applications.

In **chapter 2** a stochastic inventory model for deteriorating items under inflation and permissible delay in payments is developed. The inventory model is developed for a supplier that allows some credit period T_0 for settling the accounts of purchased quantity. The credit period is a known constant as it is settled between a supplier and a buyer at the time of the deal. The effect of inflation and time value of money was investigated under given sets of inflation and discount rates. Expressions are derived for obtaining the optimum order quantity and reorder quantity and hence the optimum cycle time. (Paper based on this chapter is **published** in an International Journal of Probability and Statistical Science (JPSS) in August 2009 issue of the journal.)

Chapter 3 discusses the impact of permissible delay in payment by introducing a provision for part payment, which is a practical aspect. It is a common practice that an instalment of payment is made during the period of the admitted delay in payments. The part to be paid and the time at which it is to be paid are mutually settled between the supplier and the buyer at the same time of purchase of goods. (Paper based on this chapter is **published** in the journal of Indian Association for Productivity, Quality and Reliability (IAPQR) in November 2011 issue of the journal.)

Part II deals with inventory models of future supply uncertainty with two suppliers. Here there are three states that correspond to the availability of at least one supplier, that is, states 0, 1 and 2 whereas state 3 denotes non-availability of either of them. Here it is assumed that one may place order to either one of the two supplier or partly to both when both the suppliers are available (i.e. state 0 of the system.) In case of two suppliers, spectral theory is used to derive explicit expressions for the transition probabilities of a four states continuous time Markov Chain representing the status of the system. These probabilities are used to compute the exact form of the average cost expression. Optimal solutions are obtained by Newton-Rapson method in R programming.

Our contribution is divided into 4 chapters: **chapter 4 to chapter 7.** In support of the results developed, in each chapter an illustrative example is given and sensitivity analysis is done with respect to some important parameters.

In **chapter 4** stochastic inventory model for two suppliers under trade credit is developed. From this model we have concluded that cost is minimum when the account is settled at credit time given by both the suppliers. Sensitivity analysis is also carried out. (Paper based on this chapter is **published** in the journal of Calcutta Statistical Association Bulletin (CSA) in September 2010 issue of the journal.)

In **chapte**r **5** stochastic inventory model for two suppliers under inflation and permissible delay in payment is developed. From this model we have concluded that as inflation rate increases, average cost also increases and cost is minimum when account is settled at credit time given by both the suppliers. (Paper based on this chapter is

published in the journal of Indian Statistical Association (JISA) in December 2010 issue)

In **chapter 6** stochastic inventory model for two suppliers under permissible delay in payment allowing partial payment is developed. From this model we have concluded that cost is minimum if part payment is not done at T_{1i} but account is cleared at T_i and the cost is maximum if part payment is done at T_{1i} but account is not cleared at T_i, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period. (Paper based on this chapter is **published** in an international journal of Engineering and Management Sciences (IJEMS) in April 2013 issue of the journal.)

In **chapter 7** stochastic inventory model for two suppliers under inflation and trade credit allowing partial payment is developed. From this model we have concluded that as inflation rate increases, average cost also increases and cost is minimum if part payment is not done at T_{1i} but account is cleared at T_i and the cost is maximum if part payment is done at T_{1i} but account is not cleared at T_i, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period. (Paper based on this chapter is **accepted** for publication in the journal of Indian Society for Probability and Statistics (ISPS)).

In **chapter 8** stochastic inventory model for multiple suppliers is developed. From this model we have concluded that when the number of suppliers becomes large, the objective function of the multiple supplier problem reduces to that of the classical EOQ model. (Paper based on this chapter is **published** in an international journal of Science, Engineering and Technology Research (IJSETR) in September 2013 issue of the journal.)

We have also thought of following problems for our post Ph.D. work.

(i) Problem of diversification when both the suppliers are available.

(ii) Demand can be considered probabilistic.

Part-I

PERISHABLE PRODUCTS STOCHASTIC INVENTORY MODELS FOR SINGLE SUPPLIER

CHAPTER 2

STOCHASTIC INVENTORY MODEL UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENT FOR SINGLE SUPPLIER

CHAPTER 2

2.1. INTRODUCTION:

In most inventory models it is implicitly assumed that the product to be ordered is always available (i.e., continuous supply availability), that is when an order is placed it is either received immediately or after a deterministic or perhaps random lead time. However if the product is purchased from outside supplier he can cut off the supply at random times for duration of random length, or the product may be unavailable as in the case of equipment breakdowns, labour strikes or other unpredictable circumstance, then the production/inventory manager would need to know how much to produce or purchase when the supply is fully available.

At any time, the state of the system can be ON or OFF. We use 0 to denote the ON state and 1 to denote the OFF state. If the supplier is available we call it ON period and if he is not available call it OFF period. Also we specifically assume that the ON and OFF periods are exponentially distributed with parameters λ and μ respectively.

Deterioration/Perishability of an item in the inventory is defined as loss of its utility. It is reasonable to note that product may be understood to have life time which ends when its utility reaches zero. There is a great deal of interest in the analysis of perishable inventory models.

From a financial standpoint, an inventory represents a capital investment and must compete with other assets for a firm’s limited capital funds. The effects of inflation are not usually considered when an inventory system is analyzed because most people think that the inflation would not influence the inventory policy to any significant degree. Due to high inflation, the financial situation has changed in many developing countries, especially in politically turmoil countries such as united Germany, Russia and Iraq; so it is necessary to consider the effects of inflation on the inventory system.

The primary benefit of taking trade credit is that one can have savings in purchase cost and opportunity cost, which become quite relevant for deteriorating items.

In such cases, one has to procure more units than required in the given cycle to account for the deteriorating effect. In particular, when unit purchase cost is high and decay is continuous, the saving due to delayed payment appears to be more significant than when the decay is continuous but without delayed payment. In order to boost up competitive spirit of the business, the small entrepreneurs are to be encouraged by giving some privileges to them. This may feasibly make them available in the business in spite of their limited finance resources.

Inventory model for non-deteriorating and deteriorating items with future supply uncertainty considering demand rate as d for single supplier was developed by Gujarathi and Kandpal [2003]. In this chapter we therefore consider a more realistic case of demand, by considering rate of demand $d \geq 1$ for a single supplier and developed stochastic inventory model for perishable products where the effect of inflation and permissible delay in payment was considered.

2.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model under inflation and permissible delay in payment for single supplier is developed on the basis of the following assumptions.

(a) Demand rate d is deterministic and it is $d \geq 1$.

(b) The status of the system is initially ON.

(c) We define X and Y to be the random variables corresponding respectively to the lengths of ON and OFF periods of the supplier. We specifically assume that $X \sim \exp(\lambda)$ and $Y \sim \exp(\mu)$. Further, X and Y are independently distributed.

(d) Ordering cost is Rs. k/order.

(e) Holding cost is Rs. h/unit/unit time.

(f) Shortage cost is Rs. π/unit.

(g) Time dependent part of the backorder cost is Rs. $\hat{\pi}$ /unit/time.

(h) q = order up to level.

(i) r = reorder level; q, r are decision variables.

(j) θ is the rate of deterioration which is constant fraction of on hand inventory. The deteriorated units can neither be replaced nor repaired during cycle period.

(k) Purchase cost is Rs. c/unit.

(l) T_0 is credit period which is a known constant and T_{00} is cycle period which is a decision variable.

(m) r_1 = discount rate representing the time value of money.

(n) f = inflation rate

(o) $R = f - r_1$ = present value of the nominal inflation rate.

(p) t_1= time period with inflation

(q) c_0 = present value of the inflated price of an item Rs. /unit = $ce^{(f-r_1)t_1} = ce^{Rt_1}$

(r) Ie = interest rate earned; Ic = interest rate charged.

(s) δ = indicator variable = 0, if account is settled completely at T_0,

= 1, otherwise.

(t) $Ie(1)$ = Interest earned over period (0 to T_0) = $dce^{Rt_1}\, T_0 T_{00}\, Ie$.

(u) $Ie(2)$ = Interest earned over period (T_0 to T_{00}) upon interest earned ($Ie(1)$) previously.

$$= [dce^{Rt_1}\, T_{00} + Ie(1)]\, \left(T_{00} - T_0\right) Ie\,.$$

(v) Ic =Interest charged by the supplier $= \delta\, dc\, e^{Rt_1}\, Ic\,(T_{00} - T_0)$, clearly ($Ic > Ie$).

$A(q,r,\theta) =$ (cost of ordering) + (cost of holding inventory) + (cost of item that deteriorate during a single interval that starts with an inventory of ($q + r$) units and ends with r units with inflation rate);

$$= k + \frac{1}{2} \cdot \frac{hq^2 e^{Rt_1}}{(d+\theta)} + \frac{hrqe^{Rt_1}}{(d+\theta)} + \frac{\theta cqe^{Rt_1}}{(d+\theta)}$$

$P_{ij}(t) = P$(being in state j at time t/starting in state i at time 0); $i, j = 0, 1$.

P_i = long run probabilities, $i = 0, 1$.

$C_{10}(r)$ = Expected cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle.

In this chapter we assume that

(i) A Supplier allows a fixed period 'T_0' to settle the account. During this fixed period no interest is charged by the supplier but beyond this period, interest Ic is charged by the supplier under the terms and conditions agreed upon.

(ii) During the fixed credit period T_0, revenue from sales is deposited in an interest bearing account.

(iii) The account is settled completely either at the end of the credit period or at the end of the cycle period.

(iv) Interest charged is usually higher than interest earned. Here we settle the account completely at T_0 as revenue generated till period T_0 may be presumably sufficient for settlement of the account completely as selling cost is greater than the purchase cost.

For inflation rate f, the continuous time inflation factor for the time period t_1 is e^{ft_1} which means that an item that costs Rs. c at time $t_1 = 0$, will cost ce^{ft_1} at time t_1. For discount rate r_1, representing the time value of money, the present value factor of an amount at time t_1 is $e^{-r_1 t_1}$. Hence the present value of the inflated amount ce^{ft_1} (net inflation factor) is $ce^{ft_1} e^{-r_1 t_1}$. For an item with initial price c (Rs./unit), at time $t_1 = 0$, the present value of the inflated price of an item is given by $c_0 = ce^{(f-r_1)t_1} = ce^{Rt_1}$ where $R = f - r_1$ in which c is inflated through time t_1 to ce^{ft_1}, $e^{-r_1 t_1}$ is the factor deflating the future worth to its present value and R is the present value of the inflation rate.

2.3. OPTIMAL POLICY DECISION FOR THE MODEL:

We use Renewal Reward Theorem (RRT) to model a stochastic inventory problem with supply interruptions where the supplier may be unavailable, since it has found wide applicability in queuing models and stochastic inventory models as exemplified in the works of Ross [1983] and Tijms [1986].

As explained in Ross [1983], RRT is a powerful tool used in optimization of stochastic systems. Once a regenerative cycle of a stochastic process is identified, one can form an average cost objective function as a ratio of the expected cycle cost to the expected cycle time.

For the inventory model under consideration the policy to be used is as follows. When inventory drops to 'r' and if the period is ON an order for 'q' units is placed which increases the inventory to the level (q + r), i.e., (q, r) policy is used. When inventory drops to r and period is OFF, then the decision maker has to wait till the supplier becomes available. Upon his availability an order can be placed for number of units which increases the inventory to the level (q + r) units. Hence in the OFF period possibility of shortages is also there. Cycle is defined to be period when inventory is replenished. Cycle is also shown in Fig.2.1. For this policy the inventory level and the status process is depicted in Fig.2.1.

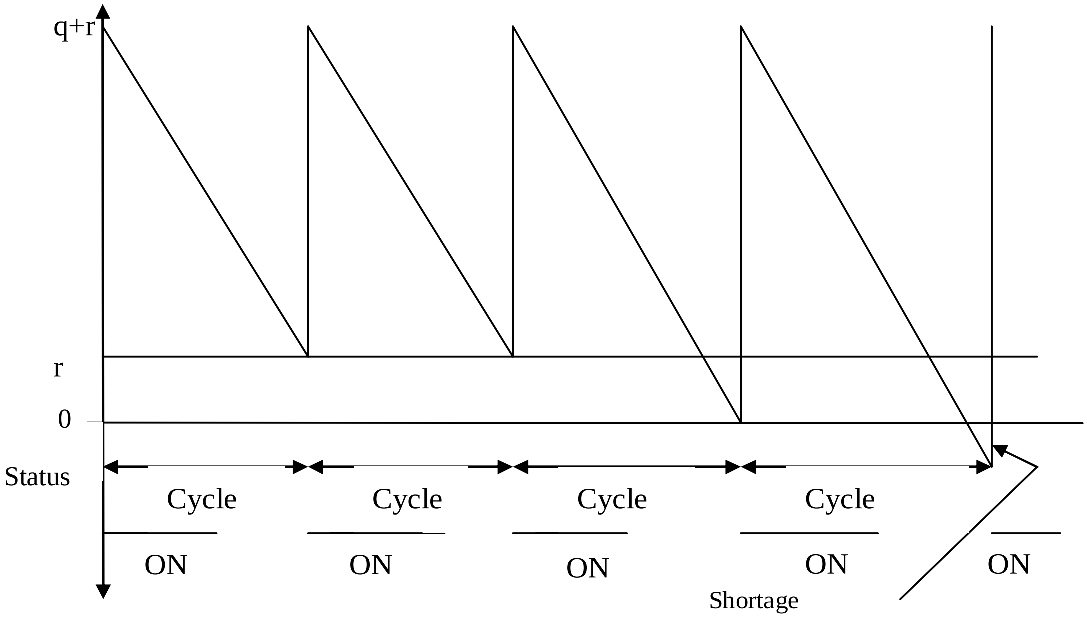

Fig.2.1 Inventory level and the status process for single supplier

Referring to Fig.2.1, we see that the cycles of this process start when the inventory goes up to a level of $(q + r)$ units. Once the cycle is identified we construct the average cost objective function as mentioned below.

$AC\,(q,r,\theta) =$ Average cost objective function.

$$= \frac{C_{00}}{T_{00}},$$ where $C_{00} = E$ (cost per cycle); $T_{00} = E$ (length of a cycle);

Now to make use of RRT we prove the following:

Lemma 2.3.1: $C_{10}(r)$ = expected cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle is obtained as

$$C_{10}(r) = \frac{1}{\mu^2} e^{\frac{-\mu r}{(d+\theta)}}\; e^{Rt_1}\left[he^{\frac{\mu r}{d+\theta}}\left(\mu r - (d+\theta)\right) + \pi\mu d + h(d+\theta) + \hat{\pi} - \theta c\mu \right] + \frac{\theta c e^{Rt_1}}{\mu}.$$

then,

$$C_{00} = A(q,r,\theta) + P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r) - [Ie(1) + Ie(2)] + Ic$$

$P_{01}\left(\frac{q}{d+\theta}\right) = P_1 - P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}}$, and $P_1 = \frac{\lambda}{\lambda+\mu}$.

Proof: Conditioning on the state of the system when inventory drops to *r*, we obtain

$$C_{00} = P_{00}\left(\frac{q}{d+\theta}\right)A(q,r,\theta) + P_{01}\left(\frac{q}{d+\theta}\right)[A(q,r,\theta) + C_{10}(r)] - [Ie(1) + Ie(2)] + Ic. \quad (2.3.1)$$

This follows because when inventory drops to *r*, the state will be 0 (ON) with probability $P_{00}\left(\frac{q}{d+\theta}\right)$ and 1 (OFF) with probability $P_{01}\left(\frac{q}{d+\theta}\right) = 1 - P_{00}\left(\frac{q}{d+\theta}\right)$. If the state is ON, the cost incurred is $A(q,r,\theta)$ which is weighted by the probability $P_{00}\left(\frac{q}{d+\theta}\right)$ of this event If on the other hand, the state is OFF when inventory drops to *r*, the expected cost is $A(q,r,\theta) + C_{10}(r)$ which is weighted by the probability $P_{01}\left(\frac{q}{d+\theta}\right)$ of the corresponding event. The transition probability $P_{00}\left(\frac{q}{d+\theta}\right)$ and $P_{01}\left(\frac{q}{d+\theta}\right)$ are obtained by CTMC, Bhat, U.N.[1984] for these states. They are given by

$$P_{00}\left(\frac{q}{d+\theta}\right) = P_0 + P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}}, \quad P_{01}\left(\frac{q}{d+\theta}\right) = P_1 - P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}}$$

$P_1 = 1 - P_0 = \frac{\lambda}{\lambda+\mu}$, and $P_0 = \frac{\mu}{\lambda+\mu}$ which are the steady state probabilities for the OFF and ON states respectively.

Now, referring to Fig.2.1, the cost incurred from the time when inventory drops to *r* and the state is OFF to the beginning of the next cycle is equal to

$$\frac{1}{2}hy^2(d+\theta)e^{Rt_1} + hy[r-(d+\theta)]e^{Rt_1} + \theta c y e^{Rt_1}, \qquad y < \frac{r}{d+\theta}$$

$$\frac{1}{2}\cdot\frac{hr^2e^{Rt_1}}{(d+\theta)} + \pi e^{Rt_1}\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}e^{Rt_1}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta c r e^{Rt_1}}{(d+\theta)}, \quad y \geq \frac{r}{d+\theta}$$

so that

$$C_{10}(r) = \int_0^{r/(d+\theta)} \left\{\frac{1}{2}hy^2(d+\theta)e^{Rt_1} + he^{Rt_1}y\big(r - y(d+\theta)\big) + \theta c e^{Rt_1} y\right\}\mu e^{-\mu y}dy.$$

$$+\int_{r/(d+\theta)}^{\infty} \left\{\frac{1}{2}\frac{hr^2e^{Rt_1}}{(d+\theta)} + \pi e^{Rt_1}\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}e^{Rt_1}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta c r e^{Rt_1}}{(d+\theta)}\right\}\mu e^{-\mu y}dy$$

$$= \frac{1}{\mu^2}e^{\frac{-\mu r}{d+\theta}}e^{Rt_1}\left[he^{\frac{\mu r}{d+\theta}}\big(\mu r - (d+\theta)\big) + \pi\mu d + h(d+\theta) + \hat{\pi} - \theta c\mu\right] + \frac{\theta c e^{Rt_1}}{\mu}$$

Lemma 2.3.2: Expected cycle length is given by

$$T_{00} = \frac{q}{d+\theta} + \frac{1}{\mu}P_{01}\left(\frac{q}{d+\theta}\right).$$

Proof: Using a conditioning argument similar to the one in Lemma (2.3.1), we obtain

$$T_{00} = \frac{q}{d+\theta}P_{00}\left(\frac{q}{d+\theta}\right) + P_{01}\left(\frac{q}{d+\theta}\right)\left(\frac{q}{d+\theta} + T_{10}\right) \tag{2.3.2}$$

where T_{10} = E(Time to reach the beginning of the next cycle when inventory drops to r and state is OFF). Clearly $T_{10} = 1/\mu$, since the OFF duration Y is distributed exponentially with parameter μ. Substituting for T_{10} in (2.3.2) and solving for T_{00} gives the desired expression for T_{00}.

Lemma 2.3.3: The function $C_{10}(r)$ is strictly convex and it is minimized at

$$\bar{r} = \frac{(d+\theta)e^{Rt_1}}{\mu}\log\left[\frac{\pi\mu d + h(d+\theta) + \hat{\pi} - \theta c\mu}{h(d+\theta)}\right]$$

Proof: The first derivative of $C_{10}(r)$ is obtained as

$$\frac{d}{dr}C_{10}(r)=\frac{e^{Rt_1}}{\mu}\left[h-\frac{e^{-\mu r/(d+\theta)}}{d+\theta}\left(\pi\mu d+h(d+\theta)+\hat{\pi}-\theta c\mu\right)\right]$$

Putting $\frac{d}{dr}C_{10}(r)=0$ and solving for r gives

$$\bar{r}=\frac{(d+\theta)e^{Rt_1}}{\mu}\log\left[\frac{\pi\mu d+h(d+\theta)+\hat{\pi}-\theta c\mu}{h(d+\theta)}\right]$$

The second derivative is

$$\frac{d^2}{dr^2}C_{10}(r)=\frac{e^{Rt_1}\,e^{-\mu r/(d+\theta)}}{(d+\theta)^2}\left(\pi\mu d+h(d+\theta)+\hat{\pi}-\theta c\mu\right)$$

which is always positive, hence $C_{10}(r)$ is strictly convex.

Proposition 2.3.1: The Average cost objective function is given by

$$AC(q,r,\theta)=\frac{C_{00}}{T_{00}}=\frac{A(q,r,\theta)+P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r)-(Ie(1)+Ie(2))+Ic}{\frac{q}{d+\theta}+\frac{1}{\mu}P_{01}\left(\frac{q}{d+\theta}\right)}. \qquad (2.3.3)$$

Proof: Proof follows using Renewal Reward Theorem (RRT). The optimal solution for *q* and *r* are obtained by using Newton Rapson method in R programming.

2.4. NUMERICAL EXAMPLE:

Case-I: Taking δ=1 i.e. account is not settled at time period T_0.

In this section we verify the results by a numerical example. We assume that k=Rs. 10/order, c=Rs. 5/unit, d= 20/units, h=Rs. 5/unit/time, π=Rs. 250/unit, R=0.05, $\hat{\pi}$=Rs. 25/unit/time, θ=5/unit/time, δ=1, *Ic*=0.15, *Ie*=0.08, T_0=0.6, t_1=6, λ=0.25, μ=2.5. The last two parameters indicate that the expected lengths of the ON and OFF periods are $1/\lambda=4$, and $1/\mu=0.4$ respectively. The long run probabilities are obtained as $P_0=0.909$ and $P_1=0.091$. The optimal solution is obtained as

q=16.198, r=15.02 and $AC=\frac{C_{00}}{T_{00}} = 266.575$

Case-II: Taking δ=0 i.e. account is settled at time period T_0.

Keeping other parameters as it is, we consider δ =0 i.e. account is settled at time period T_0. The optimal solution is obtained as

q= 18.56644, r= 14.14799 and $AC=\frac{C_{00}}{T_{00}} = 260.3604$.

Conclusion:

From the above numerical example, we conclude that the cost is minimum when account is settled at credit time given by the supplier.

2.5. SENSITIVITY ANALYSIS:

Case-I: Taking δ=1 i.e. account is not settled at time period T_0.

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and purchase cost *c* keeping other parameter values fixed. Inflation rate *R* is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=1 i.e. account is not settled at time period T_0 and the purchase cost *c* is assumed to take values 5, 15, 25. We resolve the problem to find optimal values of *q*, *r* and *AC*. The optimal values of *AC* and *R* are plotted in Fig. 2.5.1.

Table 2.5.1
Sensitivity Analysis Table by varying the parameter values of *R* & *c*

	R	q	r	AC
c=5	0.05	16.198	15.02	266.575
	0.1	14.6125	15.6275	354.425
	0.15	13.1892	16.1894	472.443
	0.2	11.9098	16.708	631.126
	0.25	10.7584	17.1857	844.637
	0.3	9.22122	17.6249	1132.09
c=15	0.05	23.1296	14.9202	333.394
	0.1	19.34682	15.5277	444.622
	0.15	14.6865	16.0896	594.196
	0.2	12.92145	16.6082	795.475
	0.25	10.97276	17.0859	1066.481
	0.3	9.64643	17.5251	1431.55
c=25	0.05	26.8942	14.8194	400.206
	0.1	25.3913	15.4269	534.81
	0.15	18.6024	15.9888	715.937
	0.2	14.98498	16.5074	959.807
	0.25	11.54741	16.9851	1288.31
	0.3	9.894619	17.4243	1730.98

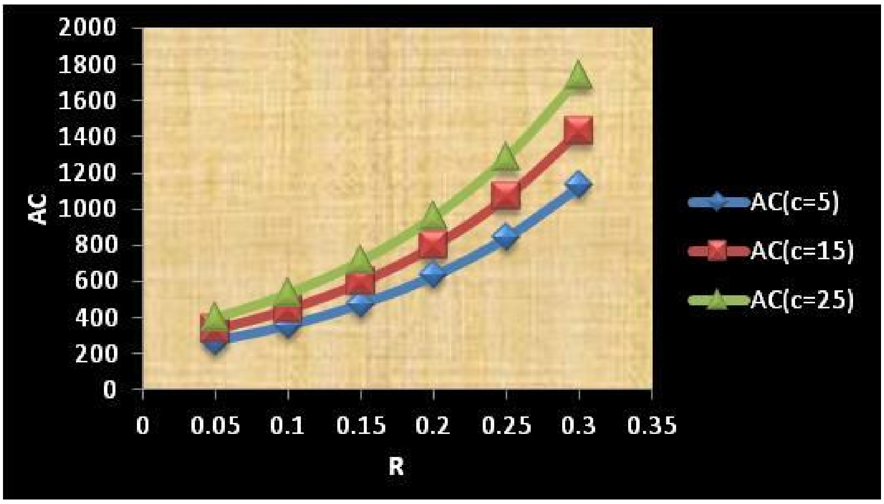

Fig. 2.5.1 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying purchase cost *c*

From the above table we see that taking inflation rate R=0.05 and increasing the value of purchase cost c i.e. c=5, 15, 25, value of q increases but the value of reorder quantity *r* decreases and hence average cost increases. Similarly when inflation rate R is increased for various values of c, we find that average cost increases.

(ii) We have also conducted Sensitivity analysis by varying the value of inflation rate *R* and length of ON period λ keeping other parameter values fixed. Inflation rate *R* is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=1 i.e. account is not settled at time period T_0 and the length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of *q*, *r* and *AC*. The optimal values of *AC* and *R* are plotted in Fig. 2.5.2.

Table 2.5.2
Sensitivity Analysis Table by varying the parameter values of *R* & λ

	R	q	r	AC
λ=0.1	0.05	16.484	6.3674	212.413
	0.1	14.8736	6.97477	281.261
	0.15	13.4272	7.5369	373.613
	0.2	12.1265	8.05632	497.633
	0.25	10.9554	8.53512	664.334
	0.3	9.90025	8.97578	888.575
λ=0.15	0.05	16.3861	10.2489	237.146
	0.1	14.7843	10.8563	314.666
	0.15	13.3458	11.4184	418.729
	0.2	12.0524	11.9375	558.563
	0.25	10.8881	12.4159	746.617
	0.3	9.83904	12.8561	999.691
λ=0.2	0.05	16.2908	12.9557	254.004
	0.1	14.6973	13.5631	337.439
	0.15	13.2665	14.1251	449.492
	0.2	11.9802	14.644	600.117
	0.25	10.8224	15.122	802.745
	0.3	9.77939	15.5616	1075.5

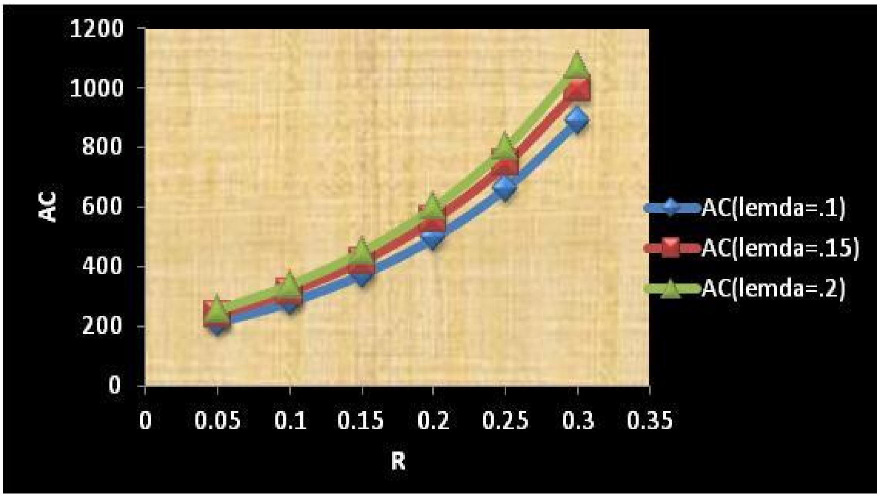

Fig. 2.5.2 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying length of ON period

We see that as inflation rate R increases and λ increases i.e. expected length of ON period decreases, value of q decreases to a smaller extent but the value of reorder quantity r increases to a larger extent and hence average cost increases.

(iii) We have also conducted Sensitivity analysis by varying the value of inflation rate R and holding cost keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=1 i.e. account is not settled at time period T_0 and the holding cost h is assumed to take values 5, 15, 20. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.3.

Table 2.5.3

Sensitivity Analysis Table by varying the parameter values of *R* & *h*

	R	q	r	AC
h=5	0.05	16.198	15.02	266.575
	0.1	14.6125	15.6275	354.425
	0.15	13.1892	16.1894	472.443
	0.2	11.9098	16.708	631.126
	0.25	10.7584	17.1857	844.637
	0.3	9.72122	17.6249	1132.09
h=15	0.05	11.1341	6.23921	479.323
	0.1	10.0597	6.69113	639.231
	0.15	9.09146	7.10621	854.276
	0.2	8.21817	7.48688	1143.67
	0.25	7.43008	7.83553	1533.32
	0.3	6.71855	8.15446	2058.21
h=20	0.05	10.1017	3.8934	545.393
	0.1	9.12928	4.30994	727.643
	0.15	8.25229	4.69198	972.76
	0.2	7.46088	5.04189	1302.677
	0.25	6.7463	5.362	1746.92
	0.3	6.10101	5.65451	2345.4

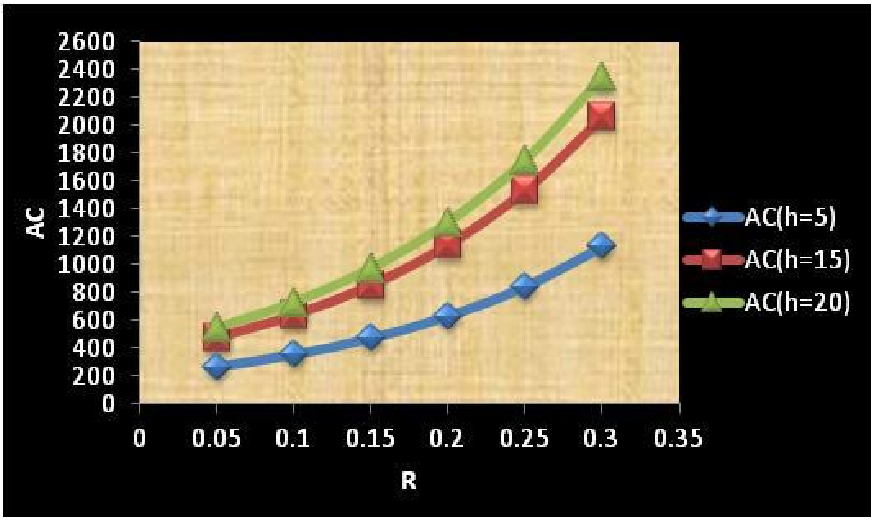

Fig. 2.5.3 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate *R* and for varying holding cost

We see that as inflation rate *R* increases and holding cost h increases, value of *q* as well as the value of reorder quantity r decreases, but average cost increases.

(iv) We have also conducted Sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed. Inflation rate *R* is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=1 i.e. account is not settled at time period T_0 and the length of OFF period μ is assumed to take values 3.5, 4.5, 5.5. We resolve the problem to find optimal values of *q*, *r* and *AC*. The optimal values of *AC* and *R* are plotted in Fig. 2.5.4.

Table 2.5.4
Sensitivity Analysis Table by varying the parameter values of *R* & μ

	R	q	r	AC
μ=3.5	0.05	8.437622	12.13917	204.1676
	0.1	7.153977	12.67328	269.7916
	0.15	6.124131	13.11492	358.0273
	0.2	5.28179	13.48485	476.7953
	0.25	4.581602	13.7983	636.7777
	0.3	3.991702	14.06654	852.3858
μ=4.5	0.05	7.526421	9.132862	172.0836
	0.1	6.4529	9.56431	226.3303
	0.15	5.577113	9.92831	299.2271
	0.2	4.849717	10.2389	397.3014
	0.25	4.236719	10.50649	529.3563
	0.3	3.713965	10.73892	707.2677
μ=5.5	0.05	6.906468	7.224054	151.4933
	0.1	5.963587	7.590872	198.4088
	0.15	5.18608	7.904698	261.4186
	0.2	4.533863	8.175977	346.1517
	0.25	3.979334	8.412433	460.1992
	0.3	3.502762	8.619933	613.8017

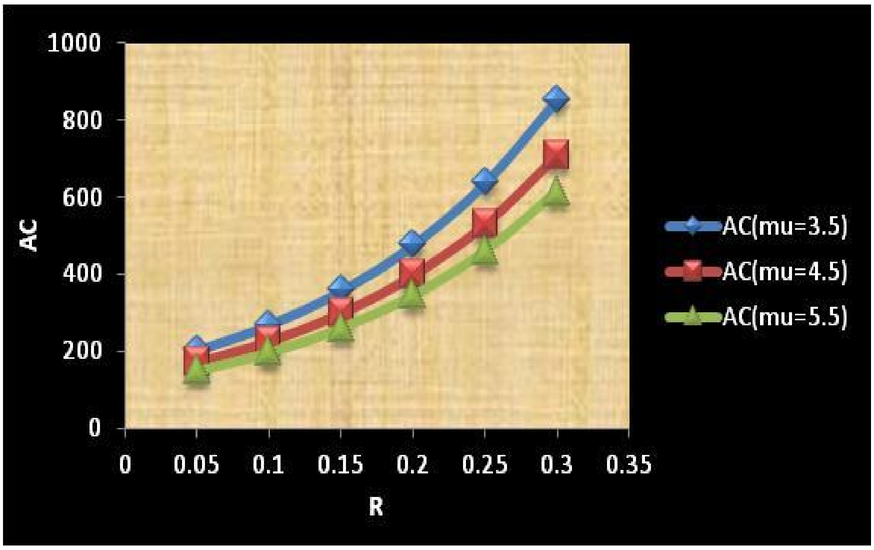

Fig. 2.5.4 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying length of OFF period

We see that as inflation rate R increases and μ increases i.e. expected length of OFF period decreases, value of q decreases and the value of reorder quantity r also decreases, as a consequence average cost also decreases.

Case-II: Taking δ=0 i.e. account is settled at time period T_0.

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and purchase cost c keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=0 i.e. account is settled at time period T_0 and the purchase cost c is assumed to take values 5, 15, 25. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.5.

Table 2.5.5
Sensitivity Analysis Table by varying the parameter values of *R* & *c*

	R	q	r	AC
c=5	0.05	18.56644	14.14799	260.3604
	0.1	16.18106	15.02638	348.3072
	0.15	14.18595	15.79423	466.0568
	0.2	12.49104	16.47081	624.0147
	0.25	11.0338	17.07047	836.2077
	0.3	9.769292	17.60432	1121.553
c=15	0.05	25.87896	11.6108	311.7744
	0.1	20.60682	13.33029	424.5098
	0.15	16.80065	14.6943	574.0795
	0.2	13.92145	15.79855	773.703
	0.25	11.67276	16.70569	1041.066
	0.3	9.874643	17.45956	1399.94
c=25	0.05	38.99424	7.968189	356.6691
	0.1	27.59133	10.99124	497.1973
	0.15	20.60244	13.23097	680.3026
	0.2	15.84982	14.95113	922.6037
	0.25	12.46741	16.27976	1245.682
	0.3	9.994619	17.30766	1678.288

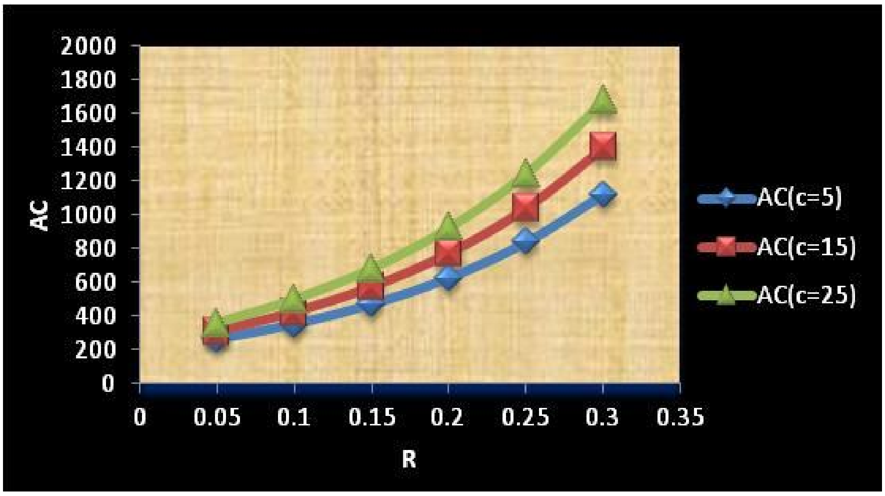

Fig. 2.5.5 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying purchase cost *c*

We see that as inflation rate *R* increases and purchase cost c increases, value of q increases but the value of reorder quantity *r* decreases and hence average cost increases.

(ii) We have also conducted Sensitivity analysis by varying the value of inflation rate *R* and length of ON period λ keeping other parameter values fixed. Inflation rate *R* is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=0 i.e. account is settled at time period T_0 and the length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of *q*, *r* and *AC*. The optimal values of *AC* and *R* are plotted in Fig. 2.5.6.

Table 2.5.6
Sensitivity Analysis Table by varying the parameter values of *R* & λ

	R	q	r	AC
λ=0.1	0.05	18.91048	5.488464	206.4616
	0.1	16.54903	6.343288	275.7315
	0.15	14.57433	7.08979	368.3015
	0.2	12.89612	7.747436	492.3067
	0.25	11.45203	8.330734	658.7104
	0.3	10.1972	8.850874	882.295
λ=0.15	0.05	18.79284	9.37233	231.1044
	0.1	16.42319	10.23511	308.9339
	0.15	14.44147	10.98883	413.0475
	0.2	12.75745	11.65294	552.6226
	0.25	11.30862	12.24187	740.0292
	0.3	10.05017	12.76677	991.9486
λ=0.2	0.05	18.67823	12.0813	247.874
	0.1	16.30061	12.95201	331.5106
	0.15	14.312	13.7129	443.4523
	0.2	12.62245	14.3832	593.582
	0.25	11.16925	14.97765	795.2222
	0.3	9.90748	15.5072	1066.338

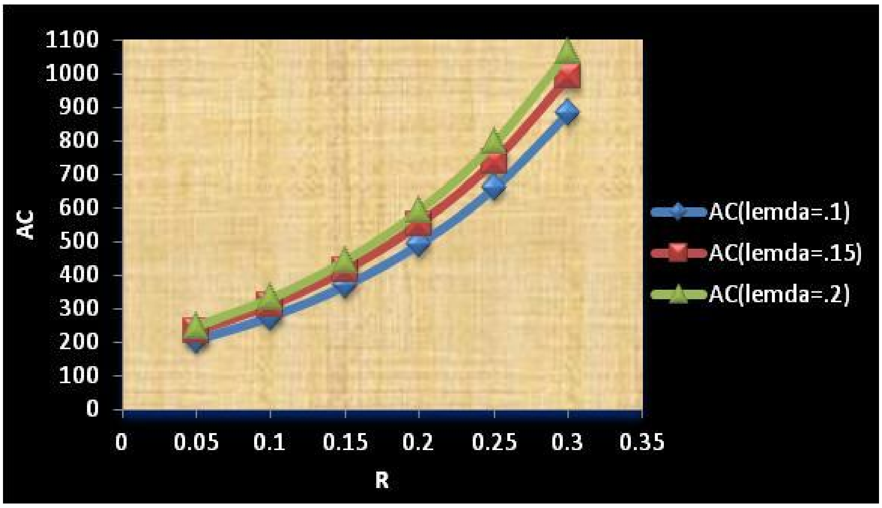

Fig. 2.5.6 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying length of ON period

We see that as inflation rate R increases and λ increases i.e. expected length of ON period decreases, value of q decreases to a smaller extent but the value of reorder quantity r increases to a larger extent and hence average cost increases.

(iii) We have also conducted Sensitivity analysis by varying the value of inflation rate R and holding cost keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=0 i.e. account is settled at time period T_0 and the holding cost h is assumed to take values 5, 15, 20. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.7.

Table 2.5.7
Sensitivity Analysis Table by varying the parameter values of *R* & *h*

	R	q	r	AC
h=5	0.05	18.56644	14.14799	260.3604
	0.1	16.18106	15.02638	348.3072
	0.15	14.18595	15.79423	466.0568
	0.2	12.49104	16.47081	624.0147
	0.25	11.0338	17.07047	836.2077
	0.3	9.769292	17.60432	1121.553
h=15	0.05	7.030882	11.97079	381.3605
	0.1	6.040024	12.41903	508.4446
	0.15	5.227961	12.79203	679.6592
	0.2	4.552189	13.10624	910.4462
	0.25	3.982188	13.37394	1221.642
	0.3	3.495849	13.60424	1641.367
h=20	0.05	11.83737	5.948259	475.0154
	0.1	10.52584	6.49393	634.6434
	0.15	9.382288	6.980734	849.05
	0.2	8.378483	7.41655	1137.362
	0.25	7.492708	7.807643	1525.319
	0.3	6.707541	8.15942	2047.697

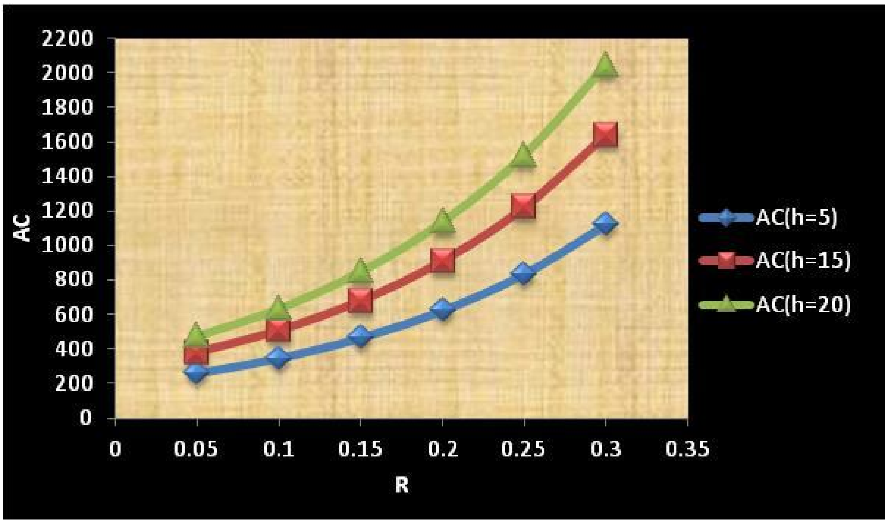

Fig. 2.5.7 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate *R* and for varying holding cost

We see that as inflation rate R increases and holding cost h increases, value of q as well as the value of reorder quantity r decreases but the average cost increases.

(iv) We have also conducted Sensitivity analysis by varying the value of inflation rate R and length of OFF period μ keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=0 i.e. account is settled at time period T_0 and the length of OFF period μ is assumed to take values 3.5, 4.5, 5.5. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.8.

Table 2.5.8
Sensitivity Analysis Table by varying the parameter values of *R* & μ

	R	q	r	AC
μ=3.5	0.05	18.67569	8.462394	233.5384
	0.1	15.82504	9.371443	313.8484
	0.15	13.5768	10.14325	420.7643
	0.2	11.74742	10.80899	563.6429
	0.25	10.22504	11.38977	755.0646
	0.3	8.937003	11.90059	1011.973
μ=4.5	0.05	15.13281	6.511063	175.8006
	0.1	13.29046	7.08039	232.7234
	0.15	11.73491	7.592297	308.5391
	0.2	10.40158	8.055119	409.8156
	0.25	9.245652	8.475048	545.4002
	0.3	8.234697	8.856968	727.2189
μ=5.5	0.05	14.20465	4.849949	156.7112
	0.1	12.48422	5.341539	206.4619
	0.15	11.03189	5.787196	272.5594
	0.2	9.787081	6.193134	360.6749
	0.25	8.707786	6.563986	478.4458
	0.3	7.763559	6.903423	636.164

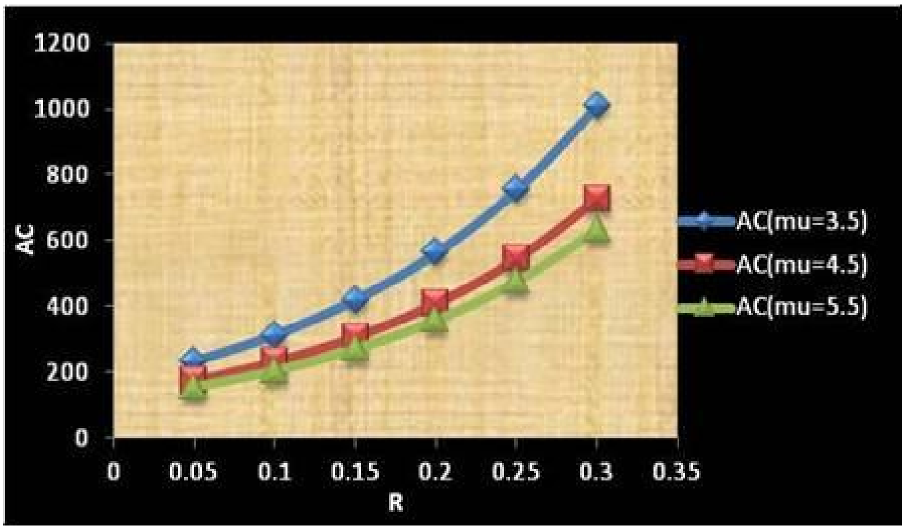

Fig. 2.5.8 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying length of OFF period

We see that as inflation rate R increases and μ increases i.e. expected length of OFF period decreases, value of q decreases and the value of reorder quantity r also decreases as a consequence average cost also decreases.

2.6. CONCLUSION:

In this chapter, on comparing the average cost value for various sensitivity analysis done by varying the various parameter values, we find that the cost is minimum if payment is done at T_0 i.e. account is settled at time period T_0 which is credit period given by supplier. This implies that we encourage the small businessmen to do the business by giving them a loan and simultaneously we want to discourage them from not clearing the account at the end of credit period.

CHAPTER 3

STOCHASTIC INVENTORY MODEL UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENT ALLOWING PARTIAL PAYMENT FOR SINGLE SUPPLIER

CHAPTER 3

3.1. INTRODUCTION:

In this chapter, we have introduced the aspect of part payment. A part of the purchased cost is to be paid during the permissible delay period. What quantity of the part is to be paid and the time at which it has to be paid can be fixed up at the time of the deal of purchasing the goods.

3.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model under inflation and permissible delay in payment allowing partial payment for single supplier is developed on the basis of the following assumptions.

(a) T_1 is the time at which α $(0< \alpha <1)$ fraction of total amount has to be paid.

(b) T $(T> T_1)$ is the time at which remaining amount has to be cleared.

(c) T_{00} is the expected cycle time. T_1 and T are known constants and T_{00} is a decision variable.

(d) U and V are indicator variables where

U= 0 if part payment is done at T_1

=1 otherwise

V= 0 if the balanced amount is cleared at T

=1 otherwise

In this chapter, we assume that supplier allows a fixed period T_1 during which α fraction of total amount has to be paid and at time T remaining amount has to be cleared. Hence up to time period T_1 no interest is charged for α fraction, but beyond that period, interest will be charged upon not doing promised payment of α fraction. Similarly for

(1- α) fraction no interest will be charged up to time period T but beyond that period interest will be charged. However, customer can sell the goods and earn interest on the sales revenue during the period of admissible delay.

Interest earned and interest charged is as follows.

(i) Interest earned on the entire amount up to time period T_1 is $dce^{Rt_1}T_{00}T_1 Ie$

(ii) Interest earned on (1-α) fraction during the period (T-T_1) is

$$(1-\alpha)\,d\,c\,e^{Rt_1}(T-T_1)T_{00}\;Ie$$

(iii) If part payment is not done at T_1 then interest will be earned over α fraction for period (T-T_1) but interest will also be charged for α fraction for (T-T_1) period.

Interest earned= $d\,c\,e^{Rt_1}\;\alpha\,(T-T_1)T_{00}\;Ie$

Interest charged= $d\,c\,e^{Rt_1}\;\alpha\,(T-T_1)T_{00}\;Ic$

To discourage not doing promised payment, we assume that Ic is quite larger than Ie.

(iv) Interest earned over the amount $dce^{Rt_1}T_{00}T_1 Ie$ over the period (T-T_1) is

$$d\,c\,e^{Rt_1}\;T_{00}\,T_1\,Ie\;\;(T-T_1)\,Ie$$

(v) If the remaining amount is not cleared at T then interest will be earned for the period (T_{00}-T) for (1-α) fraction. Simultaneously interest will be charged on the same amount for the same period.

Interest earned= $(1-\alpha)\,d\,c\,e^{Rt_1}(T_{00}-T)\,T_{00}\;Ie$

Interest charged= $(1-\alpha)\,d\,c\,e^{Rt_1}(T_{00}-T)\,T_{00}\;Ic$

Total interest earned and charged is as follows.

$$\begin{aligned}
&dce^{Rt_1}T_{00}T_1 Ie+(1-\alpha\;)\,d\,c\,e^{Rt_1}(T-T_1)T_{00}\;Ie+\{d\,c\,e^{Rt_1}\,\alpha(T-T_1)T_{00}\;Ie\\
&-d\,c\,e^{Rt_1}\;\alpha(T-T_1)T_{00}\;Ic\}+d\,c\,e^{Rt_1}\;T_{00}\,T_1\,Ie(T-T_1)\,Ie\\
&+V\,[(1-\alpha)\,d\,c\,e^{Rt_1}(T_{00}-T)T_{00}\;Ie+\;d\,c\,e^{Rt_1}\;T_{00}\,T_1\,Ie(T-T_1)\,Ie(T_{00}-T)\,Ie\\
&+d\,c\,e^{Rt_1}\;T_{00}\,T_1 Ie(T_{00}-T)\,Ie+(1-\alpha)\,d\,c\,e^{Rt_1}T_{00}\,(T-T_1)Ie(T_{00}-T)\,Ie\\
&+\{\,d\,c\,e^{Rt_1}\;\alpha\,T_{00}\;Ie(T-T_1)\,Ie\;-d\,c\,e^{Rt_1}\;\alpha(T_{00}-T)\,T_{00}\;Ic\}\\
&-(1-\alpha)\,d\,c\,e^{Rt_1}(T_{00}-T)\,T_{00}\;Ic]
\end{aligned}$$

3.3. OPTIMAL POLICY DECISION FOR THE MODEL:

We use the same policy as discussed in chapter 2.

Lemma 3.3.1: $C_{10}(r)$ = expected cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle is obtained as

$$C_{10}(r)=\frac{1}{\mu^2}e^{\frac{-\mu r}{(d+\theta)}}\ e^{Rt_1}\left[he^{\frac{\mu r}{d+\theta}}\left(\mu r-(d+\theta)\right)+\pi\mu d+h(d+\theta)+\hat{\pi}-\theta c\mu\right]+\frac{\theta c e^{Rt_1}}{\mu}.$$

Then,

$$\begin{aligned}C_{00} &= A(q,r,\theta)+P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r)-dce^{Rt_1}T_{00}T_1Ie-(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\\ &-Udce^{Rt_1}\alpha T_{00}(T-T_1)Ie+Udce^{Rt_1}\alpha T_{00}(T-T_1)Ic-dce^{Rt_1}T_{00}\,T_1Ie(T-T_1)Ie\\ &-V[(1-\alpha)dce^{Rt_1}T_{00}(T_{00}-T)Ie+dce^{Rt_1}T_{00}\,T_1Ie(T-T_1)Ie(T_{00}-T)Ie\\ &+dce^{Rt_1}T_{00}\,T_1Ie(T_{00}-T)Ie+(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\ (T_{00}-T)Ie]\\ &-V[U\ \{dce^{Rt_1}\alpha T_{00}\,Ie(T-T_1)(T_{00}-T)Ie\}]\\ &+V[U\{dce^{Rt_1}\alpha T_{00}\,Ic(T_{00}-T)+(1-\alpha)dce^{Rt_1}T_{00}\,Ic(T_{00}-T)\}]\end{aligned}$$

$$P_{01}\left(\frac{q}{d+\theta}\right)=P_1-P_1e^{\frac{-(\lambda+\mu)q}{d+\theta}},\ \text{and}\ P_1=\frac{\lambda}{\lambda+\mu}.$$

Proof: Conditioning on the state of the system when inventory drops to r, we obtain

$$\begin{aligned}C_{00} &= A(q,r,\theta)+P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r)-dce^{Rt_1}T_{00}T_1Ie-(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\\ &-Udce^{Rt_1}\alpha T_{00}(T-T_1)Ie+Udce^{Rt_1}\alpha T_{00}(T-T_1)Ic-dce^{Rt_1}T_{00}\,T_1Ie(T-T_1)Ie\\ &-V[(1-\alpha)dce^{Rt_1}T_{00}(T_{00}-T)Ie+dce^{Rt_1}T_{00}\,T_1Ie(T-T_1)Ie(T_{00}-T)Ie\\ &+dce^{Rt_1}T_{00}\,T_1Ie(T_{00}-T)Ie+(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\ (T_{00}-T)Ie]\\ &-V[U\ \{dce^{Rt_1}\alpha T_{00}\,Ie(T-T_1)(T_{00}-T)Ie\}]\\ &+V[U\{dce^{Rt_1}\alpha T_{00}\,Ic(T_{00}-T)+(1-\alpha)dce^{Rt_1}T_{00}\,Ic(T_{00}-T)\}]\quad\}\end{aligned}$$

This follows because when inventory drops to r, the state will be 0 (ON) with probability $P_{00}\left(\frac{q}{d+\theta}\right)$ and 1(OFF) with probability $P_{01}\left(\frac{q}{d+\theta}\right)=1-P_{00}\left(\frac{q}{d+\theta}\right)$. If the state is ON, the cost incurred is $A(q, r, \theta)$ which is weighted by the probability $P_{00}\left(\frac{q}{d+\theta}\right)$ of this event If on the other hand, the state is OFF when inventory drops to r, the expected cost is $A(q, r, \theta) + C_{10}(r)$ which is weighted by the probability $P_{01}\left(\frac{q}{d+\theta}\right)$ of the corresponding event. The transition probability $P_{00}\left(\frac{q}{d+\theta}\right)$ and $P_{01}\left(\frac{q}{d+\theta}\right)$ are obtained by CTMC, Bhat, U.N.[1984] for these states. They are given by

$$P_{00}\left(\frac{q}{d+\theta}\right) = P_0 + P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}},\ P_{01}\left(\frac{q}{d+\theta}\right) = P_1 - P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}},$$

$$P_1 = 1 - P_0 = \frac{\lambda}{\lambda+\mu}, \text{ and } P_0 = \frac{\mu}{\lambda+\mu}$$

which are the steady state probabilities for the OFF and ON states respectively. Now, referring to Figure 3.1, of chapter 2 the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle is equal to

$$\frac{1}{2}hy^2(d+\theta)e^{Rt_1} + hy[r-(d+\theta)]e^{Rt_1} + \theta cye^{Rt_1}, \qquad y < \frac{r}{d+\theta}$$

$$\frac{1}{2}\cdot\frac{hr^2e^{Rt_1}}{(d+\theta)} + \pi e^{Rt_1}\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}e^{Rt_1}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta cre^{Rt_1}}{(d+\theta)}, \quad y \geq \frac{r}{d+\theta}$$

so that

$$C_{10}(r) = \int_0^{r/(d+\theta)} \left\{\frac{1}{2}hy^2(d+\theta)e^{Rt_1} + he^{Rt_1}y\left(r - y(d+\theta)\right) + \theta ce^{Rt_1}y\right\}\mu e^{-\mu y}dy.$$

$$+ \int_{r/(d+\theta)}^{\infty} \left\{\frac{1}{2}\frac{hr^2e^{Rt_1}}{(d+\theta)} + \pi e^{Rt_1}\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}e^{Rt_1}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta cre^{Rt_1}}{(d+\theta)}\right\}\mu e^{-\mu y}dy$$

$$= \frac{1}{\mu^2} e^{\frac{-\mu r}{d+\theta}} e^{Rt_1} \left[he^{\frac{\mu r}{d+\theta}} \left(\mu r - (d+\theta)\right) + \pi\mu d + h(d+\theta) + \hat{\pi} - \theta c\mu \right] + \frac{\theta c e^{Rt_1}}{\mu}.$$

Proposition 3.3.2: The Average cost objective function under inflation and permissible delay in payments allowing partial payment is given by

$AC(q,r,\theta) = \dfrac{C_{00}}{T_{00}}$, where T_{00} is the same expression as in lemma (2.3.2) of chapter 2.

C_{00} is given by

$$\begin{aligned}
C_{00} &= A(q,r,\theta) + P_{01}\left(\frac{q}{d+\theta}\right) C_{10}(r) - dce^{Rt_1} T_{00} T_1 Ie - (1-\alpha) dce^{Rt_1} T_{00}(T - T_1) Ie \\
&- Udce^{Rt_1} \alpha T_{00}(T - T_1) Ie + Udce^{Rt_1} \alpha T_{00}(T - T_1) Ic - dce^{Rt_1} T_{00} T_1 Ie(T - T_1) Ie \\
&- V[(1-\alpha) dce^{Rt_1} T_{00}(T_{00} - T) Ie + dce^{Rt_1} T_{00} T_1 Ie(T - T_1) Ie(T_{00} - T) Ie \\
&+ dce^{Rt_1} T_{00} T_1 Ie(T_{00} - T) Ie + (1-\alpha) dce^{Rt_1} T_{00}(T - T_1) Ie \ (T_{00} - T) Ie] \\
&- V[U \ \{dce^{Rt_1} \alpha T_{00} \ Ie(T - T_1)(T_{00} - T) Ie\}] \\
&+ V[U\{dce^{Rt_1} \alpha T_{00} \ Ic(T_{00} - T) + (1-\alpha) dce^{Rt_1} T_{00} \ Ic(T_{00} - T)\}] \quad \}
\end{aligned}$$

Proof: Proof follows using Renewal Reward Theorem (RRT). The optimal solution for *q* and *r* are obtained by using Newton Rapson method in R programming.

3.4. NUMERICAL EXAMPLE:

There are four patterns of payments:

1. U=0, V=0 i.e. promise of doing part payment at time T_1 and clearing the remaining amount at time T both are satisfied.

2. U=0, V=1 i.e. promise of doing part payment at time T_1 is satisfied but remaining amount is not cleared at time *T*.

3. U=1, V=0 i.e. part payment is not done at time T_1 but all the amount is cleared at time *T*.

4. U=1, V=1 i.e. part payment is not done at time T_1 and also the amount is not cleared at time *T*.

Case-I: Inflation rate is less than interest charged.

In this section we verify the results by a numerical example. We assume that k = Rs. 10/order, c = Rs.5/unit, d = 20/units, h = Rs. 5/unit/time, π = Rs. 250/unit, $\hat{\pi}$ =Rs. 25/unit/time, θ=5/unit/time, Ic = 0.15, Ie = 0.08, T_1 = 0.3, T=0.6, α=0.5, R = 0.05, t_1 = 6, λ = 0.25, μ = 2.5. The last two parameters indicate that the expected lengths of the ON and OFF periods are $1/\lambda$ = 4, and $1/\mu$ = 0.4 respectively. The long run probabilities are obtained as P_0 = 0.909 and P_1 = 0.091.

The optimal solution for the above numerical example based on the above four patterns of payment is obtained as

Patterns	q	r	AC
U=0,V=0	16.19804	15.01994	261.6373
U=0,V=1	17.8344	14.41302	260.9979
U=1,V=0	16.19804	15.01994	263.0547
U=1,V=1	13.2388	16.16954	263.247

Conclusion:

From the above numerical example we conclude that cost is minimum if part payment is done at T_1 but account is not cleared at T and the cost is maximum if part payment is not done at T_1 and also account is not cleared at T, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

Case-II: Inflation rate is greater than interest charged.

In this section we verify the results by a numerical example. We assume that k = Rs. 10/order, c = Rs.5/unit, d = 20/units, h = Rs. 5/unit/time, π = Rs. 250/unit, $\hat{\pi}$ =Rs. 25/unit/time, θ=5/unit/time, Ic = 0.15, Ie = 0.08, T_1 = 0.3, T=0.6, α=0.5, R = 0.35, t_1 = 6, λ = 0.25, μ = 2.5. The last two parameters indicate that the expected lengths of the ON and OFF periods are $1/\lambda$ = 4, and $1/\mu$ = 0.4 respectively. The long run probabilities are obtained as P_0 = 0.909 and P_1 = 0.091.

The optimal solution for the above numerical example based on the above four patterns of payment is obtained as

Patterns	q	r	AC
U=0,V=0	8.786195	18.02806	1489.404
U=0,V=1	10.49453	17.29658	1496.151
U=1,V=0	8.786195	18.02806	1497.978
U=1,V=1	6.289428	19.13815	1473.322

Conclusion:

In this case we observe that average cost is minimum if part payment is not done at T_1 and also account is not cleared at T which implies that businessmen are advised not to settle the account at the end of the credit period but settle the account at the end of the cycle period. The reason for this is once the inflation rate is greater than the interest rates charged, we actually see our debt wiped out by inflation.

3.5. SENSITIVITY ANALYSIS:

We study below in the Sensitivity analysis, the effect of change in the parameter on the following four patterns of payment.

Case-I: Inflation rate is less than interest charged.

3.5.1. Sensitivity Analysis for λ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=0 and V=0. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.1.1
Sensitivity Analysis Table by varying the parameter values of *R* & *λ* when pattern of payment is (U=0, V=0)

	R	q	r	AC
λ=0.1	0.03	17.1787	6.111374	185.6628
	0.05	16.48399	6.36741	207.4757
	0.08	15.49699	6.73732	245.374
	0.1	14.87357	6.97477	274.596
	0.13	13.9873	7.31738	325.4052
λ=0.15	0.03	17.0773	9.99281	209.2525
	0.05	16.38612	10.24894	232.2091
	0.08	15.40427	10.61895	277.2296
	0.1	14.7842	10.85633	308.0017
	0.13	13.90276	11.19893	368.4761
λ=0.2	0.03	16.9786	12.69945	224.1986
	0.05	16.29084	12.95568	249.0663
	0.08	15.31407	13.3257	295.1825
	0.1	14.69726	13.56314	330.7744
	0.13	13.82053	13.90564	392.6875

From the above table we see that when both the promises are fulfilled of doing payment, average cost increases when inflation rate R increases and λ increases.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of ON period *λ* keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of ON period *λ* is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.1.2
Sensitivity Analysis Table by varying the parameter values of *R* & *λ* when pattern of payment is (U=0, V=1)

	R	q	r	AC
λ=0.1	0.03	18.81541	5.5221	184.985
	0.05	18.11683	5.77123	206.874
	0.08	17.12575	6.13076	244.926
	0.1	16.50092	6.36113	274.293
	0.13	15.61408	6.693096	325.377
λ=0.15	0.03	18.03791	9.642713	208.873
	0.05	18.02028	9.64908	231.594
	0.08	16.35819	10.25937	277.004
	0.1	16.41433	10.23841	307.679
	0.13	14.85316	10.82981	368.4328
λ=0.2	0.03	17.93968	12.34732	223.8133
	0.05	17.9261	12.35221	248.4391
	0.08	16.9475	12.7111	294.707
	0.1	16.3299	12.9414	330.4343
	0.13	15.4543	13.27214	392.614

We see that as inflation rate R increases and λ increases average cost increases, when promise of doing part payment at time T_1 is satisfied but remaining amount is not cleared at time T.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=1 and V=0. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.1.3
Sensitivity Analysis Table by varying the parameter values of *R* & *λ* when pattern of payment is (U=1, V=0)

	R	q	r	AC
λ=0.1	0.03	17.1787	6.111374	186.9198
	0.05	16.48399	6.36741	208.8931
	0.08	15.4969	6.737318	247.066
	0.1	14.87357	6.97477	276.5097
	0.13	13.9873	7.31738	327.6958
λ=0.15	0.03	17.0773	9.99281	211.0484
	0.05	16.38612	10.24894	233.6265
	0.08	15.40427	10.6189	279.6537
	0.1	14.78426	10.85633	309.9149
	0.13	13.90276	11.19893	371.7051
λ=0.2	0.03	16.9786	12.69945	225.9945
	0.05	16.29084	12.95568	250.4836
	0.08	15.31407	13.3257	296.8794
	0.1	14.69726	13.56314	332.6876
	0.13	13.8205	13.90564	394.9781

In this situation also average cost increases when inflation rate R increases and λ increases, when promise of doing part payment at time T_1 is not satisfied but remaining amount is cleared at time *T*.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of ON period *λ* keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of ON period *λ* is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.1.4
Sensitivity Analysis Table by varying the parameter values of *R* & λ when pattern of payment is (U=1, V=1)

	R	q	r	AC
λ=0.1	0.03	14.1953	7.23641	187.3716
	0.05	13.53545	7.494338	209.0099
	0.08	12.59908	7.86609	246.5389
	0.1	12.00875	8.103967	275.4387
	0.13	11.17122	8.446078	325.6036
λ=0.15	0.03	13.71008	11.27458	211.4674
	0.05	13.43388	11.38354	233.7692
	0.08	12.13923	11.90223	278.902
	0.1	11.91404	11.99377	308.8793
	0.13	10.73621	12.47908	369.0934
λ=0.2	0.03	13.6082	13.98946	226.4385
	0.05	13.33503	14.09784	250.6515
	0.08	12.4068	14.4704	296.4133
	0.1	11.82176	14.70873	331.6866
	0.13	10.99174	15.0514	392.9696

When both the promises of doing payment are not satisfied, impact of increase in inflation rate R and λ results in increase in average cost.

3.5.2. Sensitivity Analysis for μ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=0 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.2.1
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=0, V=0)

	R	q	r	AC
μ=0.5	0.03	25.64692	84.83821	819.447
	0.05	24.63514	85.3299	922.9856
	0.08	23.19264	86.03272	1103.495
	0.1	22.27871	86.47913	1243.159
	0.13	20.97547	87.11686	1486.665
μ=1.5	0.03	19.46766	27.08709	344.3221
	0.05	18.69011	27.41263	386.6923
	0.08	17.58339	27.88095	460.4715
	0.1	16.88332	28.18018	517.494
	0.13	15.88654	28.6102	616.816
μ=2.5	0.03	16.88233	14.7637	233.6852
	0.05	16.19804	15.01994	261.6373
	0.08	15.22611	15.3901	310.2421
	0.1	14.61246	15.62752	347.7608
	0.13	13.74027	15.96999	413.0359

When both the promises of doing payment are fulfilled by the businessman we find that as inflation rate R increases and μ increases, average cost decreases. This may be because unavailability of supplier is for less period of time and hence it is not necessary to stock more items which results in decrease in average cost.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.2.2
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=0, V=1)

	R	q	r	AC
μ=0.5	0.03	36.55733	79.61103	814.1644
	0.05	35.7292	80.00257	917.2922
	0.08	34.57371	80.55032	1097.127
	0.1	33.85915	80.88998	1236.297
	0.13	32.86669	81.36282	1478.994
μ=1.5	0.03	22.24361	25.94828	342.7546
	0.05	21.47833	26.25856	385.1115
	0.08	20.39274	26.70352	458.8971
	0.1	19.70851	26.9868	515.9462
	0.13	18.7378	27.39258	615.3507
μ=2.5	0.03	18.52109	14.1643	232.9749
	0.05	17.8344	14.41302	260.9979
	0.08	16.86092	14.77166	309.7512
	0.1	16.24746	15.00135	347.4029
	0.13	15.37742	15.33197	412.9406

We observe that when promise of doing part payment at T_1 is satisfied but clearing the remaining amount at T is not fulfilled, impact of increase in inflation rate R and μ results in decrease in average cost.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.2.3
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=1, V=0)

	R	q	r	AC
μ=0.5	0.03	25.6469	84.83821	820.704
	0.05	24.63514	85.3299	924.403
	0.08	23.19264	86.03272	1105.192
	0.1	22.27871	86.47913	1245.072
	0.13	20.97547	87.11686	1488.955
μ=1.5	0.03	19.46766	27.08709	345.5792
	0.05	18.69011	27.41263	388.1097
	0.08	17.58339	27.88095	462.1684
	0.1	16.88332	28.18018	519.4073
	0.13	15.88654	28.6102	619.1065
μ=2.5	0.03	16.88233	14.7637	234.9423
	0.05	16.19804	15.01994	263.0547
	0.08	15.22611	15.3901	311.939
	0.1	14.6124	15.62752	349.674
	0.13	13.74027	15.96999	415.3264

We see that as inflation rate R increases and μ increases, average cost decreases when promise of doing part payment at T_1 is not satisfied but clearing the remaining amount at *T* is fulfilled.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.2.4
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=1, V=1)

	R	q	r	AC
μ=0.5	0.03	14.32158	90.39558	826.235
	0.05	13.5173	90.79397	929.952
	0.08	12.39144	91.35254	1110.664
	0.1	11.69119	91.70023	1250.405
	0.13	10.71192	92.18692	1493.917
μ=1.5	0.03	14.87902	29.04959	347.4107
	0.05	14.14924	29.3708	389.7167
	0.08	13.11527	29.83011	463.3103
	0.1	12.46443	30.12178	520.1362
	0.13	11.54302	30.53792	619.0256
μ=2.5	0.03	13.89057	15.91055	235.4601
	0.05	13.2388	16.16954	263.247
	0.08	12.31452	16.54256	311.5025
	0.1	11.7319	16.7811	348.7066
	0.13	10.90561	17.124	413.3583

We observe that as inflation rate R increases and μ increases, average cost decreases when both the promises of doing payment are not satisfied.

3.5.3. Sensitivity Analysis for k:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=0 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.3.1
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=0, V=0)

	R	q	r	AC
k=10	0.03	16.88233	14.7637	233.6852
	0.05	16.19804	15.01994	261.6373
	0.08	15.22611	15.3901	310.2421
	0.1	14.61246	15.62752	347.7608
	0.13	13.74027	15.96999	413.0359
k=15	0.03	19.43265	13.83951	240.2752
	0.05	18.63733	14.12252	268.499
	0.08	17.5092	14.53204	317.5309
	0.1	16.79783	14.79516	355.3481
	0.13	15.78782	15.1753	421.0925
k=20	0.03	21.49331	13.1275	246.1465
	0.05	20.60615	13.43032	274.6147
	0.08	19.34934	13.86894	324.03
	0.1	18.55767	14.15115	362.1161
	0.13	17.43471	14.5594	428.2823

For the above pattern we see that increase in inflation rate R and ordering cost k results in increase in average cost. However order quantity q increases but the reorder quantity r decreases.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.3.2
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=0, V=1)

	R	q	r	AC
k=10	0.03	18.52109	14.1643	232.9749
	0.05	17.8344	14.41302	260.9979
	0.08	16.86092	14.77166	309.7512
	0.1	16.24746	15.00135	347.4029
	0.13	15.37742	15.33197	412.9406
k=15	0.03	21.08817	13.26511	239.0319
	0.05	20.28637	13.54083	267.2845
	0.08	19.15108	13.93917	316.3951
	0.1	18.43626	14.19484	354.2939
	0.13	17.42327	14.56361	420.2155
k=20	0.03	23.17008	12.57033	244.4731
	0.05	22.27303	12.866	272.9367
	0.08	21.00418	13.29378	322.3759
	0.1	20.20617	13.56868	360.502
	0.13	19.07622	13.96576	426.7775

We see that as inflation rate R increases and k increases, value of q increases and the value of reorder quantity r decreases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.3.3
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=1, V=0)

	R	q	r	AC
k=10	0.03	16.88233	14.7637	234.9423
	0.05	16.19804	15.01994	263.0547
	0.08	15.22611	15.3901	311.939
	0.1	14.6124	15.62752	349.674
	0.13	13.74027	15.96999	415.3264
k=15	0.03	19.43265	13.83951	241.5323
	0.05	18.63733	14.12252	269.9164
	0.08	17.5092	14.53204	319.2278
	0.1	16.79783	14.79516	357.2613
	0.13	15.78782	15.1753	423.3831
k=20	0.03	21.49331	13.1275	247.4036
	0.05	20.60615	13.43032	276.0321
	0.08	19.34934	13.86894	325.7276
	0.1	18.55767	14.15115	364.0294
	0.13	17.43471	14.5594	430.5728

Here we see that impact of increase in inflation rate R and ordering cost k, results in increase in average cost.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.3.4
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=1, V=1)

	R	q	r	AC
k=10	0.03	13.89057	15.91055	235.4601
	0.05	13.2388	16.16954	263.247
	0.08	12.31452	16.54256	311.5025
	0.1	11.7319	16.7811	348.7066
	0.13	10.90561	17.124	413.3583
k=15	0.03	16.32302	14.97289	243.3403
	0.05	15.56404	15.26058	271.5019
	0.08	14.48802	15.67602	320.3556
	0.1	13.81005	15.94238	357.9842
	0.13	12.84854	16.32624	423.3143
k=20	0.03	18.28978	14.24768	250.2498
	0.05	17.44311	14.55632	278.7377
	0.08	16.24353	15.00282	328.1113
	0.1	15.48803	15.28964	366.1084
	0.13	14.417	15.70376	432.0264

In this situation also increase in inflation rate R and ordering cost k, results in increase in q and decrease in r which results in increase in average cost.

3.5.4. Sensitivity Analysis for θ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=0 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of *q, r* and *AC*.

Table 3.5.4.1
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=0, V=0)

	R	q	r	AC
θ=5	0.03	16.88233	14.7637	233.6852
	0.05	16.19804	15.01994	261.6373
	0.08	15.22611	15.3901	310.2421
	0.1	14.61246	15.62752	347.7608
	0.13	13.74027	15.96999	413.0359
θ=7	0.03	17.75518	15.27922	256.1078
	0.05	17.03662	15.55068	286.8706
	0.08	16.01591	15.9426	340.3739
	0.1	15.37125	16.1939	381.6815
	0.13	14.45485	16.55638	453.5606
θ=10	0.03	19.02515	15.95638	289.0128
	0.05	18.25677	16.25005	323.9032
	0.08	17.16488	16.67385	384.6002
	0.1	16.47531	16.94548	431.4721
	0.13	15.49453	17.3371	513.0501

We observe that with increase in inflation rate R and increase in rate of deterioration θ, average cost increases even if both the promises of doing payment are fulfilled.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.4.2
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=0, V=1)

	R	q	r	AC
θ=5	0.03	18.52109	14.1643	232.9749
	0.05	17.8344	14.41302	260.9979
	0.08	16.86092	14.77166	309.7512
	0.1	16.24746	15.00135	347.4029
	0.13	15.37742	15.33197	412.9406
θ=7	0.03	19.38058	14.67849	255.5029
	0.05	18.66003	14.9425	286.3457
	0.08	17.63801	15.32323	340.0127
	0.1	16.99379	15.56698	381.4646
	0.13	16.07963	15.91792	453.6254
θ=10	0.03	20.63381	15.35375	288.5467
	0.05	19.86382	15.64016	323.5288
	0.08	18.7713	16.05299	384.4094
	0.1	18.08221	16.31729	431.4405
	0.13	17.10397	16.69774	512.8098

We see that as inflation rate R increases and θ increases, value of q increases and the value of reorder quantity r increases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.4.3
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=1, V=0)

	R	q	r	AC
θ=5	0.03	16.88233	14.7637	234.9423
	0.05	16.19804	15.01994	263.0547
	0.08	15.22611	15.3901	311.939
	0.1	14.6124	15.62752	349.674
	0.13	13.74027	15.96999	415.3264
θ=7	0.03	17.75518	15.27922	257.3648
	0.05	17.03662	15.55068	288.2879
	0.08	16.01591	15.9426	342.0708
	0.1	15.37125	16.19395	383.5947
	0.13	14.45485	16.55638	455.8512
θ=10	0.03	19.02515	15.95638	290.2699
	0.05	18.25677	16.25005	325.3206
	0.08	17.16488	16.67385	386.2971
	0.1	16.47531	16.94548	433.3853
	0.13	15.49453	17.3371	515.3406

We see that as inflation rate R increases and θ increases, value of q and the value of reorder quantity r increase which results in increase in average cost.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.4.4
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=1, V=1)

	R	q	r	AC
θ=5	0.03	13.89057	15.91055	235.4601
	0.05	13.2388	16.16954	263.247
	0.08	12.31452	16.54256	311.5025
	0.1	11.7319	16.7811	348.7066
	0.13	10.90561	17.124	413.3583
θ=7	0.03	14.74002	16.44295	257.7117
	0.05	14.05276	16.71732	288.2974
	0.08	13.07758	17.11251	341.432
	0.1	12.46279	17.3652	382.4109
	0.13	11.59046	17.72846	453.6438
θ=10	0.03	15.97927	17.14276	290.3865
	0.05	15.24029	17.43963	325.0835
	0.08	14.19148	17.86711	385.3853
	0.1	13.52994	18.14043	431.9093
	0.13	12.59087	18.53334	513.325

We see that as inflation rate R increases and θ increases, value of q and the value of reorder quantity r increase and hence average cost increases.

Conclusion:

The comparative study of the above sensitivity analysis of case-I that is when inflation rate is less than interest charged is summarized below:
Average cost is least for pattern (U=0, V=1) and highest for pattern (U=1, V=1).

AC (U=0, V=1) < AC (U=0, V=0) < AC (U=1, V=0) < AC (U=1, V=1).
It is always beneficial to keep promises especially first one. The option of part payment is very useful for enhancing business and encouraging the small entrepreneurs.

Case-II: Inflation rate is greater than interest charged.

3.5.5. Sensitivity Analysis for λ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of ON period *λ* keeping other parameter values fixed where U=0 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of ON period *λ* is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.5.1
Sensitivity Analysis Table by varying the parameter values of *R* & *λ* when pattern of payment is (U=0, V=0)

	R	q	r	AC
λ=0.1	0.15	13.42719	7.536989	364.6168
	0.2	12.12649	8.056274	485.4891
	0.25	10.95544	8.535088	647.9415
	0.3	9.90025	8.975768	866.4483
	0.35	8.94875	9.380658	1160.54
λ=0.15	0.15	13.34581	11.41842	409.7328
	0.2	12.05238	11.93749	546.419
	0.25	10.888	12.4159	730.2247
	0.3	9.83904	12.85605	977.5636
	0.35	8.893186	13.26035	1310.584
λ=0.2	0.15	13.26658	14.12509	440.4957
	0.2	11.9801	14.64394	587.9732
	0.25	10.8224	15.1219	786.3524
	0.3	9.77936	15.56166	1053.371
	0.35	8.839035	15.96537	1412.965

We see that as inflation rate R increases and λ increases, value of q decreases and the value of reorder quantity r increases and hence average cost increases, when both the promises are fulfilled of doing payment.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of ON period λ keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.5.2
Sensitivity Analysis Table by varying the parameter values of *R* & λ when pattern of payment is (U=0, V=1)

	R	q	r	AC
λ=0.1	0.15	15.05468	6.905507	364.8249
	0.2	13.75917	7.406539	486.5188
	0.25	12.5981	7.866458	650.241
	0.3	11.55748	8.287616	870.6585
	0.35	10.6249	8.672205	1167.569
λ=0.15	0.15	14.9775	10.782	409.9142
	0.2	13.6909	11.28211	547.4114
	0.25	12.53827	11.74098	732.473
	0.3	11.5055	12.1608	981.7035
	0.35	10.58041	12.54409	1317.517
λ=0.2	0.15	14.9022	13.48393	440.651
	0.2	13.6243	13.983	588.9292
	0.25	12.47977	14.4409	788.5504
	0.3	11.45476	14.85957	1057.442
	0.35	10.53698	15.24143	1419.805

We see that as inflation rate R increases and λ increases, average cost increases when promise of doing part payment at time T_1 is satisfied but remaining amount is not cleared at time *T*.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of ON period λ keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.5.3
Sensitivity Analysis Table by varying the parameter values of *R* & λ when pattern of payment is (U=1, V=0)

	R	q	r	AC
λ=0.1	0.15	13.4272	7.536989	367.1994
	0.2	12.12649	8.056274	488.9752
	0.25	10.95545	8.535088	652.6472
	0.3	9.900255	8.975768	872.8004
	0.35	8.94875	9.38065	1169.115
λ=0.15	0.15	13.3458	11.4184	412.3154
	0.2	12.05238	11.9374	549.9051
	0.25	10.88806	12.4159	734.9305
	0.3	9.83904	12.85605	983.9157
	0.35	8.89318	13.26035	1319.158
λ=0.2	0.15	13.266	14.12509	443.0783
	0.2	11.98016	14.64394	591.4593
	0.25	10.8224	15.12197	791.0582
	0.3	9.77936	15.56166	1059.723
	0.35	8.839035	15.9653	1421.54

In this situation also average cost increases when inflation rate R increases and λ increases, when promise of doing part payment at time T_1 is not satisfied but remaining amount is cleared at time *T*.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of ON period λ keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.5.4
Sensitivity Analysis Table by varying the parameter values of *R* & λ when pattern of payment is (U=1, V=1)

	R	q	r	AC
λ=0.1	0.15	10.64318	8.664582	364.2633
	0.2	9.421672	9.178524	483.2027
	0.25	8.329594	9.64799	642.6581
	0.3	7.353962	10.07542	856.6479
	0.35	6.48322	10.46331	1144.07
λ=0.15	0.15	10.5547	12.55482	409.4269
	0.2	9.339238	13.06881	544.1954
	0.25	8.252826	13.53809	725.0229
	0.3	7.282574	13.96508	967.8677
	0.35	6.416958	14.3523	1294.246
λ=0.2	0.15	10.46866	15.27013	440.2361
	0.2	9.25895	15.78422	585.8109
	0.25	8.178065	16.2533	781.2304
	0.3	7.21297	16.67991	1043.777
	0.35	6.352394	17.06647	1396.756

When both the promises of doing payment are not satisfied, impact of increase in inflation rate R and λ, results in increase in average cost.

3.5.6. Sensitivity Analysis for μ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=0 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.6.1
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=0, V=0)

	R	q	r	AC
	0.15	20.14965	87.52194	1675.081
	0.2	18.22546	88.46774	2257.823
μ=0.5	0.25	16.48615	89.32529	3044.119
	0.3	14.91364	90.10249	4105.151
	0.35	13.49172	90.80671	5537.12
	0.15	15.25575	28.88473	693.5999
	0.2	13.78838	29.53055	930.8138
μ=1.5	0.25	12.46484	30.12157	1250.464
	0.3	11.27009	30.66194	1681.334
	0.35	10.19126	31.15538	2262.274
	0.15	13.18918	16.18941	463.4471
	0.2	11.90975	16.70801	618.9821
μ=2.5	0.25	10.75833	17.1857	828.2443
	0.3	9.72121	17.62486	1109.961
	0.35	8.786195	18.02806	1489.404

When both the promises of doing payment are fulfilled by the businessman we find that as inflation rate R increases and μ increases, average cost decreases. This may be because unavailability of supplier is for less period of time and hence it is not necessary to stock more items which result in decrease in average cost.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.6.2
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=0, V=1)

	R	q	r	AC
μ=0.5	0.15	32.25563	81.65452	1682.245
	0.2	30.89473	82.30591	2264.322
	0.25	29.75355	82.85393	3049.063
	0.3	28.80833	83.30901	4107.211
	0.35	28.0364	83.68135	5534.236
μ=1.5	0.15	18.12623	27.65052	695.41
	0.2	16.71267	28.25346	931.116
	0.25	15.45126	28.79946	1250.197
	0.3	14.32767	29.29204	1682.27
	0.35	13.32927	29.73465	2265.116
μ=2.5	0.15	14.82867	15.54353	463.5771
	0.2	13.55915	16.04184	619.9026
	0.25	12.42268	16.49856	830.3931
	0.3	11.40532	16.91599	1113.965
	0.35	10.49453	17.29658	1496.151

We observe that when promise of doing part payment at T_1 is satisfied but clearing the remaining amount at *T* is not fulfilled, impact of increase in inflation rate R and μ results in decrease in average cost.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.6.3
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=1, V=0)

	R	q	r	AC
μ=0.5	0.15	20.14965	87.52194	1677.66
	0.2	18.2254	88.46774	2261.309
	0.25	16.48615	89.32529	3048.824
	0.3	14.91364	90.10249	4111.503
	0.35	13.49172	90.80671	5545.575
μ=1.5	0.15	15.25575	28.8847	696.1825
	0.2	13.78838	29.53055	934.2999
	0.25	12.46484	30.12157	1255.169
	0.3	11.27009	30.66194	1687.686
	0.35	10.19126	31.15538	2270.848
μ=2.5	0.15	13.18918	16.18941	466.0297
	0.2	11.9097	16.70801	622.4682
	0.25	10.75833	17.1857	832.95
	0.3	9.72121	17.62486	1116.313
	0.35	8.786195	18.02806	1497.978

We see that as inflation rate R increases and μ increases, average cost decreases when promise of doing part payment at T_1 is not satisfied but clearing the remaining amount at *T* is fulfilled.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.6.4
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=1, V=1)

	R	q	r	AC
μ=0.5	0.15	10.10361	92.48949	1666.819
	0.2	8.725648	93.17555	2247.877
	0.25	7.531373	93.77078	3032.146
	0.3	6.497442	94.28654	4090.728
	0.35	5.603545	94.73265	5519.602
μ=1.5	0.15	10.96332	30.80171	692.225
	0.2	9.627772	31.41513	929.829
	0.25	8.441602	31.96649	1248.263
	0.3	7.390016	32.46037	1675.188
	0.35	6.459664	32.90118	2250.115
μ=2.5	0.15	10.38479	17.34289	463.2328
	0.2	9.180637	17.85713	616.8797
	0.25	8.10508	18.32605	823.2002
	0.3	7.145135	18.75224	1100.467
	0.35	6.289428	19.13815	1473.322

We observe that as inflation rate R increases and μ increases, average cost decreases when both the promises of doing payment are not satisfied.

3.5.7. Sensitivity Analysis for k:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=0 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.7.1
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=0, V=0)

	R	q	r	AC
k=10	0.15	13.18918	16.18941	463.4471
	0.2	11.90975	16.70801	618.9821
	0.25	10.75833	17.1857	828.2443
	0.3	9.72121	17.62486	1109.961
	0.35	8.786195	18.02806	1489.404
k=15	0.15	15.15034	15.41925	471.8318
	0.2	13.67226	15.99694	628.2439
	0.25	12.34417	16.53048	838.4713
	0.3	11.14946	17.02225	1121.251
	0.35	10.07356	17.47471	1501.865
k=20	0.15	16.72656	14.82172	479.3162
	0.2	15.08645	15.44387	636.5154
	0.25	13.61499	16.01966	847.6086
	0.3	12.29256	16.5515	1131.341
	0.35	11.10301	17.04159	1513.004

For the above pattern we see that increase in inflation rate R and ordering cost k results in increase in average cost. However order quantity q increases but the reorder quantity r decreases.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.7.2
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=0, V=1)

	R	q	r	AC
k=10	0.15	14.82867	15.54353	463.5771
	0.2	13.55915	16.04184	619.9026
	0.25	12.42268	16.49856	830.3931
	0.3	11.40532	16.91599	1113.965
	0.35	10.49453	17.29658	1496.151
k=15	0.15	16.78515	14.79988	473.6709
	0.2	15.30952	15.35803	628.6383
	0.25	13.98951	15.87152	839.3724
	0.3	12.80799	16.34258	1123.736
	0.35	11.75012	16.77359	1506.763
k=20	0.15	18.3647	14.22062	482.8003
	0.2	16.72121	14.82372	638.9079
	0.25	15.25239	15.38	848.0316
	0.3	13.93839	15.89166	1132.612
	0.35	12.76223	16.36103	1516.425

We see that as inflation rate R increases and k increases, value of q increases and the value of reorder quantity r decreases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.7.3
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=1, V=0)

	R	q	r	AC
k=10	0.15	13.18918	16.18941	466.0297
	0.2	11.9097	16.70801	622.4682
	0.25	10.75833	17.1857	832.95
	0.3	9.72121	17.62486	1116.313
	0.35	8.786195	18.02806	1497.978
k=15	0.15	15.15034	15.41925	474.4144
	0.2	13.67226	15.99694	631.73
	0.25	12.34417	16.53048	843.1771
	0.3	11.14946	17.0222	1127.603
	0.35	10.07356	17.47471	1510.439
k=20	0.15	16.72656	14.82172	481.8987
	0.2	15.08645	15.44387	640.0016
	0.25	13.61499	16.01966	852.3144
	0.3	12.29256	16.5515	1137.693
	0.35	11.10301	17.04159	1521.578

Here we see that impact of increase in inflation rate R and ordering cost k results in increase in average cost.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.7.4
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=1, V=1)

	R	q	r	AC
k=10	0.15	10.38479	17.34289	463.2328
	0.2	9.180637	17.85713	616.8797
	0.25	8.10508	18.32605	823.2002
	0.3	7.145135	18.75224	1100.467
	0.35	6.289428	19.13815	1473.322
k=15	0.15	12.24252	16.57191	471.1172
	0.2	10.84128	17.15092	628.1384
	0.25	9.588645	17.6816	836.465
	0.3	8.46937	18.16621	1115.454
	0.35	7.470183	18.60712	1490.281
k=20	0.15	13.74212	15.96924	477.9238
	0.2	12.18185	16.59665	635.588
	0.25	10.78702	17.17366	847.5113
	0.3	9.54012	17.7024	1128.499
	0.35	8.426102	18.18515	1505.017

In this situation also increase in inflation rate R and ordering cost k results in increase in value of q and decrease in the value of reorder quantity r which results in increase in average cost.

3.5.8. Sensitivity Analysis for θ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=0 and V=0. Inflation rate

R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.8.1
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=0, V=0)

	R	q	r	AC
θ=5	0.15	13.18918	16.18941	463.4471
	0.2	11.90975	16.70801	618.9821
	0.25	10.75833	17.1857	828.2443
	0.3	9.72121	17.62486	1109.961
	0.35	8.786195	18.02806	1489.404
θ=7	0.15	13.87577	16.78852	509.0805
	0.2	12.53098	17.33704	680.4105
	0.25	11.32044	17.84203	910.9765
	0.3	10.22971	18.30616	1221.431
	0.35	9.246411	18.73199	1639.645
θ=10	0.15	14.87466	17.58784	576.0727
	0.2	13.43478	18.17996	770.6005
	0.25	12.13826	18.72478	1032.456
	0.3	10.96982	19.22517	1385.12
	0.35	9.915986	19.68414	1860.281

We observe that with increases in inflation rate R and increases in rate of deterioration θ, average cost increases even if both the promises of doing payment are fulfilled.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and rate of deterioration θ, keeping other parameter values fixed where U=0 and V=1. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.8.2
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=0, V=1)

	R	q	r	AC
θ=5	0.15	14.82867	15.54353	463.5771
	0.2	13.55915	16.04184	619.9026
	0.25	12.42268	16.49856	830.3931
	0.3	11.40532	16.91599	1113.965
	0.35	10.49453	17.29658	1496.151
θ=7	0.15	15.50304	16.14231	509.3848
	0.2	14.1681	16.67102	681.547
	0.25	12.9721	17.15566	913.3938
	0.3	11.90049	17.59871	1225.77
	0.35	10.94051	18.0027	1646.812
θ=10	0.15	16.4867	16.94095	576.6058
	0.2	15.05659	17.514	772.0204
	0.25	13.7741	18.03921	1035.225
	0.3	12.62372	18.51949	1389.898
	0.35	11.59204	18.95757	1867.997

We see that as inflation rate R increases and θ increases, value of q increases and the value of reorder quantity r increases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.8.3
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=1, V=0)

	R	q	r	AC
θ=5	0.15	13.18918	16.18941	466.0297
	0.2	11.9097	16.70801	622.4682
	0.25	10.75833	17.1857	832.95
	0.3	9.72121	17.62486	1116.313
	0.35	8.786195	18.02806	1497.978
θ=7	0.15	13.87577	16.78852	511.6631
	0.2	12.53098	17.33704	683.8966
	0.25	11.32044	17.84203	915.6823
	0.3	10.22971	18.30616	1227.783
	0.35	9.246411	18.73199	1648.22
θ=10	0.15	14.87466	17.58784	578.6553
	0.2	13.43478	18.17996	774.0867
	0.25	12.13826	18.72478	1037.162
	0.3	10.96982	19.22517	1391.472
	0.35	9.915986	19.68414	1868.856

We see that as inflation rate R increases and θ increases, value of q and the value of reorder quantity r increase and hence average cost increases.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.8.4
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=1, V=1)

	R	q	r	AC
θ=5	0.15	10.38479	17.34289	463.2328
	0.2	9.180637	17.85713	616.8797
	0.25	8.10508	18.32605	823.2002
	0.3	7.145135	18.75224	1100.467
	0.35	6.289428	19.13815	1473.322
θ=7	0.15	11.04048	17.96032	508.6105
	0.2	9.768346	18.50508	678.0063
	0.25	8.631103	19.00202	905.577
	0.3	7.615318	19.4538	1211.52
	0.35	6.709058	19.86311	1623.074
θ=10	0.15	11.99861	18.78409	575.2568
	0.2	10.62766	19.37339	767.7875
	0.25	9.400965	19.91109	1026.574
	0.3	8.304131	20.40015	1374.641
	0.35	7.324433	20.84353	1843.043

We see that as inflation rate R increases and θ increases, value of q and the value of reorder quantity r increase which results increase in average cost.

Conclusion:

The comparative study of the above sensitivity analysis of case-II that is when inflation rate is higher than interest charged is summarized below:

Average cost is least for pattern (U=1, V=1) and highest for pattern (U=1, V=0).

This implies that businessmen are advised not to settle the account at the end of the credit period but settle the account at the end of the cycle period. The reason for this is that once the inflation rate is greater than the interest rates charged, we actually see our debt wiped out by inflation.

3.6. CONCLUSION:

By comparing two cases that is when inflation rate is less than interest charged we conclude that cost is minimum if part payment is done at T_1 but account is not cleared at T and the cost is maximum if part payment is not done at T_1 and also account is not cleared at T. This implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period. However when inflation rate is higher than interest charged we observe that average cost is minimum if part payment is not done at T_1 and also account is not cleared at T which implies that businessmen are advised not to settle the account at the end of the credit period but settle the account at the end of the cycle period. The reason for this is that once the inflation rate is greater than the interest rates charged, we actually see our debt wiped out by inflation. Debtors are benefitted by inflation due to the reduction of real value of debt burden.

Part-II

PERISHABLE PRODUCTS STOCHASTIC INVENTORY MODELS FOR TWO SUPPLIERS

CHAPTER 4

STOCHASTIC INVENTORY MODEL UNDER PERMISSIBLE DELAY IN PAYMENT FOR TWO SUPPLIERS

CHAPTER 4

4.1. INTRODUCTION:

In the previous chapters we discussed monopolistic case. Here we consider a generalization of the model discussed in chapter 2 representing practical life situation by assuming that the supplier's market is not monopolistic as competitive spirit in the business is increased especially after induction of multinational companies. We undertake a duopolistic case which can be generalized further. In other words, it is assumed that the inventory manager may place his order with any one of two suppliers. This generalization results is a more difficult problem, however it makes the model more realistic when the manager may receive his supply from more than one source. Here we assume that the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 where as state 3 denotes the non availability of either of them. Status of both the suppliers is explained as below.

State	Status of supplier 1	Status of supplier 2
0	ON	ON
1	ON	OFF
2	OFF	ON
3	OFF	OFF

Here it is assumed that one may place order to either one of the two suppliers or partly to both when both suppliers are available (i.e. state 0 of the system).

In today's business transactions it is more and more common to see that the customer are allowed some grace period before settling the account with the supplier. This provides an advantage to the customers, due to the fact that they do not have to pay the supplier immediately after receiving the product but instead, can defer their payment until the end of the allowed period. The customer pays no interest during the fixed period, but if the payment is delayed beyond that period, interest will be charged. The customer can start to accumulate revenues on the sale or use of the product, and earn interest on that revenue. So it is to the advantage of the customer to defer the payment to the supplier until the end of the period. Shortages are very important, especially in a model that considers delay in payment due to the fact that shortages can affect the quantity ordered to benefit from the delay in payment.

4.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model for two suppliers under permissible delay in payment is developed on the basis of the following assumptions.

(a) Demand rate d is deterministic and it is d>1.

(b) We define Xi and Yi to be the random variables corresponding to the length of ON and OFF period respectively for i^{th} supplier where i=1, 2. We specifically assume that Xi~ exp (λi) and Yi~ exp (μi). Further Xi and Yi are independently distributed.

(c) q_i = order up to level i=0, 1, 2

(d) r = reorder up to level ; q_i and r are decision variables.

(e) T_{0i} is a credit period allowed by i^{th} supplier where i=1, 2 which is a known constant.

(f) T_{00} is cycle period which is a decision variable.

(g) ie_i=Interest rate earned when purchase made from i^{th} supplier where i=1, 2

ic_i=Interest rate charged by i^{th} supplier where i=1, 2

(h) α_i= Indicator variable for i^{th} supplier where i=1, 2

$\alpha_i = 0$ if account is settled completely at T_{0i}

$= 1$ otherwise

(i) $Ie(1i)$ =Interest earned over period (0 to T_{0i}) $= dcT_{00}T_{0i}ie_i$

(j) $Ie(2i)$=Interest earned over period (T_{0i} to T_{00}) upon interest earned $(Ie(1i))$ previously.

$$Ie(2i) = \left(dcT_{00} + Ie(1i)\right)(T_{00} - T_{0i})ie_i$$

(k) Interest charged by the i^th supplier clearly ($ic_i > ie_i$) i=1, 2

$$Ic_i = \alpha_i dcic_i (T_{00} - T_{0i})$$

In this chapter we assume that

- A Supplier allows a fixed period 'T_{0i}' to settle the account. During this fixed period no interest is charged by the i^th supplier but beyond this period, interest is charged by the i^th supplier under the terms and conditions agreed upon.
- Interest charged is usually higher than interest earned.
- The account is settled completely either at the end of the credit period or at the end of the cycle.
- During the fixed credit period T_{0i}, revenue from sales is deposited in an interest bearing account.

The policy we have chosen is denoted by (q_0, q_1, q_2, r). An order is placed for q_i units i=0, 1, 2, whenever inventory drops to the reorder point r and the state found is i=0, 1, 2.When both suppliers are available, q_0 is the total ordered from either one or both suppliers. If the process is found in state 3 that is both the suppliers are not available nothing can be ordered in which case the buffer stock of r units is reduced. If the process stays in state 3 for longer time then the shortages start accumulating at rate of d units/time. When the process leaves state 3 and supplier becomes available, enough units are ordered to increase the inventory to q_i +r units where i=0, 1, 2.

$A(q_i, r, \theta)$=cost of ordering+ cost of holding inventory+ cost of items that deteriorate during a single interval that starts with an inventory of q_i units and ends with r units.

$$A(q_i, r, \theta) = k + \frac{1}{2}\frac{hq_i^2}{(d+\theta)} + \frac{hrq_i}{(d+\theta)} + \frac{\theta cq_i}{(d+\theta)} \qquad i=0, 1, 2$$

$P_{ij}(t)$ =P (Being in state j at time t/starting in state i at time 0)   i, j=0, 1, 2, 3

p_i =long run probabilities   i=0, 1, 2, 3

4.3. OPTIMAL POLICY DECISION FOR THE MODEL:

For calculation of average cost objective function, we need to identify the cycles. Below given figure gives us the idea about cycles and their identification.

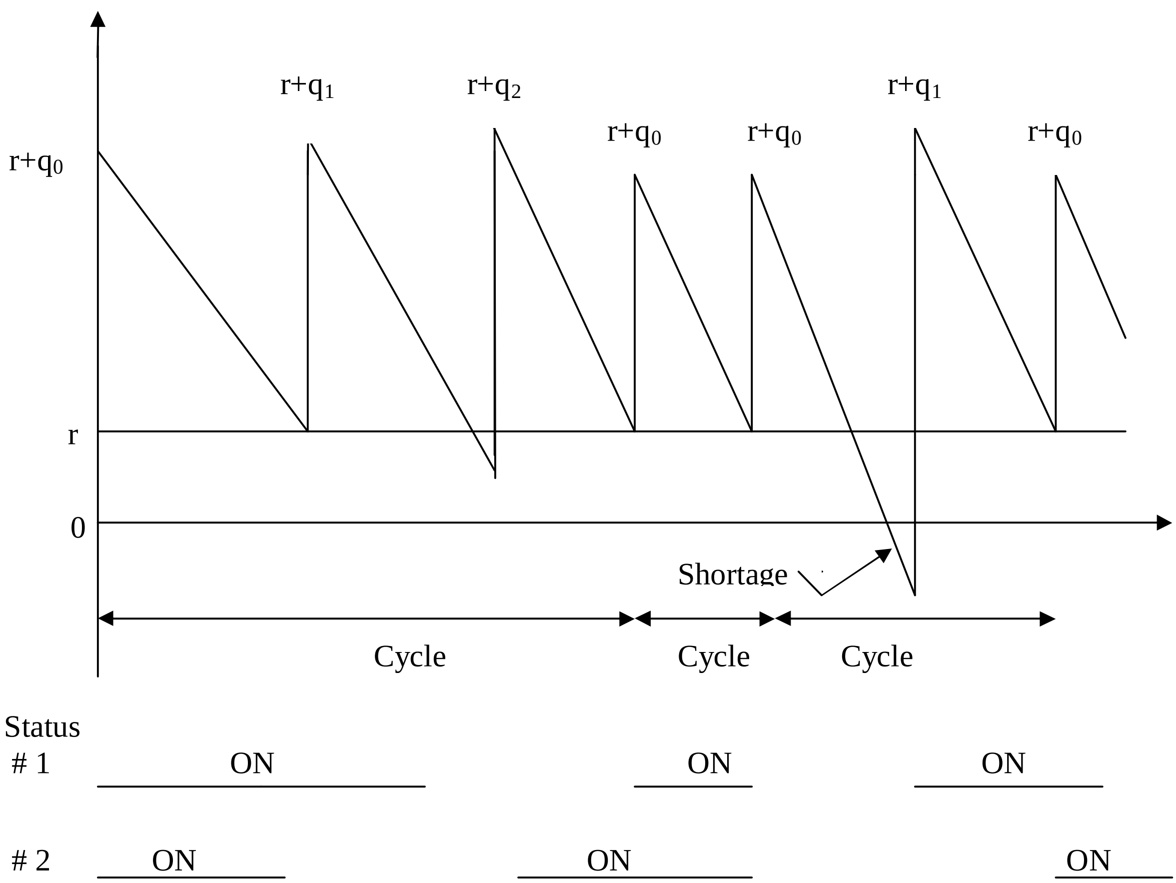

Fig. 4.1 Inventory level and status process with two suppliers

Referring to Fig.4.1, we see that the cycles of this process start when the inventory goes up to a level of q_0+r units. Once the cycle is identified, we construct the average cost objective function as a ratio of the expected cost per cycle to the expected cycle length.

i.e. AC (q_0, q_1, q_2, r) $= \dfrac{C_{00}}{T_{00}}$

where C_{00}=E (cost per cycle) and T_{00}=E (length of a cycle)

Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, i, j=0, 1, 2, 3 for the four state CTMC. The solution is provided in the following lemma.

Lemma 4.3.1: Let $P(t) = \lfloor P_{ij}(t) \rfloor$ t≥0, i, j=0, 1, 2, 3 be 4×4 matrix of transition functions for the CTMC. The exact transient solution is given as $P(t) = UD(t)U^{-1}$.where

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -\mu_2/\lambda_2 & -\mu_2/\lambda_2 \\ 1 & -\mu_1/\lambda_1 & 1 & -\mu_1/\lambda_1 \\ 1 & -\mu_1/\lambda_1 & -\mu_2/\lambda_2 & \mu_1\mu_2/\lambda_1\lambda_2 \end{bmatrix}$$

$$D(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-(\lambda_1+\mu_1)t} & 0 & 0 \\ 0 & 0 & e^{-(\lambda_2+\mu_2)t} & 0 \\ 0 & 0 & 0 & e^{-(\lambda_1+\mu_1+\lambda_2+\mu_2)t} \end{bmatrix}$$

$$U^{-1} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \begin{bmatrix} \mu_1\mu_2 & \lambda_2\mu_1 & \lambda_1\mu_2 & \lambda_1\lambda_2 \\ \lambda_1\mu_2 & \lambda_1\lambda_2 & -\lambda_1\mu_2 & -\lambda_1\lambda_2 \\ \lambda_2\mu_1 & -\lambda_2\mu_1 & \lambda_1\lambda_2 & -\lambda_1\lambda_2 \\ \lambda_1\lambda_2 & -\lambda_1\lambda_2 & -\lambda_1\lambda_2 & \lambda_1\lambda_2 \end{bmatrix}$$

Proof: We will provide a constructive proof and find the transition probabilities by solving the system of 16 ordinary linear differential equations i.e. the forward Kolmogorov equations. We will first describe the explicit derivation of the differential equations corresponding to $P_{00}(t)$ and then give the general result in matrix form.

Recall that in an infinitesimal time interval of length t we can only move from state 0 to state 0, 1 or 2. Therefore, we have

$$P_{00}(t+\Delta t) = [1-\lambda_1\Delta t + 0(\Delta t)][1-\lambda_2\Delta t + 0(\Delta t)]P_{00}(t) + [1-\lambda_1\Delta t + 0(\Delta t)][\lambda_2\Delta t + 0(\Delta t)]P_{10}(t)$$
$$+ [\lambda_1\Delta t + 0(\Delta t)][1-\lambda_2\Delta t + 0(\Delta t)]P_{20}(t)$$

Subtracting $P_{00}(t)$ from both sides, dividing by Δt, and letting Δt→0 gives a differential equation as

$$P_{00}'(t) = -(\lambda_1+\lambda_2)P_{00}(t) + \lambda_2 P_{10}(t) + \lambda_1 P_{20}(t)$$

After generating a similar set of differential equations for the other states, the resulting 16 Kolmogorov equations can be put in a more convenient matrix form as $P'(t) = QP(t)$, $P(0) = I$ where

$$Q = \begin{bmatrix} -(\lambda_1+\lambda_2) & \lambda_2 & \lambda_1 & 0 \\ \mu_2 & -(\lambda_1+\mu_2) & 0 & \lambda_1 \\ \mu_1 & 0 & -(\lambda_2+\mu_1) & \lambda_2 \\ 0 & \mu_1 & \mu_2 & -(\mu_1+\mu_2) \end{bmatrix}$$

is the infinitesimal general of the stochastic process with states 0, 1, 2 and 3 and I is the identity matrix. We now solve this system using spectral analysis (Hilderbrand (1965), Bhat(1984)).

The solution to $P'(t) = QP(t)$, $P(0) = I$ can be written in the form $P(t) = e^{Qt}$, where

$$e^{Qt} = I + \sum_{n=1}^{\infty} \frac{Q^n t^n}{n!} \qquad (4.3.1)$$

From the spectral theory of matrices (1965), we have $Q = UHU^{-1}$

Where U is a non-singular matrix formed with the right eigen vectors of Q and H is the diagonal matrix.

To find the right eigen vectors of Q, we first need to find the eigen values of Q that are obtained as the solution of the characteristic equation.

Let $(Q - wI) = 0$, solving gives $w_0=0$, $w_1=-(\lambda_1+\mu_1)$, $w_2=-(\lambda_2+\mu_2)$, $w_3=-(\lambda_1+\mu_1+\lambda_2+\mu_2)$ as the four distinct eigen values. Using the eigen values, we find the right eigen vectors and form the U matrix as

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -\mu_2/\lambda_2 & -\mu_2/\lambda_2 \\ 1 & -\mu_1/\lambda_1 & 1 & -\mu_1/\lambda_1 \\ 1 & -\mu_1/\lambda_1 & -\mu_2/\lambda_2 & \mu_1\mu_2/\lambda_1\lambda_2 \end{bmatrix}$$

If $Q = UHU^{-1}$ then $Q^n = UH^nU^{-1}$ and using (4.3.1) we get

$$P(t) = I + \sum_{n=1}^{\infty} \frac{UH^nU^{-1}t^n}{n!}$$

$$P(t) = UD(t)U^{-1} \qquad (4.3.2)$$

where $D(t) = \begin{bmatrix} e^0 & 0 & 0 & 0 \\ 0 & e^{w_1 t} & 0 & 0 \\ 0 & 0 & e^{w_2 t} & 0 \\ 0 & 0 & 0 & e^{w_3 t} \end{bmatrix}$

Because the inverse of U is formed as

$$U^{-1} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \begin{bmatrix} \mu_1\mu_2 & \lambda_2\mu_1 & \lambda_1\mu_2 & \lambda_1\lambda_2 \\ \lambda_1\mu_2 & \lambda_1\lambda_2 & -\lambda_1\mu_2 & -\lambda_1\lambda_2 \\ \lambda_2\mu_1 & -\lambda_2\mu_1 & \lambda_1\lambda_2 & -\lambda_1\lambda_2 \\ \lambda_1\lambda_2 & -\lambda_1\lambda_2 & -\lambda_1\lambda_2 & \lambda_1\lambda_2 \end{bmatrix}$$

Hence above lemma is proved.

Using lemma (4.3.1) we obtain the following transition probabilities:

$$P_{01} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_2\mu_1 + \lambda_1\lambda_2\, e^{\frac{-(\lambda_1+\mu_1)q_0}{(d+\theta)}} - \lambda_2\mu_1 e^{\frac{-(\lambda_2+\mu_2)q_0}{(d+\theta)}} - \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_0}{(d+\theta)}}\right]$$

$$P_{02} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_1\mu_2 - \lambda_1\mu_2\, e^{\frac{-(\lambda_1+\mu_1)q_0}{(d+\theta)}} + \lambda_1\lambda_2 e^{\frac{-(\lambda_2+\mu_2)q_0}{(d+\theta)}} - \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_0}{(d+\theta)}}\right]$$

$$P_{03} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_1\lambda_2 - \lambda_1\lambda_2\, e^{\frac{-(\lambda_1+\mu_1)q_0}{(d+\theta)}} - \lambda_1\lambda_2 e^{\frac{-(\lambda_2+\mu_2)q_0}{(d+\theta)}} + \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_0}{(d+\theta)}}\right]$$

$$P_{11} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_2\mu_1 + \lambda_1\lambda_2\, e^{\frac{-(\lambda_1+\mu_1)q_1}{(d+\theta)}} + \mu_1\mu_2 e^{\frac{-(\lambda_2+\mu_2)q_1}{(d+\theta)}} + \mu_2\lambda_1 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_1}{(d+\theta)}}\right]$$

$$P_{12} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_1\mu_2 - \lambda_1\mu_2\, e^{\frac{-(\lambda_1+\mu_1)q_1}{(d+\theta)}} - \mu_2\lambda_1 e^{\frac{-(\lambda_2+\mu_2)q_1}{(d+\theta)}} + \mu_2\lambda_1 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_1}{(d+\theta)}}\right]$$

$$P_{13} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_1\lambda_2 - \lambda_1\lambda_2\, e^{\frac{-(\lambda_1+\mu_1)q_1}{(d+\theta)}} + \mu_2\lambda_1 e^{\frac{-(\lambda_2+\mu_2)q_1}{(d+\theta)}} - \lambda_1\mu_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_1}{(d+\theta)}}\right]$$

$$P_{21} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_2\mu_1 - \mu_1\lambda_2\, e^{\frac{-(\lambda_1+\mu_1)q_2}{(d+\theta)}} - \lambda_2\mu_1 e^{\frac{-(\lambda_2+\mu_2)q_2}{(d+\theta)}} + \mu_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_2}{(d+\theta)}}\right]$$

$$P_{22} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_1\mu_2 + \mu_1\mu_2\, e^{\frac{-(\lambda_1+\mu_1)q_2}{(d+\theta)}} + \lambda_1\lambda_2 e^{\frac{-(\lambda_2+\mu_2)q_2}{(d+\theta)}} + \mu_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_2}{(d+\theta)}}\right]$$

$$P_{23} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}\left[\lambda_1\lambda_2 + \mu_1\lambda_2\, e^{\frac{-(\lambda_1+\mu_1)q_2}{(d+\theta)}} - \lambda_1\lambda_2 e^{\frac{-(\lambda_2+\mu_2)q_2}{(d+\theta)}} - \mu_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_2}{(d+\theta)}}\right]$$

Corollary 4.3.1: The long run probabilities $P_j = \lim_{t\to\infty} P_{ij}(t)$ are

$$[p_0, p_1, p_2, p_3] = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}[\mu_1\mu_2, \lambda_2\mu_1, \lambda_1\mu_2, \lambda_1\lambda_2]$$

Proof: As $t \to \infty$, we have

$$\lim_{t\to\infty} D(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence above corollary is proved.

Define C_{i0}=E (cost incurred to the beginning of the next cycle from the time when inventory drops to r at state i=0, 1, 2, 3 and q_i units are ordered if i=0, 1 or 2)

Lemma 4.3.2: C_{i0} is given by

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right)A(q_i, r, \theta) + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[A(q_i, r, \theta) + C_{j0}\right] \text{ i=0, 1, 2} \quad (4.3.3)$$

$$C_{30} = \overline{C} + \sum_{i=1}^{2} \rho_i C_{i0} \quad (4.3.4)$$

Where $\rho_i = \frac{\mu_i}{\delta}$ with $\delta = \mu_1 + \mu_2$ and

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}\left(\delta r - (d+\theta)\right) + \left(\pi\delta d + h(d+\theta) + \hat{\pi}\right) - \theta c\delta\right] + \frac{\theta c}{\delta} \quad (4.3.5)$$

Proof: First consider i=0. Conditioning on the state of the supplier availability process when inventory drops to r, we obtain

$$C_{00} = P_{00}\left(\frac{q_0}{d+\theta}\right)A(q_0, r, \theta) + \sum_{j=1}^{3} P_{0j}\left(\frac{q_0}{d+\theta}\right)\left[A(q_0, r, \theta) + C_{j0}\right] \quad (4.3.6)$$

The equation follows because $q_0 + r$ being the initial inventory, when q_0 units are used up we either observe state 0, 1, 2 or 3 with probabilities

$P_{00}\left(\frac{q_0}{d+\theta}\right), P_{01}\left(\frac{q_0}{d+\theta}\right), P_{02}\left(\frac{q_0}{d+\theta}\right)$ and $P_{03}\left(\frac{q_0}{d+\theta}\right)$ respectively. If we are in state 0 when r is reached, we must have incurred a cost of $A(q_0, r, \theta)$. On the other hand, if state j=1, 2, 3 is observed when inventory drops to r, then the expected cost will be $A(q_0, r, \theta) + C_{j0}$ with probability $P_{0j}\left(\frac{q_0}{d+\theta}\right)$. The equation relating C_{10} and C_{20} are very similar but C_{30} is obtained as

$$C_{30} = \left[\overline{C} + C_{10}\right]\frac{\mu_1}{\mu_1 + \mu_2} + \left[\overline{C} + C_{20}\right]\frac{\mu_2}{\mu_1 + \mu_2} \tag{4.3.7}$$

Here, $\overline{C}$ is defined as the expected cost from the time inventory drops to r until either supplier 1 or 2 becomes available, $\overline{C}$ is computed as follows:

Now referring to Fig 3.1., note that the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of next cycle is equal to

$$\frac{1}{2}hy^2(d+\theta) + hy\left[r - y(d+\theta)\right] + \theta cy \qquad y < \frac{r}{d+\theta}$$

$$\frac{1}{2}\frac{hr^2}{(d+\theta)} + \pi\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta cr}{(d+\theta)} \qquad y \geq \frac{r}{d+\theta}$$

Hence,

$$\overline{C} = \int_0^{r/(d+\theta)} \left\{\frac{1}{2}hy^2(d+\theta) + hy\left(r - y(d+\theta) + \theta c y\right\}\delta e^{-\delta y}\right.$$

$$+ \int_{r/(d+\theta)}^{\infty} \left\{\frac{1}{2}\frac{hr^2}{(d+\theta)} + \pi\left[y - \frac{r}{(d+\theta)}\right]d + \frac{\hat{\pi}}{2}\left[y - \frac{r}{(d+\theta)}\right]^2 + \frac{\theta cr}{(d+\theta)}\right\}\delta e^{-\delta y}$$

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}\left(\delta r - (d+\theta)\right) + \left(\pi\delta d + h(d+\theta) + \hat{\pi}\right) - \theta c\delta \right] + \frac{\theta c}{\delta}$$

with $\delta = \mu_1 + \mu_2$ as the rate of departure from state 3. This follows because if supplier availability process is in state 3 (OFF for both suppliers) when inventory drops to r, then the expected holding and backorder costs are equal to $\overline{C}$. If the process makes a transition to state 1, the total expected cost would then be $\overline{C} + C_{10}$. The probability of a transition from state 3 to state 1 is

$$P(Y_1 < Y_2) = \int_0^\infty P(Y_1 < Y_2 / Y_2 = t)\mu_2 e^{-\mu_2 t} dt = \frac{\mu_1}{\mu_1 + \mu_2}$$

Multiplying this probability with the expected cost term above gives the first term of (4.3.7). The second term is obtained in a similar manner. Combining the results proves the lemma.

The following lemma provides a simpler means of expressing C_{00} in an exact manner. To simplify the notation, we let $A_i = A(q_i, r, \theta)$, i=0, 1, 2 and $P_{ij} = P_{ij}\left(\frac{q_i}{d+\theta}\right)$ i, j=0, 1, 2, 3.

Lemma 4.3.3: The exact expression for C_{00} is

$$C_{00} = A_0 + P_{01}C_{10} + P_{02}C_{20} + P_{03}\left(\overline{C} + \rho_1 C_{10} + \rho_2 C_{20}\right) \tag{4.3.8}$$

where the pair [C_{10}, C_{20}] solves the system

$$\begin{bmatrix} 1 - P_{11} - P_{13}\rho_1 & -(P_{12} + P_{13}\rho_2) \\ -(P_{21} + P_{23}\rho_1) & 1 - P_{22} - P_{23}\rho_2 \end{bmatrix} \begin{bmatrix} C_{10} \\ C_{20} \end{bmatrix} = \begin{bmatrix} A_1 + P_{13}\overline{C} \\ A_2 + P_{23}\overline{C} \end{bmatrix} \tag{4.3.9}$$

Proof: Rearranging the linear system of four equations in lemma (4.3.2) in matrix form gives

$$\begin{bmatrix} 1 & -P_{01} & -P_{02} & -P_{03} \\ 0 & 1-P_{11} & -P_{12} & -P_{13} \\ 0 & -P_{21} & 1-P_{22} & -P_{23} \\ 0 & -\rho_1 & -\rho_2 & 1 \end{bmatrix} \begin{bmatrix} C_{00} \\ C_{10} \\ C_{20} \\ C_{30} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \overline{C} \end{bmatrix} \qquad (4.3.10)$$

We have $C_{30} = \overline{C} + \rho_1 C_{10} + \rho_2 C_{20}$ from the last row of the system. Substituting this result in rows two and three and rearranging gives the system in (4.3.9), with (C_{10}, C_{20}).

From the first row of (4.3.10) we obtain $C_{00} = A_0 + \sum_{j=1}^{3} P_{0j} C_{j0}$.

Hence above lemma is proved.

Define, T_{i0}=E [Time to the beginning of the next cycle from the time when inventory drops to r at state i=0, 1, 2, 3 and q_i units are ordered if i=0, 1, 2]

Lemma 4.3.4: Expected cycle length is given by

$$T_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right)\frac{q_i}{d+\theta} + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[\frac{q_i}{d+\theta} + T_{j0}\right] \qquad i = 0,\ 1\ ,2$$

$$T_{30} = \overline{T} + \sum_{j=1}^{2} \rho_i T_{i0}$$

where $\overline{T} = \dfrac{1}{\mu_1 + \mu_2}$ is the expected time from the time inventory drops to r until either supplier 1 or 2 becomes available.

Lemma 4.3.5: The exact expression for T_{00} is

$$T_{00} = \frac{q_0}{d+\theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\overline{T} + \rho_1 T_{10} + \rho_2 T_{20})$$

where the pair [T_{10}, T_{20}] solves the system.

$$\begin{bmatrix} 1-P_{11}-P_{13}\rho_1 & -(P_{12}+P_{13}\rho_2) \\ -(P_{21}+P_{23}\rho_1) & (1-P_{22}-P_{23}\rho_2) \end{bmatrix} \begin{bmatrix} T_{10} \\ T_{20} \end{bmatrix} = \begin{bmatrix} q_1 + P_{13}\overline{T} \\ q_2 + P_{23}\overline{T} \end{bmatrix}$$

The proof of the above two lemmas i.e. (4.3.4) and (4.3.5) are very similar to lemma (4.3.2) and (4.3.3)

Proposition 4.3.1: The Average cost objective function for two suppliers when delay in payment is considered is given by

$$AC = \frac{C_{00}}{T_{00}} = \frac{\begin{array}{l} A(q_0, r, \theta) + P_{01}(C_{10} - (Ie(11) + Ie(21) + Ic_1) + P_{02}(C_{20} - (Ie(12) + Ie(22) + Ic_2) \\ + P_{03}(\overline{C} + \rho_1(C_{10} - (Ie(11) + Ie(21) + Ic_1) + \rho_2(C_{20} - (Ie(12) + Ie(22)) + Ic_2)) \end{array}}{\frac{q_0}{d+\theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\overline{T} + \rho_1 T_{10} + \rho_2 T_{20})}$$

Proof: Proof follows using Renewal reward theorem (RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

4.4. NUMERICAL EXAMPLE:

In this section we verify the results by a numerical example. We assume that

(i) k =Rs. 5/order, c=Rs.1/unit, d=20/units, θ=4, h=Rs. 5/unit/time, π=Rs. 350/unit, $\hat{\pi}$ =Rs.25/unit/time, ic_1=0.11, ie_1=0.02, ic_2=0.13, ie_2=0.04, T_{01}=0.6, T_{02}=0.8, (α_1=1 and α_2=1) i.e. businessmen do not settle the account at the respective credit time given by both the suppliers, λ_1=0.58, λ_2=0.45, μ_1=3.4, μ_2=2.5.

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are $1/\lambda_1$=1.72413794, $1/\lambda_2$=2.2222, $1/\mu_1$=.2941176 and $1/\mu_2$=.4 respectively. The long run probabilities are obtained as p_0=0.7239588, p_1=0.1303126, p_2 =0.1234989 and p_3=0.02222. The optimal solution is obtained as q_0=4.92015, q_1=33.130502, q_2=32.90077, r=0.8978675 and AC=$\frac{C_{00}}{T_{00}}$ = 6.291478.

(ii) Keeping other parameters as it is, we consider (α_1=0 and α_2=0) i.e. businessmen settle the account at the respective credit time given by both the suppliers.

The optimal solution is obtained as q_0=9.21634, q_1=41.82183, q_2=41.9396, r=0.76247 and AC=$\frac{C_{00}}{T_{00}}$ = 5.900553.

(iii) Keeping other parameters as it is, we consider (α_1=1 and α_2=0) i.e. businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

The optimal solution is obtained as q_0= 6.919021, q_1= 36.68451, q_2= 36.68383,

r= 0.925376 and AC= $\frac{C_{00}}{T_{00}}$ = 6.091088.

(iv) Keeping other parameters as it is, we consider (α_1=0 and α_2=1) i.e. when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

The optimal solution is obtained as q_0= 6.573681, q_1= 35.95015, q_2= 35.9173,

r= 0.938723 and AC= $\frac{C_{00}}{T_{00}}$ = 6.142968

Conclusion:

From the above numerical example, we conclude that the cost is minimum when businessmen settle the account at the respective credit time given by both the suppliers i.e. when (α_1=0 and α_2=0).

4.5. SENSITIVITY ANALYSIS:

To observe the effects of varying parameter values on the optimal solution we have conducted sensitivity analysis, by varying λ_1, λ_2, μ_1, μ_2, h, k.

4.5.1. Sensitivity Analysis for λ_1:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 that is length of ON period for 1st supplier and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.1.1.

Table 4.5.1.1
Sensitivity Analysis Table by varying the parameter values of λ_1
(α_1=1and α_2=1)

λ_1	q_0	q_1	q_2	r	AC
0.5	4.21939	32.3257	32.1097	0.303045	6.346889
0.52	4.32861	32.41094	32.176	0.458388	6.336898
0.54	4.46853	32.54743	32.30188	0.609183	6.324458
0.56	4.65496	32.76605	32.5205	0.755719	6.30943
0.58	4.92015	33.1305	32.9007	0.897868	6.291478

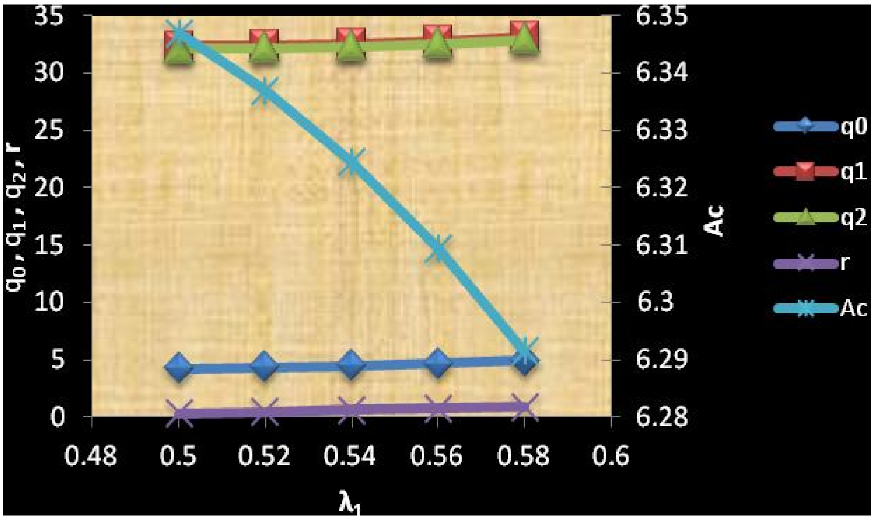

Fig. 4.5.1.1 Sensitivity analysis graph for λ_1

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases but since $1/\lambda_2$ =1/0.45=2.2 that is expected length of ON period for 2nd supplier is fixed, which results in decrease in average cost. Decreasing the expected length of ON period for 1st supplier we see there is a decrease in average cost, when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where α_1=0 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.1.2.

Table 4.5.1.2
Sensitivity Analysis Table by varying the parameter values of λ_1
(α_1=0and α_2=0)

λ_1	q_0	q_1	q_2	r	AC
0.5	4.21939	32.3257	32.1097	0.303045	6.346889
0.52	4.32861	32.41094	32.176	0.458388	6.336898
0.54	4.46853	32.54743	32.30188	0.609183	6.324458
0.56	4.65496	32.76605	32.5205	0.755719	6.30943
0.58	4.92015	33.1305	32.9007	0.897868	6.291478

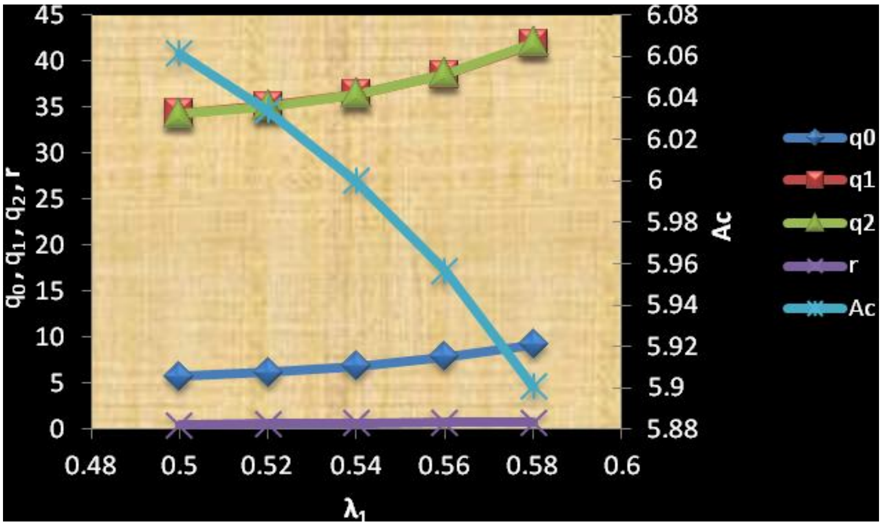

Fig. 4.5.1.2 Sensitivity analysis graph for λ_1

Increasing λ_1 i.e. decreasing expected length of ON period for 1st supplier results in decrease in average cost when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where α_1=1 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.1.3.

Table 4.5.1.3
Sensitivity Analysis Table by varying the parameter values of λ_1
(α_1=1and α_2=0)

λ_1	q_0	q_1	q_2	r	AC
0.5	4.875214	33.10239	32.96599	0.391809	6.202606
0.52	5.124579	33.43298	33.29805	0.548381	6.182258
0.54	5.472377	33.95825	33.84087	0.69646	6.157839
0.56	6.001515	34.8682	34.79183	0.82975	6.128266
0.58	6.919021	36.68451	36.68383	0.925376	6.091088

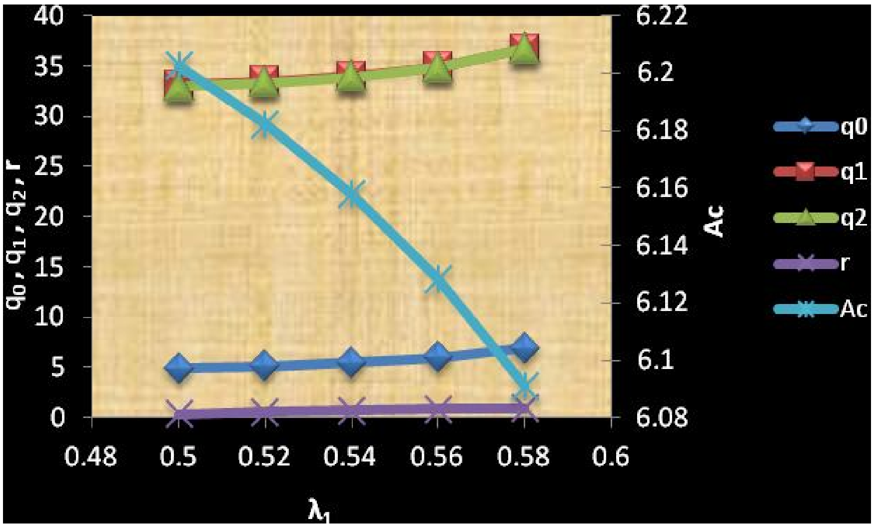

Fig.4.5.1.3 Sensitivity analysis graph for λ_1

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases, average cost decreases when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where α_1=0 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.1.4.

Table 4.5.1.4
Sensitivity Analysis Table by varying the parameter values of λ_1
(α_1=0 and α_2=1)

λ_1	q_0	q_1	q_2	r	Ac
0.5	4.817789	32.99064	32.84593	0.388688	6.22261
0.52	5.028656	33.26011	33.11216	0.543481	6.20912
0.54	5.322365	33.68757	33.55083	0.691404	6.191908
0.56	5.770063	34.4272	34.32367	0.828704	6.170542
0.58	6.573681	35.95015	35.9173	0.938723	6.142968

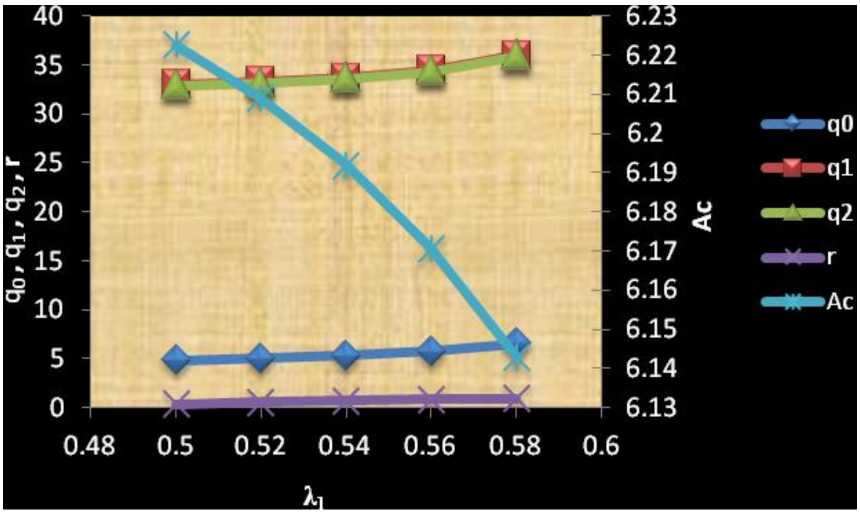

Fig. 4.5.1.4 Sensitivity analysis graph for λ_1

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases, average cost decreases when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.2. Sensitivity Analysis for λ_2:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 that is length of ON period for 2nd supplier and keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.2.1.

Table 4.5.2.1
Sensitivity Analysis Table by varying the parameter values of λ_2
(α_1=1and α_2=1)

λ_2	q_0	q_1	q_2	r	AC
0.41	4.51758	32.8025	32.3632	0.532947	6.301836
0.43	4.68582	32.90287	32.56939	0.71933	6.298501
0.45	4.92015	33.1305	32.90077	0.897868	6.291478
0.47	5.284412	33.61563	33.4926	1.06763	6.280406
0.49	6.03914	34.91542	34.91874	1.21790	6.263854

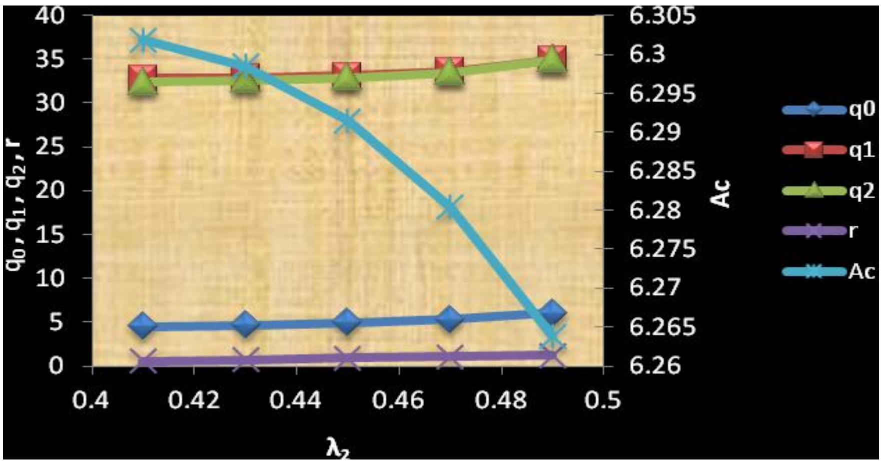

Fig. 4.5.2.1 Sensitivity analysis graph for λ_2

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases but since $1/\lambda_1$ =1/0.58=1.72 that is expected length of ON period for 1st supplier is fixed, which results in decrease in average cost when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where α_1=0 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.2.2.

Table 4.5.2.2
Sensitivity Analysis Table by varying the parameter values of λ_2
(α_1=0and α_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	6.97535	36.76276	36.67674	0.62583	5.9735
0.43	7.93252	38.75428	38.79006	0.73206	5.94342
0.45	9.21634	41.82184	41.93962	0.762478	5.90055
0.47	10.66972	45.76968	45.91412	0.720598	5.842058
0.49	12.13478	50.24337	50.37507	0.631777	5.766487

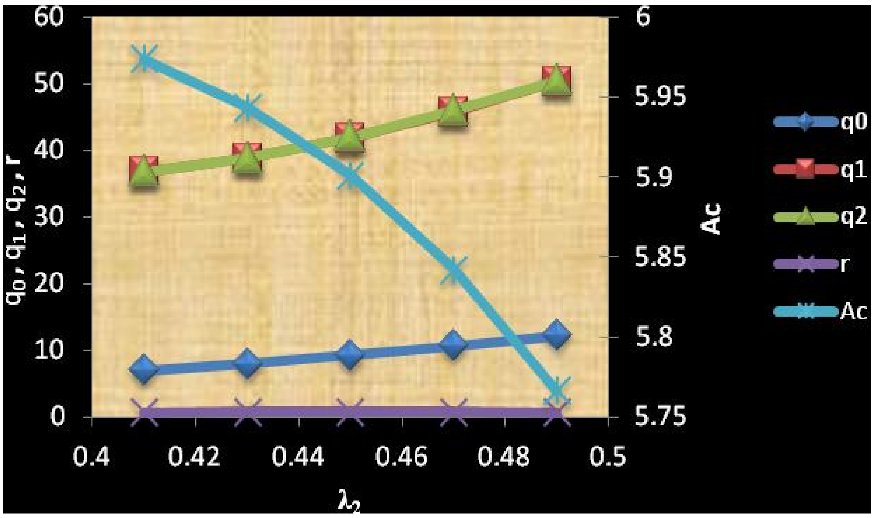

Fig. 4.5.2.2 Sensitivity analysis graph for λ_2

Increasing λ_2 i.e. decreasing expected length of ON period for 2[nd] supplier results in decrease in average cost when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.2.3.

Table 4.5.2.3
Sensitivity Analysis Table by varying the parameter values of λ_2
(α_1=1and α_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	5.648678	34.42982	34.17833	0.621036	6.12024
0.43	6.122379	35.17788	35.04982	0.790781	6.1092
0.45	6.919021	36.68451	36.68383	0.925376	6.091088
0.47	8.37173	39.95611	40.06992	0.967174	6.061918
0.49	10.33746	45.19611	45.34853	0.871826	6.014996

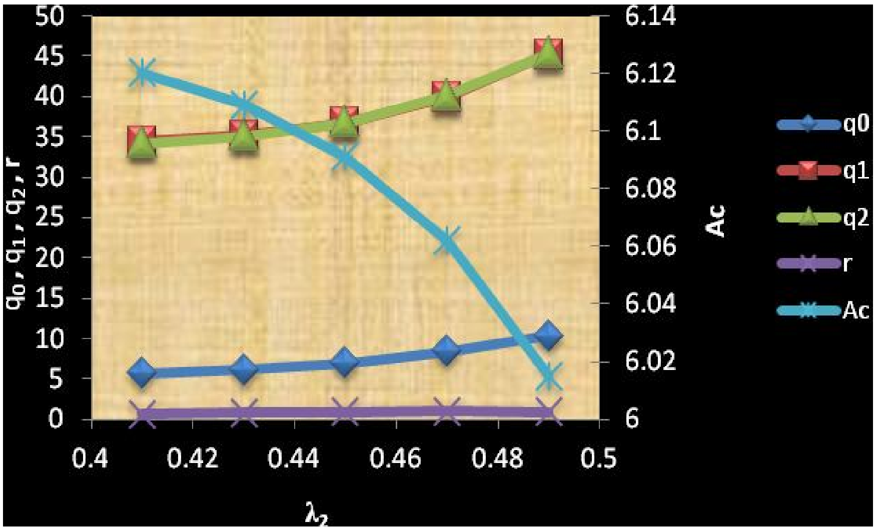

Fig. 4.5.2.3 Sensitivity analysis graph for λ_2

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases, which results in decrease in average cost when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where α_1=0 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.2.4.

Table 4.5.2.4
Sensitivity Analysis Table by varying the parameter values of λ_2
(α_1=0 and α_2=1)

λ_2	q_0	q_1	q_2	r	AC
0.41	5.306492	33.85771	33.55111	0.607382	6.180515
0.43	5.761708	34.5091	34.33355	0.788504	6.16539
0.45	6.573681	35.95015	35.9173	0.938723	6.142968
0.47	8.30733	39.74837	39.8591	0.976085	6.107925
0.49	10.61271	45.88837	46.03926	0.840191	6.050721

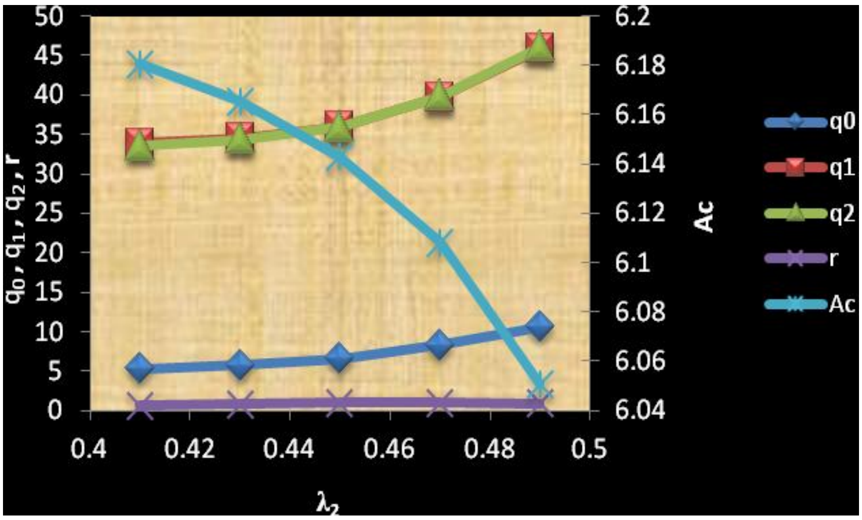

Fig. 4.5.2.4 Sensitivity analysis graph for λ_2

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases, which results in decrease in average cost when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.3. Sensitivity Analysis for μ_1:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 that is length of OFF period for 1st supplier and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.3.1.

Table 4.5.3.1
Sensitivity Analysis Table by varying the parameter values of μ_1
(α_1=1and α_2=1)

μ_1	q_0	q_1	q_2	r	AC
3.4	4.92015	33.1305	32.90077	0.897868	6.291478
3.6	4.63996	32.37693	32.13742	0.75245	6.23645
3.8	4.47092	31.82851	31.59548	0.62584	6.184397
4	4.35657	31.3834	31.16678	0.514433	6.13567
4.2	4.274396	31.00246	30.80907	0.415555	6.090279

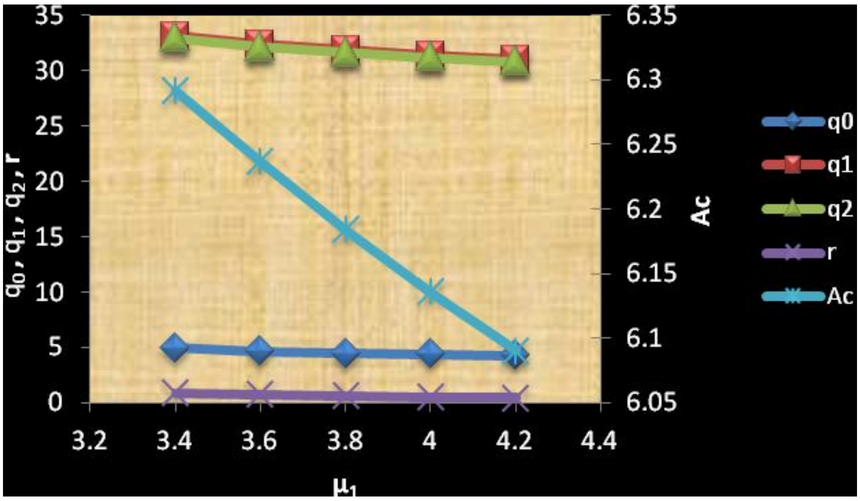

Fig. 4.5.3.1 Sensitivity analysis graph for μ_1

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, average cost decreases when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where α_1=0 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.3.2.

Table 4.5.3.2
Sensitivity Analysis Table by varying the parameter values of μ_1
(α_1=0and α_2=0)

μ_1	q_0	q_1	q_2	r	AC
3.4	9.21634	41.82184	41.93962	0.762478	5.90055
3.6	7.38513	37.1336	37.20425	0.78833	5.8883
3.8	6.4575	34.90709	34.93798	0.71046	5.8604
4	5.93738	33.64032	33.65347	0.61157	5.82773
4.2	5.606021	32.78604	32.79784	0.513805	5.79421

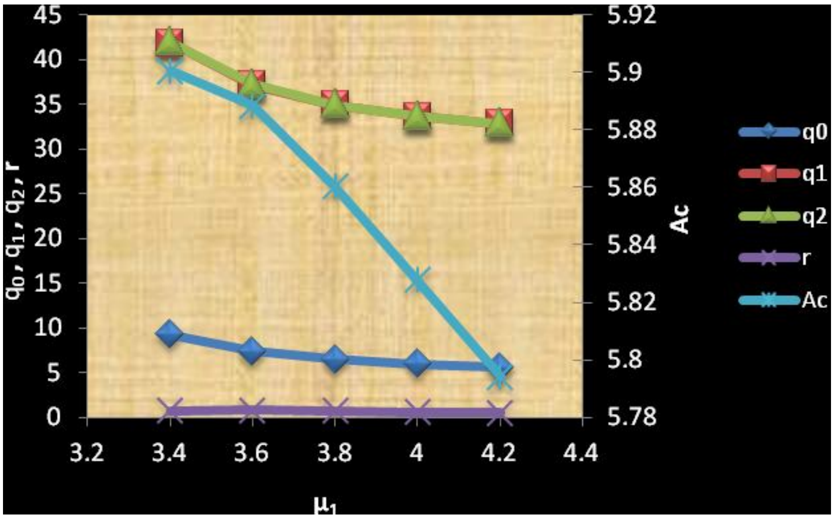

Fig. 4.5.3.2 Sensitivity analysis graph for μ_1

We see that as μ_1 increases i.e. expected length of OFF period for 1^{st} supplier decreases, which results in decrease in average cost when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where α_1=1 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.3.3.

Table 4.5.3.3
Sensitivity Analysis Table by varying the parameter values of μ_1
(α_1=1and α_2=0)

μ_1	q_0	q_1	q_2	r	AC
3.4	6.919021	36.68451	36.68383	0.925376	6.091088
3.6	5.83538	34.22192	34.14971	0.817911	6.054619
3.8	5.353431	33.07427	32.97752	0.693801	6.013849
4	5.068892	32.32638	32.22671	0.578918	5.973334
4.2	4.877779	31.76316	31.67317	0.475274	5.934428

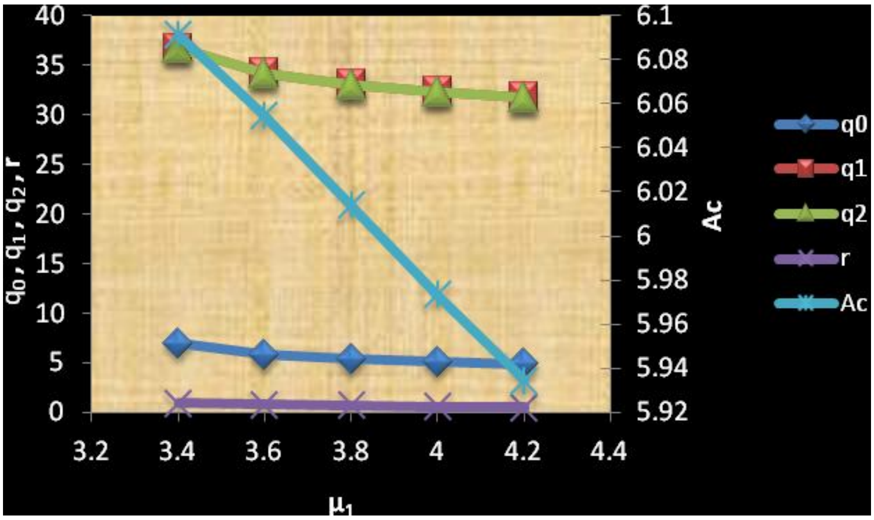

Fig. 4.5.3.3 Sensitivity analysis graph for μ_1

We see that as μ_1 increases i.e. expected length of OFF period for 1^{st} supplier decreases, which results in decrease in average cost when businessmen do not settle the account at the credit time given by the 1^{st} supplier but they settle the account at the credit time given by the 2^{nd} supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where α_1=0 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.3.4.

Table 4.5.3.4
Sensitivity Analysis Table by varying the parameter values of μ_1
(α_1=0 and α_2=1)

μ_1	q_0	q_1	q_2	r	AC
3.4	6.573681	35.95015	35.9173	0.938723	6.142968
3.6	5.657289	33.87504	33.77881	0.817562	6.098598
3.8	5.250657	32.87008	32.75627	0.692375	6.052388
4	5.00937	32.19269	32.08098	0.578698	6.007538
4.2	4.847366	31.6709	31.57269	0.476728	5.964934

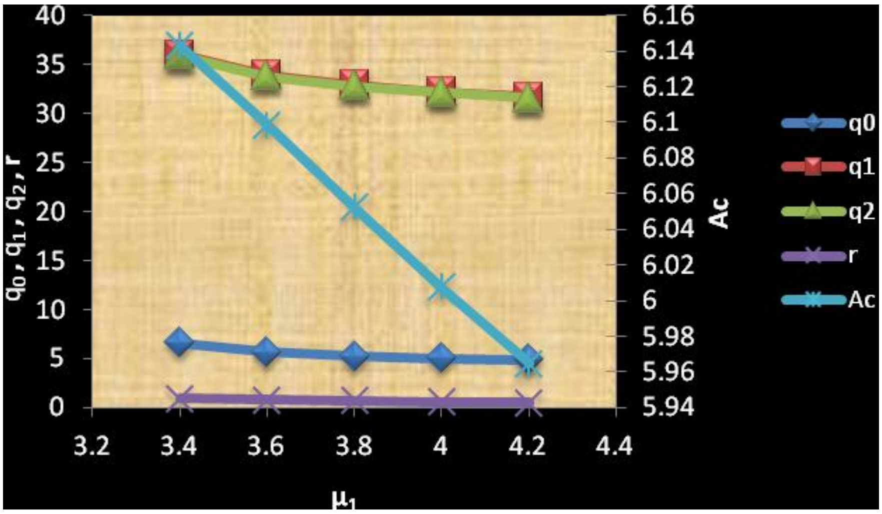

Fig. 4.5.3.4 Sensitivity analysis graph for μ_1

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, which results in decrease in average cost when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.4. Sensitivity Analysis for μ_2:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_2 that is length of OFF period for 2nd supplier and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.4.1.

Table 4.5.4.1
Sensitivity Analysis Table by varying the parameter values of μ_2
(α_1=1and α_2=1)

μ_2	q_0	q_1	q_2	r	AC
2.5	4.92015	33.1305	32.90077	0.897868	6.291478
2.7	4.65914	32.42439	32.18495	0.714823	6.22704
2.9	4.50296	31.89978	31.67263	0.558189	6.16661
3.1	4.40044	31.47235	31.26477	0.42312	6.11043
3.3	4.329983	31.10838	30.92135	0.305699	6.058361

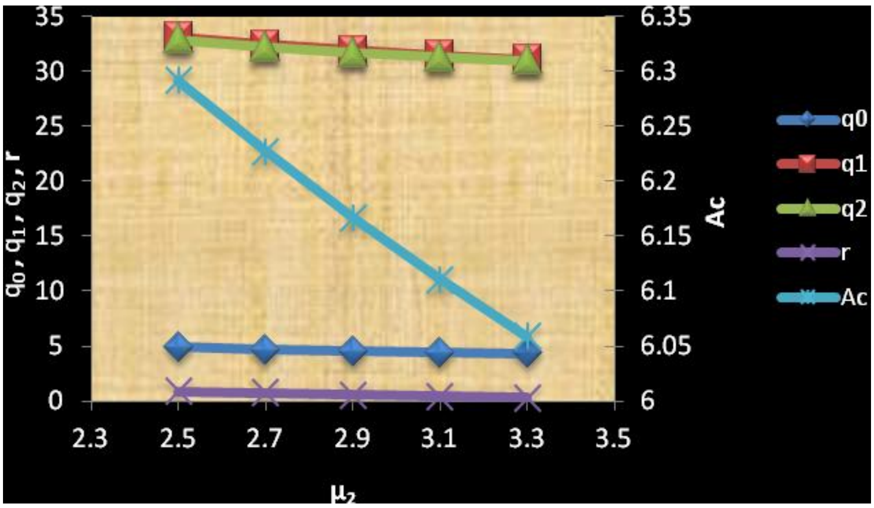

Fig. 4.5.4.1 Sensitivity analysis graph for μ_2

We see that as μ_2 increases i.e. expected length of OFF period for 2nd supplier decreases, which results in decrease in average cost when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_2 and keeping other parameter values fixed where α_1=0 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.4.2.

Table 4.5.4.2
Sensitivity Analysis Table by varying the parameter values of μ_2
(α_1=0and α_2=0)

μ_2	q_0	q_1	q_2	r	AC
2.5	9.21634	41.8218	41.93962	0.76247	5.90055
2.7	7.60633	37.7112	37.72316	0.740037	5.87412
2.9	6.71613	35.5491	35.49144	0.63948	5.83621
3.1	6.19726	34.2676	34.17347	0.52256	5.79468
3.3	5.86514	33.3992	33.2874	0.40938	5.75344

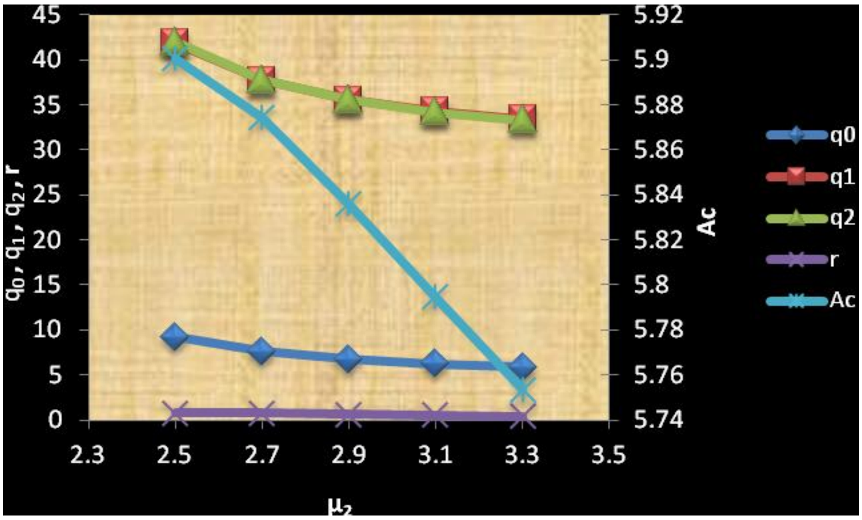

Fig. 4.5.4.2 Sensitivity analysis graph for μ_2

We see that as μ_2 increases i.e. expected length of OFF period for 2^{nd} supplier decreases, which results in decrease in average cost when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_2 and keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.4.3.

Table 4.5.4.3
Sensitivity Analysis Table by varying the parameter values of μ_2
(α_1=1and α_2=0)

μ_2	q_0	q_1	q_2	r	AC
2.5	6.919021	36.68451	36.68383	0.925376	6.091088
2.7	6.014855	34.59541	34.49889	0.782994	6.039226
2.9	5.579188	33.5183	33.38583	0.635233	5.986091
3.1	5.320454	32.79904	32.6555	0.501161	5.93477
3.3	5.149895	32.25601	32.11205	0.381983	5.886177

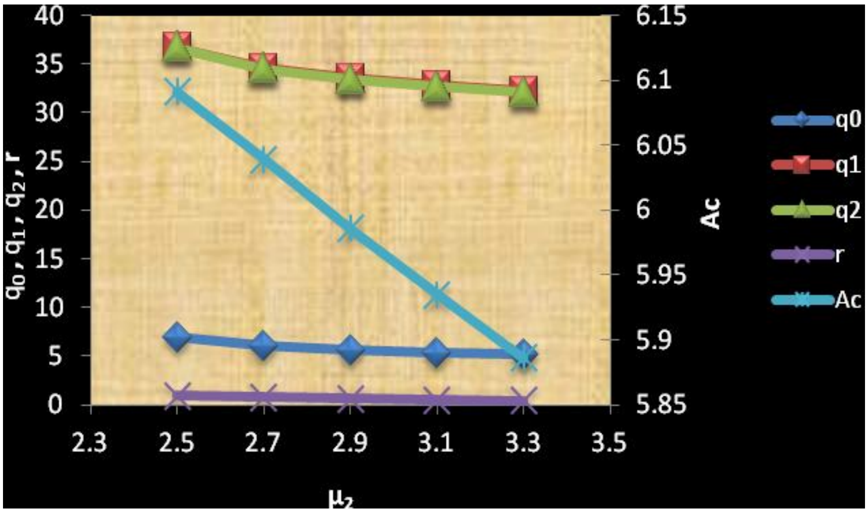

Fig. 4.5.4.3 Sensitivity analysis graph for μ_2

We see that as μ_2 increases i.e. expected length of OFF period for 2nd supplier decreases, which results in decrease in average cost when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_2 and keeping other parameter values fixed where α_1=0 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.4.4.

Table 4.5.4.4
Sensitivity Analysis Table by varying the parameter values of μ_2
(α_1=0 and α_2=1)

μ_2	q_0	q_1	q_2	r	AC
2.5	6.573681	35.95015	35.9173	0.938723	6.142968
2.7	5.663003	33.94184	33.8112	0.779756	6.092003
2.9	5.250152	32.95139	32.79107	0.623286	6.039782
3.1	5.007076	32.28605	32.12098	0.484359	5.9895
3.3	4.846396	31.77769	31.61777	0.362246	5.942063

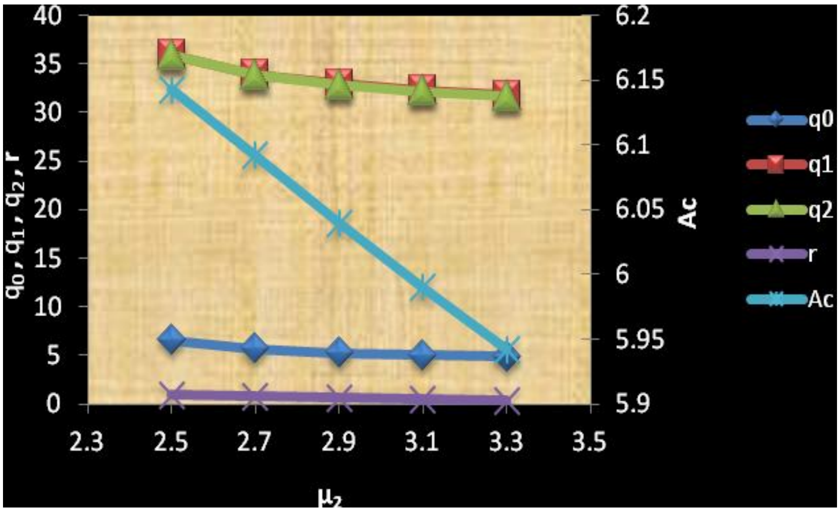

Fig. 4.5.4.4 Sensitivity analysis graph for μ_2

We see that as μ_2 increases i.e. expected length of OFF period for 2nd supplier decreases, which results in decrease in average cost when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.5. Sensitivity Analysis for h:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.5.1.

Table 4.5.5.1
Sensitivity Analysis Table by varying the parameter values of h
(α_1=1and α_2=1)

h	q_0	q_1	q_2	r	AC
5	4.92015	33.1305	32.90077	0.897868	6.29147
5.2	4.51023	32.13459	31.81629	0.736602	6.44817
5.4	4.22727	31.4038	31.01222	0.582429	6.59807
5.6	4.01276	30.81565	30.36028	0.435484	6.74237
5.8	3.840995	30.31773	29.80502	0.295231	6.88182

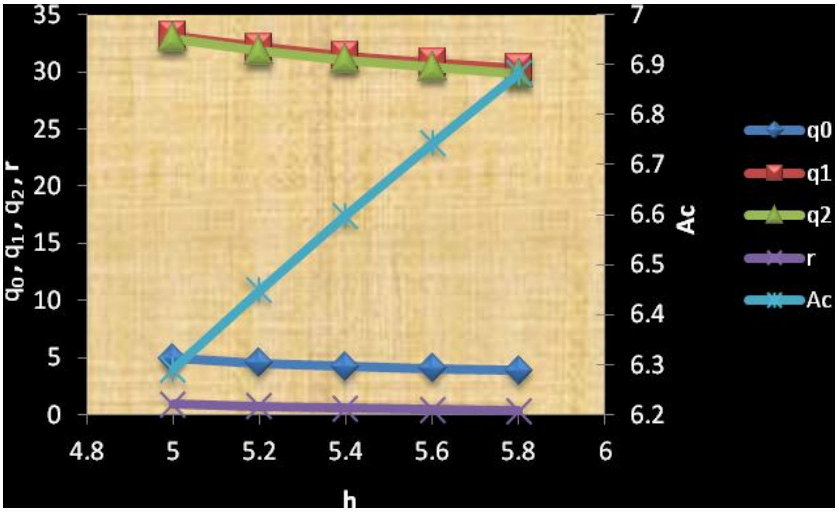

Fig. 4.5.5.1 Sensitivity analysis graph for h

We see that as holding cost h increases, average cost increases, when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed where α_1=0 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.5.2.

Table 4.5.5.2
Sensitivity Analysis Table by varying the parameter values of h
(α_1=0and α_2=0)

h	q_0	q_1	q_2	r	AC
5	9.21634	41.82184	41.93962	0.762478	5.90055
5.2	7.608994	37.48657	37.52147	0.775403	6.097604
5.4	6.458036	34.71559	34.63385	0.702893	6.275461
5.6	5.711362	33.02134	32.8299	0.58585	6.439946
5.8	5.204992	31.89054	31.60519	0.454638	6.594901

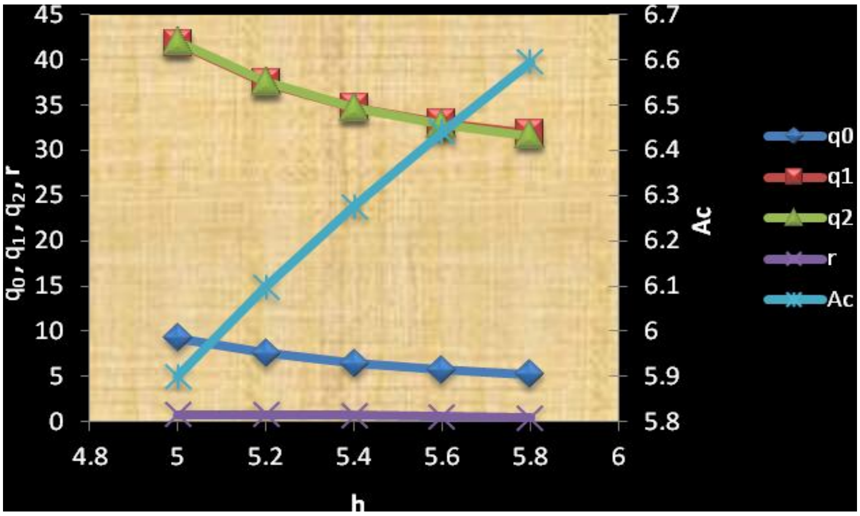

Fig. 4.5.5.2 Sensitivity analysis graph for h

We see that as holding cost h increases, average cost increases, when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed where α_1=1 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.5.3.

Table 4.5.5.3
Sensitivity Analysis Table by varying the parameter values of h
(α_1=1and α_2=0)

h	q_0	q_1	q_2	r	AC
5	6.919021	36.68451	36.68383	0.925376	6.091088
5.2	5.80463	34.07729	33.94275	0.816696	6.265027
5.4	5.181679	32.65309	32.41251	0.675464	6.426575
5.6	4.771607	31.69855	31.37156	0.530788	6.579658
5.8	4.472563	30.97895	30.57812	0.389227	6.726147

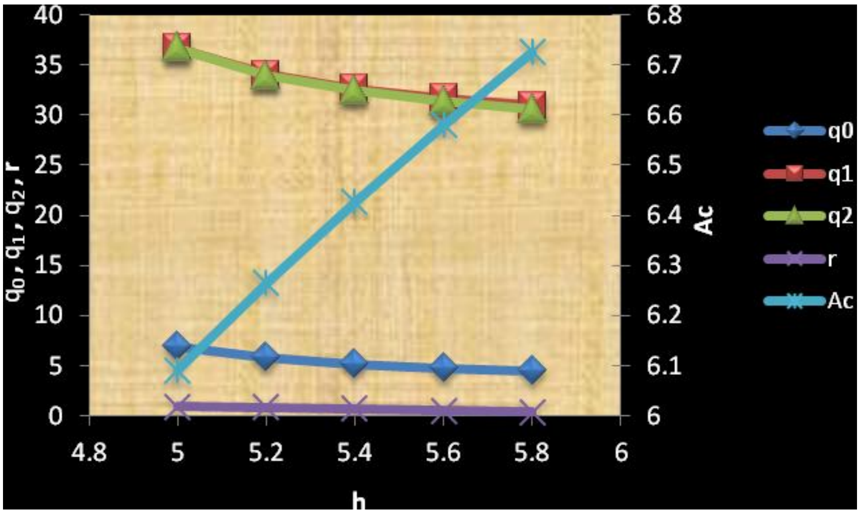

Fig. 4.5.5.3 Sensitivity analysis graph for h

Increasing the holding cost h, results in increase in average cost, when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.5.4.

Table 4.5.5.4
Sensitivity Analysis Table by varying the parameter values of h
(α_1=0 and α_2=1)

h	q_0	q_1	q_2	r	AC
5	6.573681	35.95015	35.9173	0.938723	6.142968
5.2	5.521374	33.5696	33.39709	0.812252	6.313326
5.4	4.958185	32.29829	32.02236	0.663447	6.472119
5.6	4.587592	31.43345	31.07425	0.51553	6.623014
5.8	4.31594	30.77104	30.34097	0.372561	6.767689

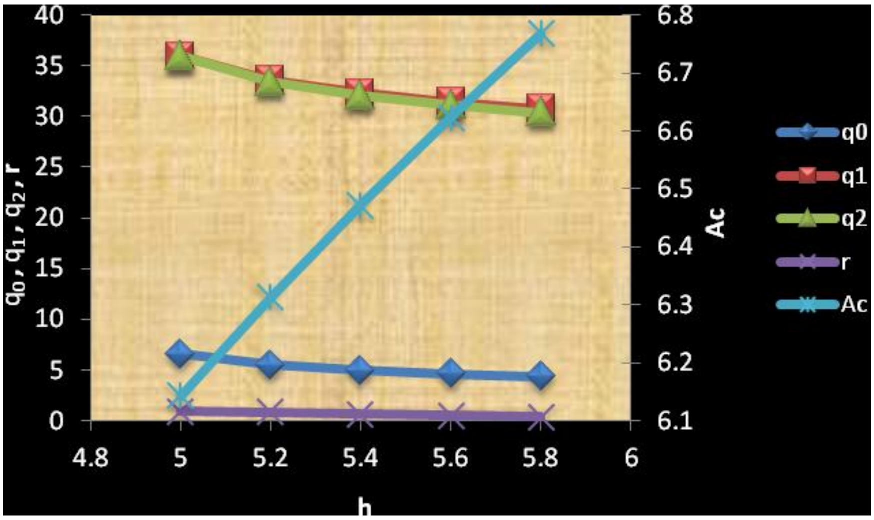

Fig. 4.5.5.4 Sensitivity analysis graph for h

Increasing the holding cost h, results in increase in average cost, when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.6. Sensitivity Analysis for k:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of ordering cost k and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.6.1.

Table 4.5.6.1
Sensitivity Analysis Table by varying the parameter values of k
(α_1=1and α_2=1)

k	q_0	q_1	q_2	r	AC
4.5	4.309901	31.82929	31.47698	0.875967	6.198703
5	4.92015	33.1305	32.90077	0.867868	6.291478
5.5	5.64599	34.71506	34.59938	0.842023	6.37395
6	6.494584	36.65105	36.63384	0.819756	6.44698
6.5	7.3987	38.84879	38.90353	0.82776	6.51175

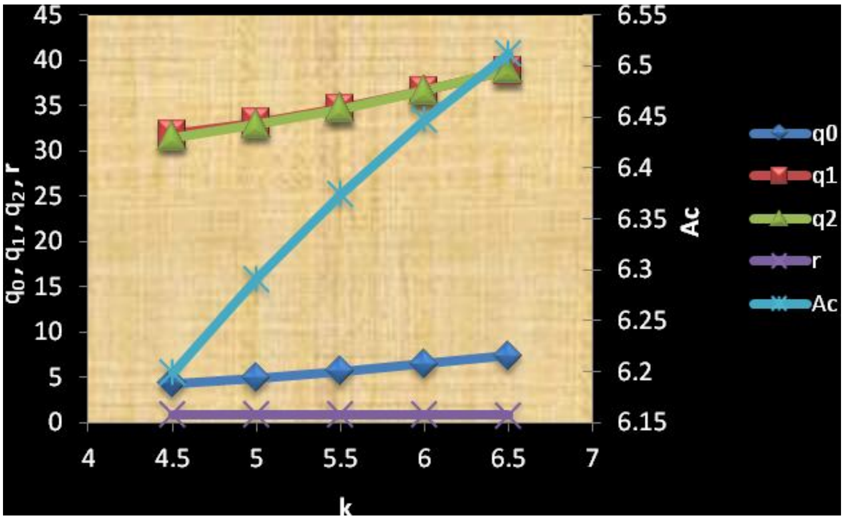

Fig. 4.5.6.1 Sensitivity analysis graph for k

We see that as ordering cost k increases, average cost increases, when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of ordering cost k and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.6.2.

Table 4.5.6.2
Sensitivity Analysis Table by varying the parameter values of k
(α_1=0and α_2=0)

k	q_0	q_1	q_2	r	AC
4.5	8.3278	39.33008	39.41338	0.86655	5.845945
5	9.21634	41.82184	41.93962	0.762478	5.90055
5.5	9.88241	43.83543	43.96582	0.672766	5.950828
6	10.41764	45.54794	45.68173	0.594338	5.998093
6.5	10.86781	47.05556	47.18837	0.524441	6.04307

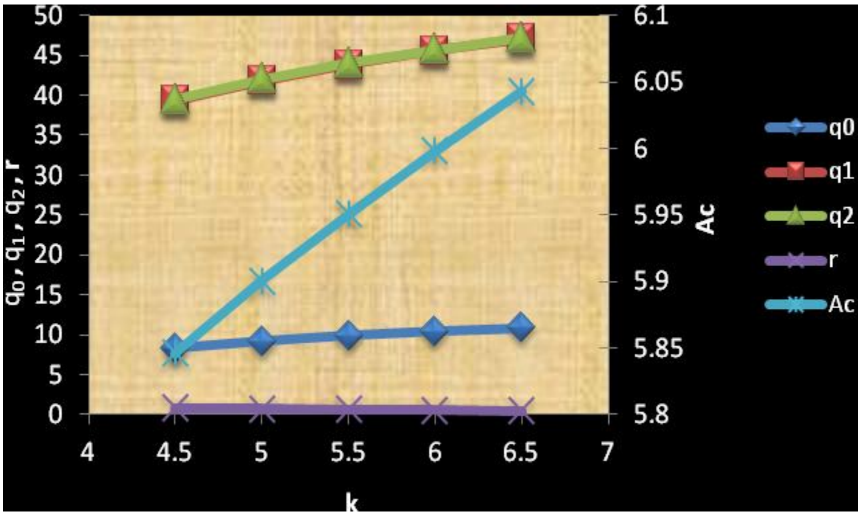

Fig. 4.5.6.2 Sensitivity analysis graph for k

We see that increasing the ordering cost k, results in increase in average cost, when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of ordering cost k and keeping other parameter values fixed where α_1=1 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.6.3.

Table 4.5.6.3
Sensitivity Analysis Table by varying the parameter values of k
(α_1=1and α_2=0)

k	q_0	q_1	q_2	r	AC
4.5	5.816673	34.16042	34.03268	0.97103	6.019221
5	6.919021	36.68451	36.68383	0.925376	6.091088
5.5	7.948582	39.27446	39.34869	0.845075	6.153469
6	8.783016	41.56245	41.6723	0.757834	6.209351
6.5	9.45033	43.5245	43.64968	0.675901	6.260824

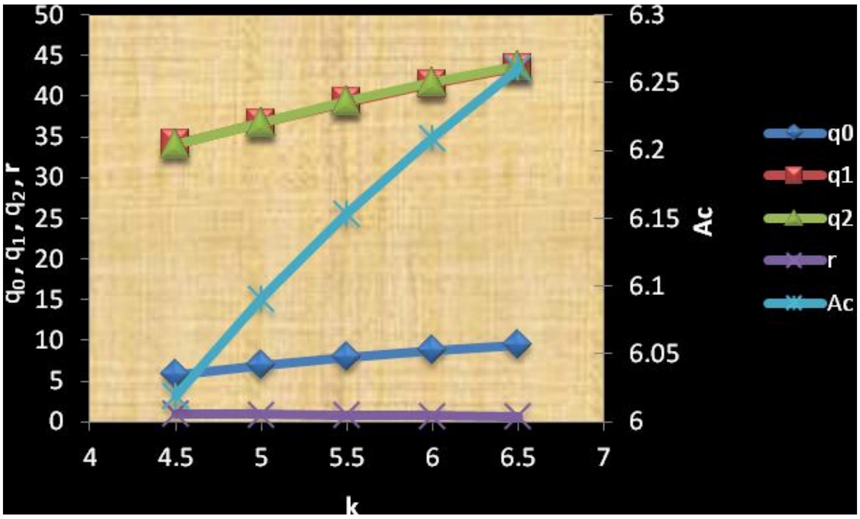

Fig. 4.5.6.3 Sensitivity analysis graph for k

We see that as ordering cost k increases, average cost increases, when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

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(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of ordering cost k and keeping other parameter values fixed where α_1=0 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.6.4.

Table 4.5.6.4
Sensitivity Analysis Table by varying the parameter values of k
(α_1=0 and α_2=1)

k	q_0	q_1	q_2	r	AC
4.5	5.44694	33.48098	33.30266	0.965965	6.067404
5	6.573681	35.95015	35.9173	0.938723	6.142968
5.5	7.743695	38.77173	38.83411	0.861564	6.207476
6	8.698424	41.30941	41.41623	0.768249	6.264253
6.5	9.436885	43.43414	43.55884	0.68039	6.316019

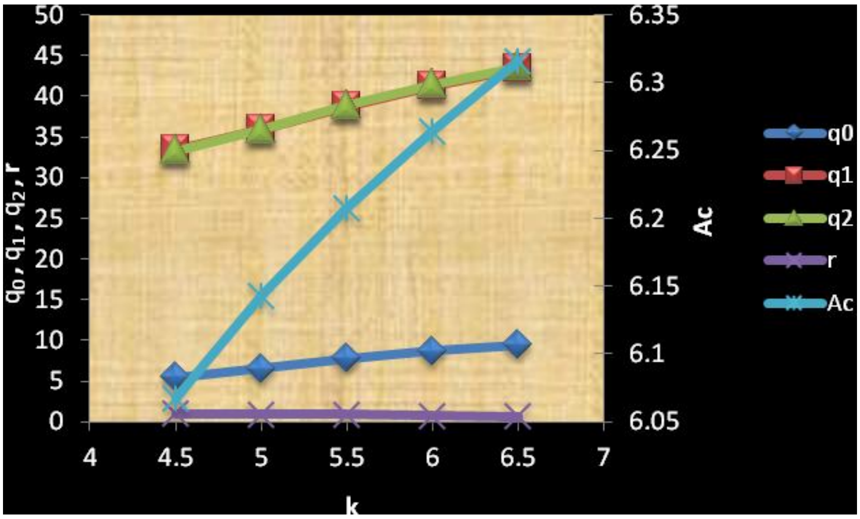

Fig. 4.5.6.4 Sensitivity analysis graph for k

Increasing the ordering cost k, results in increase in average cost, when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.6. CONCLUSION:

From the above sensitivity analysis, we conclude that in all the situations cost is minimum when account is settled by businessman at the respective credit time given by both the suppliers i.e. when (α_1=0 and α_2=0). The privilege offered proves to be bliss for entrepreneurs, so they are in the business resulting keen competition due to increased number of entrepreneurs.

CHAPTER 5

STOCHASTIC INVENTORY MODEL UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENT FOR TWO SUPPLIERS

CHAPTER 5

5.1. INTRODUCTION:

In this chapter we have introduced the effect of inflation and time value of money was investigated under given sets of inflation and discount rates.

5.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model for two suppliers under inflation and permissible delay in payment is developed on the basis of the following assumptions.

(a) r_1 = discount rate representing the time value of money.

(b) f = inflation rate

(c) $R = f - r_1$ = present value of the nominal inflation rate.

(d) t_1= time period with inflation

(e) c_0 = present value of the inflated price of an item Rs. /unit = $ce^{(f-r_1)t_1} = ce^{Rt_1}$

(f) $Ie(1i)$ =Interest earned over period (0 to T_{0i}) $= dce^{Rt_1} T_{00}\, T_{0i}\, ie_i$

(g) $Ie(2i)$ =Interest earned over period (T_{0i} to T_{00}) upon interest earned $(Ie(1i))$ previously.

$$Ie(2i) = \left(dce^{Rt_1} T_{00} + Ie(1i)\right)(T_{00} - T_{0i})ie_i$$

(h) Interest charged by the ith supplier clearly ($ic_i > ie_i$) i=1, 2

$$Ic_i = \alpha i\, dce^{Rt_1} ic_i (T_{00} - T_{0i})$$

$A(q_i, r, \theta)$ =(cost of ordering)+(cost of holding inventory)+ (cost of item that deteriorate during a single interval that starts with an inventory of (q_i+r) units and ends with r units with inflation rate);

$$A(q_i, r, \theta) == k + \frac{1}{2} \cdot \frac{hq_i^2 e^{Rt_1}}{(d+\theta)} + \frac{hrq_i e^{Rt_1}}{(d+\theta)} + \frac{\theta c q_i e^{Rt_1}}{(d+\theta)} \qquad \text{i=0, 1, 2}$$

C_{00} =E (cost per cycle); and T_{00} =E (length of a cycle);

$P_{ij}(t)$ =P (Being in state j at time t/starting in state i at time 0) , i, j=0, 1, 2, 3 ;

p_i =long run probabilities, i=0, 1, 2, 3

5.3. OPTIMAL POLICY DECISION FOR THE MODEL:

Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, i, j=0, 1, 2, 3 for the four state CTMC. The lemma which is used to obtain the transition probabilities is same as discussed in chapter 4, (lemma (4.3.1)) hence we omit it here.

Define C_{i0}=E (cost incurred to the beginning of the next cycle from the time when inventory drops to r at state i=0, 1, 2, 3 and q_i units are ordered if i=0, 1 or 2)

Lemma 5.3.1: C_{i0} is given by

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right) A(q_i, r, \theta) + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[A(q_i, r, \theta) + C_{j0}\right] \quad \text{i=0, 1, 2} \tag{5.3.1}$$

$$C_{30} = \overline{C} + \sum_{i=1}^{2} \rho_i C_{i0} \tag{5.3.2}$$

Where $\rho_i = \frac{\mu_i}{\delta}$ with $\delta = \mu_1 + \mu_2$ and

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}} e^{Rt_1}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}\left(\delta r - (d+\theta)\right) + \left(\pi \delta d + h(d+\theta) + \hat{\pi}\right) - \theta c \delta \right] + \frac{\theta c e^{Rt_1}}{\delta} \tag{5.3.3}$$

Proof: First consider i=0. Conditioning on the state of the supplier availability process when inventory drops to r, we obtain

$$C_{00} = P_{00}\left(\frac{q_0}{d+\theta}\right)A(q_0,r,\theta) + \sum_{j=1}^{3} P_{0j}\left(\frac{q_0}{d+\theta}\right)\left[A(q_0,r,\theta)+C_{j0}\right] \tag{5.3.4}$$

The equation follows because $q_0 + r$ being the initial inventory, when q_0 units are used up we either observe state 0, 1, 2 or 3 with probabilities

$P_{00}\left(\frac{q_0}{d+\theta}\right), P_{01}\left(\frac{q_0}{d+\theta}\right), P_{02}\left(\frac{q_0}{d+\theta}\right)$ and $P_{03}\left(\frac{q_0}{d+\theta}\right)$ respectively. If we are in state 0 when r is reached, we must have incurred a cost of $A(q_0,r,\theta)$. On the other hand, if state j=1, 2, 3 is observed when inventory drops to r, then the expected cost will be $A(q_0,r,\theta)+C_{j0}$ with probability $P_{0j}\left(\frac{q_0}{d+\theta}\right)$. The equation relating C_{10} and C_{20} are very similar but C_{30} is obtained as

$$C_{30} = \left[\overline{C}+C_{10}\right]\frac{\mu_1}{\mu_1+\mu_2} + \left[\overline{C}+C_{20}\right]\frac{\mu_2}{\mu_1+\mu_2} \tag{5.3.5}$$

Here, $\overline{C}$ is defined as the expected cost from the time inventory drops to r until either supplier 1 or 2 becomes available and it is computed as follows:

Now referring to Fig 4.1., note that the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of next cycle is equal to

$$\frac{1}{2}hy^2e^{Rt_1}(d+\theta) + hye^{Rt_1}\left[r-y(d+\theta)\right] + \theta c e^{Rt_1}y \qquad y < \frac{r}{d+\theta}$$

$$\frac{1}{2}\frac{hr^2e^{Rt_1}}{(d+\theta)} + \pi e^{Rt_1}\left(y-\frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}e^{Rt_1}}{2}\left(y-\frac{r}{(d+\theta)}\right)^2 + \frac{\theta c r e^{Rt_1}}{(d+\theta)} \qquad y \geq \frac{r}{d+\theta}$$

Hence,

$$\overline{C} = \int_0^{r/(d+\theta)} \left\{ \frac{1}{2} hy^2 (d+\theta) e^{R\,t_1} + h e^{R\,t_1} y (r - y(d+\theta) + \theta c e^{R\,t_1} \right\} \delta e^{-\delta y} dy$$

$$+ \int_{r/(d+\theta)}^{\infty} \left\{ \frac{1}{2} \frac{h r^2 e^{R\,t_1}}{(d+\theta)} + \pi e^{R\,t_1} \left[y - \frac{r}{(d+\theta)} \right] d + \frac{\hat{\pi} e^{R\,t_1}}{2} \left[y - \frac{r}{(d+\theta)} \right]^2 + \frac{\theta c r e^{R\,t_1}}{(d+\theta)} \right\} \delta e^{-\delta y} dy$$

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}} e^{R\,t_1}}{\delta^2} \left[h e^{\frac{\delta r}{(d+\theta)}} \left(\delta r - (d+\theta) \right) + \left(\pi \delta d + h(d+\theta) + \hat{\pi} \right) - \theta c \delta \right] + \frac{\theta c e^{R\,t_1}}{\delta}$$

with $\delta = \mu_1 + \mu_2$ as the rate of departure from state 3. This follows because if supplier availability process is in state 3 (OFF for both suppliers) when inventory drops to r, then the expected holding and backorder costs are equal to $\overline{C}$. If the process makes a transition to state 1, the total expected cost would then be $\overline{C} + C_{10}$. The probability of a transition from state 3 to state 1 is

$$P(Y_1 < Y_2) = \int_0^{\infty} P(Y_1 < Y_2 / Y_2 = t) \mu_2 e^{-\mu_2 t} dt = \frac{\mu_1}{\mu_1 + \mu_2}$$

Multiplying this probability with the expected cost term above gives the first term of (5.3.5). The second term is obtained in a similar manner. Combining the results proves the lemma.

The following lemma provides a simpler means of expressing C_{00} in an exact manner. To simplify the notation, we let $A_i = A(q_i, r, \theta)$, i=0, 1, 2 and $P_{ij} = P_{ij}\left(\frac{q_i}{d+\theta} \right)$ i, j=0, 1, 2, 3.

Lemma 5.3.2: The exact expression for C_{00} is

$$C_{00} = A_0 + P_{01} C_{10} + P_{02} C_{20} + P_{03} \left(\overline{C} + \rho_1 C_{10} + \rho_2 C_{20} \right) \qquad (5.3.6)$$

where the pair [C_{10}, C_{20}] solves the system

$$\begin{bmatrix} 1-P_{11}-P_{13}\rho_1 & -(P_{12}+P_{13}\rho_2) \\ -(P_{21}+P_{23}\rho_1) & 1-P_{22}-P_{23}\rho_2 \end{bmatrix} \begin{bmatrix} C_{10} \\ C_{20} \end{bmatrix} = \begin{bmatrix} A_1+P_{13}\overline{C} \\ A_2+P_{23}\overline{C} \end{bmatrix} \tag{5.3.7}$$

Proof: Rearranging the linear system of four equations in lemma (5.3.1) in matrix form gives

$$\begin{bmatrix} 1 & -P_{01} & -P_{02} & -P_{03} \\ 0 & 1-P_{11} & -P_{12} & -P_{13} \\ 0 & -P_{21} & 1-P_{22} & -P_{23} \\ 0 & -\rho_1 & -\rho_2 & 1 \end{bmatrix} \begin{bmatrix} C_{00} \\ C_{10} \\ C_{20} \\ C_{30} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \overline{C} \end{bmatrix} \tag{5.3.8}$$

We have $C_{30} = \overline{C} + \rho_1 C_{10} + \rho_2 C_{20}$ from the last row of the system. Substituting this result in rows two and three and rearranging gives the system in (5.3.7), with (C_{10}, C_{20}). From the first row of (5.3.8) we obtain $C_{00} = A_0 + \sum_{j=1}^{3} P_{0j} C_{j0}$.

Hence above lemma is proved.

The lemma (4.3.4) and (4.3.5) are same as discussed in chapter 4, hence we omit it here.

Proposition 5.3.1: The Average cost objective function for two suppliers when inflation and delay in payment is considered is given by

$$AC = \frac{C_{00}}{T_{00}} = \frac{\begin{array}{c} A(q_0, r, \theta) + P_{01}(C_{10} - (Ie(11) + Ie(21) + Ic_1) + P_{02}(C_{20} - (Ie(12) + Ie(22) + Ic_2) \\ + P_{03}(\overline{C} + \rho_1(C_{10} - (Ie(11) + Ie(21) + Ic_1) + \rho_2(C_{20} - (Ie(12) + Ie(22)) + Ic_2)) \end{array}}{\dfrac{q_0}{d+\theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\overline{T} + \rho_1 T_{10} + \rho_2 T_{20})}$$

Proof: Proof follows using Renewal reward theorem (RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

5.4. NUMERICAL EXAMPLE:

In this section we verify the results by a numerical example. We assume that

(i) k =Rs. 5/order, c=Rs.1/unit, d=20/units, θ=4, h=Rs. 5/unit/time, π=Rs. 350/unit, $\hat{\pi}$=Rs.25/unit/time, ic_1=0.11, ie_1=0.02, ic_2=0.13, ie_2=0.04, R=0.05, t_1=6, T_{01}=0.6, T_{02}=0.8, (α_1=1 and α_2=1) i.e. businessmen do not settle the account at the respective credit time given by both the suppliers, λ_1=0.58, λ_2=0.45, μ_1=3.4, μ_2=2.5.

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are $1/\lambda_1$=1.72413794, $1/\lambda_2$=2.2222, $1/\mu_1$=.2941176 and $1/\mu_2$=.4 respectively. The long run probabilities are obtained as p_0=0.7239588, p_1=0.1303126, p_2 =0.1234989 and p_3=0.02222. The optimal solution is obtained as

q_0=3.506669, q_1=30.128739, q_2=29.56780, r=0.81358 and AC=$\frac{C_{00}}{T_{00}}$ = 8.1358.

(ii) Keeping other parameters as it is, we consider (α_1=0 and α_2=0) i.e. businessmen settle the account at the respective credit time given by both the suppliers.

The optimal solution is obtained as q_0=6.106844, q_1=33.97769, q_2=33.8575, r=1.026170 and AC=$\frac{C_{00}}{T_{00}}$ = 7.750814.

(iii) Keeping other parameters as it is, we consider (α_1=1 and α_2=0) i.e. businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

The optimal solution is obtained as q_0= 4.384248, q_1= 31.17163, q_2= 30.78434 r= 0.95295 and AC=$\frac{C_{00}}{T_{00}}$ = 7.935795.

(iv) Keeping other parameters as it is, we consider (α_1=0 and α_2=1) i.e. when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

The optimal solution is obtained as q_0= 4.12906, q_1= 30.80062, q_2= 30.3622, r= 0.925938 and AC=$\frac{C_{00}}{T_{00}}$ = 7.9908.

Conclusion:

From the above numerical example, we conclude that the cost is minimum when account is settled at the credit time given by the i^{th} supplier. Comparing the above results with that of chapter 4 we observe that cost is more here due to inflation. So in this situation also businessmen are advised to settle the account at the credit time given by the respective suppliers.

5.5. SENSITIVITY ANALYSIS:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R keeping other parameter values fixed where α_1=1 and α_2=1. Inflation rate R is assumed to take values 0.05, 0.08, 0.1, 0.12, 0.15. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 5.5.1
Sensitivity Analysis Table by varying the parameter values of R
(α_1=1 and α_2=1)

R	q_0	q_1	q_2	r	AC
0.05	3.50667	30.1287	29.5678	0.81888	8.1358
0.08	2.97984	29.0031	28.2686	0.7625	9.48985
0.1	2.69918	28.3961	27.5549	0.72583	10.5174
0.12	2.4587	27.8708	26.9295	0.69054	11.6591
0.15	2.15475	27.1996	26.1194	0.64053	13.6164

We see that as inflation rate R increases values of q_0, q_1, q_2 and value of reorder quantity r decreases and hence average cost increases.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R keeping other parameter values fixed where α_1=0 and α_2=0. Inflation rate R is assumed to take values 0.05, 0.08, 0.1, 0.12, 0.15. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 5.5.2
Sensitivity Analysis Table by varying the parameter values of R
(α_1=0 and α_2=1)

R	q_0	q_1	q_2	r	AC
0.05	6.10684	33.9777	33.8578	1.02617	7.75081
0.08	4.46811	30.6747	30.2617	1.01785	9.10662
0.1	3.80797	29.4459	28.8683	0.97442	10.1314
0.12	3.32578	28.5693	27.8497	0.92544	11.2683
0.15	2.79119	27.6076	26.7053	0.85213	13.2162

We see that as inflation rate R increases values of q_0, q_1, q_2 and value of reorder quantity r decreases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R keeping other parameter values fixed where α_1=1 and α_2=0. Inflation rate R is assumed to take values 0.05, 0.08, 0.1, 0.12, 0.15. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 5.5.3
Sensitivity Analysis Table by varying the parameter values of R
(α_1=1 and α_2=0)

R	q_0	q_1	q_2	r	AC
0.05	4.12907	30.8006	30.3623	0.92594	7.9908
0.08	3.39002	29.3407	28.6986	0.86341	9.34255
0.1	3.02527	28.6195	27.8568	0.82053	10.3676
0.12	2.72496	28.0219	27.1484	0.7788	11.5063
0.15	2.35853	27.286	26.2617	0.71948	13.4583

We see that as inflation rate R increases values of q_0, q_1, q_2 and value of reorder quantity r decreases and hence average cost increases.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=1$. Inflation rate R is assumed to take values 0.05, 0.08, 0.1, 0.12, 0.15. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 5.5.4
Sensitivity Analysis Table by varying the parameter values of R
($\alpha_1=0$ and $\alpha_2=1$)

R	q_0	q_1	q_2	r	AC
0.05	4.384242	31.17162	30.78432	0.95295	7.935795
0.08	3.57047	29.5611	28.9627	0.8954	9.28645
0.1	3.17314	28.7834	28.0601	0.85285	10.3108
0.12	2.84871	28.1486	27.3108	0.8104	11.4489
0.15	2.45606	27.3774	26.3844	0.74903	13.4005

We see that as inflation rate R increases values of q_0, q_1, q_2 and value of reorder quantity r decreases and hence average cost increases.

5.6. CONCLUSION:

From the above sensitivity analysis, in all the various situations of settling the account we conclude that the cost is minimum when account is settled at credit time given by the i^{th} supplier where i=1, 2. Comparing the above results with that of chapter 4 we observe that cost is more here due to inflation. So in this situation also businessmen are advised to settle the account at the credit time given by respective suppliers.

Comparing the above results with that of chapter 2 we observe the following:

Here, the long run probability of non-availability of both suppliers case is 0.02222 and in a single supplier case is 0.091. In this favorable condition of reduced probability we find that average cost is lower here than that obtained in chapter 2 i.e. in case of single supplier case. The moral that follows is that it is always advisable to go for two suppliers or multiple suppliers for reduced average cost.

CHAPTER 6

STOCHASTIC INVENTORY MODEL UNDER PERMISSIBLE DELAY IN PAYMENT ALLOWING PARTIAL PAYMENT FOR TWO SUPPLIERS

CHAPTER 6

6.1. INTRODUCTION:

In this chapter, we have introduced the aspect of part payment. A part of the purchased cost is to be paid during the permissible delay period. What quantity of the part is to be paid and the time at which it has to be paid can be fixed up at the time of the deal of purchasing the goods.

6.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model for two suppliers under permissible delay in payment allowing partial payment is developed on the basis of the following assumptions.

(a) T_{1i} is the time allowed by i^{th} supplier where i=1, 2 at which α_i ($0 < \alpha_i < 1$) fraction of total amount has to be paid to the i^{th} supplier where i=1, 2.

(b) T_i ($T_i > T_{1i}$) is the time at which remaining amount has to be cleared.

(c) T_{00} is the expected cycle time. T_{1i} and T_i are known constants and T_{00} is a decision variable.

(d) Ie_i=Interest rate earned when purchase made from i^{th} supplier where i=1, 2

Ic_i=Interest rate charged by i^{th} supplier where i=1, 2.

(e) U_i and V_i are indicator variables for i^{th} supplier where i=1, 2

$U_1=0$ if part payment is done at T_{11} to the first supplier by the businessmen
$= 1$ otherwise

$U_2=0$ if part payment is done at T_{12} to the second supplier by the businessmen
$= 1$ otherwise

$V_1=0$ if the balanced amount is cleared at T_1 for the 1st supplier by the businessmen
$= 1$ otherwise

$V_2=0$ if the balanced amount is cleared at T_2 for the 2nd supplier by the businessmen
$= 1$ otherwise

In this chapter, we assume that supplier allows a fixed period T_{1i} during which α_i fraction of total amount has to be paid and remaining amount i.e. (1- α_i) fraction has to be cleared up to time T_i. Hence up to time period T_{1i} no interest is charged for α_i fraction, but beyond that period, interest will be charged upon not doing promised payment of α_i fraction. Similarly for (1- α_i) fraction no interest will be charged up to time period T_i but beyond that period interest will be charged. However, customer can sell the goods and earn interest on the sales revenue during the period of admissible delay.

Interest earned and interest charged is as follows.

(f) Interest earned on the entire amount up to time period T_{1i} is $dcT_{1i}\, T_{00}\, Ie_i$

(g) Interest earned on (1- α_i) fraction during the period (T_i- T_{1i}) is

$(1-\alpha_i)\, d\, c(T_i - T_{1i})\, T_{00}\, Ie_i$

(h) If part payment is not done at T_{1i} then interest will be earned over α_i fraction for period $(T_i - T_{1i})$ but interest will also be charged for α_i fraction for $(T_i - T_{1i})$ period.

Interest earned= $d\, c\, \alpha_i\, T_{00}\, (T_i - T_{1i})\, Ie_i$

Interest charged= $d\, c\, \alpha_i\, T_{00}\, (T_i - T_{1i})\, Ic_i$

To discourage not doing promised payment, we assume that Ic_i is quite larger than Ie_i .

(i) Interest earned over the amount $d\, c\, T_{00} T_{1i} Ie_i$ over the period $(T_i - T_{1i})$ is

$d\, c\, T_{00}\, T_{1i}\, Ie_i\, (T_i - T_{1i})\, Ie_i$

(j) If the remaining amount is not cleared at T_i then interest will be earned for the period $(T_{00} - T_i)$ for $(1-\alpha_i)$ fraction simultaneously interest will be charged on the same amount for the same period.

Interest earned= $d\, c\, (1-\alpha_i)\, T_{00}\, (T_{00} - T_i)\, Ie_i$

Interest charged= $d\, c\, (1-\alpha_i)\, T_{00}\, (T_{00} - T_i)\, Ic_i$

Total interest earned = $dcT_{1i}\,T_{00}\,Ie_i + (1-\alpha_i)\,d\,c(T_i - T_{1i})T_{00}\,Ie_i + d\,c\,\alpha_i\,T_{00}\,(T_i - T_{1i})\,Ie_i$ +

$d\,c\,T_{00}\,T_{1i}\,Ie_i\,(T_i - T_{1i})\,Ie_i$ + V_i

$[\,d\,c\,(1-\alpha_i)\,T_{00}\,(T_{00} - T_i)\,Ie_i + dcT_{1i}\,T_{00}\,Ie_i\,(T_i - T_{1i})\;Ie_i\,(T_{00} - T_i)\,Ie_i + dcT_{1i}\,T_{00}\,Ie_i\,(T_{00} - T_i)\,Ie_i$

$+\,d\,c\,(1-\alpha_i)\,T_{00}(T_i - T_{1i})\,Ie_i(T_{00} - T_i)\,Ie_i + \{\,d\,c\,\alpha_i\,T_{00}Ie_i\,(T_i - T_{1i})\,Ie_i$ - $d\,c\,\alpha_i\,T_{00}Ic_i\,(T_{00} - T_i)\;\}]$

Total Interest charged= $d\,c\,\alpha_i\,T_{00}\,(T_i - T_{1i})\,Ic_i$ + $V_i[\,d\,c\,(1-\alpha_i)\,T_{00}\,(T_{00} - T_i)\,Ic_i\,]$

Total interest earned and charged is as follows.

$dcT_{1i}\,T_{00}\,Ie_i + (1-\alpha_i)\,d\,c(T_i - T_{1i})T_{00}\,Ie_i + \{\,d\,c\,\alpha_i\,T_{00}\,(T_i - T_{1i})\,Ie_i$ -

$d\,c\,\alpha_i\,T_{00}\,(T_i - T_{1i})\,Ic_i\,\} + d\,c\,T_{00}\,T_{1i}\,Ie_i\,(T_i - T_{1i})\,Ie_i + V_i$

$[\,d\,c\,(1-\alpha_i)\,T_{00}\,(T_{00} - T_i)\,Ie_i + dcT_{1i}\,T_{00}\,Ie_i\,(T_i - T_{1i})\;Ie_i\,(T_{00} - T_i)\,Ie_i + dcT_{1i}\,T_{00}\,Ie_i\,(T_{00} - T_i)\,Ie_i$

$+\,d\,c\,(1-\alpha_i)\,T_{00}(T_i - T_{1i})\,Ie_i(T_{00} - T_i)\,Ie_i + \{\,d\,c\,\alpha_i\,T_{00}Ie_i\,(T_i - T_{1i})\,Ie_i$

$-\,d\,c\,\alpha_i\,T_{00}Ic_i\,(T_{00} - T_i)\;\} - d\,c\,(1-\alpha_i)\,T_{00}\,(T_{00} - T_i)\,Ic_i\,]$

6.3. OPTIMAL POLICY DECISION FOR THE MODEL:

Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, i, j=0, 1, 2, 3 for the four state CTMC. The lemma which is used to obtain the transition probabilities is same as discussed in chapter 4, (lemma (4.3.1)) also lemma 4.3.2 to 4.3.5 are also same hence we omit it here.

Proposition 6.3.1: The Average cost objective function for two suppliers when delay in payment allowing partial payment is given by AC= $\frac{C_{00}}{T_{00}}$

C_{00} is given by

$$
\begin{aligned}
C_{00} = {} & A(q_0,r)+P_{01}\left\{C_{10}-dcT_{00}T_{11}Ie_1-(1-\alpha_1)dcT_{00}(T_1-T_{11})Ie_1-U_1dc\alpha_1T_{00}(T_1-T_{11})Ie_1\right. \\
& +U_1dc\alpha_1T_{00}(T_1-T_{11})Ic_1-dcT_{00}T_{11}Ie_1(T_1-T_{11})Ie_1 \\
& -V_1\begin{bmatrix}(1-\alpha_1)dcT_{00}(T_{00}-T_1)Ie_1+dcT_{00}T_{11}Ie_1(T_1-T_{11})Ie_1(T_{00}-T_1)Ie_1\\ +dcT_{00}T_{11}Ie_1(T_{00}-T_1)Ie_1+(1-\alpha_1)dcT_{00}(T_1-T_{11})Ie_1(T_{00}-T_1)Ie_1\end{bmatrix} \\
& -V_1\left[U_1\left\{dc\alpha_1T_{00}Ie_1(T_1-T_{11})(T_{00}-T_1)Ie_1\right\}\right] \\
& \left.+V_1\left[U_1\left\{dc\alpha_1T_{00}Ic_1(T_{00}-T_1)+(1-\alpha_1)dcT_{00}Ic_1(T_{00}-T_1)\right\}\right]\right\} \\
& +P_{02}\left\{C_{20}-dcT_{00}T_{12}Ie_2-(1-\alpha_2)dcT_{00}(T_2-T_{12})Ie_2-U_2dc\alpha_2T_{00}(T_2-T_{12})Ie_2\right. \\
& +U_2dc\alpha_2T_{00}(T_2-T_{12})Ic_2-dcT_{00}T_{12}Ie_1(T_2-T_{12})Ie_2 \\
& -V_2\begin{bmatrix}(1-\alpha_2)dcT_{00}(T_{00}-T_2)Ie_2+dcT_{00}T_{12}Ie_2(T_2-T_{12})Ie_2(T_{00}-T_2)Ie_2\\ +dcT_{00}T_{12}Ie_2(T_{00}-T_2)Ie_2+(1-\alpha_2)dcT_{00}(T_2-T_{12})Ie_2(T_{00}-T_2)Ie_2\end{bmatrix} \\
& -V_2\left[U_2\left\{dc\alpha_2T_{00}Ie_2(T_2-T_{12})(T_{00}-T_2)Ie_2\right\}\right] \\
& \left.+V_2\left[U_2\left\{dc\alpha_2T_{00}Ic_2(T_{00}-T_2)+(1-\alpha_2)dcT_{00}Ic_2(T_{00}-T_2)\right\}\right]\right\} \\
& +P_{03}\left\{\overline{C}+\rho_1\begin{pmatrix}C_{10}-dcT_{00}T_{11}Ie_1-(1-\alpha_1)dcT_{00}(T_1-T_{11})Ie_1-U_1dc\alpha_1T_{00}(T_1-T_{11})Ie_1\\ +U_1dc\alpha_1T_{00}(T_1-T_{11})Ic_1-dcT_{00}T_{11}Ie_1(T_1-T_{11})Ie_1\\ -V_1\begin{bmatrix}(1-\alpha_1)dcT_{00}(T_{00}-T_1)Ie_1+dcT_{00}T_{11}Ie_1(T_1-T_{11})Ie_1(T_{00}-T_1)Ie_1\\ +dcT_{00}T_{11}Ie_1(T_{00}-T_1)Ie_1+(1-\alpha_1)dcT_{00}(T_1-T_{11})Ie_1(T_{00}-T_1)Ie_1\end{bmatrix}\\ -V_1\left[U_1\left\{dc\alpha_1T_{00}Ie_1(T_1-T_{11})(T_{00}-T_1)Ie_1\right\}\right]\\ +V_1\left[U_1\left\{dc\alpha_1T_{00}Ic_1(T_{00}-T_1)+(1-\alpha_1)dcT_{00}Ic_1(T_{00}-T_1)\right\}\right]\end{pmatrix}\right. \\
& \left.+\rho_2\begin{pmatrix}C_{20}-dcT_{00}T_{12}Ie_2-(1-\alpha_2)dcT_{00}(T_2-T_{12})Ie_2-U_2dc\alpha_2T_{00}(T_2-T_{12})Ie_2\\ +U_2dc\alpha_2T_{00}(T_2-T_{12})Ic_2-dcT_{00}T_{12}Ie_1(T_2-T_{12})Ie_2\\ -V_2\begin{bmatrix}(1-\alpha_2)dcT_{00}(T_{00}-T_2)Ie_2+dcT_{00}T_{12}Ie_2(T_2-T_{12})Ie_2(T_{00}-T_2)Ie_2\\ +dcT_{00}T_{12}Ie_2(T_{00}-T_2)Ie_2+(1-\alpha_2)dcT_{00}(T_2-T_{12})Ie_2(T_{00}-T_2)Ie_2\end{bmatrix}\\ -V_2\left[U_2\left\{dc\alpha_2T_{00}Ie_2(T_2-T_{12})(T_{00}-T_2)Ie_2\right\}\right]\\ +V_2\left[U_2\left\{dc\alpha_2T_{00}Ic_2(T_{00}-T_2)+(1-\alpha_2)dcT_{00}Ic_2(T_{00}-T_2)\right\}\right]\end{pmatrix}\right\}
\end{aligned}
$$

and $$T_{00}=\frac{q_0}{d+\theta}+P_{01}T_{10}+P_{02}T_{20}+P_{03}(\overline{T}+\rho_1T_{10}+\rho_2T_{20})$$

Proof: Proof follows using Renewal reward theorem (RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

6.4. NUMERICAL EXAMPLE:

There are sixteen different patterns of payments, some of them we consider here.

1. Ui=0 and Vi=0 where i=1, 2 that is promise of doing part payment at time T_{1i} and clearing the remaining amount at time T_i, the time period given by i^{th} supplier where i=1, 2, both are satisfied.

2. Ui=0 and Vi=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i , the time period given by i^{th} supplier where i=1, 2.

3. Ui=1 and Vi=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is not satisfied for both the suppliers but all the amount are cleared at time T_i , the time period given by i^{th} supplier where i=1, 2.

4. Ui=0, V_1=0 and V_2=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and clearing the remaining amount at time T_1 for 1^{st} supplier is satisfied, but remaining amount is not cleared at time T_2 for 2^{nd} supplier.

5. Ui=0, V_1=1 and V_2=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and promise of clearing the remaining amount at time T_2 for 2^{nd} supplier is satisfied, but remaining amount is not cleared at time T_1 for 1^{st} supplier.

6. U_1=0, U_2=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is kept for 1^{st} supplier but promise of doing part payment at time T_{12} is not satisfied for 2^{nd} supplier however clearing the remaining amount at time T_i, the time period given by i^{th} supplier where i=1, 2, are satisfied for both the suppliers.

7. U_1=1, U_2=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is not satisfied for 1^{st} supplier but promise of doing part payment at time T_{12} is satisfied for 2^{nd} supplier however clearing the remaining amount at time $T_{i,}$ the time period given by i^{th} supplier where i=1, 2 are satisfied for both the suppliers.

In this section we verify the results by a numerical example. We assume that k=Rs. 5/order, c=Rs.1/unit, d=20/units, θ=4, h=Rs. 5/unit/time, π=Rs. 350/unit, T_{11}=0.6, $\hat{\pi}$=Rs. 25/unit/time, α_1=0.5, α_2=0.6, Ic_1=0.11, Ie_1=0.02, Ic_2=0.13, Ie_2=0.04, T_{12}=0.8, T_1=0.9, T_2=1.1, λ_1=0.58, λ_2=0.45, μ_1=3.4, μ_2=2.5.

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are $1/\lambda_1$=1.72413794, $1/\lambda_2$=2.2222, $1/\mu_1$=.2941176 and $1/\mu_2$=.4 respectively. The long run probabilities are obtained as p_0=0.7239588, p_1=0.1303126, p_2 =0.1234989 and p_3=0.02222979. The optimal solution for the above numerical example based on the seven patterns of payment is obtained as

(U_1,U_2,V_1,V_2)	q_0	q_1	q_2	r	AC
(0,0,0,0)	3.2899	30.17858	29.58059	0.745935	6.406068
(0,0,1,1)	2.9496	29.82422	29.14462	0.664672	6.50769
(1,1,0,0)	3.34668	30.15484	29.56186	0.766788	6.37324
(0,0,0,1)	3.04876	29.91408	29.25791	0.690931	6.475395
(0,0,1,0)	3.15503	30.04058	29.41159	0.714835	6.443119
(0,1,0,0)	3.32203	30.16482	29.56969	0.757816	6.386726
(1,0,0,0)	3.31408	30.16817	29.5723	0.75489	6.392686

Conclusion:

From this we conclude that the cost is minimum if part payment is not done at T_{1i} but account is cleared at T_i and the cost is maximum if part payment is done at T_{1i} but account is not cleared at T_i, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

6.5. SENSITIVITY ANALYSIS:

To observe the effects of varying parameter values on the optimal solution we have conducted sensitivity analysis, by varying μ_1, λ_2, h and k on the following seven patterns of payment.

6.5.1. Sensitivity Analysis for μ_1:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where U_i=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} and clearing the remaining amount at time T_i, the time period given by i^{th} supplier where i=1, 2, both are satisfied. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.1.1
Sensitivity Analysis Table by varying the parameter values of μ_1 when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=0)

μ_1	q_0	q_1	q_2	r	AC
2.4	3.1989	31.742	31.195	1.6671	6.8755
3	3.25	30.764	30.153	1.0378	6.5665
3.4	3.289	30.178	29.58	0.7459	6.406
4.4	3.3954	28.947	28.514	0.2633	6.1107
4.8	3.4374	28.539	28.201	0.1312	6.0228

We see that increasing μ_1 i.e. decreasing expected length of OFF period for 1^{st} supplier, results in decrease in average cost when the businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where U_i=0 and V_i=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.1.2
Sensitivity Analysis Table by varying the parameter values of μ_1 when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=1)

μ_1	q_0	q_1	q_2	r	AC
2.4	2.7897	31.375	30.797	1.5588	6.9929
3	2.8875	30.408	29.729	0.9462	6.6734
3.4	2.9496	29.824	29.144	0.6646	6.5076
4.4	3.094	28.586	28.058	0.2048	6.2026
4.8	3.1492	28.174	27.741	0.0805	6.1116

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, average cost decreases when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where U_i=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is not satisfied but all the amount is cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.1.3
Sensitivity Analysis Table by varying the parameter values of μ_1 when patterns of payment is (U_1=1, U_2=1, V_1=0, V_2=0)

μ_1	q_0	q_1	q_2	r	AC
2.4	3.2628	31.715	31.168	1.6931	6.8421
3	3.3091	30.739	30.131	1.0607	6.5335
3.4	3.3466	30.154	29.561	0.7667	6.3732
4.4	3.4478	28.927	28.534	0.2796	6.0783
4.8	3.4883	28.521	28.189	0.1459	5.9912

We see that increasing μ_1 i.e. decreasing expected length of OFF period for 1st supplier, results in decrease in average cost when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where Ui=0, V_1=0 and V_2=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and clearing the remaining amount at time T_1 for 1st supplier is satisfied, but remaining amount is not cleared at time T_2 for 2nd supplier. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.1.4
Sensitivity Analysis Table by varying the parameter values of μ_1 when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=1)

μ_1	q_0	q_1	q_2	r	AC
2.4	2.894807	31.459918	30.889826	1.589321	6.95821
3	2.9883564	30.495828	29.835451	0.9743761	6.640378
3.4	3.048766	29.91408	29.25791	0.690931	6.475395
4.4	3.1923548	28.685818	28.187664	0.2262752	6.171573
4.8	3.2463929	28.277973	27.876554	0.1000576	6.080889

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, average cost decreases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_1 for 1st supplier, however remaining amount is not cleared at time T_2 for 2nd supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where Ui=0, V_1=1 and V_2=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and promise of clearing the remaining amount at time T_2 for 2nd supplier is satisfied, but remaining amount is not cleared at time T_1 for 1st supplier. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.1.5
Sensitivity Analysis Table by varying the parameter values of μ_1 when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=0)

μ_1	q_0	q_1	q_2	r	AC
2.4	3.04146	31.596199	31.037312	1.627847	6.916945
3	3.108438	30.625285	29.98824	1.003636	6.604978
3.4	3.155037	30.04058	29.41159	0.714835	6.443119
4.4	3.2711593	28.806351	28.336233	0.2394774	6.14556
4.8	3.3159297	28.395869	28.020877	0.109958	6.056989

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, average cost decreases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_2 for 2nd supplier, however remaining amount is not cleared at time T_1 for 1st supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where U_1=0, U_2=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is satisfied for 1st supplier but promise of doing part payment at time T_{12} is not cleared for 2nd supplier however clearing the remaining amount at time T_i, the time period given by ith supplier where i=1, 2, are satisfied for both the suppliers. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.1.6
Sensitivity Analysis Table by varying the parameter values of μ_1 when patterns of payment is (U_1=0, U_2=1, V_1=0, V_2=0)

μ_1	q_0	q_1	q_2	r	AC
2.4	3.237649	31.725811	31.17901	1.682961	6.855258
3	3.284455	30.749471	30.14012	1.051278	6.546867
3.4	3.322036	30.16482	29.5696	0.757816	6.386726
4.4	3.4229698	28.936749	28.5068	0.2719976	6.09227
4.8	3.463274	28.529834	28.195133	0.1387996	6.004668

We see that increasing μ_1 i.e. decreasing expected length of OFF period for 1^{st} supplier, results in decrease in average cost, when part payment at time T_{11} is done for 1^{st} supplier but part payment at time T_{12} is not cleared for 2^{nd} supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where U_1=1, U_2=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is not satisfied for 1^{st} supplier but promise of doing part payment at time T_{12} is satisfied for 2^{nd} supplier however clearing the remaining amount at time T_i, the time period given by i^{th} supplier where i=1, 2 are satisfied for both the suppliers. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.1.7
Sensitivity Analysis Table by varying the parameter values of μ_1 when patterns of payment is (U_1=1, U_2=0, V_1=0, V_2=0)

μ_1	q_0	q_1	q_2	R	AC
2.4	3.223528	31.731817	31.184855	1.677208	6.86253
3	3.274242	30.753851	30.143832	1.047325	6.553339
3.4	3.31408	30.16817	29.5723	0.75489	6.392686
4.4	3.420041	28.93784	28.50754	0.271084	6.096975
4.8	3.462193	28.530216	28.195383	0.138487	6.008877

We see that increasing μ_1 i.e. decreasing expected length of OFF period for 1^{st} supplier, results in decrease in average cost, when part payment at time T_{11} is not done for 1^{st} supplier but part payment at time T_{12} is cleared for 2^{nd} supplier however remaining amount is cleared at the respective time given by both the suppliers.

6.5.2. Sensitivity Analysis for λ_2:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where U_i=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} and clearing the remaining amount at time T_i, the time period given by i^{th}

supplier where i=1, 2, both are satisfied. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.2.1
Sensitivity Analysis Table by varying the parameter values of λ_2 when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	3.318411	30.617133	29.833753	0.4008284	6.39465
0.43	3.303763	30.395773	29.708538	0.5773873	6.401486
0.45	3.28921	30.1781	29.58231	0.745911	6.406121
0.47	3.2767399	29.965521	29.450673	0.9071424	6.408698
0.49	3.264202	29.756447	29.756447	1.061605	6.409636

We see that increasing λ_2 i.e. decreasing expected length of ON period for 2nd supplier, results in increase in average cost when the businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where U_i=0 and V_i=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i, the time period given by ith supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.2.2
Sensitivity Analysis Table by varying the parameter values of λ_2 when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=1)

λ_2	q_0	q_1	q_2	r	AC
0.41	2.9996152	30.281891	29.409401	0.3235322	6.490541
0.43	2.9742574	30.050881	29.2783	0.4980911	6.500238
0.45	2.949612	29.8241	29.1442	0.664632	6.507126
0.47	2.9255607	29.601849	29.009157	0.8239513	6.513199
0.49	2.9020726	29.383642	28.872557	0.9765192	6.517024

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases, average cost increases when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where U_i=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is not satisfied but all the amount is cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.2.3
Sensitivity Analysis Table by varying the parameter values of λ_2 when patterns of payment is (U_1=1, U_2=1, V_1=0, V_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	3.3723279	30.593768	29.815691	0.4209089	6.36274
0.43	3.3591025	30.372193	29.690152	0.5978603	6.369118
0.45	3.3466	30.154	29.561	0.7667	6.3732
0.47	3.3349783	29.941525	29.431656	0.9283884	6.375412
0.49	3.323915	29.732249	29.30012	1.083232	6.37589

We see that increasing λ_2 i.e. decreasing expected length of ON period for 2nd supplier, results in increase in average cost when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where Ui=0, V_1=0 and V_2=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and clearing the remaining amount at time T_1 for 1st supplier is satisfied, but remaining amount is not cleared at time T_2 for 2nd supplier. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.2.4
Sensitivity Analysis Table by varying the parameter values of λ_2 when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=1)

λ_2	q_0	q_1	q_2	r	AC
0.41	3.0876767	30.362482	29.514204	0.3471289	6.461766
0.43	3.0678308	30.13612	29.387407	0.5230215	6.469714
0.45	3.048766	29.91408	29.25791	0.690931	6.475395
0.47	3.030395	29.696262	29.126538	0.851541	6.479113
0.49	3.012644	29.48258	28.9939	1.005435	6.481126

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases, average cost increases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_1 for 1st supplier, however remaining amount is not cleared at time T_2 for 2nd supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where Ui=0, V_1=1 and V_2=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and promise of clearing the remaining amount at time T_2 for 2nd supplier is satisfied, but remaining amount is not cleared at time T_1 for 1st supplier. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.2.5
Sensitivity Analysis Table by varying the parameter values of λ_2 when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	3.200164	30.495511	29.680571	0.373045	6.427482
0.43	3.177307	30.266018	29.547438	0.5479518	6.43641
0.45	3.155037	30.04058	29.41159	0.714835	6.443119
0.47	3.1332771	29.819173	29.273825	0.8743763	6.447911
0.49	3.111951	29.60169	29.13481	1.027171	6.451045

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases, average cost increases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_2 for 2nd supplier, however remaining amount is not cleared at time T_1 for 1st supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where U_1=0, U_2=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is satisfied for 1st supplier but promise of doing part payment at time T_{12} is not cleared for 2nd supplier however clearing the remaining amount at time T_i, the time period given by ith supplier where i=1, 2, are satisfied for both the suppliers. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.2.6
Sensitivity Analysis Table by varying the parameter values of λ_2 when patterns of payment is (U_1=0, U_2=1, V_1=0, V_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	3.350144	30.603112	29.822842	0.412714	6.375137
0.43	3.3356959	30.38188	29.697632	0.5892718	6.382061
0.45	3.322036	30.16482	29.56969	0.757816	6.386726
0.47	3.3090783	29.951878	29.439786	0.9190188	6.389437
0.49	3.296745	29.74292	29.30857	1.073475	6.390453

We see that increasing λ_2 i.e. decreasing expected length of ON period for 2nd supplier, results in increase in average cost, when part payment at time T_{11} is done for 1st supplier but part payment at time T_{12} is not cleared for 2nd supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where U_1=1, U_2=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is not satisfied for 1st supplier but promise of doing part payment at time T_{12} is satisfied for 2nd supplier however clearing the remaining amount at time T_i, the time period given by i[th] supplier where i=1, 2 are satisfied for both the suppliers. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.2.7
Sensitivity Analysis Table by varying the parameter values of λ_2 when patterns of payment is (U_1=1, U_2=0, V_1=0, V_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	3.3401819	30.607429	29.82618	0.4090033	6.382346
0.43	3.3267296	30.385702	29.700613	0.5859545	6.388642
0.45	3.31408	30.16817	29.5723	0.75489	6.392686
0.47	3.3021433	29.954737	29.442052	0.916488	6.394781
0.49	3.290846	29.74532	29.31048	1.071338	6.395187

We see that increasing λ_2 i.e. decreasing expected length of ON period for 2[nd] supplier, results in increase in average cost, when part payment at time T_{11} is not done for 1[st] supplier but part payment at time T_{12} is cleared for 2[nd] supplier however remaining amount is cleared at the respective time given by both the suppliers.

6.5.3. Sensitivity Analysis for h:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost h and keeping other parameter values fixed where U_i=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} and clearing the remaining amount at time T_i, the time period given by i[th] supplier where i=1, 2, both are satisfied. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.3.1
Sensitivity Analysis Table by varying the parameter values of h when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=0)

h	q_0	q_1	q_2	r	AC
5	3.289	30.178	29.58	0.7459	6.406
5.2	3.2362706	29.854723	29.221549	0.6022174	6.545112
5.4	3.1855529	29.54831	28.8808	0.4640401	6.680748
5.6	3.137486	29.257809	28.556833	0.331034	6.813115
5.8	3.0918631	28.981893	28.248148	0.2028219	6.942342

We see that increasing holding cost, results in increase in average cost when the businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost h and keeping other parameter values fixed where U_i=0 and V_i=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.3.2
Sensitivity Analysis Table by varying the parameter values of h when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=1)

h	q_0	q_1	q_2	r	AC
5	2.94963	29.8241	29.14412	0.6646	6.5076
5.2	2.9175047	29.526532	28.814907	0.5252968	6.643017
5.4	2.8863705	29.243731	28.500818	0.3911335	6.77517
5.6	2.8561698	28.974583	28.201073	0.2618246	6.904264
5.8	2.8268737	28.717979	27.914578	0.137043	7.030412

We see that as holding cost h increases, average cost increases when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost h and keeping other parameter values fixed where U_i=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is not satisfied but all the amount is cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.3.3
Sensitivity Analysis Table by varying the parameter values of h when patterns of payment is (U_1=1, U_2=1, V_1=0, V_2=0)

h	q_0	q_1	q_2	r	AC
5	3.34667	30.1548	29.561	0.766756	6.37329
5.2	3.290283	29.831422	29.203236	0.6223978	6.512695
5.4	3.237025	3.237025	29.525489	0.483581	6.648723
5.6	3.1866242	29.235481	28.539221	0.3499659	6.781464
5.8	3.1388387	28.960003	28.230933	0.2211845	6.911049

We see that increasing holding cost h, results in increase in average cost when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost h and keeping other parameter values fixed where Ui=0, V_1=0 and V_2=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and clearing the remaining amount at time T_1 for 1st supplier is satisfied, but remaining amount is not cleared at time T_2 for 2nd supplier. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.3.4
Sensitivity Analysis Table by varying the parameter values of h when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=1)

h	q_0	q_1	q_2	r	AC
5	3.048766	29.91408	29.25791	0.690931	6.475395
5.2	3.011016	29.610617	28.921667	0.5502039	6.611792
5.4	2.974681	29.32252	28.60153	0.414787	6.744952
5.6	2.939686	29.048488	28.296173	0.2843193	6.875
5.8	2.905965	28.78737	28.00445	0.158462	7.00205

We see that as holding cost h increases, results in increase in average cost when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_1 for 1st supplier, however remaining amount is not cleared at time T_2 for 2nd supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost h and keeping other parameter values fixed where Ui=0, V_1=1 and V_2=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and promise of clearing the remaining amount at time T_2 for 2nd supplier is satisfied, but remaining amount is not cleared at time T_1 for 1st supplier. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.3.5
Sensitivity Analysis Table by varying the parameter values of h when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=0)

h	q_0	q_1	q_2	r	AC
5	3.155037	30.04058	29.41159	0.714835	6.443119
5.2	3.1108146	29.72814	29.065332	0.5728694	6.580717
5.4	3.068563	29.4319	28.73604	0.436307	6.715007
5.6	3.028145	29.150475	28.422401	0.304777	6.846118
5.8	2.989451	28.8827	28.12302	0.177931	6.97417

We see that as holding cost h increases, results in increase in average cost when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_2 for 2nd supplier, however remaining amount is not cleared at time T_1 for 1st supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost h and keeping other parameter values fixed where U_1=0, U_2=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is satisfied for 1st supplier but promise of doing part payment at time T_{12} is not cleared for 2nd supplier however clearing the remaining amount at time T_i, the time period given by ith supplier where i=1, 2, are satisfied for both the suppliers. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.3.6
Sensitivity Analysis Table by varying the parameter values of h when patterns of payment is (U_1=0, U_2=1, V_1=0, V_2=0)

h	q_0	q_1	q_2	r	AC
5	3.322036	30.16482	29.56969	0.757816	6.386726
5.2	3.266879	29.84124	29.210885	0.6137228	6.526002
5.4	3.214758	29.53511	28.87036	0.475192	6.66186
5.6	3.1654016	29.244897	28.546563	0.3418463	6.79444
5.8	3.118576	28.96922	28.23815	0.21333	6.92387

We see that increasing holding cost h, results in increase in average cost, when part payment at time T_{11} is done for 1st supplier but part payment at time T_{12} is not cleared for 2nd supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost h and keeping other parameter values fixed where U_1=1, U_2=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is not satisfied for 1st supplier but promise of doing part payment at time T_{12} is satisfied for 2nd supplier however clearing the remaining amount at time T_i, the time period given by ith supplier where i=1, 2 are satisfied for both the suppliers. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.3.7
Sensitivity Analysis Table by varying the parameter values of h when patterns of payment is (U_1=1, U_2=0, V_1=0, V_2=0)

h	q_0	q_1	q_2	r	AC
5	3.31408	30.16817	29.5723	0.75489	6.392686
5.2	3.2592393	29.844537	29.213475	0.6108684	6.531904
5.4	3.207414	29.53836	28.87292	0.472404	6.667706
5.6	3.1583346	29.248112	28.54909	0.3391224	6.800232
5.8	3.111767	28.97239	28.24064	0.210667	6.92961

We see that increasing holding cost h, results in increase in average cost, when part payment at time T_{11} is not done for 1^{st} supplier but part payment at time T_{12} is cleared for 2^{nd} supplier however remaining amount is cleared at the respective time given by both the suppliers.

6.5.4. Sensitivity Analysis for k:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost k and keeping other parameter values fixed where U_i=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.4.1
Sensitivity Analysis Table by varying the parameter values of k when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=0)

k	q_0	q_1	q_2	r	AC
4.5	3.111032	29.671291	29.006302	0.737815	6.279806
5	3.28943	30.17821	29.5867	0.745923	6.40678
5.5	3.4598931	30.669571	30.131509	0.7515865	6.525952
6	3.6221036	31.145901	30.66168	0.7551955	6.640325
6.5	3.777383	31.609071	31.173463	0.757111	6.749874

We see that increasing ordering cost k, results in increase in average cost when the businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost k and keeping other parameter values fixed where U_i=0 and V_i=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.4.2
Sensitivity Analysis Table by varying the parameter values of k when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=1)

k	q_0	q_1	q_2	r	AC
4.5	2.8054216	29.35804	28.614707	0.6604661	6.370295
5	2.9496	29.824	29.144	0.6646	6.5076
5.5	3.085467	30.273758	29.651465	0.6670386	6.638618
6	3.214123	30.708484	30.138066	0.6678945	6.763939
6.5	3.3364355	31.129981	30.606673	0.6675279	6.884337

We see that as ordering cost k increases, average cost increases when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost k and keeping other parameter values fixed where U_i=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is not satisfied but all the amount is cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.4.3
Sensitivity Analysis Table by varying the parameter values of k when patterns of payment is (U_1=1, U_2=1, V_1=0, V_2=0)

k	q_0	q_1	q_2	r	AC
4.5	3.1652013	29.646864	28.987165	0.7589051	6.248338
5	3.3466	30.154	29.561	0.7667	6.3732
5.5	3.5190507	30.646433	30.113229	0.7721612	6.49187
6	3.6834178	31.12346	30.6439	0.7754561	6.605076
6.5	3.840672	31.587317	31.156156	0.777025	6.713537

We see that increasing ordering cost k, results in increase in average cost when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost k and keeping other parameter values fixed where Ui=0, V_1=0 and V_2=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and clearing the remaining amount at time T_1 for 1st supplier is satisfied, but remaining amount is not cleared at time T_2 for 2nd supplier. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.4.4
Sensitivity Analysis Table by varying the parameter values of k when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=1)

k	q_0	q_1	q_2	r	AC
4.5	2.894766	29.43805	28.71733	0.685312	6.341442
5	3.048766	29.91408	29.25791	0.690931	6.475395
5.5	3.194302	30.37343	29.7753	0.694534	6.602912
6	3.3324755	30.817963	30.272255	0.6964828	6.724851
6.5	3.464168	31.24919	30.751	0.697085	6.841897

We see that as ordering cost k increases, results in increase in average cost when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_1 for 1st supplier, however remaining amount is not cleared at time T_2 for 2nd supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost k and keeping other parameter values fixed where Ui=0, V_1=1 and V_2=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and promise of clearing the remaining amount at time T_2 for 2nd supplier is satisfied, but remaining amount is not cleared at time T_1 for 1st supplier. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.4.5
Sensitivity Analysis Table by varying the parameter values of k when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=0)

k	q_0	q_1	q_2	r	AC
4.5	2.990814	29.55069	28.85612	0.708247	6.312748
5	3.155037	30.04058	29.41159	0.714835	6.443119
5.5	3.310505	30.51383	29.94361	0.719203	6.567096
6	3.458317	30.97222	30.454996	0.721744	6.685543
6.5	3.599366	31.41726	30.94793	0.722792	6.799144

We see that as ordering cost k increases, results in increase in average cost when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_2 for 2nd supplier, however remaining amount is not cleared at time T_1 for 1st supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost k and keeping other parameter values fixed where U_1=0, U_2=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is satisfied for 1st supplier but promise of doing part payment at time T_{12} is not cleared for 2nd supplier however clearing the remaining amount at time T_i, the time period given by ith supplier where i=1, 2, are satisfied for both the suppliers. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.4.6
Sensitivity Analysis Table by varying the parameter values of k when patterns of payment is (U_1=0, U_2=1, V_1=0, V_2=0)

k	q_0	q_1	q_2	r	AC
4.5	3.141795	29.65712	28.99513	0.749866	6.26124
5	3.322036	30.16482	29.56969	0.757816	6.38672
5.5	3.493264	30.65622	30.12091	0.763269	6.50589
6	3.656581	31.13297	30.65136	0.766664	6.61960
6.5	3.812865	31.59656	31.16344	0.768352	6.728543

We see that increasing ordering cost k, results in increase in average cost, when part payment at time T_{11} is done for 1st supplier but part payment at time T_{12} is not cleared for 2nd supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost k and keeping other parameter values fixed where U_1=1, U_2=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is not satisfied for 1st supplier but promise of doing part payment at time T_{12} is satisfied for 2nd supplier however clearing the remaining amount at time T_i, the time period given by ith supplier where i=1, 2 are satisfied for both the suppliers. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 6.5.4.7
Sensitivity Analysis Table by varying the parameter values of k when patterns of payment is (U_1=1, U_2=0, V_1=0, V_2=0)

k	q_0	q_1	q_2	r	AC
4.5	3.133973	29.66065	28.99789	0.74682	6.267006
5	3.31408	30.16817	29.5723	0.75489	6.392686
5.5	3.485211	30.65937	30.12339	0.760469	6.51203
6	3.6484766	31.135938	30.653712	0.7639863	6.625898
6.5	3.804734	31.59936	31.16566	0.765793	6.734975

We see that increasing ordering cost k, results in increase in average cost, when part payment at time T_{11} is not done for 1st supplier but part payment at time T_{12} is cleared for 2nd supplier however remaining amount is cleared at the respective time given by both the suppliers.

6.6. CONCLUSION:

From this we conclude that the cost is minimum if part payment is not done at T_{1i} but account is cleared at T_i and the cost is maximum if part payment is done at T_{1i} but account is not cleared at T_i, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

CHAPTER 7

STOCHASTIC INVENTORY MODEL UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENT ALLOWING PARTIAL PAYMENT FOR TWO SUPPLIERS

CHAPTER 7

7.1. INTRODUCTION:

In this chapter, we have introduced the aspect of part payment. A part of the purchased cost is to be paid during the permissible delay period. What quantity of the part is to be paid and the time at which it has to be paid can be fixed up at the time of the deal of purchasing the goods. We have also introduced the effect of inflation and time value of money was investigated under given sets of inflation and discount rates.

7.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model for two suppliers under inflation and permissible delay in payment allowing partial payment is developed on the basis of the following assumptions.

- Interest earned and interest charged is as follows.

(a) Interest earned on the entire amount up to time period T_{1i} is $dce^{Rt_1} T_{00} T_{1i} Ie_i$

(b) Interest earned on (1-α_i) fraction during the period (T_i-T_{1i}) is $(1-\alpha_i)\, d\,c\,e^{Rt_1}(T_i - T_{1i})\, T_{00}\, Ie_i$

(c) If part payment is not done at T_{1i} then interest will be earned over α_i fraction for period (T_i-T_{1i}) but interest will also be charged for α_i fraction for (T_i-T_{1i}) period.

Interest earned= $d\,c\,e^{Rt_1}\ \alpha_i (T_i - T_{1i}) T_{00}\ Ie_i$

Interest charged= $d\,c\,e^{Rt_1}\ \alpha_i (T_i - T_{1i}) T_{00}\ Ic_i$

To discourage not doing promised payment, we assume that Ic_i is quite larger than Ie_i .

(d) Interest earned over the amount $dce^{Rt_1} T_{00} T_{1i} Ie_i$ over the period (T_i-T_{1i}) is $d\,c\,e^{Rt_1}\ T_{00}\, T_{1i}\, Ie_i\, (T_i - T_{1i})\, Ie_i$

(e) If the remaining amount is not cleared at T_i then interest will be earned for the period (T_{00}-T_i) for (1-α_i) fraction simultaneously interest will be charged on the same amount for the same period.

Interest earned= $(1-\alpha_i)\, d\, c\, e^{Rt_1}(T_{00}-T_i)T_{00}\, Ie_i$

Interest charged= $(1-\alpha_i)\, d\, c\, e^{Rt_1}(T_{00}-T_i)T_{00}\, Ic_i$

Total interest earned and charged is as follows.

$$
\begin{aligned}
& dce^{Rt_1}T_{00}T_{1i}Ie_i + (1-\alpha_i)\, d\, c\, e^{Rt_1}(T_i - T_{1i})T_{00}\, Ie_i + \{d\, c\, e^{Rt_1}\,\alpha_i (T_i - T_{1i})T_{00}\, Ie_i \\
& - d\, c\, e^{Rt_1}\,\alpha_i (T_i - T_{1i})T_{00}\, Ic_i\} + d\, c\, e^{Rt_1}\, T_{00}\, T_{1i}\, Ie_i\, (T_i - T_{1i})\, Ie_i \\
& + V_i\, [(1-\alpha_i)\, d\, c\, e^{Rt_1}(T_{00}-T_i)T_{00}\, Ie_i + \, d\, c\, e^{Rt_1}\, T_{00}\, T_{1i}\, Ie_i\, (T_i - T_{1i})\, Ie_i\, (T_{00}-T_i)\, Ie_i \\
& + d\, c\, e^{Rt_1}\, T_{00}\, T_{1i}\, Ie_i\, (T_{00}-T_i)\, Ie_i + (1-\alpha_i)\, d\, c\, e^{Rt_1} T_{00}(T_i - T_{1i}) Ie_i (T_{00}-T_i)\, Ie_i \\
& + \{\, d\, c\, e^{Rt_1}\,\alpha_i\, T_{00}\, Ie_i (T_i - T_{1i})\, Ie_i - d\, c\, e^{Rt_1}\,\alpha_i (T_{00}-T_i)T_{00}\, Ic_i\} \\
& - (1-\alpha_i)\, d\, c\, e^{Rt_1}(T_{00}-T_i)T_{00}\, Ic_i\,]
\end{aligned}
$$

$A(q_i, r, \theta)$= (cost of ordering) + (cost of holding inventory) + (cost of item that deteriorate during a single interval that starts with an inventory of (q_i+r) units and ends with r units with inflation rate);

$$
A(q_i, r, \theta) = k + \frac{1}{2}\frac{hq_i^{\,2}\, e^{Rt_1}}{(d+\theta)} + \frac{hrq_i\, e^{Rt_1}}{(d+\theta)} + \frac{\theta c q_i\, e^{Rt_1}}{(d+\theta)} \qquad i = 0,1,2
$$

7.3. OPTIMAL POLICY DECISION FOR THE MODEL:

Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, i, j=0, 1, 2, 3 for the four state CTMC. The lemma which is used to obtain the transition probabilities is same as discussed in chapter 4, (lemma (4.3.1)) hence we omit it here also lemma 4.3.4, 4.3.5, 5.3.2 and5.3.3 are also same hence we omit it here.

Proposition 7.3.1: The Average cost objective function for two suppliers under inflation and permissible delay in payments allowing partial payment is given by AC=$\frac{C_{00}}{T_{00}}$

C_{00} is given by

$$
\begin{aligned}
C_{00} =& A(q_0,r)+P_{01}\{C_{10}-dce^{Rt_1}T_{00}T_{11}Ie_1-(1-\alpha_1)dce^{Rt_1}T_{00}(T_1-T_{11})Ie_1-U_1dce^{Rt_1}\alpha_1T_{00}(T_1-T_{11})Ie_1\\
&\quad +U_1dce^{Rt_1}\alpha_1T_{00}(T_1-T_{11})Ic_1-dce^{Rt_1}T_{00}\,T_{11}Ie_1(T_1-T_{11})Ie_1-V_1[(1-\alpha_1)dce^{Rt_1}T_{00}(T_{00}-T_1)Ie_1\\
&+dce^{Rt_1}T_{00}\,T_{11}Ie_1(T_1-T_{11})Ie_1(T_{00}-T_1)Ie_1+dce^{Rt_1}T_{00}\,T_{11}Ie_1(T_{00}-T_1)Ie_1\\
&+(1-\alpha_1)dce^{Rt_1}T_{00}(T_1-T_{11})Ie_1(T_{00}-T_1)Ie_1]-V_1[U_1\{dce^{Rt_1}\alpha_1T_{00}\,Ie_1(T_1-T_{11})(T_{00}-T_1)Ie_1\}]\\
&+V_1[U_1\{dce^{Rt_1}\alpha_1T_{00}\,Ic_1(T_{00}-T_1)+(1-\alpha_1)dce^{Rt_1}T_{00}\,Ic_1(T_{00}-T_1)\}]\quad\}\\
&+P_{02}\{C_{20}-dce^{Rt_1}T_{00}T_{12}Ie_2-(1-\alpha_2)dce^{Rt_1}T_{00}(T_2-T_{12})Ie_1-U_2dce^{Rt_1}\alpha_2T_{00}(T_2-T_{12})Ie_2\\
&\quad +U_2dce^{Rt_1}\alpha_2T_{00}(T_2-T_{12})Ic_2-dce^{Rt_1}T_{00}\,T_{12}Ie_2(T_2-T_{12})Ie_2-V_2[(1-\alpha_2)dce^{Rt_1}T_{00}(T_{00}-T_2)Ie_2\\
&+dce^{Rt_1}T_{00}\,T_{12}Ie_2(T_2-T_{12})Ie_2(T_{00}-T_2)Ie_2+dce^{Rt_1}T_{00}\,T_{12}Ie_2(T_{00}-T_2)Ie_2\\
&+(1-\alpha_2)dce^{Rt_1}T_{00}(T_2-T_{12})Ie_2(T_{00}-T_2)Ie_2]-V_2[U_2\{dce^{Rt_1}\alpha_2T_{00}\,Ie_2(T_2-T_{12})(T_{00}-T_2)Ie_2\}]\\
&+V_2[U_2\{dce^{Rt_1}\alpha_2T_{00}\,Ic_2(T_{00}-T_2)+(1-\alpha_2)dce^{Rt_1}T_{00}\,Ic_2(T_{00}-T_2)\}]\quad\}\\
&+P_{03}\{\overline{C}+\rho_1\left[\begin{array}{l}
C_{10}-dce^{Rt_1}T_{00}T_{11}Ie_1-(1-\alpha_1)dce^{Rt_1}T_{00}(T_1-T_{11})Ie_1-U_1dce^{Rt_1}\alpha_1T_{00}(T_1-T_{11})Ie_1\\
+U_1dce^{Rt_1}\alpha_1T_{00}(T_1-T_{11})Ic_1-dce^{Rt_1}T_{00}\,T_{11}Ie_1(T_1-T_{11})Ie_1\\
-V_1[(1-\alpha_1)dce^{Rt_1}T_{00}(T_{00}-T_1)Ie_1+dce^{Rt_1}T_{00}\,T_{11}Ie_1(T_1-T_{11})Ie_1(T_{00}-T_1)Ie_1\\
+dce^{Rt_1}T_{00}\,T_{11}Ie_1(T_{00}-T_1)Ie_1+(1-\alpha_1)dce^{Rt_1}T_{00}(T_1-T_{11})Ie_1(T_{00}-T_1)Ie_1]\\
-V_1[U_1\{dce^{Rt_1}\alpha_1T_{00}\,Ie_1(T_1-T_{11})(T_{00}-T_1)Ie_1\}]\\
+V_1[U_1\{dce^{Rt_1}\alpha_1T_{00}\,Ic_1(T_{00}-T_1)+(1-\alpha_1)dce^{Rt_1}T_{00}\,Ic_1(T_{00}-T_1)\}]\quad\}
\end{array}\right]\\
&\rho_2\left[\begin{array}{l}
C_{20}-dce^{Rt_1}T_{00}T_{12}Ie_2-(1-\alpha_2)dce^{Rt_1}T_{00}(T_2-T_{12})Ie_1-U_2dce^{Rt_1}\alpha_2T_{00}(T_2-T_{12})Ie_2\\
+U_2dce^{Rt_1}\alpha_2T_{00}(T_2-T_{12})Ic_2-dce^{Rt_1}T_{00}\,T_{12}Ie_2(T_2-T_{12})Ie_2\\
-V_2[(1-\alpha_2)dce^{Rt_1}T_{00}(T_{00}-T_2)Ie_2+dce^{Rt_1}T_{00}\,T_{12}Ie_2(T_2-T_{12})Ie_2(T_{00}-T_2)Ie_2\\
+dce^{Rt_1}T_{00}\,T_{12}Ie_2(T_{00}-T_2)Ie_2+(1-\alpha_2)dce^{Rt_1}T_{00}(T_2-T_{12})Ie_2(T_{00}-T_2)Ie_2]\\
-V_2[U_2\{dce^{Rt_1}\alpha_2T_{00}\,Ie_2(T_2-T_{12})(T_{00}-T_2)Ie_2\}]\\
+V_2[U_2\{dce^{Rt_1}\alpha_2T_{00}\,Ic_2(T_{00}-T_2)+(1-\alpha_2)dce^{Rt_1}T_{00}\,Ic_2(T_{00}-T_2)\}]\quad\}
\end{array}\right]\}
\end{aligned}
$$

and $$T_{00}=\frac{q_0}{d+\theta}+P_{01}T_{10}+P_{02}T_{20}+P_{03}(\overline{T}+\rho_1T_{10}+\rho_2T_{20})$$

Proof: Proof follows using Renewal reward theorem (RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

7.4. NUMERICAL EXAMPLE:

There are sixteen different patterns of payments, some of them we consider here.

1. Ui=0 and Vi=0 where i=1, 2 that is promise of doing part payment at time T_{1i} and clearing the remaining amount at time T_i, the time period given by i^{th} supplier where i=1, 2, both are satisfied.

2. Ui=0 and Vi=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i , the time period given by i^{th} supplier where i=1, 2.

3. Ui=1 and Vi=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is not satisfied for both the suppliers but all the amount are cleared at time T_i , the time period given by i^{th} supplier where i=1, 2.

4. Ui=0, V_1=0 and V_2=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and clearing the remaining amount at time T_1 for 1st supplier is satisfied, but remaining amount is not cleared at time T_2 for 2nd supplier.

5. Ui=0, V_1=1 and V_2=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and promise of clearing the remaining amount at time T_2 for 2nd supplier is satisfied, but remaining amount is not cleared at time T_1 for 1st supplier.

6. U_1=0, U_2=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is kept for 1st supplier but promise of doing part payment at time T_{12} is not satisfied for 2nd supplier however clearing the remaining amount at time T_i, the time period given by i^{th} supplier where i=1, 2, are satisfied for both the suppliers.

7. U_1=1, U_2=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is not satisfied for 1st supplier but promise of doing part payment at time T_{12} is satisfied for 2nd supplier however clearing the remaining amount at time T_i, the time period given by i^{th} supplier where i=1, 2 are satisfied for both the suppliers.

In this section we verify the results by a numerical example. We assume that k=Rs. 5/order, c=Rs.1/unit, d=20/units, θ=4, h=Rs. 5/unit/time, π=Rs. 350/unit, T_{11}=0.6, $\hat{\pi}$=Rs. 25/unit/time, α_1=0.5, α_2=0.6, Ic_1=0.11, Ie_1=0.02, Ic_2=0.13, Ie_2=0.04, T_{12}=0.8, T_1=0.9, T_2=1.1, R=0.05, t_1=6 λ_1=0.58, λ_2=0.45, μ_1=3.4, μ_2=2.5.

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are $1/\lambda_1$=1.72413794, $1/\lambda_2$=2.2222, $1/\mu_1$=.2941176 and $1/\mu_2$=.4 respectively. The long run probabilities are obtained as p_0=0.7239588, p_1=0.1303126, p_2 =0.1234989 and p_3=0.02222979. The optimal solution for the above numerical example based on the seven patterns of payment is obtained as

(U_1,U_2,V_1,V_2)	q_0	q_1	q_2	r	AC
(0,0,0,0)	2.8044	28.8243	28.035	0.71827	8.18389
(0,0,1,1)	2.55527	28.5755	27.715	0.64861	8.28186
(1,1,0,0)	2.8538	28.799	28.0153	0.73958	8.14469
(0,0,0,1)	2.6286	28.6399	27.8001	0.67088	8.2503
(0,0,1,0)	2.7077	28.7305	27.9146	0.69175	8.2195
(0,1,0,0)	2.8326	28.8095	28.0235	0.73052	8.1607
(1,0,0,0)	2.8251	28.813	28.026	0.7272	8.168011

Conclusion:

From this we conclude that the cost is minimum if part payment is not done at T_{1i} but account is cleared at T_i and the cost is maximum if part payment is done at T_{1i} but account is not cleared at T_i, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

7.5. SENSITIVITY ANALYSIS:

To observe the effects of varying parameter values on the optimal solution we have conducted sensitivity analysis, by varying value of inflation rate R on the following seven patterns of payment.

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and keeping other parameter values fixed where U_i=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2 and 0.25. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r, AC and R are plotted in Fig. 7.5.1.

Table 7.5.1
Sensitivity Analysis Table by varying the parameter values of R when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=0)

R	q_0	q_1	q_2	r	AC
0.05	2.8044	28.8243	28.035	0.71827	8.18389
0.1	2.38808	27.72073	26.7435	0.67913	10.50586
0.15	2.0325	26.8227	25.6677	0.63246	13.5493
0.2	1.72991	26.0919	24.7733	0.58148	17.5515
0.25	1.472827	25.49633	24.03038	0.528973	22.83057

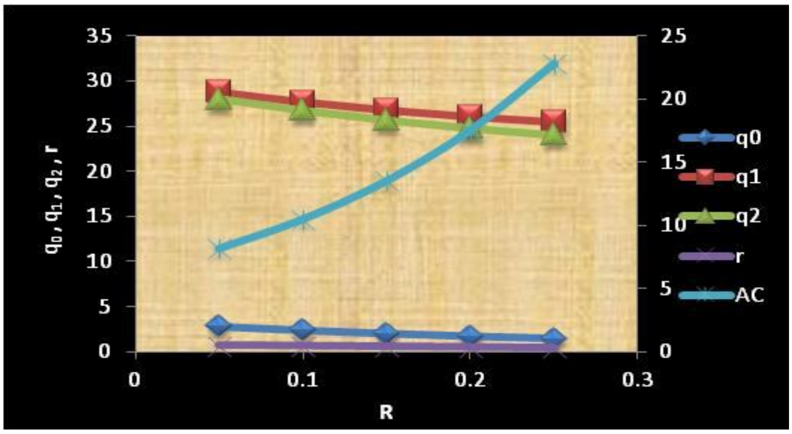

Fig. 7.5.1 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R

We see that as inflation rate R increases, results in increase in average cost, when businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and keeping other parameter values fixed where U_i=0 and V_i=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2 and 0.25. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r, AC and R are plotted in Fig. 7.5.2.

Table 7.5.2
Sensitivity Analysis Table by varying the parameter values of R when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=1)

R	q_0	q_1	q_2	r	AC
0.05	2.55527	28.5755	27.715	0.64861	8.28186
0.1	2.20818	27.5466	26.5099	0.6217	10.5985
0.15	1.904439	26.70043	25.4976	0.58679	13.63506
0.2	1.63989	26.00519	24.649	0.5464	17.6286
0.25	1.41041	25.43376	23.93952	0.50306	22.89708

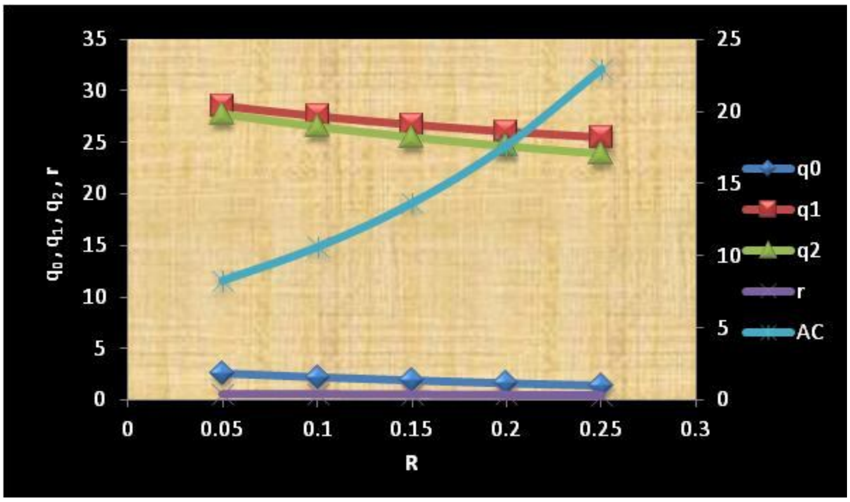

Fig. 7.5.2 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R

Increasing the value of inflation rate R, results in increase in average cost, when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and keeping other parameter values fixed where U_i=1 and V_i=0 where i=1, 2 that promise of doing part payment at time T_{1i} is not satisfied but all the amount is cleared at time T_i, the time period given by i^{th} supplier where i=1, 2. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2 and 0.25. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r, AC and R are plotted in Fig. 7.5.3.

Table 7.5.3
Sensitivity Analysis Table by varying the parameter values of R when patterns of payment is (U_1=1, U_2=1, V_1=0, V_2=0)

R	q_0	q_1	q_2	r	AC
0.05	2.8538	28.799	28.0153	0.73958	8.14469
0.1	2.43058	27.6947	26.7234	0.70022	10.45932
0.15	2.0687	26.79694	25.6477	0.6527	13.49431
0.2	1.76056	26.0669	24.75399	0.60075	17.48683
0.25	1.498653	25.47265	24.01201	0.546858	22.75464

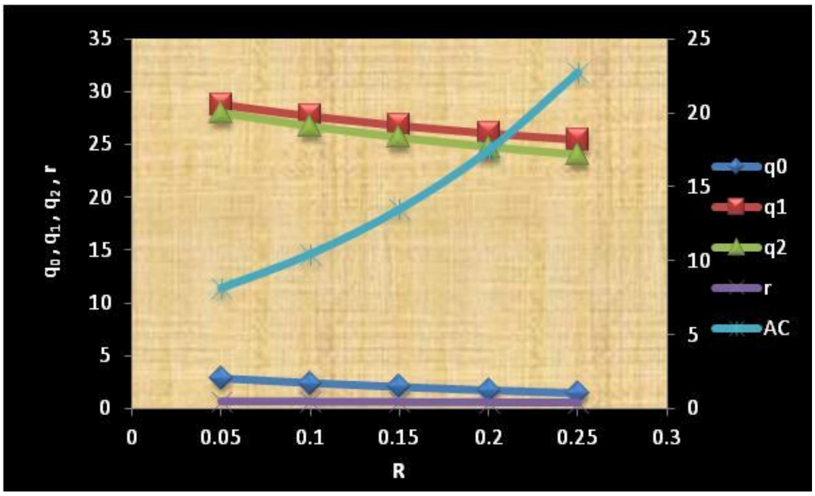

Fig. 7.5.3 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R

We see that as inflation rate R increases, average cost increases, when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and keeping other parameter values fixed where Ui=0, V_1=0 and V_2=1 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and clearing the remaining amount at time T_1 for 1st supplier is satisfied, but remaining amount is not cleared at time T_2 for 2nd supplier. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2 and 0.25. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r, AC and R are plotted in Fig. 7.5.4.

Table 7.5.4
Sensitivity Analysis Table by varying the parameter values of R when patterns of payment is (U_1=0, U_2=0, V_1=0, V_2=1)

R	q_0	q_1	q_2	r	AC
0.05	2.6286	28.6399	27.8001	0.67088	8.2503
0.1	2.261854	27.59264	26.57341	0.640006	10.56836
0.15	1.943228	26.73339	25.54476	0.601397	13.60659
0.2	1.667597	26.02892	24.68429	0.557797	17.60238
0.25	1.429954	25.45107	23.96543	0.51159	22.87361

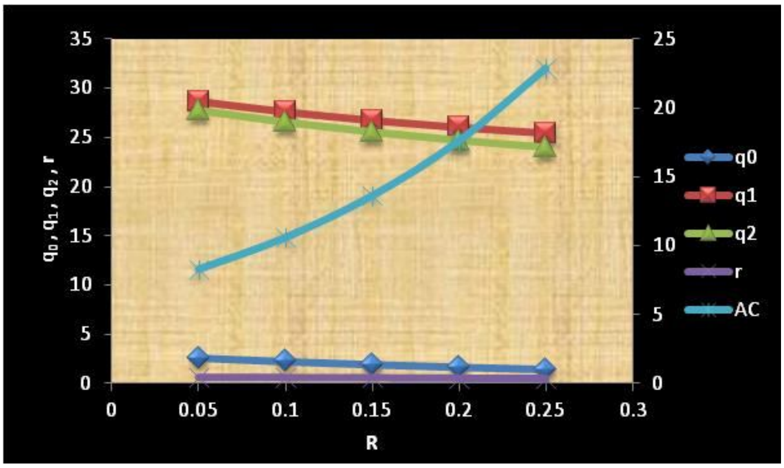

Fig. 7.5.4 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R

We see that as inflation rate R increases, average cost increases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_1 for 1st supplier, however remaining amount is not cleared at time T_2 for 2nd supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and keeping other parameter values fixed where Ui=0, V_1=1 and V_2=0 where i=1, 2 that is promise of doing part payment at time T_{1i} is satisfied for both suppliers and promise of clearing the remaining amount at time T_2 for 2nd supplier is satisfied, but remaining amount is not cleared at time T_1 for 1st supplier. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2 and 0.25. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r, AC and R are plotted in Fig. 7.5.5.

Table 7.5.5
Sensitivity Analysis Table by varying the parameter values of R when patterns of payment is (U_1=0, U_2=0, V_1=1, V_2=0)

R	q_0	q_1	q_2	r	AC
0.05	2.7077	28.7305	27.9146	0.69175	8.2195
0.1	2.319587	27.65739	26.65832	0.657395	10.53958
0.15	1.984613	26.77981	25.60752	0.615251	13.58067
0.2	1.696712	26.06255	24.73068	0.568359	17.58012
0.25	1.450045	25.4759	24.00002	0.519217	22.85588

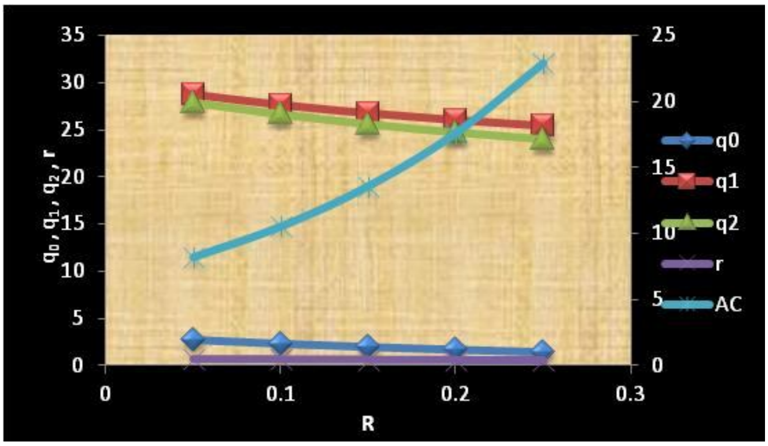

Fig. 7.5.5 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R

Increasing the inflation rate R, results in increase in average cost, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time T_2 for 2nd supplier, however remaining amount is not cleared at time T_1 for 1st supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and keeping other parameter values fixed where U_1=0, U_2=1 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is satisfied for 1st supplier but promise of doing part payment at time T_{12} is not cleared for 2nd supplier however clearing the remaining amount at time T_i, the time period given by ith supplier where i=1, 2, are satisfied for both the suppliers. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2 and 0.25. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r, AC and R are plotted in Fig. 7.5.6.

Table 7.5.6
Sensitivity Analysis Table by varying the parameter values of R when patterns of payment is (U_1=0, U_2=1, V_1=0, V_2=0)

R	q_0	q_1	q_2	r	AC
0.05	2.8326	28.8095	28.0235	0.73052	8.1607
0.1	2.412552	27.70555	26.73171	0.691337	10.47825
0.15	2.053535	26.80758	25.6559	0.644287	13.51658
0.2	1.74778	26.07718	24.76184	0.592769	17.51294
0.25	1.48795	25.48231	24.01946	0.539494	22.78518

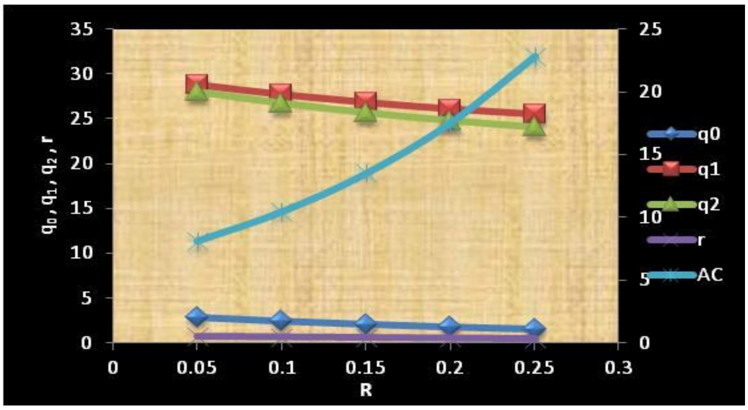

Fig. 7.5.6 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R

We see that as inflation rate R increases, average cost increases, when part payment at time T_{11} is done for 1st supplier but part payment at time T_{12} is not cleared for 2nd supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and keeping other parameter values fixed where U_1=1, U_2=0 and V_i=0 where i=1, 2 that is promise of doing part payment at time T_{11} is not satisfied for 1st supplier but promise of doing part payment at time T_{12} is satisfied for 2nd supplier however clearing the remaining amount at time $T_{i,}$ the time period given by ith supplier where i=1, 2 are satisfied for both the suppliers. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2 and 0.25. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r, AC and R are plotted in Fig. 7.5.7.

Table 7.5.7
Sensitivity Analysis Table by varying the parameter values of R when patterns of payment is (U_1=1, U_2=0, V_1=0, V_2=0)

R	q_0	q_1	q_2	r	AC
0.05	2.8251	28.813	28.026	0.7272	8.168011
0.1	2.405692	27.70974	26.73496	0.687934	10.4871
0.15	2.047394	26.81195	25.65929	0.640833	13.52723
0.2	1.74237	26.08159	24.76526	0.589369	17.52567
0.25	1.48324	25.48662	24.02281	0.536231	22.80031

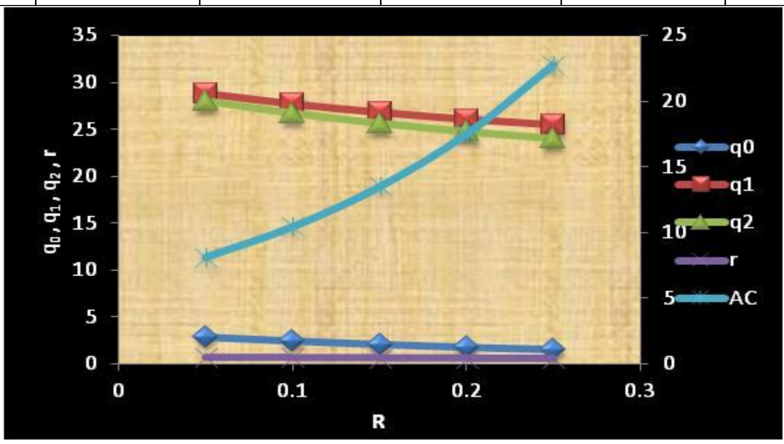

Fig. 7.5.7 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R

We see that as inflation rate R increases, average cost increases, when part payment at time T_{11} is not done for 1st supplier but part payment at time T_{12} is cleared for 2nd supplier and remaining amount is cleared at the respective time given by both the suppliers.

7.6. CONCLUSION:

From this we conclude that the cost is minimum if part payment is not done at T_{1i} but account is cleared at T_i and the cost is maximum if part payment is done at T_{1i} but the account is not cleared at T_i, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period. The option of part payment is very useful for enhancing business and encouraging the small entrepreneurs. Comparing the average cost with that of chapter 3 in all the situations, we find that cost is less here as there are two suppliers, so here also we can conclude that two suppliers help in reducing the average cost.

CHAPTER 8

STOCHASTIC INVENTORY MODEL FOR MULTIPLE SUPPLIERS

CHAPTER 8

8.1. INTRODUCTION:

In this chapter, we have generalized the model and consider the case where there are M suppliers, and at any time suppliers may be available or not available which we represent as ON or OFF state. The stochastic process representing the supplier availabilities would have 2^M states: $0, 1, 2, \ldots\ldots\ldots, 2^M-1$. State 0 would correspond to the situation where all the suppliers being ON, state 1 would correspond to only the M^{th} supplier being OFF etc. and finally state 2^M-1 would correspond to all being OFF. The transition probabilities $P_{ij}(t)$, i, $j=0, 1, 2, \ldots\ldots, 2^M-1$, decision variables q_i and costs C_{i0}, $i=0, 1, 2, \ldots\ldots, 2^M-1$ are defined in a manner similar to chapter 4.

The system of equations for C_{i0} is obtained as

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right)A(q_i, r, \theta) + \sum_{j=1}^{2^M-1} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[A(q_i, r, \theta) + C_{j0}\right] \qquad i = 0, 1, \ldots\ldots\ldots, 2^M - 2$$

$$C_{2^M-1,0} = \overline{C} + \sum_{i=1}^{M} \rho_i C_{i0} \ , \qquad \text{where } \rho_i = \frac{\mu_i}{\sum_{j=1}^{M}\mu_j}$$

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}\left(\delta r - (d+\theta)\right) + \left(\pi\delta d + h(d+\theta) + \hat{\pi}\right) - \theta c\delta\right] + \frac{\theta c}{\delta}$$

$$\text{with } \delta = \sum_{j=i}^{M}\mu_j$$

Equations for T_{i0} are written in a similar way as in Lemma (4.3.4).

Solving the above equations require the exact solution for the transient probabilities $P_{ij}(t)$ of the CTMC with the 2^M states which appears to be a formidable task, because we would first need the exact solution for the transient probabilities $P_{ij}(t)$ of the CTMC with the 2^M states. It would also be necessary to solve explicitly for the quantities C_{00} and T_{00} using the system of 2^M equations in 2^M unknowns.

As the number of suppliers is very large, that is we have a situation approximating a free market, we can develop a much simpler model by assuming that if an order needs to be placed and at least one of the suppliers is available, then the order quantity will be q units regardless of which supplier is available. We combine the first 2^M-1 states where at least one supplier is available and define a super state denoted by o. The last state denoted by $\bar{\imath}$, is the state where all the suppliers are OFF. We also assume that for any supplier the ON and OFF periods are exponential with parameters λ and μ, respectively.

With these assumptions the expected cost and the expected length of a cycle are obtained as

$$C_{oo} = A(q, r, \theta) + P_{o\bar{\imath}}(q/(d+\theta))\, C_{\bar{\imath}o}(r),$$

$$T_{oo} = q/(d+\theta) + P_{o\bar{\imath}}(q/(d+\theta))/M_{\mu}$$

Therefore, the average cost function is given by

$$AC = C_{oo}/T_{oo}$$

where $A(q, r, \theta), P_{o\bar{\imath}}(q/(d+\theta))$ and $C_{\bar{\imath}o}(r)$ have the same meaning as in chapter 2.

8.2. CONCLUSION:

When the number of suppliers become large, the objective function of multiple suppliers problem reduces to that of classical EOQ model. This can be shown by arguing that as the length of stay in state $\bar{\imath}$ is exponential with parameter M_{μ} , it becomes a degenerate random variable with mass at 0; that is the process never visits or stays in state $\bar{\imath}$.

APPENDIX

// Newton Rapson method for two variables (q, r) in R Programming for chapter 2 //

```
AC=function(x)
{
 q=x[1]
 r=x[2]
 k=10;  h=5;  d=20;  theata=5; c=5
 mu=2.5; lemda=0.25; pic=25;  pi=250
 de=0; theata=d+theata;  ie=0.08
 ic=0.15;  T0=0.6; R=0.05;  t1=6
 p0=mu/(lemda+mu)
 p1=lemda/(lemda+mu)
 u=exp((-mu*r)/dtheata)
 v=exp((mu*r)/dtheata)
 w=exp(R*t1)
 C10=(u*w/(mu^2))*((h*v*(mu*r-(dtheata))+((pi*mu*d)+(h*dtheata)+pic)
-(theata*c*mu)))+((theata*c)/mu)
 w1=exp(-(lemda+mu)*(q/dtheata))
 P01=p1-p1*w1
 A0=k+((.5*h*q^2*w)/dtheata)+((h*r*q*w)/dtheata)+((theata*c*q*w)/dtheata)
 T00=(q/dtheata)+((1/mu)*P01)
 Ie1=d*c*T00*T0*Ie
 Ie2=((d*c*T00+Ie1)*(T00-T0)*Ie)
 Ic=de*d*c*ic*(T00-T0)
 A=A0/T00
 B0=C10*P01
 B=B0/T00
 C0=(Ie1+Ie2)
 C=C0/T00
 D=Ic/T00
 return(A+B-C+D)
}
```

Output of R-code

```
nlm(AC, p=c(8, 9))
$minimum
[1] 260.3604
$estimate
[1] 18.56644    14.14799
```

// Newton Rapson method for two variables (q, r) in R Programming for chapter 3 //

```
AC=function(x)
{
 q=x[1]
 r=x[2]
 k=10; h=5; d=20;  theata=5; c=5;  mu=2.5; lemda=0.25;  pic=25;  pi=250
 T1=0.3; T=0.6; alpha=0.5; Ie=0.08; Ic=0.15; R=0.05; t1=6; U=0; V=1
 dtheata=d+theata
 p0=mu/(lemda+mu)
 p1=lemda/(lemda+mu)
 a=exp((-mu*r)/dtheata)
 b=exp((mu*r)/dtheata)
 w=exp(R*t1)
 C10=(a*w/(mu^2))*((h*b*(mu*r-(dtheata))+((pi*mu*d)+(h*dtheata)+pic)
- (theata*c*mu)))+((theata*c*w)/mu)
w1=exp(-(lemda+mu)*(q/dtheata))
 P01=p1-p1*w1
 A0=k+((.5*h*w*q^2)/dtheata)+((h*r*w*q)/dtheata)+((theata*c*w*q)/dtheata)
 T00=(q/dtheata)+((1/mu)*P01)
 E1=(-c*d*w*T00*T1*Ie)-((1-alpha)*c*d*w*T00*(T-T1)*Ie)
- (U*d*c*w*alpha*T00*(T-T1)*Ie)+(U*d*c*w*alpha*T00*(T-T1)*Ic)
-(c*d*w*T00*T1*Ie*(T-T1)*Ie)
 E2=(-V*(((1-alpha)*c*d*w*T00*(T00-T)*Ie)+(c*d*w*T00*T1*Ie*(T-T1)
*Ie*(T00-T)*Ie)+(c*d*w*T00*T1*Ie*(T00-T)*Ie)+((1-alpha)*c*d*w*T00*(T-T1)
*Ie*(T00-T)*Ie)))
 E3=(-V*(U*(d*c*w*alpha*T00*Ie*(T-T1)*(T00-T)
*Ie)))+(V*(U*((d*c*w*alpha*T00*Ic*(T00-T))+((1-alpha)*c*d*w*T00*Ic*(T00-T)))))
 A=A0/T00
 B0=C10*P01
 B=B0/T00
 D=(E1+E2+E3)
 E=D/T00
 return(A+B+E)
}
```

Output of R-code

```
nlm(AC, p=c(8,9))
$minimum
[1] 260.9979

$estimate
[1] 17.83440 14.41302
```

// Newton Rapson method for four variables (q_0, q_1, q_2, r) in R Programming for chapter 4 //

```
AC = function(x)
{
 q0 = x[1]
 q1 = x[2]
 q2 = x[3]
  r  = x[4]
 k=5;  h=5;  d=20;  theata=4;  c=1;  mu1=3.4;  mu2=2.5;  lemda1=0.58;  lemda2=0.45;
pi=350; T01=.6; T02=.8 Ie1=0.02; Ie2=0.04; ic1=0.11; ic2=0.13; alpha1=0; alpha2=0
delta=mu1+mu2
dtheata=d+theata
row1=mu1/delta
row2=mu2/delta
Tbar=1/(mu1+mu2)
p0=(mu1*mu2)/((lemda1+mu1)*(lemda2+mu2))
p1=(lemda2*mu1)/((lemda1+mu1)*(lemda2+mu2))
p2=(lemda1*mu2)/((lemda1+mu1)*(lemda2+mu2))
p3=(lemda1*lemda2)/((lemda1+mu1)*(lemda2+mu2))
u=exp(-(delta*r)/dtheata)
v=exp((delta*r)/dtheata)
cbar=(u/(delta^2))*((h*v*(delta*r-dtheata)+((pi*delta*d)+(h*dtheata)+pic)-
(theata*c*delta)))+(theata*c)/delta
w1=exp(-(lemda1+mu1)*(q0/dtheata))
w2=exp(-(lemda2+mu2)*(q0/dtheata))
w3=exp(-(lemda1+mu1+lemda2+mu2)*(q0/dtheata))
P01=p1+(p3*w1)-(p1*w2)-(p3*w3)
P02=p2-(p2*w1)+(p3*w2)-(p3*w3)
P03=p3-(p3*w1)-(p3*w2)+(p3*w3)
y1=exp(-(lemda1+mu1)*(q1/dtheata))
y2=exp(-(lemda2+mu2)*(q1/dtheata))
y3=exp(-(lemda1+mu1+lemda2+mu2)*(q1/dtheata))
P11=p1+(p3*y1)+(p0*y2)+(p2*y3)
P12=p2-(p2*y1)-(p2*y2)+(p2*y3)
P13=p3-(p3*y1)+(p2*y2)-(p2*y3)
z1=exp(-(lemda1+mu1)*(q2/dtheata))
z2=exp(-(lemda2+mu2)*(q2/dtheata))
z3=exp(-(lemda1+mu1+lemda2+mu2)*(q2/dtheata))
P21=p1-(p1*z1)-(p1*z2)+(p1*z3)
```

```
P22=p2+(p0*z1)+(p3*z2)+(p1*z3)
P23=p3+(p1*z1)-(p3*z2)-(p1*z3)
A0=k+((.5*h*q0^2)/dtheata)+((h*r*q0)/dtheata)+((theata*c*q0)/dtheata)
A1=k+((.5*h*q1^2)/dtheata)+((h*r*q1)/dtheata)+((theata*c*q1)/dtheata)
A2=k+((.5*h*q2^2)/dtheata)+((h*r*q2)/dtheata)+((theata*c*q2)/dtheata)
Nu=((A2+P23*cbar)*(1-P11-P13*row1))+((P21+P23*row1)*(A1+P13*cbar))
Den=((1-P22-P23*row2)*(1-P11-P13*row1))-((P21+P23*row1)*(P12+P13*row2))
C20=Nu/Den
Num1=(A1+P13*cbar)+((P12+P13*row2)*C20)
Den1=(1-P11-P13*row1)
C10=Num1/Den1
C30=cbar+(row1*C10+row2*C20)
Num2=((q2+P23*Tbar)*(1-P11-P13*row1))+((P21+P23*row1)*(q1+P13*Tbar))
Den2=((1-P22-P23*row2)*(1-P11-P13*row1))-((P21+P23*row1)*(P12+P13*row2))
T20=Num2/Den2
Num3=(q1+P13*Tbar)+((P12+P13*row2)*T20)
Den3=(1-P11-P13*row1)
T10=Num3/Den3
T30=Tbar+(row1*T10+row2*T20)
T00=(q0/dtheata)+(P01*T10)+(P02*T20)+(P03*(Tbar+row1*T10+row2*T20))
Ie11=(d*c*T00*T01*Ie1)
Ie12=(d*c*T00*T02*Ie2)
Ie21=((d*c*T00+Ie11)*(T00-T01)*Ie1)
Ie22=((d*c*T00+Ie12)*(T00-T02)*Ie2)
Ic1=(alpha1*d*c*ic1*(T00-T01))
Ic2=(alpha2*d*c*ic2*(T00-T02))
C101=C10-(Ie11+Ie21)+Ic1
C201=C20-(Ie12+Ie22)+Ic2
C301=cbar+(row1*C101+row2*C201)
B0=P01*C101
C0=P02*C201
D0=P03*C301
A=A0/T00
B=B0/T00
C=C0/T00
D=D0/T00
return (A+B+C+D)
}
```

Output of R-code

```
nlm(AC, p=c(8,9,1,4))
$minimum
[1] 5.900553
$estimate
[1]  9.216337 41.8218088 41.9396503  0.7624755
```

// Newton Rapson method for four variables (q_0, q_1, q_2, r) in R Programming for chapter 5 //

```
AC = function(x)
{
q0 = x[1]
 q1 = x[2]
 q2 = x[3]
  r  = x[4]
 k=5;  h=5;  d=20; theata=4;  c=1;  mu1=3.4;  mu2=2.5;  lemda1=.58;  lemda2=.45
 pic=25;  pi=350;  T01=.6; T02=.8;  ie1=0.02;  ie2=0.04;  ic1=.11 ;  ic2=.13 R=0.05
 t1=6; alpha1=0; alpha2=0
 delta=mu1+mu2
 dtheata=d+theata
 row1=mu1/delta
 row2=mu2/delta
 Tbar=1/(mu1+mu2)
 p0=(mu1*mu2)/((lemda1+mu1)*(lemda2+mu2))
 p1=(lemda2*mu1)/((lemda1+mu1)*(lemda2+mu2))
 p2=(lemda1*mu2)/((lemda1+mu1)*(lemda2+mu2))
 p3=(lemda1*lemda2)/((lemda1+mu1)*(lemda2+mu2))
u=exp(-(delta*r)/dtheata)
v=exp((delta*r)/dtheata)
w=exp(R*t1)
cbar=(u*w/(delta^2))*((h*v*(delta*r-dtheata)+((pi*delta*d)+(h*dtheata)+pic)-
(theata*c*delta)))+(theata*c*w)/delta
w1=exp(-(lemda1+mu1)*(q0/dtheata))
w2=exp(-(lemda2+mu2)*(q0/dtheata))
w3=exp(-(lemda1+mu1+lemda2+mu2)*(q0/dtheata))
P01=p1+(p3*w1)-(p1*w2)-(p3*w3)
P02=p2-(p2*w1)+(p3*w2)-(p3*w3)
P03=p3-(p3*w1)-(p3*w2)+(p3*w3)
y1=exp(-(lemda1+mu1)*(q1/dtheata))
y2=exp(-(lemda2+mu2)*(q1/dtheata))
```

```
y3=exp(-(lemda1+mu1+lemda2+mu2)*(q1/dtheata))
P11=p1+(p3*y1)+(p0*y2)+(p2*y3)
P12=p2-(p2*y1)-(p2*y2)+(p2*y3)
P13=p3-(p3*y1)+(p2*y2)-(p2*y3)
z1=exp(-(lemda1+mu1)*(q2/dtheata))
z2=exp(-(lemda2+mu2)*(q2/dtheata))
z3=exp(-(lemda1+mu1+lemda2+mu2)*(q2/dtheata))
P21=p1-(p1*z1)-(p1*z2)+(p1*z3)
P22=p2+(p0*z1)+(p3*z2)+(p1*z3)
P23=p3+(p1*z1)-(p3*z2)-(p1*z3)
A0=k+((.5*h*q0^2*w)/dtheata)+((h*r*q0*w)/dtheata)+((theata*c*q0*w)/dtheata)
A1=k+((.5*h*q1^2*w)/dtheata)+((h*r*q1*w)/dtheata)+((theata*c*q1*w)/dtheata)
A2=k+((.5*h*q2^2*w)/dtheata)+((h*r*q2*w)/dtheata)+((theata*c*q2*w)/dtheata)
Nu=((A2+P23*cbar)*(1-P11-P13*row1))+((P21+P23*row1)*(A1+P13*cbar))
Den=((1-P22-P23*row2)*(1-P11-P13*row1))-((P21+P23*row1)*(P12+P13*row2))
C20=Nu/Den
Num1=(A1+P13*cbar)+((P12+P13*row2)*C20)
Den1=(1-P11-P13*row1)
C10=Num1/Den1
C30=cbar+(row1*C10+row2*C20)
Num2=((q2+P23*Tbar)*(1-P11-P13*row1))+((P21+P23*row1)*(q1+P13*Tbar))
Den2=((1-P22-P23*row2)*(1-P11-P13*row1))-((P21+P23*row1)*(P12+P13*row2))
T20=Num2/Den2
Num3=(q1+P13*Tbar)+((P12+P13*row2)*T20)
Den3=(1-P11-P13*row1)
T10=Num3/Den3
T30=Tbar+(row1*T10+row2*T20)
T00=(q0/dtheata)+(P01*T10)+(P02*T20)+(P03*(Tbar+row1*T10+row2*T20))
Ie11=(d*c*T00*w*T01*ie1)
Ie12=(d*c*T00*w*T02*ie2)
Ie21=((d*c*w*T00+Ie11)*(T00-T01)*ie1)
Ie22=((d*c*w*T00+Ie12)*(T00-T02)*ie2)
Ic1=(alpha1*d*c*w*ic1*(T00-T01))
Ic2=(alpha2*d*c*w*ic2*(T00-T02))
C101=C10-(Ie11+Ie21)+Ic1
C201=C20-(Ie12+Ie22)+Ic2
C301=cbar+(row1*C101+row2*C201)
B0=P01*C101
C0=P02*C201
D0=P03*C301
```

```
 A=A0/T00
 B=B0/T00
 C=C0/T00
 D=D0/T00
 return (A+B+C+D)
}
```

Output of R-code

```
nlm(AC,  p=c(8,9,3,2))
$minimum
[1] 7.750814
$estimate
[1]  6.106851  33.977701 33.857779  1.026171
```

// Newton Rapson method for four variables (q_0, q_1, q_2, r) in R Programming for chapter 6//

```
 AC = function(x)
{
q0 = x[1]
q1 = x[2]
q2 = x[3]
 r  = x[4]
 k=5;  h=5;  d=20;  theata=4;  c=1;  mu1=3.4;  mu2=2.5; lemda1=0.58; lemda2=0.45
 pic=25;  pi=350;  T11=0.6; T12=0.8; Ie1=0.02;  Ie2=0.04; Ic1=0.11; Ic2=0.13;
T1=0.9;  T2=1.1; alpha1=0.5; alpha2=0.6;  U1=1; U2=1; V1=0;  V2=0
delta=mu1+mu2
dtheata=d+theata
row1=mu1/delta
row2=mu2/delta
Tbar=1/(mu1+mu2)
p0=(mu1*mu2)/((lemda1+mu1)*(lemda2+mu2))
p1=(lemda2*mu1)/((lemda1+mu1)*(lemda2+mu2))
p2=(lemda1*mu2)/((lemda1+mu1)*(lemda2+mu2))
p3=(lemda1*lemda2)/((lemda1+mu1)*(lemda2+mu2))
u=exp(-(delta*r)/dtheata)
v=exp((delta*r)/dtheata)
cbar=(u/(delta^2))*((h*v*(delta*r-dtheata)+((pi*delta*d)+(h*dtheata)+pic)-
(theata*c*delta)))+(theata*c)/delta
w1=exp(-(lemda1+mu1)*(q0/dtheata))
w2=exp(-(lemda2+mu2)*(q0/dtheata))
w3=exp(-(lemda1+mu1+lemda2+mu2)*(q0/dtheata))
```

```
P01=p1+(p3*w1)-(p1*w2)-(p3*w3)
P02=p2-(p2*w1)+(p3*w2)-(p3*w3)
P03=p3-(p3*w1)-(p3*w2)+(p3*w3)
y1=exp(-(lemda1+mu1)*(q1/dtheata))
y2=exp(-(lemda2+mu2)*(q1/dtheata))
 y3=exp(-(lemda1+mu1+lemda2+mu2)*(q1/dtheata))
 P11=p1+(p3*y1)+(p0*y2)+(p2*y3)
 P12=p2-(p2*y1)-(p2*y2)+(p2*y3)
 P13=p3-(p3*y1)+(p2*y2)-(p2*y3)
 z1=exp(-(lemda1+mu1)*(q2/dtheata))
 z2=exp(-(lemda2+mu2)*(q2/dtheata))
 z3=exp(-(lemda1+mu1+lemda2+mu2)*(q2/dtheata))
 P21=p1-(p1*z1)-(p1*z2)+(p1*z3)
 P22=p2+(p0*z1)+(p3*z2)+(p1*z3)
 P23=p3+(p1*z1)-(p3*z2)-(p1*z3)
 A0=k+((.5*h*q0^2)/dtheata)+((h*r*q0)/dtheata)+((theata*c*q0)/dtheata)
 A1=k+((.5*h*q1^2)/dtheata)+((h*r*q1)/dtheata)+((theata*c*q1)/dtheata)
 A2=k+((.5*h*q2^2)/dtheata)+((h*r*q2)/dtheata)+((theata*c*q2)/dtheata)
 Nu=((A2+P23*cbar)*(1-P11-P13*row1))+((P21+P23*row1)*(A1+P13*cbar))
 Den=((1-P22-P23*row2)*(1-P11-P13*row1))-((P21+P23*row1)*(P12+P13*row2))
 C20=Nu/Den
 Num1=(A1+P13*cbar)+((P12+P13*row2)*C20)
 Den1=(1-P11-P13*row1)
 C10=Num1/Den1
 C30=cbar+(row1*C10+row2*C20)
 Num2=((q2+P23*Tbar)*(1-P11-P13*row1))+((P21+P23*row1)*(q1+P13*Tbar))
 Den2=((1-P22-P23*row2)*(1-P11-P13*row1))-((P21+P23*row1)*(P12+P13*row2))
 T20=Num2/Den2
 Num3=(q1+P13*Tbar)+((P12+P13*row2)*T20)
 Den3=(1-P11-P13*row1)
 T10=Num3/Den3
 T30=Tbar+(row1*T10+row2*T20)
 T00=(q0/dtheata)+(P01*T10)+(P02*T20)+(P03*(Tbar+row1*T10+row2*T20))
 E1=(c*d*T00*T11*Ie1)-((1-alpha1)*c*d*T00*(T1-T11)*Ie1)-
(U1*d*c*alpha1*T00*(T1-T11)*Ie1)+(U1*d*c*alpha1*T00*(T1-T11)*Ic1)-
(c*d*T00*T11*Ie1*(T1-T11)*Ie1)
 E2=(-V1*(((1-alpha1)*c*d*T00*(T00-T1)*Ie1)+(c*d*T00*T11*Ie1*(T1- T11)
*Ie1*(T00-T1)*Ie1)+(c*d*T00*T11*Ie1*(T1-T11)*Ie1*(T00-
T1)*Ie1)+(c*d*T00*T11*Ie1*(T00-T1)*Ie1)+((1-alpha1)*c*d*T00*(T1- T11)
*Ie1*(T00-T1)*Ie1)))
```

```
 E3=(-V1*(U1*(d*c*alpha1*T00*Ie1*(T1-T11)*(T00- T1)
*Ie1)))+(V1*(U1*((d*c*alpha1*T00*Ic1*(T00-T1))+((1- alpha1) *c*d*T00*Ic1*
(T00-T1)))))
F1=(c*d*T00*T12*Ie2)-((1-alpha2)*c*d*T00*(T2-T12)*Ie2)-
(U2*d*c*alpha2*T00*(T2-T12)*Ie2)+(U2*d*c*alpha2*T00*(T2-T12)*Ic2)-
(c*d*T00*T12*Ie2*(T2-T12)*Ie2)
 F2=(-V2*(((1-alpha2)*c*d*T00*(T00-T2)*Ie2)+(c*d*T00*T12*Ie2*(T2- T12)
*Ie2*(T00-T2)*Ie2)+(c*d*T00*T12*Ie2*(T2-T12)*Ie2*(T00-
T2)*Ie2)+(c*d*T00*T12*Ie2*(T00-T2)*Ie2)+((1-alpha2)*c*d*T00*(T2- T12)
*Ie2*(T00-T2)*Ie2)))
 F3=(-V2*(U2*(d*c*alpha2*T00*Ie2*(T2-T12)*(T00- T2)
*Ie2)))+(V2*(U2*((d*c*alpha2*T00*Ic2*(T00-T2))+((1- alpha2)*c*d*T00*Ic2*
 (T00-T2)))))
 C101=C10-(E1+E2+E3)
 C201=C20-(F1+F2+F3)
 C301=cbar+(row1*C101+row2*C201)
 B0=P01*C101
 C0=P02*C201
 D0=P03*C301
 A=A0/T00
 B=B0/T00
 C=C0/T00
 D=D0/T00
 return (A+B+C+D)
}
```

Output of R-code

```
nlm(AC, p=c(8,9,4,3))
$minimum
[1] 6.37324
$estimate
[1]  3.3466827 30.1547987 29.5618813  0.7667965
```

// Newton Rapson method for four variables (q_0, q_1, q_2, r) in R Programming for chapter 7 //

```
AC = function(x)
{
 q0 = x[1]
 q1 = x[2]
 q2 = x[3]
  r  = x[4]
  k=5;  h=5;  d=20;  theata=4;  c=1;  mu1=3.4; mu2=2.5;  lemda1=0.58; lemda2=0.45
  pic=25;  pi=350;  T11=0.6;  T12=0.8; Ie1=0.02; Ie2=0.04; Ic1=0.11; Ic2=0.13
  T1=0.9; T2=1.1; alpha1=0.5;  alpha2=0.6; R=0.05; t1=6; U1=1;  U2=1; V1=0;  V2=0
  delta=mu1+mu2
```

```
dtheata=d+theata
row1=mu1/delta
row2=mu2/delta
Tbar=1/(mu1+mu2)
p0=(mu1*mu2)/((lemda1+mu1)*(lemda2+mu2))
p1=(lemda2*mu1)/((lemda1+mu1)*(lemda2+mu2))
p2=(lemda1*mu2)/((lemda1+mu1)*(lemda2+mu2))
p3=(lemda1*lemda2)/((lemda1+mu1)*(lemda2+mu2))
u=exp(-(delta*r)/dtheata)
v=exp((delta*r)/dtheata)
w=exp(R*t1)
cbar=(u*w/(delta^2))*((h*v*(delta*r-dtheata)+((pi*delta*d)+(h*dtheata)+pic)-
(theata*c*delta)))+(theata*c*w)/delta
w1=exp(-(lemda1+mu1)*(q0/dtheata))
w2=exp(-(lemda2+mu2)*(q0/dtheata))
w3=exp(-(lemda1+mu1+lemda2+mu2)*(q0/dtheata))
P01=p1+(p3*w1)-(p1*w2)-(p3*w3)
P02=p2-(p2*w1)+(p3*w2)-(p3*w3)
P03=p3-(p3*w1)-(p3*w2)+(p3*w3)
y1=exp(-(lemda1+mu1)*(q1/dtheata))
y2=exp(-(lemda2+mu2)*(q1/dtheata))
y3=exp(-(lemda1+mu1+lemda2+mu2)*(q1/dtheata))
P11=p1+(p3*y1)+(p0*y2)+(p2*y3)
P12=p2-(p2*y1)-(p2*y2)+(p2*y3)
P13=p3-(p3*y1)+(p2*y2)-(p2*y3)
z1=exp(-(lemda1+mu1)*(q2/dtheata))
z2=exp(-(lemda2+mu2)*(q2/dtheata))
z3=exp(-(lemda1+mu1+lemda2+mu2)*(q2/dtheata))
P21=p1-(p1*z1)-(p1*z2)+(p1*z3)
P22=p2+(p0*z1)+(p3*z2)+(p1*z3)
P23=p3+(p1*z1)-(p3*z2)-(p1*z3)
A0=k+((.5*h*q0^2*w)/dtheata)+((h*r*q0*w)/dtheata)+((theata*c*q0*w)/dtheata)
A1=k+((.5*h*q1^2*w)/dtheata)+((h*r*q1*w)/dtheata)+((theata*c*q1*w)/dtheata)
A2=k+((.5*h*q2^2*w)/dtheata)+((h*r*q2*w)/dtheata)+((theata*c*q2*w)/dtheata)
Nu=((A2+P23*cbar)*(1-P11-P13*row1))+((P21+P23*row1)*(A1+P13*cbar))
Den=((1-P22-P23*row2)*(1-P11-P13*row1))-((P21+P23*row1)*(P12+P13*row2))
C20=Nu/Den
Num1=(A1+P13*cbar)+((P12+P13*row2)*C20)
Den1=(1-P11-P13*row1)
C10=Num1/Den1
C30=cbar+(row1*C10+row2*C20)
Num2=((q2+P23*Tbar)*(1-P11-P13*row1))+((P21+P23*row1)*(q1+P13*Tbar))
Den2=((1-P22-P23*row2)*(1-P11-P13*row1))-((P21+P23*row1)*(P12+P13*row2))
```

```
 T20=Num2/Den2
 Num3=(q1+P13*Tbar)+((P12+P13*row2)*T20)
 Den3=(1-P11-P13*row1)
 T10=Num3/Den3
 T30=Tbar+(row1*T10+row2*T20)
 T00=(q0/dtheata)+(P01*T10)+(P02*T20)+(P03*(Tbar+row1*T10+row2*T20))
 E1=(c*d*w*T00*T11*Ie1)-((1-alpha1)*c*d*w*T00*(T1-T11)*Ie1)
- (U1*d*c*w*alpha1*T00*(T1-T11)*Ie1)+(U1*d*c*w*alpha1*T00*(T1-T11)*Ic1)
-(c*d*w*T00*T11*Ie1*(T1-T11)*Ie1)
 E2=(-V1*(((1-alpha1)*c*d*w*T00*(T00-T1)*Ie1)+(c*d*w*T00*T11*Ie1*(T1- T11)
*Ie1*(T00-T1)*Ie1)+(c*d*w*T00*T11*Ie1*(T00-T1)*Ie1)+((1- alpha1)
*c*d*w*T00*(T1-T11)*Ie1*(T00-T1)*Ie1)))
 E3=(-V1*(U1*(d*c*w*alpha1*T00*Ie1*(T1-T11)*(T00- T1)
*Ie1)))+(V1*(U1*((d*c*w*alpha1*T00*Ic1*(T00-T1))+((1- alpha1)
*c*d*w*T00*Ic1*(T00-T1)))))
 F1=(c*d*w*T00*T12*Ie2)-((1-alpha2)*c*d*w*T00*(T2-T12)*Ie2)
- (U2*d*c*w*alpha2*T00*(T2-T12)*Ie2)+(U2*d*c*w*alpha2*T00*(T2-T12)*Ic2)
-(c*d*w*T00*T12*Ie2*(T2-T12)*Ie2)
 F2=(-V2*(((1-alpha2)*c*d*w*T00*(T00-T2)*Ie2)+(c*d*w*T00*T12*Ie2*(T2- T12)
*Ie2*(T00-T2)*Ie2)+(c*d*w*T00*T12*Ie2*(T00-T2)*Ie2)+((1- alpha2)
*c*d*w*T00*(T2-T12)*Ie2*(T00-T2)*Ie2)))
F3=(-V2*(U2*(d*c*w*alpha2*T00*Ie2*(T2-T12)*(T00- T2)
*Ie2)))+(V2*(U2*((d*c*w*alpha2*T00*Ic2*(T00-T2))+((1- alpha2)
*c*d*w*T00*Ic2*(T00-T2)))))
C101=C10-(E1+E2+E3)
C201=C20-(F1+F2+F3)
C301=cbar+(row1*C101+row2*C201)
B0=P01*C101
C0=P02*C201
D0=P03*C301
A=A0/T00
B=B0/T00
C=C0/T00
 D=D0/T00
 return (A+B+C+D)
}
```

Output of R-code

```
nlm (AC, p=c(4,7,8,2))
$minimum
[1] 8.144696
$estimate
[1]  2.8538783 28.7990363 28.0153335  0.7395786
```

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LIST OF PUBLICATIONS

Sr. No.	Title	Name of the Journal, Vol. No., Year with ISSN No.
1.	Future Supply Uncertainty Model for Deteriorating items under Inflation and Permissible delay in payment	Journal of Probability and Statistical Science (JPSS) ,August 2009 Vol.7, No.2.(Page No: 245-259) ISSN-1726-3328
2.	Future Supply Uncertainty Model for Deteriorating items under Permissible delay in payment for two suppliers	Calcutta Statistical Association Bulletin (CSA) , Published in December 2010, Vol.62, Nos .247-248.(Page No: 228-246) ISSN-0008-0683
3.	Future Supply Uncertainty Model for Deteriorating items under Inflation and Permissible delay in payment for two suppliers	Journal of Indian Statistical Association(JISA) December 2010 Vol.48, No.2.(Page No: 243-261) ISSN-0537-2585
4.	Perishable-Inventory Model Under Inflation and Delay in Payment Allowing Partial Payment	Journal of the Indian Association for Productivity, Quality & Reliability (IAPQR) December 2011, Vol.36, No.2. (Page No: 111-131) ISSN-0970-0102
5.	Stochastic Inventory Model for Ameliorating Items under Supplier's Trade Credit Policy	International Journal of Engineering and Management Sciences (IJEMS-SFSN) Vol 4(2), 2013 Published in April - 2013 (Page No: 203- 211) ISSN- 2229-600X
6.	Comparative Study of Inventory Model for Duopolistic Market under Trade Credit	International Journal of Statistika and Mathematika (IJSAM) Vol 6, Issue 1 of 2013 Page No: 22 to 29) P-ISSN - 2277-2790 E-ISSN- 2249-8605
7.	Inflationary inventory model under trade credit subject to supply uncertainty	International Journal of Management (IJM) Vol 4, Issue 4, Published in July-August (2013) (Page No:111 to 118) P-ISSN-0976-6502,E-ISSN-0976-6510

Sr. No.	Title	Name of the Journal, Vol. No., Year with ISSN No.
8.	Deteriorating Items Stochastic Inventory Model for Oligopoly Market	"International Journal of Science, Engineering and Technology Research" (IJSETR) Vol. 2, Issue-9 published in Sept. 2013, Page No. 1720-1726 (E-ISSN:)-2278-7798
9.	Optimization of inventory model for uncertain Supplier under trade credit and inflationary environment	Centre for Promotion of Multidisciplinary Research (CPMR) Special issue of Opinion – International Journal of Business Management(IJBM), Vol. 3 No. 2 published in Dec. 2013, P-ISSN-2277-4637 E-ISSN-2231-5470
10.	Future Supply Uncertainty Model with Deteriorating Items for two suppliers under inflation and permissible delay in payments allowing partial payment	Journal of Indian Society for Probability and Statistics(ISPS) Vol. 14 Published in 2013, Page No. 38 to 58 ISSN-2277-9620

LIST OF PAPERS PRESENTED IN SEMINAR/ NATIONAL/ INTERNATIONAL CONFERENCE

1. Participated and **presented** a paper on “Future Supply Uncertainty Model for Deteriorating items under Inflation and Permissible delay in payments for two Suppliers” at the **4th International Conference** on Quality, Reliability and Info com Technology (ICQRIT) held at **University of Delhi** during December, 18-20, 2009.

2. Participated and **presented** a paper on “Future Supply Uncertainty Model for Deteriorating items under Inflation and Permissible delay in payments allowing partial payments for two Suppliers” at the **International Conference** on Development and Applications of Statistics in Emerging Areas of Science and Technology held at **University of Jammu** during December, 8-10, 2010.

3. Participated and **presented** a paper on “Future Supply Uncertainty Model for Deteriorating items under Permissible delay in payments for two Suppliers” at the **International Conference** on Applied Mathematics and Statistics (ICAPMS-2011) held at **Gujarat University of Ahmedabad** during December, 16-18, 2011.

4. Participated and **presented** a paper on “Stochastic Inventory Model for Perishable items under Permissible Delay in Payment allowing Partial Payment” at the Regional Science Congress on Science for Shaping the Future of India Conference held at **The Maharaja Sayajirao University,Vadodara** on September,15-16,2012.

5. Participated and **presented** a paper on “Optimization of Stochastic Inventory Model under Inflation and Trade Credit for Duopolistic Market” at the **International Conference** on Role of Statistics in the Advancement of Science and Technology held at **University of Pune** during December, 16-18, 2013.

6. Participated and **presented** a paper on "Perishable products Stochastic Inventory model under inflation and Trade credit" during National Seminar on Biostatistics and Data Analytics held at **The Maharaja Sayajirao University, Vadodara** on December, 23-24, 2013.

7. Participated and **presented** a paper on "Comparative study of Stochastic Inventory Model under Trade Credit for Duopolistic Market" at **Science Excellence-2014** held at **Gujarat University of Ahmedabad** on 4^{th} January 2014 and **awarded First prize**.

8. Participated and **presented** a paper on "Stochastic Inventory model for Perishable products for Oligopoly market" during National Seminar on Research in Statistical Science**:** Present and Future held at **Sardar Patel University, Vallabh Vidyanagar** on February, 21-22, 2014.