

# Contents

|                                                                                |             |
|--------------------------------------------------------------------------------|-------------|
| <b>Declaration</b>                                                             | <b>i</b>    |
| <b>Certificate</b>                                                             | <b>ii</b>   |
| <b>Certificate</b>                                                             | <b>iii</b>  |
| <b>Preface</b>                                                                 | <b>vi</b>   |
| <b>Acknowledgements</b>                                                        | <b>ix</b>   |
| <b>Contents</b>                                                                | <b>xiii</b> |
| <b>1 Introduction</b>                                                          | <b>1</b>    |
| 1.1 Definitions, Notations and Formulae . . . . .                              | 3           |
| 1.2 Scalar Form: . . . . .                                                     | 3           |
| 1.2.1 Other Generalized Functions . . . . .                                    | 5           |
| 1.2.2 Inverse Series Relation . . . . .                                        | 8           |
| 1.2.3 Polynomials and Their Inverse Series Relations . . . . .                 | 10          |
| 1.2.4 Riordan's Inversion Pairs . . . . .                                      | 14          |
| 1.3 Matrix Form: . . . . .                                                     | 17          |
| 1.3.1 Preliminaries . . . . .                                                  | 17          |
| 1.3.2 Examples . . . . .                                                       | 24          |
| 1.4 Matrix Polynomials . . . . .                                               | 27          |
| 1.5 Operators $E$ and $\Delta$ . . . . .                                       | 29          |
| 1.5.1 Auxiliary result . . . . .                                               | 29          |
| 1.6 Fractional Order Integral and Derivatives . . . . .                        | 31          |
| <b>2 A General Inverse Matrix Series Relation and Associated Polynomials-I</b> | <b>33</b>   |
| 2.1 Introduction . . . . .                                                     | 34          |
| 2.2 Lemma . . . . .                                                            | 35          |
| 2.3 Main Result . . . . .                                                      | 38          |
| 2.4 General Class of Matrix Polynomials . . . . .                              | 42          |

|          |                                                                                   |            |
|----------|-----------------------------------------------------------------------------------|------------|
| 2.5      | Particular Cases . . . . .                                                        | 44         |
| 2.6      | Wilson Matrix Polynomial and Racah<br>Matrix Polynomial . . . . .                 | 50         |
| 2.7      | Application . . . . .                                                             | 52         |
| 2.7.1    | Generating Function Relations . . . . .                                           | 52         |
| 2.7.2    | Summation Formulas . . . . .                                                      | 55         |
| 2.8      | Matrix Analogues of Riordan's Inverse Pairs . . . . .                             | 57         |
| <b>3</b> | <b>A General Matrix Inverse Series Relation and Associated<br/>Polynomials-II</b> | <b>59</b>  |
| 3.1      | Introduction . . . . .                                                            | 60         |
| 3.2      | Lemma . . . . .                                                                   | 62         |
| 3.3      | Main result . . . . .                                                             | 64         |
| 3.4      | Particular cases . . . . .                                                        | 69         |
| 3.5      | Alternative Forms of Theorem -I . . . . .                                         | 72         |
| 3.6      | Wilson Matrix Polynomials and Racah Matrix Polynomials . . . . .                  | 74         |
| 3.7      | Application . . . . .                                                             | 79         |
| 3.7.1    | Generating Function Relations . . . . .                                           | 80         |
| 3.7.2    | Summation Formulas . . . . .                                                      | 82         |
| 3.8      | Matrix analogues of Riordan's Inverse Pairs . . . . .                             | 85         |
| <b>4</b> | <b>Generalized Konhauser Matrix Polynomial and its Properties</b>                 | <b>90</b>  |
| 4.1      | Introduction . . . . .                                                            | 91         |
| 4.2      | Generalized Konhauser Matrix Polynomial . . . . .                                 | 91         |
| 4.3      | Inverse Series Relation . . . . .                                                 | 93         |
| 4.4      | Mittag-Leffler Matrix Function . . . . .                                          | 96         |
| 4.5      | Generating Function Relations . . . . .                                           | 97         |
| 4.6      | Summation Formulas . . . . .                                                      | 101        |
| 4.7      | Differential Recurrence Relation . . . . .                                        | 103        |
| 4.8      | General Order Derivative . . . . .                                                | 105        |
| 4.9      | Mixed Relation . . . . .                                                          | 106        |
| 4.10     | Contour Integral . . . . .                                                        | 107        |
| 4.11     | Matrix Integral Transform . . . . .                                               | 108        |
| <b>5</b> | <b>Generalized Mittag-Leffler Matrix Function</b>                                 | <b>110</b> |
| 5.1      | Introduction . . . . .                                                            | 111        |
| 5.2      | Generalized Mittag-Leffler Matrix Function . . . . .                              | 111        |
| 5.3      | Differential Equation . . . . .                                                   | 113        |
| 5.4      | Eigen Function Property . . . . .                                                 | 116        |
| 5.5      | Mixed Relation . . . . .                                                          | 118        |
| 5.6      | Particular Cases . . . . .                                                        | 121        |
| 5.6.1    | Differential Equation . . . . .                                                   | 121        |
| 5.6.2    | Eigen Function Property . . . . .                                                 | 123        |
| 5.7      | Extended Konhauser Matrix Polynomial . . . . .                                    | 124        |
| 5.8      | Inverse Series Relations . . . . .                                                | 126        |

|          |                                                                               |            |
|----------|-------------------------------------------------------------------------------|------------|
| 5.9      | Generating Function Relations . . . . .                                       | 130        |
| 5.10     | Summation Formulas. . . . .                                                   | 133        |
| <b>6</b> | <b>A Generalized Matrix Function <math>{}_pR_q(A, B : (C), (D); z)</math></b> | <b>136</b> |
| 6.1      | Introduction . . . . .                                                        | 137        |
| 6.2      | The Matrix Function ${}_pR_q(A, B : (C), (D); z)$ . . . . .                   | 138        |
| 6.3      | Particular Cases . . . . .                                                    | 138        |
| 6.4      | Contiguous Matrix Function Relations . . . . .                                | 142        |
| 6.5      | Differential Equation . . . . .                                               | 147        |
| 6.6      | Matrix Differential Formulas . . . . .                                        | 150        |
| 6.7      | Integral Representation . . . . .                                             | 154        |
| 6.8      | Fractional Calculus of ${}_pR_q(A, B : (C), (D); z)$ . . . . .                | 157        |
| <b>7</b> | <b>Bibliography</b>                                                           | <b>159</b> |
|          | <b>Reprints of Publied/Accepted Research Papers</b>                           | <b>170</b> |
|          | <b>Papers Presented at the National/International Conferences</b>             | <b>171</b> |
|          | <b>Lists of Communicated Paper</b>                                            | <b>172</b> |