Chapter 6

Relativistic Stellar Models Admitting a Quadratic Equation of State

Contents

In this chapter a class of solutions describing the interior of a static spherically symmetric compact anisotropic star based on paraboloidal spacetime is reported. Based on physical grounds appropriate bounds on the model parameters have been obtained and it has been shown that the model admits an equation of state which is quadratic in nature.

6.1 Introduction

A neutron star is assumed to be the final stage of a collapsing star whose gravitational attraction is counter balanced by its constituent degenerate neutron gas. However, observational studies in the recent past strongly point towards the existence of exotic class of compact stars which are more compact than ordinary neutron stars (see for example, [18], [19], [51], [52], [53], [104], [105]). To have a proper understanding of such ultra-compact objects, it is imperative to know the exact composition and nature of particle interactions at extremely high density regime. From general relativistic perspective, if the equation of state of the material composition of a compact star is known, one can easily integrate the Tolman-Oppenheimer-Volkoff (TOV) equations to analyze the physical features of the star. The problem is that we still lack reliable information about physics of particle interactions beyond nuclear density. In the class of stars having a density regime exceeding nuclear matter density, many exotic phases may exist in the interior, including a possible transition from hadronic to quark degrees of free ([1], [2], [11], [24], [101], [103]). However, even in the extreme case of a quantum cromo dynamics inspired model based equation of state, by and large, remains phenomenological till date.

The objective of this chapter is to construct models of equilibrium configurations of relativistic ultra-compact objects when no reliable information about the composition and nature of particle interactions are available. This can be achieved by generating exact solutions of Einsteins field equations describing the interior of a static spherically symmetric relativistic star. However, finding exact solutions of Einsteins field equations is extremely difficult due to highly non-linear nature of the governing field equations. Consequently, many simplifying assumptions are often made to tackle the problem. Since general relativity provides a mutual correspondence between the material composition of a relativistic star and its associated spacetime, we will adopt a geometric approach to deal with such a situation. In this approach, a suitable ansatz with a clear geometric characterization for one of the metric potentials of the associated spacetime metric will be prescribed to determine the other. Such a method was initially proposed by Vaidya and Tikekar [99]; subsequently the method was utilized by many to generate and analyze physically viable models of compact astrophysical objects (see for example, [43], [45], [58], [64], [81], [82], [83], [88] and references therein). In present work we consider paraboloidal spacetime metric described by Tikekar and Jotania [89].

We have incorporated a general anisotropic term in the stress-energy tensor representing the material composition of the star. Impact of anisotropy on stellar configurations may be found in the pioneering works of Bowers and Liang [7] and Herrera and Santos [37]. Local anisotropy at the interior of an extremely dense object may occur due various factors such as the existence of type 3A superfluid ([7], [76], [44]), phase transition ([84]), presence of electromagnetic field ([41]), etc. Mathematically, anisotropy provides an extra degree of freedom in our system of equations. Therefore, on top of paraboloidal spacetime metric, we shall utilize this freedom to assume a particular pressure profile to solve the system. In the past, a large class of exact solutions corresponding to spherically symmetric anisotropic matter distributions have been found and analyzed (see for example, [5], [28], [36], [50], [59], [61], [80]). Maharaj and Chaisi [55] have prescribed an algorithm to generate anisotropic models from known isotropic solutions. Dev and Gleiser ([14],[15],[17]) have studied the effects of anisotropy on the properties of spherically symmetric gravitationally bound objects and also investigated stability of such configurations. It has been shown that if the tangential pressure $p_\perp$ is greater than the radial pressure p_r of a stellar configuration, the system becomes more stable. Impact of anisotropy has also been investigated by Ivanov [40]. In an anisotropic stellar model for strange stars developed by Paul *et. al.* [71], it has been shown that the value of the bag constant depends on the anisotropic parameter. For a charged anisotropic stellar model governed by the Massachusetts Institute of Technology (MIT) bag model equation of state, Rahaman *et. al.* [74] have shown that the bag constant depends on the compactness of the star. A core-envelope type model describing a gravitationally bound object with an anisotropic fluid distribution has been obtained in [86], [87], [95].

In this chapter, we have constructed a non-singular anisotropic stellar model on paraboloidal spacetime, satisfying all the necessary conditions of a realistic compact star. Based on physical grounds, bounds on the model parameters are prescribed and the relevant equation of state for the system is worked out. An interesting feature of this model is that the solution admits a quadratic equation of state. Due to complexity, it is often very difficult to generate an equation of state $(p = p(\rho))$ from known solutions of Einsteins field equations. In fact, in most of the models involving an equation of state, the equation of state is prescribed a priori to generate the solutions. For example, Sharma and Maharaj [80] have obtained an analytic solution for compact anisotropic stars where a linear equation of state was assumed. Thirukkanesh and Maharaj [85] have assumed a linear equation of state to obtain solutions of an anisotropic fluid distribution. Feroze and Siddiqui [25] and Maharaj and Takisa [60] have separately utilized a quadratic equation of state to generate solutions for static anisotropic spherically symmetric charged distributions. A general approach to deal with anisotropic charged fluid systems admitting a linear or non-linear equation of state have been discussed by Varela *et. al.* [100]. In present model, we do not prescribe the equation of state; rather the solution imposes a constraint on the equation of state corresponding to the material composition of the highly dense system.

In section 6.2, the relevant field equations describing a gravitationally bound spherically symmetric anisotropic stellar configuration in equilibrium have been laid down. Solution to the system of equations is obtained in section 6.3 and analyzed bounds on the model parameters based on physical grounds are analyzed. Physical features of the model have been discussed in section 6.4. We have also generated an approximated equation of state in this section which has been found to be quadratic in nature. In section 6.5, we have concluded by pointing out some interesting features of our model.

6.2 Field Equations

We write the interior spacetime of a static spherically symmetric stellar configuration in the standard form

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{6.2.1}$$

where $\nu(r)$ and $\lambda(r)$ are yet to be determined. We assume that the material composition of the configuration is anisotropic in nature and accordingly we write the energy-momentum tensor in the form

$$T_{ij} = (\rho + p)\, u_i u_j - p g_{ij} + \pi_{ij}, \tag{6.2.2}$$

where ρ and p represent energy-density and isotropic pressure of the system and u^i is the 4-velocity of fluid. The anisotropic stress-tensor π_{ij} is assumed to be of the form

$$\pi_{ij} = \sqrt{3}S\left[C_iC_j - \frac{1}{3}\left(u_iu_j - g_{ij}\right)\right], \tag{6.2.3}$$

where $S = S(r)$ denotes the magnitude of anisotropy and $C^i = \left(0, -e^{-\lambda/2}, 0, 0\right)$ is a radially directed vector. We Calculate non-vanishing components of the energy-momentum tensor as

$$T_0^0 = \rho, \quad T_1^1 = -\left(p + \frac{2S}{\sqrt{3}}\right), \quad T_2^2 = T_3^3 = -\left(p - \frac{S}{\sqrt{3}}\right). \tag{6.2.4}$$

This implies that the radial presure and the tangential pressure can be obtained in the following forms

$$p_r = p + \frac{2S}{\sqrt{3}}, \tag{6.2.5}$$

$$p_\perp = p - \frac{S}{\sqrt{3}}, \tag{6.2.6}$$

respectively. Therefore, magnitude of the anisotroy is obtained as

$$p_r - p_\perp = \sqrt{3}S. \tag{6.2.7}$$

The Einstein's field equations corresponding to the spacetime metric (6.2.1) and the energy-momentum tensor (6.2.3) are obtained as

$$8\pi\rho = \frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right), \tag{6.2.8}$$

$$8\pi p_r = e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2}, \tag{6.2.9}$$

$$8\pi p_\perp = \frac{e^{-\lambda}}{4}\left[2\nu'' + (\nu' - \lambda')\left(\nu' + \frac{2}{r}\right)\right]. \tag{6.2.10}$$

Defining the mass within a radius r as

$$m(r) = \frac{1}{2}\int_0^r \tilde{r}^2\rho(\tilde{r})d\tilde{r}. \tag{6.2.11}$$

We rewrite the field equations (6.2.8) - (6.2.10) in the form

$$e^{-\lambda} = 1 - \frac{2m}{r}, \tag{6.2.12}$$

$$r\left(r - 2m\right)\nu' = 8\pi p_r r^3 + 2m, \tag{6.2.13}$$

$$\left(8\pi\rho + 8\pi p_r\right)\nu' + 2(8\pi p_r') = -\frac{4}{r}\left(8\pi\sqrt{3}S\right). \tag{6.2.14}$$

6.3 Interior Solution

To solve the system of equations (6.2.12) - (6.2.14), we make use of the ansatz for metric potential $e^{\lambda(r)}$ as

$$e^{\lambda(r)} = 1 + \frac{r^2}{R^2}, \tag{6.3.1}$$

where R is the curvature parameter. The $t = constant$ sections of (6.2.1) for ansatz (6.3.1) represent paraboloidal spacetimes immersed in 4-Euclidean spacetime.

The energy density and mass function are then obtained as

$$8\pi\rho = \frac{3 + \frac{r^2}{R^2}}{R^2\left(1 + \frac{r^2}{R^2}\right)^2}, \tag{6.3.2}$$

$$m(r) = \frac{r^3}{2R^2\left(1 + \frac{r^2}{R^2}\right)}. \tag{6.3.3}$$

Combining equations (6.2.13) and (6.3.3), we get

$$\nu' = \left(8\pi p_r\right) r\left(1 + \frac{r^2}{R^2}\right) + \frac{r}{R^2}. \tag{6.3.4}$$

To integrate equation (6.3.4), we assume $8\pi p_r$ in the form

$$8\pi p_r = \frac{p_0\left(1 - \frac{r^2}{R^2}\right)}{R^2\left(1 + \frac{r^2}{R^2}\right)^2}, \tag{6.3.5}$$

where $p_0 > 0$ is a parameter such that $\frac{p_0}{R^2}$ denotes the central pressure. The particular form of the radial pressure profile assumed here is reasonable due to the following facts:

1. Differentiation of equation (6.3.5) yields

$$8\pi\frac{dp_r}{dr}=\frac{-2rp_0\left(3-\frac{r^2}{R^2}\right)}{R^4\left(1+\frac{r^2}{R^2}\right)^3}. \tag{6.3.6}$$

For $p_0>0$, equation (6.3.6) implies that $\frac{dp_r}{dr}<0$, i.e., the radial pressure is a decreasing function of radial parameter r. At a finite radial distance $r=R$ the radial pressure vanishes which is an essential criterion for the construction of a realistic compact star. The curvature parameter R is then identified as the radius of the star.

2. The particular choice (6.3.5) makes equation (6.3.4) integrable.

Substituting equation (6.3.5) in equation (6.3.4), we obtain

$$\nu'=\frac{2p_0r}{R^2\left(1+\frac{r^2}{R^2}\right)}+(1-p_0)\frac{r}{R^2}, \tag{6.3.7}$$

which is integrable and yields

$$e^{\nu}=C\left(1+\frac{r^2}{R^2}\right)^{p_0}e^{(1-p_0)r^2/2R^2}, \tag{6.3.8}$$

where C is a constant of integration. Thus the interior spacetime of the configuration takes the form

$$\begin{aligned} ds^2 &= C\left(1+\frac{r^2}{R^2}\right)^{p_0}e^{(1-p_0)r^2/2R^2}dt^2-\left(1+\frac{r^2}{R^2}\right)dr^2 \\ &\quad -r^2\left(d\theta^2+\sin^2\theta d\phi^2\right), \end{aligned} \tag{6.3.9}$$

which is non-singular at $r=0$.

Making use of equations (6.2.14), (6.3.2), (6.3.5) and (6.3.7), the anisotropy can be determined as follows

$$\begin{aligned} 8\pi\sqrt{3}S &= -\frac{\frac{r^2}{R^2}}{4R^2\left(1+\frac{r^2}{R^2}\right)^3}\left[\left((3+p_0)+(1-p_0)\frac{r^2}{R^2}\right)\right. \\ &\quad\left.\times\left(2p_0+(1-p_0)\left(1+\frac{r^2}{R^2}\right)\right)+4p_0\left(\frac{r^2}{R^2}-3\right)\right]. \end{aligned} \tag{6.3.10}$$

Note that the anisotropy vanishes at the centre ($r = 0$) as expected. The tangential pressure takes the form

$$8\pi p_{\perp} = 8\pi p_r - 8\pi\sqrt{3}S = \frac{4p_0\left(1 - \frac{r^4}{R^4}\right) + \frac{r^2}{R^2} f\left(r, p_0, R\right)}{4R^2\left(1 + \frac{r^2}{R^2}\right)^3}, \tag{6.3.11}$$

where,

$$f\left(r, p_0, R\right) = \left[\left(3 + p_0 + (1 - p_0)\frac{r^2}{R^2}\right)\left(2p_0 + (1 - p_0)\left(1 + \frac{r^2}{R^2}\right)\right) + 4p_0\left(\frac{r^2}{R^2} - 3\right)\right].$$

This model has three independent parameters, namely, p_0, C and R. The requirement that the interior metric (6.3.9) should be matched to the Schwarzschild exterior spacetime metric

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{6.3.12}$$

across the boundary $r = R$ of the star together with the condition that the radial pressure should vanish at the surface ($p_r\left(r = R\right) = 0$) help us to determine these constants. Note that the form of the radial pressure profile is such that the condition ($p_r\left(r = R\right) = 0$) itself becomes the definition of the radius R of the star in this construction. Matching the relevant metric coefficients across the boundary R then yields

$$R = 4m, \tag{6.3.13}$$

$$C = \frac{e^{-(1-p_0)/2}}{2^{p_0+1}}, \tag{6.3.14}$$

where m is the total mass enclosed within the radius R from the centre of the star. If the radius of the star R is known, equation (6.3.13) can be utilized to determine the total mass m of the star and vice-versa. For a given value of p_0, equation (6.3.14) determines C. In this model $\frac{p_0}{R^2}$ corresponds to the central pressure. Therefore, for a given mass (m) or radius (R), if the central pressure is specified, the system is completely determined.

Following Delgaty and Lake [13], we impose the following conditions on our system so that it becomes a realistic and physical acceptable model.

(i) $\rho(r),\ p_r(r),\ p_\perp(r) \geq 0$, for $0 \leq r \leq R$.

(ii) $\rho - p_r - 2p_\perp \geq 0$, for $0 \leq r \leq R$.

(iii) $\frac{d\rho}{dr},\ \frac{dp_r}{dr},\ \frac{dp_\perp}{dr} < 0$, for $0 \leq r \leq R$.

(iv) $0 \leq \frac{dp_r}{d\rho} \leq 1$, $0 \leq \frac{dp_\perp}{d\rho} \leq 1$, for $0 \leq r \leq R$.

The requirements (i) and (ii) imply that the weak and dominant energy conditions are satisfied. Condition (iii) ensures regular behaviour of the energy density and the two pressures $(p_r,\ p_\perp)$. The condition (iv) is invoked to ensure that the sound speed does not exceed speed of light. In addition, for regularity, we demand that the anisotropy should vanish at the centre, i.e., $p_r = p_\perp$ at $r = 0$. From equation (6.3.10), we observe that the anisotropy vanishes at $r = 0$ and $S(r) > 0$ for $0 < r < R$. Interestingly, for the particular choice $p_0 = 1$, the anisotropy also vanishes at the boundary $r = R$ in this construction. From equation (6.3.2), it is obvious that $\rho > 0$, and

$$8\pi\frac{d\rho}{dr} = \frac{-2r\left(5+\frac{r^2}{R^2}\right)}{R^4\left(1+\frac{r^2}{R^2}\right)^3}, \tag{6.3.15}$$

which estabilishes that ρ decreases in the radially outward direction. We have already stated that $\frac{p_0}{R^2}$ corresponds to the central pressure which implies that $p_0 > 0$. It can be shown that for $p_\perp > 0$, we must have $p_0 < 1$. Thus the bounds on p_0 can be obtained as

$$0 < p_0 \leq 1. \tag{6.3.16}$$

To obtain a more stringent bound on p_0, we evaluate

$$8\pi\frac{dp_\perp}{dr} = \frac{r\left[(3-20p_0+p_0^2)+(2+12p_0-6p_0^2)\frac{r^2}{R^2}+(-1-4p_0+5p_0^2)\frac{r^4}{R^4}\right]}{2R^4\left(1+\frac{r^2}{R^2}\right)^4}, \tag{6.3.17}$$

at two different points. At the centre of the star $(r = 0)$ it takes the following form

$$\left(8\pi\frac{dp_\perp}{dr}\right)_{(r=0)} = 0, \tag{6.3.18}$$

and at the boundary of the star $(r = R)$, it takes the form

$$\left(8\pi\frac{dp_\perp}{dr}\right)_{(r=R)} = \frac{1-3p_0}{8R^3}, \tag{6.3.19}$$

which will be negative if $p_0 > \frac{1}{3}$. Therefore, a more stringent bound on the parameter p_0 is obtained as

$$\frac{1}{3} < p_0 \leq 1. \tag{6.3.20}$$

To verify whether the bound on p_0 satisfies the causality condition $0 < \frac{dp_r}{d\rho} < 1$, we combine equations (6.3.6) and (6.3.15), to yield

$$\frac{dp_r}{d\rho} = \frac{p_0\left(3 - \frac{r^2}{R^2}\right)}{5 + \frac{r^2}{R^2}}. \tag{6.3.21}$$

Now, at the centre of the star ($r = 0$), $\frac{dp_r}{d\rho} < 1$ if the condition $p_0 < 1.6667$ is satisfied and at the boundary of the star ($r = R$), $\frac{dp_r}{d\rho} < 1$ if the condition $p_0 < 3$ is satisfied. Both these restrictions are consistent with the requirement given in (6.3.20).

Similarly, we can obtain as

$$\frac{dp_\perp}{d\rho} = \frac{(-3 + 20p_0 - p_0^2) + (-2 - 12p_0 + 6p_0^2)\frac{r^2}{R^2} + (1 + 4p_0 - 5p_0^2)\frac{r^4}{R^4}}{4\left(1 + \frac{r^2}{R^2}\right)\left(5 + \frac{r^2}{R^2}\right)}, \tag{6.3.22}$$

At the centre ($r = 0$), the requirement $\frac{dp_\perp}{d\rho} < 1$ puts a constraint on p_0 such that $p_0 < 1.2250$. At the boundary of the star the corresponding requirement is given by $p_0 < 4.3333$. Both these requirements are also consistent with the bound $\frac{1}{3} < p_0 \leq 1$.

We now investigate the bound on the model parameters based on stability. To check stability of our model, we shall use Herrera's [35] overtuning technique which states that the region for which radial speed of sound is greater than the tangential speed of sound, is a potentially stable region. The radial and tangential sound speeds in our model are obtained as

$$v_{sr}^2 = \frac{dp_r}{d\rho} = \frac{p_0\left(3 - \frac{r^2}{R^2}\right)}{5 + \frac{r^2}{R^2}}, \tag{6.3.23}$$

$$v_{st}^2 = \frac{dp_\perp}{d\rho} = \frac{(-3 + 20p_0 - p_0^2) + (-2 - 12p_0 + 6p_0^2)\frac{r^2}{R^2} + (1 + 4p_0 - 5p_0^2)\frac{r^4}{R^4}}{4\left(1 + \frac{r^2}{R^2}\right)\left(5 + \frac{r^2}{R^2}\right)} \tag{6.3.24}$$

Herrera's [35] prescription demands that we must have $v_{st}^2 - v_{sr}^2 < 0$ throughout the star. Now, at the centre of the star

$$\left(v_{st}^2 - v_{sr}^2\right)_{(r=0)} = \frac{-3 + 8p_0 - p_0^2}{20}, \tag{6.3.25}$$

for $(v_{st}^2 - v_{sr}^2)_{(r=0)} < 0$, it is required that $-3 + 8p_0 - p_0^2 < 0$ i.e. $p_0 < 0.3944$. At the boundary of the star, we have

$$\left(v_{st}^2 - v_{sr}^2\right)_{(r=R)} = -\frac{(1 + p_0)}{12}, \tag{6.3.26}$$

which is negative for $\frac{1}{3} < p_0 < 0.3944$. Therefore, our model is physically reasonable and stable if the following bound is imposed: $\frac{1}{3} < p_0 < 0.3944$.

6.4 Physical Analysis

To check whether our model can accommodate realistic ultra-compact stars, let us first analyze the gross behaviour of the physical parameters such as energy density and pressure. For a particular choice $p_0 = 0.36$ (consistent with the bound), plugging in c and G at appropriate places, we have calculated the mass m (in terms of $M_\odot$), central density ρ_c (in MeV fm^{-3}), surface density ρ_R in (MeV fm^{-3}) of a star of radius R (in kilometers). This have been shown in Table 6.1.

Table 6.1: Values of physical parameters for different radii with $p_0 = 0.36$.

Case	R	M	ρ_c	ρ_R
I	6.55	1.11	2108.46	702.82
II	6.7	1.14	2015.11	671.70
III	7.07	1.20	1809.71	603.24
IV	8	1.36	1413.41	471.14
V	9	1.53	1116.77	372.26
VI	10	1.69	904.58	301.53
VII	11	1.86	747.59	249.20
VIII	12	2.03	628.18	209.39

We note that the central density in each case (except VIII, where we have assumed a comparatively larger radius which in turn has generated a bigger mass) lies above the deconfinement density [34] $\sim$ 700 MeV fm^{-3} which implies that quark phases may exist at the interiors of such configurations. Variations of the physical parameters in (MeV fm^{-3}) for a particular case VI have been shown in Figures 6.1 - 6.5.

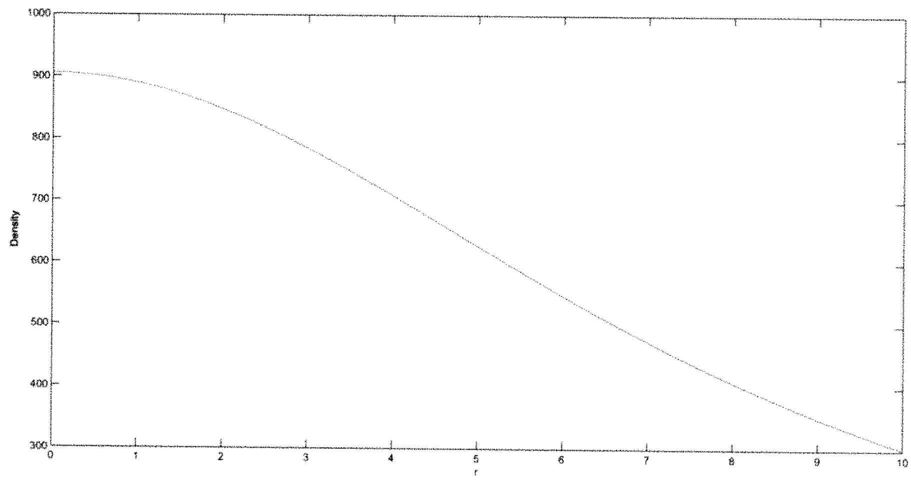

Figure 6.1: Variation of density (ρ) against the radial parameter r.

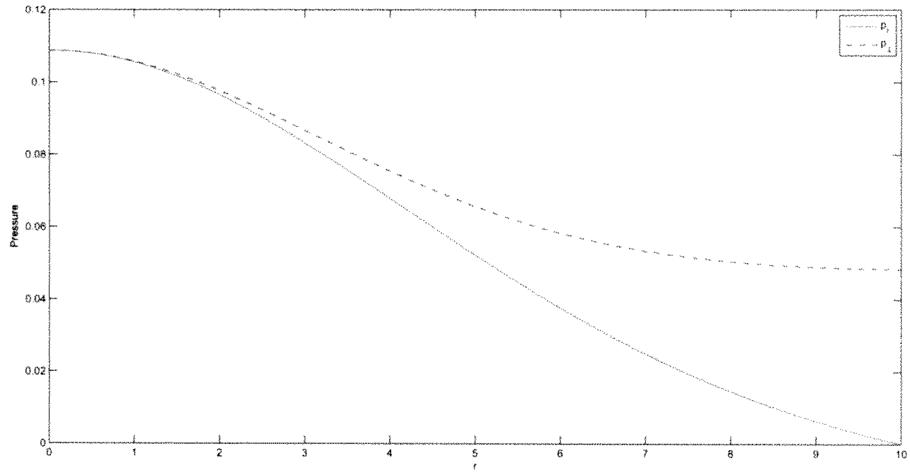

Figure 6.2: Variation of pressure (p_r and $p_\perp$) against the radial parameter r.

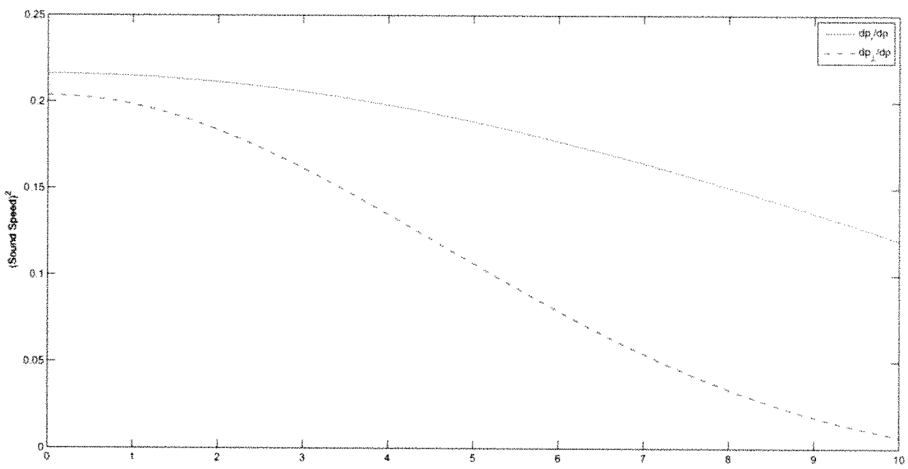

Figure 6.3: Variation of $\frac{dp_r}{d\rho}$ against the radial parameter r.

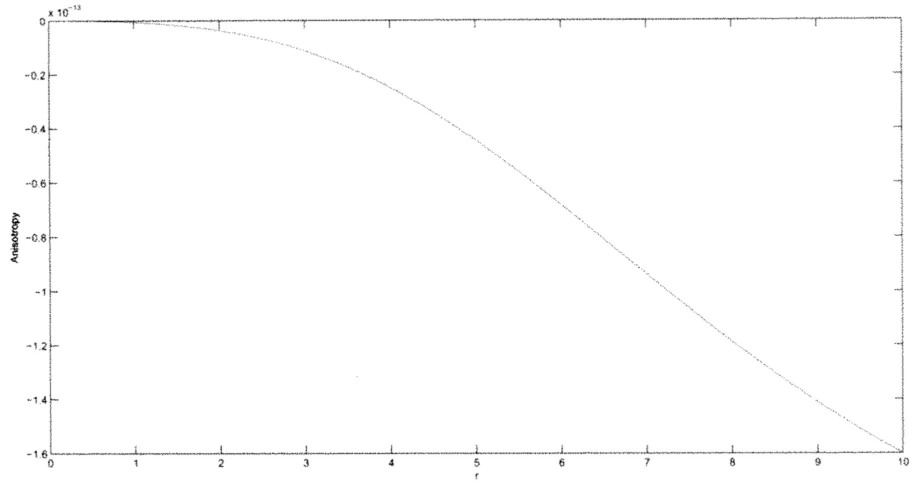

Figure 6.4: Variation of anisotropic parameter $S(r)$ against the radial parameter r.

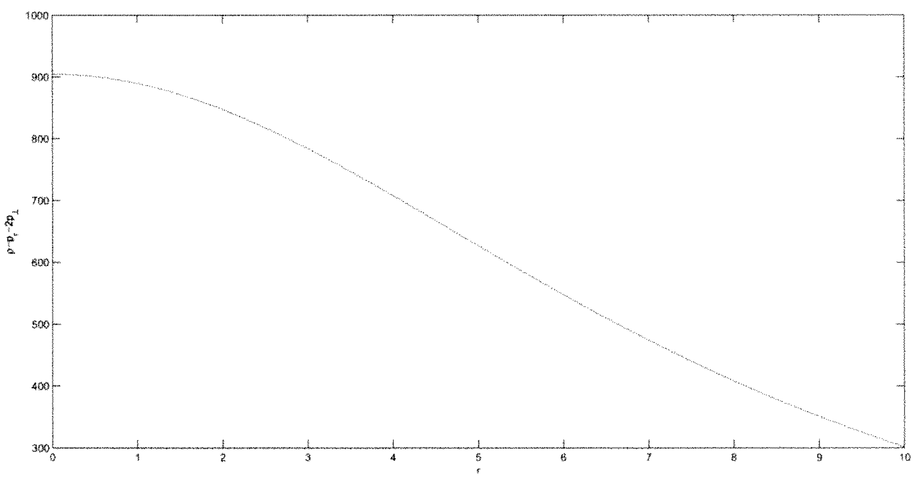

Figure 6.5: Variation of $\rho - p_r - 2p_{\perp}$ against the radial parameter r.

The figures clearly indicate that the physical parameters are well-behaved and all the regularity conditions discussed above are satisfied at all interior points of the star. Moreover, the assumed parameters generate a stable configuration as shown in Figure 6.6.

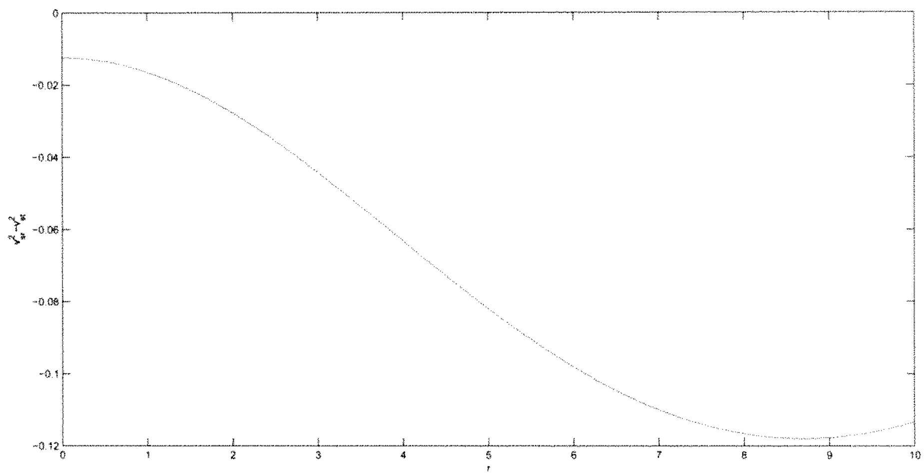

Figure 6.6: Variation of $v_{sr}^2 - v_{sp}^2$ against the radial parameter r.

Having derived a physically acceptable model, question to be asked is, what kind of material composition can be predicted for the stellar configuration admissible in this model? In other words, what would be the equation of state corresponding to the material compositions of the configurations constructed from the model? Though construction of equation of state is essentially governed by the physical laws of the system, one can parametrically relate energy-density and the radial pressure from the mathematical model which may be useful in predicting the composition of the system. Making use of equations (6.3.2) and (6.3.5), we have plotted variation of the radial pressure against the energy-density as shown by the solid curve in Figure 6.7. Our intention now is to prescribe an approximate equation of state which can produce similar kind of curve. Though, in principle, a barotropic equation of state ($p_r = p_r(\rho)$) can be generated from equations (6.3.2) and (6.3.5) by eliminating r, however we have tried curve fitting approach to find equation of state, we have tried linear equation of state $p_r = \rho_0 + \alpha\rho$ and quadratic equation of state $p_r = \rho_0 + \alpha\rho + \beta\rho^2$, where ρ_0, α and β are constants. We found that linear equation of state has norm of residuals 0.021983 and quadratic equation of state has norm of residuals 0.0027629. Hence we consider that the relevant equation of state has the form

$$p_r = \rho_0 + \alpha\rho + \beta\rho^2, \tag{6.4.1}$$

where ρ_0, α and β are constants. We make use of this equation of state to plot ρ Vs p_r (dashed curve) in Figure 6.7, which turns out to be almost identical to the curve generated from the analytic model if we set $\rho_0 = -0.36$, $\alpha = 9.6 \times 10^{-5}$ and $\beta = 7.2 \times 10^{-8}$. Though this has been shown to be true for a particular choice (case VI), it can be shown that the model admits the quadratic equation of state (6.4.1) for different choices of the parameters as well.

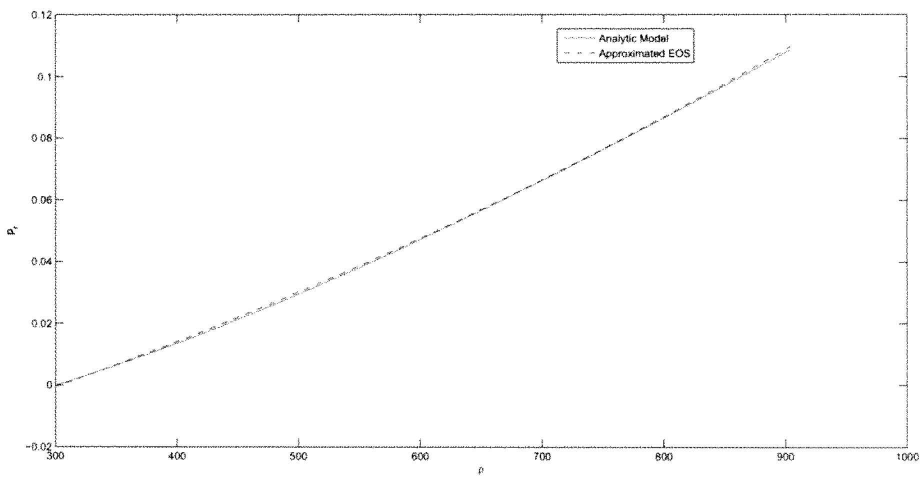

Figure 6.7: Variation of radial pressure against density.

6.5 Discussion

Making use of paraboloidal spacetime metric, we have generated exact solution of Einstein's field equations representing a static spherically symmetric anisotropic stellar configuration. Bounds on the model parameters have been obtained on physical grounds and it has been shown that model is stable for $\frac{1}{3} < p_0 < 0.3944$. In this model $\frac{p_0}{R^2}$ denotes the central density and, therefore, the bound indicates that for a given radius or mass arbitrary choice of the central density is not permissible in this model. We have shown that the model admits an equation of state which is quadratic in nature. Mathematically, this may be understood in the following manner. The ansatz (6.3.1), together with the assumption (6.3.5), generates an anisotropic stellar model whose composition may be described by the equation of state of the form (6.4.1). In [25], [60], quadratic equation of state have been assumed a priori to obtain exact solutions of Einstein's field equations. We have shown that such an assumption is consistent with an analytical model which has been constructed by making use of paraboloidal spacetime metric. In cosmology, a non-linear quadratic equation of state has been shown to be relevant for the descreption of dark energy and dark matter [3]. What type of matter can generate such an equation of state in the ultra-high density regime of an astrophysical object is an open question.