Chapter 2

Core-Envelope Models of Superdense Star with Anisotropic Envelope

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In this chapter, a core-envelope model for superdense matter distribution with the feature - core consisting of isotropic fluid distribution and envelope with anisotropic fluid distribution - is studied on the background of pseudo spheroidal spacetime. In the case of superdense stars core-envelope models, with isotropic pressure in the core and anisotropic pressure in the envelope, may not be unphysical. Physical plausibility of the models have been examined analytically and using programming.

2.1 Introduction

Theoretical investigations of Ruderman [76] and Canuto [8] on compact stars having densities much higher than nuclear densities led to the conclusion that matter may be anisotropic at the central region of the distribution. Maharaj and Maartens [59] have obtained models of spherical anisotropic distribution with uniform density. Gokhroo and Mehra [28] have extended this model to include anisotropic distributions with variable density. Dev and Gleiser [14] have obtained exact solutions for various forms of equation of state connecting the radial and tangential pressure.

When matter density of spherical objects are much higher than nuclear density, it is difficult to have a definite description of matter in the form of an equation of state. The uncertainty about the equation of state of matter beyond nuclear regime led to the consideration of a complementary approach called core-envelope models. In this approach, a relativistic stellar configuration is made up of two regions - a core surrounded by an envelope - containing matter distribution with different physical features. The first core-envelope model was obtained by Bondi [6] in 1964. A detailed analysis of such models are discussed by Hartle [32], Iyer and Vishveshwara [42]. A common feature of the core-envelope models reported in literature was that their core and envelope regions contain distributions of perfect fluids in equilibrium with different density distributions. Negi, Pande and Durgapal [65] have developed core-envelope models where both pressure and density are continuous across the core boundary.

Core-envelope models with anisotropic pressure distribution in the core and isotropic pressure distribution in the envelope are available [95]. We shall investigate whether the prescription of isotropic pressure in the core and anisotropic pressure in the envelope leads to a solution consistent with the physical requirements. Such an assumption may not be unphysical because in the case of superdense stars with core consisting of degenerate fermi fluid, the core can be considered as isotropic while its outer envelope may consist of fluid with anisotropic pressure. Further, the study of glitches and starquakes are important in stars having thin envelopes. It is, therefore, pertinent to investigate the physical viability of spherical distributions of matter with isotropic pressure in the core and anisotropic pressure in the envelope.

2.2 The Field Equations

We begin with a static spherically symmetric spacetime described by the metric

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{2.2.1}$$

with an ansatz

$$e^{\lambda(r)} = \frac{1 + K\frac{r^2}{R^2}}{1 + \frac{r^2}{R^2}}, \tag{2.2.2}$$

where K and R are geometric parameters. The $t = const.$ section of (2.2.1) has the geometry of a 3-pseudo spheroid.

We consider the energy-momentum tensor of the form

$$T_{ij} = (\rho + p)\, u_i u_j - p g_{ij} + \pi_{ij}, \qquad u_i u^j = 1 \tag{2.2.3}$$

where ρ, p, u_i respectively denote matter density, isotropic pressure, unit four velocity field of matter. The anisotropic stress tensor π_{ij} is given by

$$\pi_{ij} = \sqrt{3}S\left[C_i C_j - \frac{1}{3}\left(u_i u_j - g_{ij}\right)\right]. \tag{2.2.4}$$

For radially symmetric anisotropic fluid distribution of matter, $S = S(r)$ denotes the magnitude of the anisotropic stress tensor and $C^i = \left(0, -e^{-\lambda/2}, 0, 0\right)$, which is a radial vector. For equilibrium models,

$$u_i = \left(e^{\nu/2}, 0, 0, 0\right) \tag{2.2.5}$$

and the energy-momentum tensor (2.2.3) has non-vanishing components

$$T_0^0 = \rho \quad T_1^1 = -\left(p + \frac{2S}{\sqrt{3}}\right) \quad T_2^2 = T_3^3 = -\left(p - \frac{S}{\sqrt{3}}\right). \tag{2.2.6}$$

The pressure along the radial direction

$$p_r = p + \frac{2S}{\sqrt{3}}, \tag{2.2.7}$$

is different from the pressure along the tangential direction

$$p_\perp = p - \frac{S}{\sqrt{3}}. \tag{2.2.8}$$

The magnitude of anisotropy is given by

$$S = \frac{p_r - p_\perp}{\sqrt{3}}. \tag{2.2.9}$$

The field equation (1.1.2) corresponding to the metric (2.2.1) using ansatz (2.2.2) is given by a set of three equations:

$$8\pi\rho = \frac{K-1}{R^2}\left(3 + K\frac{r^2}{R^2}\right)\left(1 + K\frac{r^2}{R^2}\right)^{-2}, \tag{2.2.10}$$

$$8\pi p_r = \left[\left(1 + \frac{r^2}{R^2}\right)\frac{\nu'}{r} - \frac{K-1}{R^2}\right]\left(1 + K\frac{r^2}{R^2}\right)^{-1}, \tag{2.2.11}$$

$$\begin{aligned} 8\pi\sqrt{3}S &= -\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'}{2r}\right)\left(1 + \frac{r^2}{R^2}\right)\left(1 + K\frac{r^2}{R^2}\right)^{-1} + \\ &\quad \frac{(K-1)}{R^2}r\left(\frac{\nu'}{2} + \frac{1}{r}\right)\left(1 + K\frac{r^2}{R^2}\right)^{-2} + \\ &\quad \frac{K-1}{R^2}\left(1 + K\frac{r^2}{R^2}\right)^{-1}. \end{aligned} \tag{2.2.12}$$

Equation (2.2.10) provides the law of variation of density of matter from which it follows that the density gradient

$$\frac{d\rho}{dr} = -\frac{2K(K-1)}{8\pi R^4}r\left(5 + K\frac{r^2}{R^2}\right)\left(1 + K\frac{r^2}{R^2}\right)^{-3} \tag{2.2.13}$$

is negative.

We consider a star with isotropic core and anisotropic envelope with radial pressure p_r and tangential pressure $p_\perp$. The anisotropy starts developing from the core boundary having radius $r = b$. The radial pressure decreases in the enveloping region and it becomes zero at the surface (say $r = a$, where a is the radius of the star under consideration). We describe the core upto the radius $r = b$, throughout which $S(r) = 0$. The radius of the star is taken as a and we divide it into two parts:

(i) $0 \leq r \leq b$ as the core of the star described by a fluid distribution with isotropic pressure.

(ii) $b \leq r \leq a$ as the outer envelope of the core which can be described by a fluid distribution with anisotropic pressure.

2.3 The Core of the Star

The core of the distribution is characterized by the isotropic distribution of matter. So throughout the core region $0 \leq r \leq b$ the radial pressure p_r is equal to the tangential pressure $p_\perp$ and hence $S(r) = 0$. Then equation (2.2.12) reduces to

$$\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'}{2r}\right)\left(1 + \frac{r^2}{R^2}\right)\left(1 + K\frac{r^2}{R^2}\right) - \frac{(K-1)}{R^2}r\left(\frac{\nu'}{2} + \frac{1}{r}\right) - \frac{K-1}{R^2}\left(1 + K\frac{r^2}{R^2}\right) = 0. \quad (2.3.1)$$

Which is a non-linear differential equation, if we choose new independent variable z and dependent variable F defined by:

$$z = \sqrt{1 + \frac{r^2}{R^2}}, \quad (2.3.2)$$

$$F = e^{\nu/2}, \quad (2.3.3)$$

equation (2.3.1) takes the linear form

$$\left(1 - K + Kz^2\right)\frac{d^2F}{dz^2} - Kz\frac{dF}{dz} + K(K-1)F = 0. \quad (2.3.4)$$

We again make the transformation

$$x = \sqrt{\frac{K}{K-1}}z, \quad (2.3.5)$$

which reduces the equation (2.3.4) in the form

$$\left(1-x^2\right)\frac{d^2F}{dx^2}+x\frac{dF}{dx}-(K-1)F=0. \tag{2.3.6}$$

We observe that $x=\pm1$ are two regular singular points. We assume the solution in the form $F=\sum\limits_{n=0}^{\infty}C_nx^n$. Substituting this in the equation (2.3.6), gives the recurrence relation

$$(n+2)+(n+1)C_{n+2}-\left(n^2-2n+K-1\right)C_n=0. \tag{2.3.7}$$

If $n^2-2n+K-1=0$, then the integral values of n forms a solution of one of two sets of coefficients $(C_0, C_2, C_4.....)$, $(C_1, C_3, C_4.....)$ containing finite number of terms for equation (2.3.6), n is a positive integer only when $K=2$ and hence $F_1=C_1x=Ax$ (without loss of generality replacing C_1 by A) is a finite polynomial solution of equation (2.3.6). For finding second linearly independent solution to the second order linear differential equation (2.3.6), we use the method of variation of parameters and assume $F_2=y(x)x$ as the other linearly independent solution and substitute F_2 in equation (2.3.6) which results into

$$x\left(1-x^2\right)\frac{d^2y}{dx^2}+\left(2-x^2\right)\frac{dy}{dx}=0, \tag{2.3.8}$$

which admits the solutions

$$y=\left[ln\left(x+\sqrt{x^2-1}\right)-\frac{\sqrt{x^2-1}}{x}\right]. \tag{2.3.9}$$

Hence the general solution of (2.3.6) is

$$F=F_1+F_2=Ax+B\left[x\ ln\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}\right]. \tag{2.3.10}$$

The backsubstitution of variable x gives closed form solution of equation (2.3.4) as

$$F=e^{\nu/2}=A\sqrt{1+\frac{r^2}{R^2}}+B\left[\sqrt{1+\frac{r^2}{R^2}}\mathbb{L}(r)-\frac{1}{\sqrt{2}}\sqrt{1+2\frac{r^2}{R^2}}\right], \tag{2.3.11}$$

for $K = 2$, where A and B are constants of integration and

$$\mathbb{L}(r) = ln\left(\sqrt{2}\sqrt{1+\frac{r^2}{R^2}}+\sqrt{1+2\frac{r^2}{R^2}}\right). \tag{2.3.12}$$

Thus the spacetime metric of the core region $0 \le r \le b$ is described by:

$$\begin{aligned} ds^2 &= \left\{A\sqrt{1+\frac{r^2}{R^2}}+B\left[\sqrt{1+\frac{r^2}{R^2}}\mathbb{L}(r)-\frac{1}{\sqrt{2}}\sqrt{1+2\frac{r^2}{R^2}}\right]\right\}^2 dt^2 - \\ &\left(\frac{1+2\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}\right)dr^2 - r^2d\theta^2 - r^2sin^2\theta d\phi^2. \end{aligned} \tag{2.3.13}$$

The density and pressure of the distribution are given by:

$$8\pi\rho = \frac{1}{R^2}\left(3+2\frac{r^2}{R^2}\right)\left(1+2\frac{r^2}{R^2}\right)^{-2}, \tag{2.3.14}$$

$$8\pi p = \frac{A\sqrt{1+\frac{r^2}{R^2}}+B\left[\sqrt{1+\frac{r^2}{R^2}}\mathbb{L}(r)+\frac{1}{\sqrt{2}}\sqrt{1+\frac{r^2}{R^2}}\right]}{R^2\left(1+2\frac{r^2}{R^2}\right)\left\{A\sqrt{1+\frac{r^2}{R^2}}+B\left[\sqrt{1+\frac{r^2}{R^2}}\mathbb{L}(r)-\frac{1}{\sqrt{2}}\sqrt{1+2\frac{r^2}{R^2}}\right]\right\}}. \tag{2.3.15}$$

The constants A and B are to be determined by requiring that the pressure and metric coefficients must be continuous across the core boundary $r = b$; and this is done in section 2.5.

2.4 The Envelope of the Star

The envelope of the star is characterized by the anisotropic distribution of matter. So throughout the enveloping region $b \le r \le a$ the radial pressure p_r is different from the tangential pressure $p_\perp$, and hence $S(r) \neq 0$. To obtain the solution of equation (2.2.12), in this case, we introduce new variables z and ψ defined by:

$$z = \sqrt{1+\frac{r^2}{R^2}} \qquad \psi = \frac{e^{\nu/2}}{(1-K+Kz^2)^{1/4}} \tag{2.4.1}$$

in terms of which equation (2.2.12) assumes the form:

$$\frac{d^2\psi}{dz^2} + \left[\frac{2K\left(2K-1\right)\left(1-K+Kz^2\right)-5K^2z^2}{4\left(1-K+Kz^2\right)^2} + \frac{8\sqrt{3}\pi R^2 S\left(1-K+Kz^2\right)}{z^2-1}\right]\psi = 0 \tag{2.4.2}$$

On prescribing

$$8\pi\sqrt{3}S = -\frac{\left(z^2-1\right)\left[2K\left(2K-1\right)\left(1-K+Kz^2\right)-5K^2z^2\right]}{4R^2\left(1-K+Kz^2\right)^3} \tag{2.4.3}$$

the second term of (2.4.2) vanishes and the resulting equation is

$$\frac{d^2\psi}{dz^2} = 0, \tag{2.4.4}$$

which has solution of the form

$$\psi = Cz + D \tag{2.4.5}$$

where C and D are constants of integration. From equation (2.4.1) we get

$$e^{\nu/2} = \left(1+K\frac{r^2}{R^2}\right)^{\frac{1}{4}}\left(C\sqrt{1+\frac{r^2}{R^2}}+D\right). \tag{2.4.6}$$

Thus the spacetime metric of the enveloping region $b \leq r \leq a$ is described by:

$$\begin{aligned} ds^2 &= \sqrt{1+K\frac{r^2}{R^2}}\left(C\sqrt{1+\frac{r^2}{R^2}}+D\right)^2 dt^2 - \left(\frac{1+K\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}\right)dr^2 - \\ & r^2\left(d\theta^2+sin^2\theta d\phi^2\right). \end{aligned} \tag{2.4.7}$$

The radial pressure p_r and anisotropy parameter $S(r)$ having explicit expressions:

$$8\pi p_r = \frac{C\sqrt{1+\frac{r^2}{R^2}}\left[3+2K\frac{r^2}{R^2}+K\left(2-K\right)\frac{r^2}{R^2}\right]+D\left[1+K\left(2-K\right)\frac{r^2}{R^2}\right]}{R^2\left(1+K\frac{r^2}{R^2}\right)^2\left(C\sqrt{1+\frac{r^2}{R^2}}+D\right)} \tag{2.4.8}$$

$$8\pi\sqrt{3}S = -\frac{\frac{r^2}{R^2}\left[2K\left(2K-1\right)\left(1+K\frac{r^2}{R^2}\right)-5K^2\left(1+\frac{r^2}{R^2}\right)\right]}{4R^2\left(1+K\frac{r^2}{R^2}\right)^3} \tag{2.4.9}$$

In order to have the same 3-space geometry throughout the distribution, we shall set $K=2$.

The matter density, fluid pressure and anisotropy parameter take the simple forms:

$$8\pi\rho = \frac{3+2\frac{r^2}{R^2}}{R^2\left(1+2\frac{r^2}{R^2}\right)^2}, \tag{2.4.10}$$

$$8\pi p_r = \frac{C\sqrt{1+\frac{r^2}{R^2}}\left(3+4\frac{r^2}{R^2}\right)+D}{R^2\left(1+2\frac{r^2}{R^2}\right)^2\left(C\sqrt{1+\frac{r^2}{R^2}}+D\right)}, \tag{2.4.11}$$

$$8\pi p_\perp = \frac{C\sqrt{1+\frac{r^2}{R^2}}\left(3+4\frac{r^2}{R^2}\right)+D}{R^2\left(1+2\frac{r^2}{R^2}\right)^2\left(C\sqrt{1+\frac{r^2}{R^2}}+D\right)} - \frac{\frac{r^2}{R^2}\left(2-\frac{r^2}{R^2}\right)}{R^2\left(1+2\frac{r^2}{R^2}\right)^3}, \tag{2.4.12}$$

$$8\pi\sqrt{3}S = \frac{\frac{r^2}{R^2}\left(2-\frac{r^2}{R^2}\right)}{R^2\left(1+2\frac{r^2}{R^2}\right)^3}. \tag{2.4.13}$$

The constants C and D are to be determined by matching the solution with the Schwarzschild exterior spacetime metric:

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \left(1-\frac{2m}{r}\right)^{-1}dr^2 - r^2\left(d\theta^2+sin^2\theta d\phi^2\right) \tag{2.4.14}$$

across the boundary $r = a$ of the star, where $p_r(a) = 0$.
The continuity of metric coefficients and pressure along radial direction across $r = a$ imply the following relations:

$$e^{\nu(a)} = \frac{1+\frac{a^2}{R^2}}{1+2\frac{a^2}{R^2}} = 1-\frac{2m}{a}, \tag{2.4.15}$$

$$C\sqrt{1+\frac{a^2}{R^2}}\left(3+4\frac{a^2}{R^2}\right)+D = 0. \tag{2.4.16}$$

Equations (2.4.15) and (2.4.16) determine constants m, C and D as:

$$m = \frac{a^3}{2R^2\left(1+2\frac{a^2}{R^2}\right)} \tag{2.4.17}$$

$$C = -\frac{1}{2}\left(1+2\frac{a^2}{R^2}\right)^{-\frac{7}{4}} \tag{2.4.18}$$

$$D = \frac{1}{2}\sqrt{1+\frac{a^2}{R^2}}\left(3+4\frac{a^2}{R^2}\right)\left(1+2\frac{a^2}{R^2}\right)^{-\frac{7}{4}} \tag{2.4.19}$$

substituting for C and D in the expression (2.4.11) and (2.4.12), we get

$$8\pi p_r = \frac{\sqrt{1+\frac{a^2}{R^2}}\left(3+4\frac{a^2}{R^2}\right)-\sqrt{1+\frac{r^2}{R^2}}\left(3+4\frac{r^2}{R^2}\right)}{R^2\left(1+2\frac{r^2}{R^2}\right)\left[\sqrt{1+\frac{a^2}{R^2}}\left(3+4\frac{r^2}{R^2}\right)-\sqrt{1+\frac{r^2}{R^2}}\right]} \tag{2.4.20}$$

$$\begin{aligned} 8\pi p_\perp &= \frac{\sqrt{1+\frac{a^2}{R^2}}\left(3+4\frac{a^2}{R^2}\right)-\sqrt{1+\frac{r^2}{R^2}}\left(3+4\frac{r^2}{R^2}\right)}{R^2\left(1+2\frac{r^2}{R^2}\right)\left[\sqrt{1+\frac{a^2}{R^2}}\left(3+4\frac{r^2}{R^2}\right)-\sqrt{1+\frac{r^2}{R^2}}\right]} - \\ &\quad \frac{\frac{r^2}{R^2}\left(2-\frac{r^2}{R^2}\right)}{R^2\left(1+2\frac{r^2}{R^2}\right)^3} \end{aligned} \tag{2.4.21}$$

2.5 Physical Plausibility

This approach does not assume any equation of state for matter. Hence it is pertinent to examine the physical plausibility of the solution. A physically plausible solution for the core-envelope model is expected to fulfil the following requirements in its region of validity.

(i) The spacetime metric (2.3.13) in the core should continuously match with the spacetime metric (2.4.7) in the envelope across the core boundary $r = b$.

(ii) $\rho > 0$, $\frac{d\rho}{dr} < 0$ for $0 \le r \le a$.

(iii) $p > 0$, $\frac{dp}{dr} < 0$, $\frac{dp}{d\rho} < 1$, $\rho - 3p > 0$ for $0 \le r \le b$.

(iv) $p_r \ge 0$, $p_\perp > 0$, $\frac{dp_r}{dr} < 0$ for $b \le r \le a$.

(v) $\frac{dp_r}{d\rho} < 1$, $\frac{dp_\perp}{d\rho} < 1$, $\rho - p_r - 2p_\perp \ge 0$ for $b \le r \le a$.

At the core boundary $r = b$, the anisotropy parameter vanishes. And consequently from equation (2.4.13) we get $\frac{b^2}{R^2} = 2$. Further the positivity of the tangential pressure demands that $\frac{a^2}{R^2} > 2$. The continuity of metric coefficients and the continuity of pressure across $r = b$ of the distribution lead to:

$$\sqrt{3}A + B\left[\sqrt{3}L(b) + \sqrt{2.5}\right] = 5^{-\frac{3}{4}}\left[11\sqrt{3}C + D\right], \tag{2.5.1}$$

$$\sqrt{3}A + B\left[\sqrt{3}L(b) - \sqrt{2.5}\right] = 5^{\frac{1}{4}}\left[\sqrt{3}C + D\right], \tag{2.5.2}$$

where

$$L(b) = ln\left(\sqrt{5} + \sqrt{6}\right). \tag{2.5.3}$$

Equations (2.5.1) and (2.5.2) determine A and B in terms of C and D as:

$$A = \frac{\sqrt{2}}{5^{\frac{5}{4}}}\left[3\sqrt{3}C - 2D\right] \tag{2.5.4}$$

$$B = \frac{\left[5\sqrt{5} - 3\sqrt{6}\left(\sqrt{3}L(b) - \sqrt{2.5}\right)\right]C + \left[5\sqrt{5} + 2\sqrt{2}\left(\sqrt{3}L(b) - \sqrt{2.5}\right)\right]D}{5^{\frac{5}{4}}} \tag{2.5.5}$$

substituting these values of A and B in (2.3.15), we get pressure in the core of the star. The requirement (ii) is satisfied in the light of equation (2.3.14) and (2.2.13). Verification of the requirements (iii) analytically is highly difficult due to the complicated expression for pressure given by (2.3.15). Hence we adopt programming approach.

It is evident from (2.4.20) that p_r is positive throughout the envelope. The tangential pressure $p_\perp \left(= p_r - \sqrt{3}S\right)$ is positive if $a \geq \sqrt{2}R$. After a lengthy but straight forward computation one finds that $\rho - p_r - 2p_\perp > 0$ throughout the envelope. Owing to the complexity of expressions, programming is used to verify $\frac{dp_r}{dr} < 0$, $\frac{dp_r}{d\rho} < 1$ and $\frac{dp_\perp}{d\rho} < 1$ in the envelope.

2.6 Discussion

The scheme given by Tikekar [88], for estimating the mass and size of the fluid spheres on the background of spheroidal spacetime can be used to determine the mass and size of the fluid distribution consisting of core and envelope.

Following this scheme we adopt $\rho(a) = 2 \times 10^{14} gm/cm^3$, and introduce a density variation parameter λ given by:

$$\lambda = \frac{\rho(a)}{\rho(0)} = \frac{1 + \frac{2}{3}\frac{a^2}{R^2}}{\left(1 + 2\frac{a^2}{R^2}\right)^2} \tag{2.6.1}$$

Since ρ is a decreasing function of r, $\lambda < 1$. The condition $p_\perp \geq 0$ in the envelope

$$i.e.\ \frac{a^2}{R^2} = \frac{1 - 6\lambda + \sqrt{24\lambda + 1}}{12\lambda} \geq 2 \tag{2.6.2}$$

then restricts λ to comply with the requirement $\lambda \leq 0.093$. Thus the introduction of anisotropy in the envelope results in a high degree of density variation as one moves from the centre to the boundary.

Equation (2.3.14) implies that the matter density at the centre is explicitly related with the curvature parameter R as

$$8\pi\rho(0) = \frac{3}{R^2}. \tag{2.6.3}$$

Equation (2.6.3) determines R in terms of $\rho(a)$ and λ. The size of the configuration can be obtained from (2.6.2) in terms of surface density $\rho(a)$ and density variation parameter λ.

We take the matter density on the boundary $r = a$ of the star as $\rho(a) = 2 \times 10^{14} gm/cm^3$. Choosing different values of $\lambda \leq 0.093$, we determine the boundary radius (in kilometers) using (2.6.2) and the total mass of the star (in kilometers) using (2.4.17).

The mass of the star in grams is obtained using $M = \frac{mc^2}{G}$. Results of these calculations, the thickness of the envelope (in kilometers) together with some relevant quantities are given in table 2.1. It has been noticed that the thickness of the envelope decreases as λ increases in the range $0 < \lambda \leq 0.093$ of table 2.1. The plots showing radial variations of pressure, density and sound speed for the model with $\lambda = 0.05$ are shown in Figure 2.1, Figure 2.2 and Figure 2.3 respectively.

Table 2.1: Masses and equilibrium radii of superdense star models corresponding to $K = 2$, $\rho(a) = 2 \times 10^{14} gm/cm^3$.

λ	R	a	$\frac{a^2}{R^2}$	m	C	D	Thickness of the envelope
0.010000	2.838109	11.740629	17.112941	2.851833	-0.000982	0.298517	7.726937
0.020000	4.013692	11.862912	8.735635	2.805169	-0.003038	0.359674	6.186694
0.030000	4.915749	11.961340	5.920799	2.757472	-0.008891	0.402905	5.009422
0.040000	5.676218	12.041076	4.500000	2.709242	-0.008891	0.437896	4.013692
0.050000	6.346204	12.105678	3.638733	2.660798	-0.012378	0.468003	3.130790
0.060000	6.951919	12.157716	3.058403	2.612351	-0.016124	0.494825	2.326219
0.070000	7.508930	12.199111	2.639370	2.564047	-0.020077	0.519260	1.579880
0.080000	8.027384	12.231340	2.321667	2.515986	-0.024198	0.541869	0.878905
0.090000	8.514327	12.255567	2.071888	2.468242	-0.028459	0.563027	0.214491
0.093000	8.655069	12.261420	2.006968	2.453988	-0.029762	0.569132	0.021304

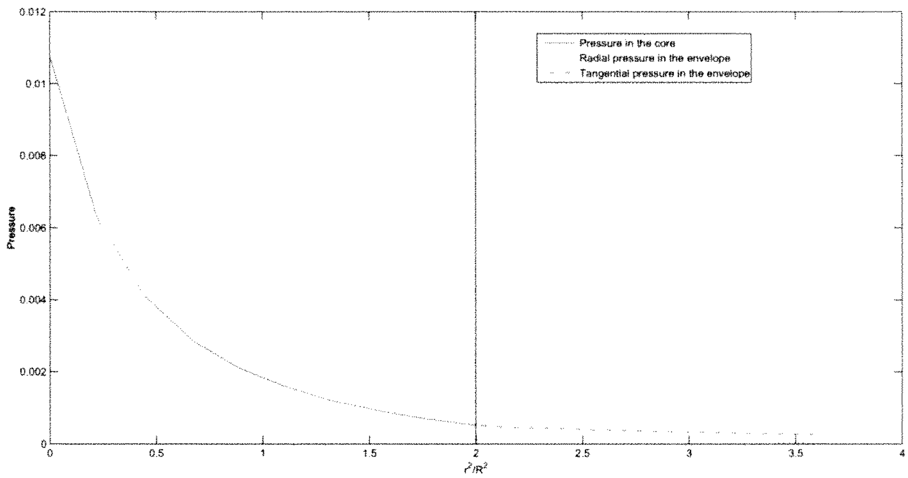

Figure 2.1: Variation of p against $\frac{r^2}{R^2}$ in the core and variation of p_r, $p_\perp$ against $\frac{r^2}{R^2}$ in the envelope.

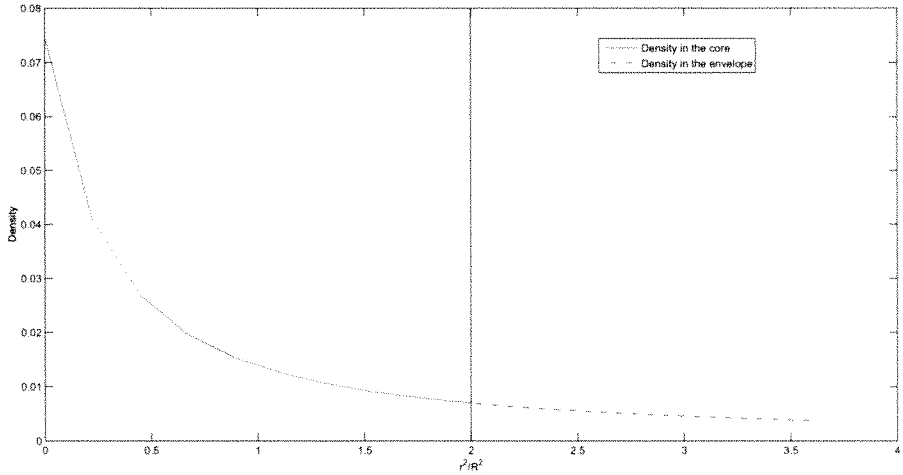

Figure 2.2: Variation of ρ against $\frac{r^2}{R^2}$ throughout the distribution.

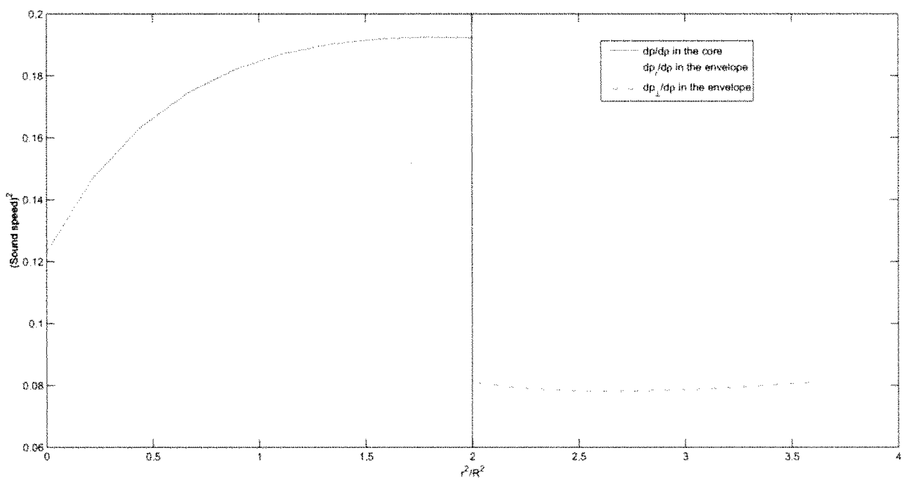

Figure 2.3: Variation of $\frac{dp}{d\rho}$ against $\frac{r^2}{R^2}$ in the core and variation of $\frac{dp_r}{d\rho}$, $\frac{dp_\perp}{d\rho}$ against $\frac{r^2}{R^2}$ in the envelope.

All the requirements stipulated in section 2.5 for the physical plausibility of the distribution are satisfied in the core as well as in the envelope.

The core-envelope models described on the background of a pseudo spheroidal space-time have the following salient features:

1. The core region contains a distribution of isotropic fluid and is surrounded by an envelope of anisotropic fluid at rest.
2. The density profile is continuous throughout, even at the core boundary.
3. The pressure is continuous throughout and continuously join across the core boundary to the anisotropic pressure of the fluid in the envelope.
4. The models admit a high degree of density variation from centre to boundary.