Chapter 5

Newly Designed Double Porous Layered Axially Undefined Journal Bearing with Anisotropic Permeability, Slip Velocity and Squeeze Velocity

Contents

5.1 Introduction

Journal bearings are used in machines like steam turbines and internal combustion engines where there are vibrations [12, 48]. Each of them consists of a circular shaft called journal rotating inside a circular bush which also may rotate. In the above machines vibrations predominate over the rotation speeds.

Prakash and Vij [68] simplified the analysis of a porous journal bearing by assuming that it was undefined in the axial direction [13], thus extending the analysis of Pinkus and Sternlicht [63] by introducing a porous facing of uniform thickness inside the bearing surface. The load capacity and response time increased or decreased as the eccentricity ratio or the permeability parameter increased. Shah *et. al.* [77] studied axially undefined journal bearing using ferrofluid (FF) as lubricant including effects of anisotropic permeability, slip velocity and squeeze velocity for a single porous layer. Here, it was shown that the bearing characteristics decreases owing to slip velocity at the film porous interface and was countered by suitably choosing anisotropic permeability of the porous facing. Recently Shah *et. al.* [91] studied squeeze film based on FF in curved porous circular plates with various porous structures.

In all above investigations, none of the authors considered double porous layered axially undefined journal bearing with FF as lubricant in their study. FF [74] is used because it generates magnetic body force $\mu_0(\mathbf{M}\cdot\nabla)\mathbf{H}$ in the presence of magnetic field $\mathbf{H}$; μ_0, $\mathbf{M}$ are free space permeability, magnetization vector respectively and the porous layer (or matrix or region) is because of its advantageous property of self lubrication. With this motivation the present study proposes the analysis of newly designed double porous layered axially undefined journal bearing using FF as lubricant including combined effects of anisotropic permeability, slip velocity and squeeze velocity. The FF lubricant used here is of water based and the flow model is due to R. E. Rosensweig [74]. The magnetic field used here is oblique to the lower surface.

5.2 Mathematical Formulation

The configuration of the journal bearing as shown in Figure 5.1 consists of a journal of radius R and the bearing with double porous facing (in which inner porous layer is of uniform thickness l_2 and outer porous layer is of uniform thickness l_1) backed by solid wall. Choose the origin O at the top of the circumference of the journal, the x- axis along its circumference, z- axis perpendicular to it and assuming that it is infinite along its axis lying along the y- axis. Assuming that there is a slip velocity at the interface of the film and porous regions and $\dot{h} = dh/dt$ represents the squeeze velocity, where h is the central film thickness and t is time. Figure 5.2 is the configuration opened up at O. It shows that the journal circumference lies in the interval $0 \leq \theta \leq 2\pi$ on the θ- axis and the film is symmetrical about the line $\theta = \pi$.

The expression for the central film thickness h is given by

$$h = c(1 + e\cos\theta), \qquad \dots (5.1)$$

where c and e are being radial clearance and the eccentricity ratio, respectively.

Here, the lubricant is FF, so magnetic field vector **H** is applied such that it vanishes at the ends of the bearing (refer Figure 5.2) and the magnitude H of magnetic field is given by

$$H^2 = Kx(L - x), \qquad \dots (5.2)$$

where $K = 10^{12}$ to 10^{14} is chosen so as to have a magnetic field of strength H between the order of 10^4 to 10^5 and $L = 2\pi R$ is the bearing length in x- direction.

With the usual assumption of lubrication and neglecting inertia terms, from using equations (2.11) to (2.15), the x- dimensional equation governing the lubricant flow in the film region yields

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta}\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right), \qquad \dots (5.3)$$

where u is the film fluid velocity component in the x- direction, p is fluid pressure in fluid film region, μ_0 is free space permeability, $\bar{\mu}$ is magnetic susceptibility, η is fluid viscosity and x, z are axial coordinates.

Solving equation (5.3) under the slip boundary conditions given by Sparrow *et.al.* [96]

$$u = 0 \ \text{ when } z = 0 \ \text{ and } \ u = -\frac{1}{s}\frac{\partial u}{\partial z} \ \text{ when } z = h, \qquad \ldots (5.4)$$

where s is the slip constant defined as $s = \frac{5}{\sqrt{\varphi_x \eta_x}}$; φ_x and η_x are permeability and porosity of lower porous region l_2 in x- direction respectively, one obtains

$$u = \frac{(shz^2 - sh^2 z + z^2 - 2hz)}{2\eta(sh+1)} \frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right). \qquad \ldots (5.5)$$

Using Darcy's law, the velocity components of the fluid in the porous matrix are given as follow:

For upper porous region l_1:

$$\bar{u}_1 = -\frac{\psi_x}{\eta}\frac{\partial}{\partial x}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right), \qquad \ldots (5.6)$$

$$\bar{w}_1 = -\frac{\psi_z}{\eta}\frac{\partial}{\partial z}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right), \qquad \ldots (5.7)$$

where $\bar{u}_1$, $\bar{w}_1$ are fluid velocity components in the porous region and ψ_x, ψ_z are permeabilities in x and z- direction respectively, and P_1 is fluid pressure in the porous region.

For lower porous region l_2:

$$\bar{u}_2 = -\frac{\varphi_x}{\eta}\frac{\partial}{\partial x}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right), \qquad \ldots (5.8)$$

$$\bar{w}_2 = -\frac{\varphi_z}{\eta}\frac{\partial}{\partial z}\left(P_2 - \frac{1}{2}\mu_0\bar{\mu}H^2\right),$$

… (5.9)

where $\bar{u}_2$, $\bar{w}_2$ are fluid velocity components in the porous region and φ_x, φ_z are permeabilities in x and z- direction respectively, and P_2 is fluid pressure in the porous region.

Assuming the boundary conditions between two porous regions l_1 and l_2 as

$$-\frac{\varphi_z}{\eta}\frac{\partial}{\partial z}\left(P_2 - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=h+l_2} = -\frac{\psi_z}{\eta}\frac{\partial}{\partial z}\left(P_1 - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=h+l_2},$$

… (5.10)

and

$$\frac{\psi_z}{\eta}\frac{\partial}{\partial z}\left(P_1 - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=h+l_2+l_1} = 0,$$

… (5.11)

because of impermeable surface at $z = h + l_2 + l_1$.

Substituting equations (5.6) and (5.7) in the continuity equation for upper porous region l_1

$$\frac{\partial \bar{u}_1}{\partial x} + \frac{\partial \bar{w}_1}{\partial z} = 0,$$

… (5.12)

yields

$$\psi_x\frac{\partial^2}{\partial x^2}\left(P_1 - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \psi_z\frac{\partial^2}{\partial z^2}\left(P_1 - \frac{1}{2}\mu_0\bar{\mu}H^2\right) = 0,$$

which on integration with respect to z across the upper porous matrix $(h + l_2,\ h + l_2 + l_1)$ and using equation (5.10), one obtains

$$\psi_z \frac{\partial}{\partial z}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\Big|_{z=h+l_2} = \psi_x l_1 \frac{\partial^2}{\partial x^2}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right),$$

… (5.13)

using Morgan-Cameron approximation [67, 81] and that the surface $z = h + l_2 + l_1$ is non-porous (or impermeable).

Substituting equations (5.8) and (5.9) in the continuity equation for lower porous region l_2

$$\frac{\partial \bar{u}_2}{\partial x} + \frac{\partial \bar{w}_2}{\partial z} = 0,$$

… (5.14)

yields

$$\varphi_x \frac{\partial^2}{\partial x^2}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right) + \varphi_z \frac{\partial^2}{\partial z^2}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right) = 0,$$

which on integration with respect to *z* across the lower porous matrix $(h,\ h + l_2)$ one obtains

$$\varphi_z \frac{\partial}{\partial z}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\Big|_{z=h+l_2} = -\varphi_x l_2 \frac{\partial^2}{\partial x^2}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right) + \varphi_z \frac{\partial}{\partial z}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\Big|_{z=h}.$$

… (5.15)

Using equations (5.10), (5.13) and (5.15), the following relation is obtained.

$$\varphi_z \frac{\partial}{\partial z}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\Big|_{z=h} = \left(\psi_x l_1 + \varphi_x l_2\right) \frac{\partial^2}{\partial x^2}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right).$$

… (5.16)

The equation of continuity in film region is

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,$$

… (5.17)

where w is film fluid velocity component in z- direction.

Integrating continuity equation (5.17) in film region over $(0, h)$,

$$\frac{\partial}{\partial x}\int_0^h u\,dz + w_h - w_0 = 0, \qquad \dots (5.18)$$

where $w_h = \dot{h} + \overline{w}_2|_{z=h}$ because of squeeze velocity and $w_0 = w|_{z=0} = \overline{w}_2|_{z=0} = 0$ because solid surface, one obtains

$$\frac{\partial}{\partial x}\left[\left\{\frac{h^3(sh+4)}{(sh+1)} + 12\left(\psi_x l_1 + \varphi_x l_2\right)\right\}\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\right] = 12\eta c\dot{e}cos\theta, \qquad \dots (5.19)$$

which is known as Reynold’s type equation.

Introducing dimensionless quantities

$$x = R\theta,\ \ \overline{\psi}_x = \frac{l_1\psi_x}{c^3},\ \ \overline{\varphi}_x = \frac{l_2\varphi_x}{c^3},\ \ \bar{s} = sc,\ \ \bar{h} = \frac{h}{c},$$
$$\mu^* = \frac{K\mu_0\bar{\mu}c^2}{2\eta\dot{e}},\ \ \bar{p} = \frac{pc^2}{R^2\eta\dot{e}},$$

one obtain the dimensionless form of equation (5.19) as under

$$\frac{\partial}{\partial\theta}\left\{G^*\frac{\partial}{\partial\theta}\left(\bar{p} - \mu^*\theta(2\pi - \theta)\right)\right\} = 12cos\theta, \qquad \dots (5.20)$$

where

$$G^* = 12\left(\overline{\varphi}_x + \overline{\psi}_x\right) + \frac{\bar{h}^3\left(\bar{s}\bar{h} + 4\right)}{\left(\bar{s}\bar{h} + 1\right)}.$$

5.3 Solution

Solving equation (5.20) for pressure under the appropriate boundary conditions

$$\frac{d\bar{p}}{d\theta} = 0, \text{ when } \theta = \pi \text{ and } \bar{p} = 0, \text{ when } \theta = 0,$$

… (5.21)

the dimensionless film pressure $\bar{p}$ obtained as

$$\bar{p} = \mu^* \theta(2\pi - \theta) + 12 \int_0^\theta \frac{sin\theta}{G^*} d\theta.$$

… (5.22)

If W_x and W_z are the components of the load carrying capacity W of the bearing,

$$W_x = LR \int_0^{2\pi} psin\theta d\theta = 0,$$

$$W_z = LR \int_0^{2\pi} pcos\theta d\theta,$$

and

$$W = \sqrt{W_x^2 + W_z^2},$$

which can be expressed in dimensionless form as

$$\bar{W} = \frac{Wc^2}{LR^3 \eta \dot{e}} = 4\pi\mu^* + 12 \int_0^{2\pi} \frac{sin^2\theta}{G^*} d\theta.$$

… (5.23)

5.4 Results and Discussion

From equation (5.23), dimensionless load carrying capacity for no slip case can be obtained by setting $\bar{s} \to \infty$ as

$$\overline{W} = \frac{Wc^2}{LR^3\eta\dot{e}} = 4\pi\mu^* + 12\int_0^{2\pi} \frac{sin^2\theta}{G^*} d\theta,$$

where

$$G^* = 12\left(\bar{\varphi}_x + \bar{\psi}_x\right) + \bar{h}^3.$$

The values of the dimensionless load capacity $\overline{W}$ are calculated for the following value of the parameters using Simpson's 1/3 rule with step size 11/14.

$$\eta = 0.012(Ns/m^2),\ \ \mu_0 = 4\pi \times 10^{-7}(N/A^2),\ \ L = 0.08(m),$$
$$\bar{\mu} = 0.05,\ \ \eta_x = 0.81,\ \ c = 2.5 \times 10^{-5}(m),\ \ R = 0.01(m),$$
$$\dot{e} = 0.1(m/s),\ \ \rho = 1400(Ns^2/m^4).$$

Case I The results of dimensionless load capacity $\overline{W}$ using conventional lubricant (or without FF effect) can be obtained as follows:

	$l_1(3\times10^{-9}) > l_2(4\times10^{-10})$	$l_1(4\times10^{-10}) = l_2(4\times10^{-10})$	$l_1(4\times10^{-10}) < l_2(3\times10^{-9})$
$\varphi_x(0.0001) > \psi_x(0.00001)$	0.650013	0.992334	0.158553
$\varphi_x(0.0001) = \psi_x(0.0001)$	0.142036	0.573926	0.142036
$\varphi_x(0.00001) < \psi_x(0.0001)$	0.158620	0.994791	0.651088

Table 5.1 The values of $\overline{W}$ using conventional lubricant (or without FF effect)

In all these three cases $\varphi_x > \psi_x$, $\varphi_x = \psi_x$ and $\varphi_x < \psi_x$ of Table 5.1 when $l_1 = l_2$ the value of dimensionless load capacity $\overline{W}$ is obtained better than the cases of $l_1 > l_2$ and $l_1 < l_2$. Also, we conclude that when $l_1 = l_2$ and $\varphi_x < \psi_x$, the value of dimensionless load capacity $\overline{W}$ is more better.

Case II The results of dimensionless load capacity $\overline{W}$ with FF effect can be obtained as follows:

	$l_1(3\times10^{-9})> l_2(4\times10^{-10})$	$l_1(4\times10^{-10})= l_2(4\times10^{-10})$	$l_1(4\times10^{-10})<l_2(3\times10^{-9})$
$\varphi_x(0.0001) > \psi_x(0.00001)$	2.707836	3.050160	2.216376
$\varphi_x(0.0001) = \psi_x(0.0001)$	2.199859	2.631750	2.199859
$\varphi_x(0.00001) < \psi_x(0.0001)$	2.216442	3.052614	2.708911

Table 5.2 The values of $\overline{W}$ using FF as lubricant (or with FF effect)

In all these three cases $\varphi_x > \psi_x$, $\varphi_x = \psi_x$ and $\varphi_x < \psi_x$ of Table 5.2 when $l_1 = l_2$ the value of dimensionless load capacity $\overline{W}$ is obtained better than the cases of $l_1 > l_2$ and $l_1 < l_2$. Also, we conclude that when $l_1 = l_2$ and $\varphi_x < \psi_x$, the value of dimensionless load capacity $\overline{W}$ is more better.

Thus, the behavior of the results obtained in both the cases remains same. Also, from both the above cases we conclude that the value of dimensionless load capacity $\overline{W}$ in the presence of FF effect is much better than in the case of conventional fluid (or without FF effect).

It is also found that the value of dimensionless load capacity $\overline{W}$ increases up to 206.86% with the addition of FF as lubricant.

5.6 Conclusion

In designing of FF lubricated double porous layered axially undefined journal bearing with anisotropic permeability, slip velocity and squeeze velocity, the decrease in the load carrying capacity can be countered by suitably more chosen values of ψ_x as compared to φ_x and $l_1 = l_2$. Moreover, with the use of FF as lubricant, it is found that the bearing performance is much better than that of a conventionally lubricated bearing.

5.6 Figures

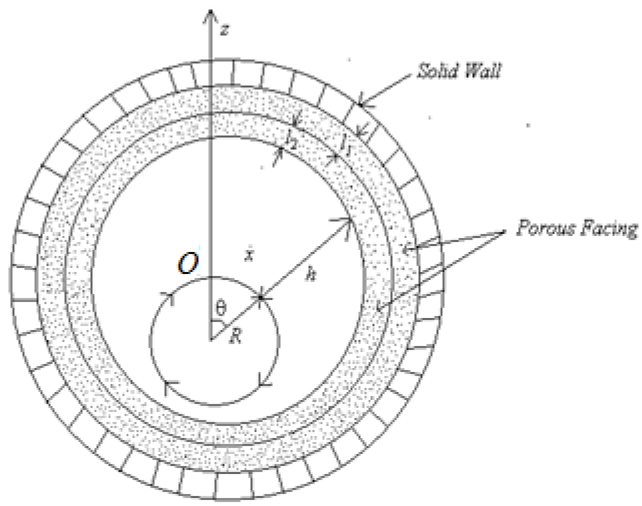

Figure 5.1 Porous journal bearing

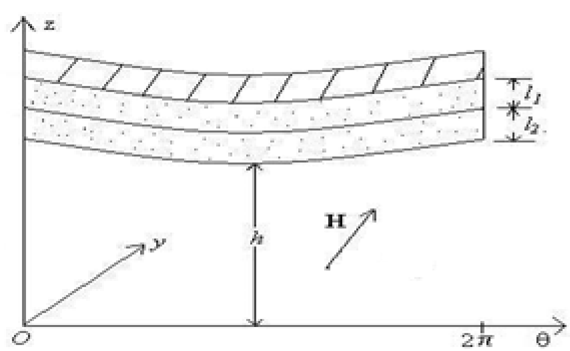

Figure 5.2 Configuration of the bearing opened up at *O*