Chapter 7

Lubrication of Porous Pivoted Slider Bearing with Slip and Squeeze Velocity

Contents

7.1 Introduction

Agrawal [1] studied magnetic fluid (MF) based porous inclined slider bearing using Neuringer-Rosensweig's model. Shah and Bhat in [85, 87] considered respectively squeeze film and slider bearing in their study using Neuringer-Rosensweig's model. Recently Ahmad *et. al*. [2] studied "MF lubrication of porous-pivoted slider bearing with slip velocity" and they have ignored the term

$$\rho\alpha^2\nabla\times\left(\frac{\mathbf{M}}{M}\times\mathbf{M}^*\right),$$

where

$$\mathbf{M}^*=\frac{D\mathbf{M}}{Dt}+\frac{1}{2}(\nabla\times\mathbf{q})\times\mathbf{M},$$

in the governing system of equations. In this study we have recapitulated the above problem [2] including the ignored term which is given by Jenkin's [34] and worked on by Ram and Verma [71], Shah and Bhat [82] in their study from different viewpoint.

With the addition of the above term and under an oblique magnetic field, it is found that the dimensionless load carrying capacity can be improved substantially with and without squeeze effect. The study also includes the detail about the effects of squeeze velocity and sliding velocity. It is observed that dimensionless load carrying capacity increases when squeeze velocity increases and sliding velocity decreases.

7.2 The Mathematical Analysis

The configuration of the porous-pivoted slider bearing with squeeze velocity is displayed in Figure 7.1 consists of a slider having a convex pad surface (or plate or disc) of length A with central thickness H_c and moving with uniform velocity U in the x- direction. The stator has a porous matrix (or region or layer) with uniform thickness l_2 backed by a solid wall. The porous flat lower plate is normally approached by the upper plate with a uniform velocity $\dot{h}=dh/dt$, where h is the central film thickness and t is time. The expression for the central film thickness h is given by [2, 82]

$$h = H_c\left\{4\left(\frac{x}{A}-\frac{1}{2}\right)^2-1\right\}+h_1\left\{a^*-\frac{a^*}{A}x+\frac{x}{A}\right\};\ \ a^*=\frac{h_2}{h_1},$$

… (7.1)

where h_1 and h_2 are minimum and maximum film thickness respectively, and x is the axial coordinate.

The above bearing is lubricated with water based ferrofluid (FF) and the equations governing the flow of FF by Jenkin's model [2, 34, 82] are

$$\rho\left\{\frac{\partial\mathbf{q}}{\partial t}+(\mathbf{q}\cdot\nabla)\mathbf{q}\right\}=-\nabla p+\eta\nabla^2\mathbf{q}+\mu_0(\mathbf{M}\cdot\nabla)\mathbf{H}+\rho\alpha^2\nabla\times\left(\frac{\mathbf{M}}{M}\times\mathbf{M}^*\right),$$

… (7.2)

$$\nabla\cdot\mathbf{q}=0,$$

… (7.3)

$$\nabla\times\mathbf{H}=0,$$

… (7.4)

$$\nabla\cdot(\mathbf{H}+4\pi\mathbf{M})=0,$$

… (7.5)

$$\gamma\frac{D^2\mathbf{M}}{Dt^2}=\ -4\pi\rho\frac{M_s}{\bar{\mu}_0}\frac{\mathbf{M}}{M_s-M}-\frac{2\alpha^2}{M}\mathbf{M}^*+\mathbf{H},$$

… (7.6)

with

$$\mathbf{M}^*=\frac{D\mathbf{M}}{Dt}+\frac{1}{2}(\nabla\times\mathbf{q})\times\mathbf{M}.$$

… (7.7)

where ρ is fluid density, p is film fluid pressure, η is fluid viscosity, $\mathbf{q}$ is fluid velocity, μ_0 is free space permeability, $\mathbf{M}$ is the magnetization vector, $\mathbf{H}$ is magnetic field vector, M is the magnitude of magnetization vector, $\mathbf{M}^*$ is corotational derivative of $\mathbf{M}$, α^2 is material constant, $\bar{\mu}_0$ is initial susceptibility of fluid, M_s is the saturation magnetization and γ is another material constant of Jenkin's model.

In the present discussion, equation (7.6) is replaced by

$$\mathbf{M} = \bar{\mu}\mathbf{H} \text{ (} \bar{\mu} \text{ is magnetic susceptibility),}$$

… (7.8)

as suggested by Maugin [44] and

$$\mathbf{M}^* = \frac{1}{2}(\nabla \times \mathbf{q}) \times \mathbf{M}.$$

… (7.9)

The lubricant is FF, so a magnetic field vector **H** is applied such that it vanishes at the ends of the bearing. The magnitude H of magnetic field is given by

$$H^2 = Kx(A - x),$$

… (7.10)

where K being a quantity chosen to suit the dimensions of both sides of equation (7.10).

The equation of continuity in the film region is

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,$$

… (7.11)

where u and w are components of film fluid velocity in x- direction and z- direction respectively, and z is the axial coordinate.

The equation of continuity in porous region is

$$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{w}}{\partial z}=0,$$

… (7.12)

where $\bar{u}$ and $\bar{w}$ are components of fluid velocity in the porous region in x- direction and z- direction respectively.

Referring the work of Agrawal [1] and Shah *et*. *al*. [82], using equations (7.2) to (7.9), one obtains

$$\frac{\partial^2 u}{\partial z^2}=\frac{1}{\eta\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\frac{\partial}{\partial x}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right).$$

… (7.13)

The velocity components of fluid in the porous region are

$$\bar{u}=-\frac{\varphi}{\eta}\left\{\frac{\partial}{\partial x}\left(P-\frac{1}{2}\mu_0\bar{\mu}H^2\right)+\frac{\rho\alpha^2}{2}\bar{\mu}\frac{\partial}{\partial z}\left(H\frac{\partial u}{\partial z}\right)\right\},$$

… (7.14)

$$\bar{w}=-\frac{\varphi}{\eta}\left\{\frac{\partial}{\partial z}\left(P-\frac{1}{2}\mu_0\bar{\mu}H^2\right)-\frac{\rho\alpha^2}{2}\bar{\mu}\frac{\partial}{\partial x}\left(H\frac{\partial u}{\partial z}\right)\right\},$$

… (7.15)

where φ and P are permeability and fluid pressure in the porous region respectively.

Substituting equations (7.14) and (7.15) into equation (7.12), one obtains

$$\frac{\partial^2}{\partial x^2}\left(P-\frac{1}{2}\mu_0\bar{\mu}H^2\right)+\frac{\partial^2}{\partial z^2}\left(P-\frac{1}{2}\mu_0\bar{\mu}H^2\right)=0,$$

… (7.16)

which on integration with respect to z across the porous region $(-l_2,0)$, yields

$$\frac{\partial}{\partial z}\left(P-\frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=0}=-l_2\frac{\partial^2}{\partial x^2}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right),$$

… (7.17)

using Morgan-Cameron approximation [67, 81, 82] and that the surface $z=-l_2$ is non-porous.

The relevant boundary conditions for the velocity field [7] in the lubricant region is

$$u=\frac{1}{s}\frac{\partial u}{\partial z} \quad \text{at } z=0,$$

… (7.18)

and

$$u=U \quad \text{at } z=h,$$

… (7.19)

where $\frac{1}{s}=\frac{\sqrt{\varphi}}{k}$; s is slip parameter and k is slip coefficient, which depends on the structure of the porous material.

Solving equation (7.13) with boundary conditions (7.18) and (7.19), one obtains

$$u=\frac{1}{2\eta\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\left\{z^2-\frac{sh^2z}{(sh+1)}-\frac{h^2}{(sh+1)}\right\}\frac{\partial}{\partial x}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right)+\frac{U(sz+1)}{(sh+1)}.$$

… (7.20)

Integrating continuity equation (7.11) in film region over $(0,h)$, one obtains

$$\frac{\partial}{\partial x}\int_0^h u\,dz+w_h-w_0=0.$$

… (7.21)

Using $w_h = V = -\dot{h}$ because of squeeze velocity is in the downward direction and $w_0 = w|_{z=0} = \overline{w}|_{z=0}$ because of continuity of velocity component at $z = 0$ of film region and porous region respectively, equations (7.17), (7.20), (7.21), gives

$$\frac{d}{dx}\left\{g^* \frac{d}{dx}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\right\} = \frac{df^*}{dx},$$

… (7.22)

where

$$g^* = 12\varphi l_2 + \frac{h^3(sh+4) - \left(\frac{3\rho\alpha^2\bar{\mu}\varphi s h^2 H}{\eta}\right)}{(sh+1)\left(1 - \frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)},$$

$$f^* = \frac{6\eta U h(sh+2) - 6U\rho\alpha^2\bar{\mu}\varphi s H}{(sh+1)} + 12\eta V x\,.$$

Equation (7.22) is known as Reynolds's type equation.

Introducing following dimensionless quantities

$$X = \frac{x}{A},\ \ \bar{h} = \frac{h}{h_1},\ \ \bar{s} = sh_1,\ \ \bar{p} = \frac{ph_1^2}{\eta AU},\ \ \mu^* = \frac{\mu_0\bar{\mu}h_1^2 AK}{\eta U},$$

$$\beta^2 = \frac{\rho\alpha^2\bar{\mu}A\sqrt{K}}{2\eta},\ \ \bar{\varphi} = \frac{12\varphi l_2}{h_1^3},\ \ S = -\frac{2VA}{Uh_1},\ \ \gamma^* = \frac{6\varphi}{h_1^2},$$

the dimensionless form of equation (7.22) is

$$\frac{d}{dX}\left\{G^* \frac{d}{dX}\left[\bar{p} - \frac{1}{2}\mu^* X(1-X)\right]\right\} = \frac{dE^*}{dX},$$

… (7.23)

where

$$G^* = \bar{\varphi} + \frac{\bar{h}^3\left(\bar{s}\bar{h}+4\right) - \beta^2\gamma^*\bar{s}\bar{h}^2\sqrt{X(1-X)}}{\left(\bar{s}\bar{h}+1\right)\left[1 - \beta^2\sqrt{X(1-X)}\right]},$$

$$E^* = \frac{6\bar{h}(\bar{s}\bar{h}+2) - 2\beta^2\gamma^*\bar{s}\sqrt{X(1-X)}}{(\bar{s}\bar{h}+1)} - 6SX,$$

which is known as dimensionless form of Reynolds's type equation.

7.3 Solution

Solving equation (7.23) for pressure under the appropriate boundary conditions

$$\bar{p} = 0 \text{ at } X = 0,\ 1,$$

yields

$$\bar{p} = \frac{1}{2}\mu^* X(1-X) + \int_0^X \left(\frac{E^* - Q}{G^*}\right) dX\,,$$

… (7.24)

where

$$Q = \int_0^1 \left(\frac{E^*}{G^*}\right) dX \Big/ \int_0^1 \left(\frac{1}{G^*}\right) dX.$$

The dimensionless form of equation (7.1) is

$$\bar{h} = 4\delta X^2 - (4\delta + a^* - 1)X + a^*,$$

… (7.25)

where

$$\delta = \frac{H_c}{h_1}.$$

… (7.26)

The dimensionless form of load carrying capacity using (7.24) can be obtained as

$$\overline{W} = \int_0^1 \bar{p}\, dX = \frac{\mu^*}{12} - \int_0^1 \left(\frac{E^* - Q}{G^*}\right) X\, dX.$$

… (7.27)

7.4 Results and Discussion

The problem on "MF lubrication of porous-pivoted slider bearing with slip velocity by [2]" is recapitulated here for its optimum performance.

During the course of investigation it is observed from equation (7.13) that a uniform magnetic field does not enhance the bearing characteristics in Rosensweig's model of FF flow.

The values of the dimensionless load carrying capacity $\overline{W}$ has been calculated for the following values [37] of the parameters using Simpson's 1/3 rule with step size 0.1.

$$h_1 = 0.000005(m),\ \ h_2 = 0.00001(m),\ \ \bar{\mu} = 0.05,$$
$$A = 0.02(m),\ \ \eta = 0.012(Ns/m^2),\ \ \mu_0 = 4\pi \times 10^{-7}(N/A^2),$$
$$H_c = 0.0000015(m),\ \ l_2 = 0.0001(m),\ \ \rho = 1400(Ns^2/m^4),\ \ k = 0.1.$$

The FF used here is water based. The magnetic field considered here is oblique to the stator and its strength is of $O(10^3)$ in order to get maximum magnetic field at $x = A/2$ for the calculation of $\overline{W}$ in Figure 7.10. For remaining Figures, magnetic field strength is indicated there.

The calculation of magnetic field strength is shown below [77]:

From equation (7.10),

$$H^2 = Kx(A - x)$$
$$\text{Max } H^2 = 10^{-4}K,$$
$$\text{For } H = O(10^3), K = O(10^{10})$$

The calculated values of $\overline{W}$ are presented graphically as shown in Figures 7.2 to 7.10 for various cases.

Figures 7.2 and 7.3 indicates the study of the effect of squeeze velocity ($\dot{h} \neq 0$) when $\alpha^2 \neq 0$ (Jenkin's model) and $\alpha^2 = 0$ (Rosensweig's model) respectively with respect to order of magnetic field strength (H is obtained from K as per above calculation).

From Figure 7.2, it is observed that, for $\alpha^2 \neq 0$, $\bar{W}$ increases considerably in the presence of squeeze velocity. Also, as K increases (that is, as order magnetic field strength increases), $\bar{W}$ increases. From Figure 7.3, it is observed that, for $\alpha^2 = 0$, again $\bar{W}$ increases considerably in the presence of squeeze velocity, but it does not affect much when the order of magnetic field strength increases.

Figures 7.4 and 7.5 shows the comparative study of Jenkin's model and Rosensweig's model when $\dot{h} \neq 0$ and $\dot{h} = 0$ respectively with respect to order of magnetic field strength.

From Figure 7.4 it is observed that when $\dot{h} \neq 0$; that is, when squeeze velocity is present, $\bar{W}$ increases considerably in the case of $\alpha^2 \neq 0$. Also, $\bar{W}$ has an increasing behavior with the increase of order of magnetic field strength. Whereas the behavior of $\bar{W}$ is consistent with respect to increase of order of magnetic field strength for $\alpha^2 = 0$. The same behavior of $\bar{W}$ can be observed from Figure 7.5 when $\dot{h} = 0$, that is, when there is no squeeze velocity.

Figures 7.6 and 7.7 shows the study of effect of squeeze velocity ($\dot{h} \neq 0$) when $\alpha^2 \neq 0$ (Jenkin's model) and $\alpha^2 = 0$ (Rosensweig's model) respectively with respect to permeability φ of the porous medium. From both the Figures it is observed that, $\bar{W}$ increases with the decrease of permeability φ. Also, when $\dot{h} \neq 0$, $\bar{W}$ is increases more as compared to $\dot{h} = 0$.

Figures 7.8 and 7.9 shows the comparative study of Jenkin's model and Rosensweig's model when $\dot{h} \neq 0$ and $\dot{h} = 0$ respectively with respect to permeability φ. From both the Figures it is observed that, $\bar{W}$ increases with the decrease of permeability φ.

Figure 7.10 displays values of $\bar{W}$ for various values of $\dot{h}$ and U for $\alpha^2 \neq 0$, and from it the following observations can be made:

(1) $\bar{W}$ increases with the increases of $\dot{h}$.

(2) $\bar{W}$ increases with the dereases of U.

From Figures 7.2 to 7.9, it is observed that the values of $\overline{W}$ increases substantially in the case of Jenkin's model; that is, with the consideration of the ignored term of Ahmed *et. al.* [2] as $\rho\alpha^2\nabla\times\left(\frac{\mathbf{M}}{M}\times\mathbf{M}^*\right)$ with $\mathbf{M}^*=\frac{1}{2}(\nabla\times\mathbf{q})\times\mathbf{M}$ and $\mathbf{M}=\bar{\mu}\mathbf{H}$ for $\dot{h}=0$ and $\dot{h}\neq 0$ rather than Rosensweig's case (Ahmed *et. al*. [2] for $\dot{h}=0$).

7.5 Conclusion

The problem on "MF lubrication of porous-pivoted slider bearing with slip velocity by [2]" is recapitulated here for its optimum performance with the inclusion of the ignored term $\rho\alpha^2\nabla\times\left(\frac{\mathbf{M}}{M}\times\mathbf{M}^*\right)$ with $\mathbf{M}^*=\frac{1}{2}(\nabla\times\mathbf{q})\times\mathbf{M}$ and $\mathbf{M}=\bar{\mu}\mathbf{H}$. The FF used here is water based and magnetic field strength considered is of as shown in Figures in order to get maximum magnetic field at $x=A/2$.

The design of the pivoted slider bearing can be made with the considerations of the following observations:

Under an oblique magnetic field to the stator, the dimensionless load carrying capacity can be improved substantially by considering following features:

1. FF flow behavior given by Jenkin's model.
2. Presence of the squeeze velocity.
3. Smaller values of permeability parameter φ.
4. Increasing values of H^2 up to $O(10^5)$ as per [98].

It should be noted from equation (7.13) that a uniform magnetic field does not enhance the bearing characteristics in Rosensweig's model of FF flow.

7.6 Figures

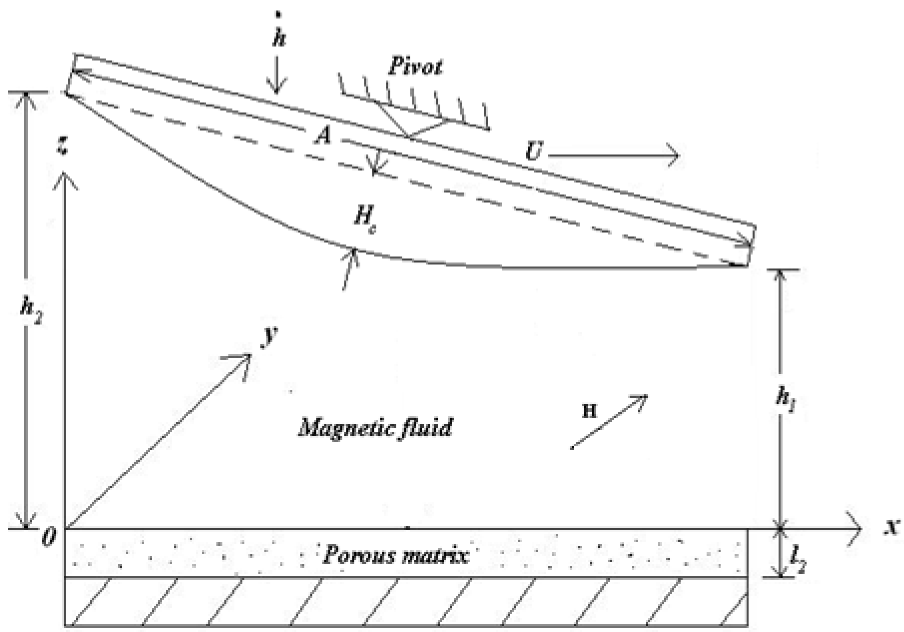

Figure 7.1 Porous-pivoted slider bearing with a convex pad surface

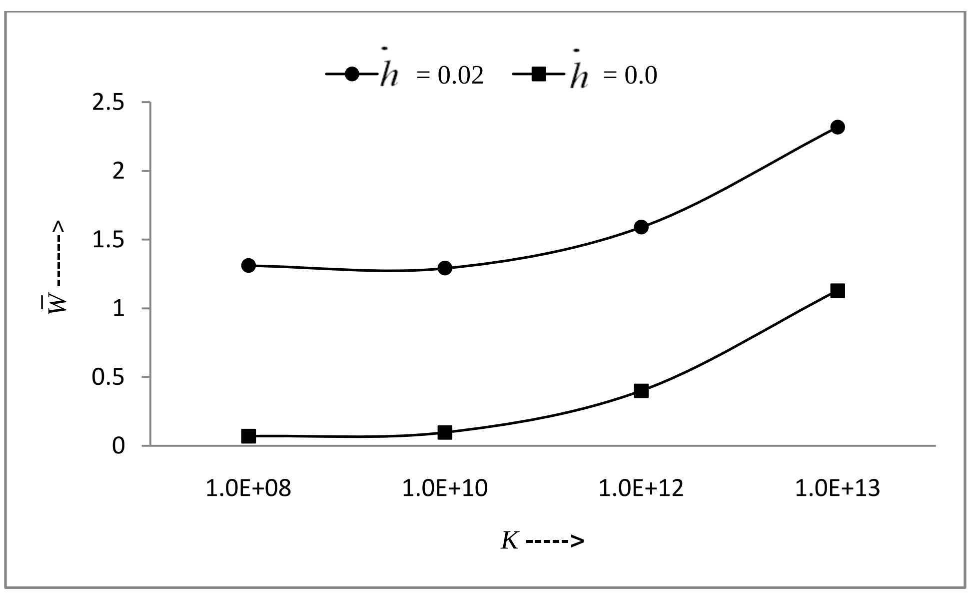

Figure 7.2 Values of $\bar{W}$ for various values of K when $\alpha^2 = 0.0001, \varphi = 10^{-12}$ and $U = 6.28$

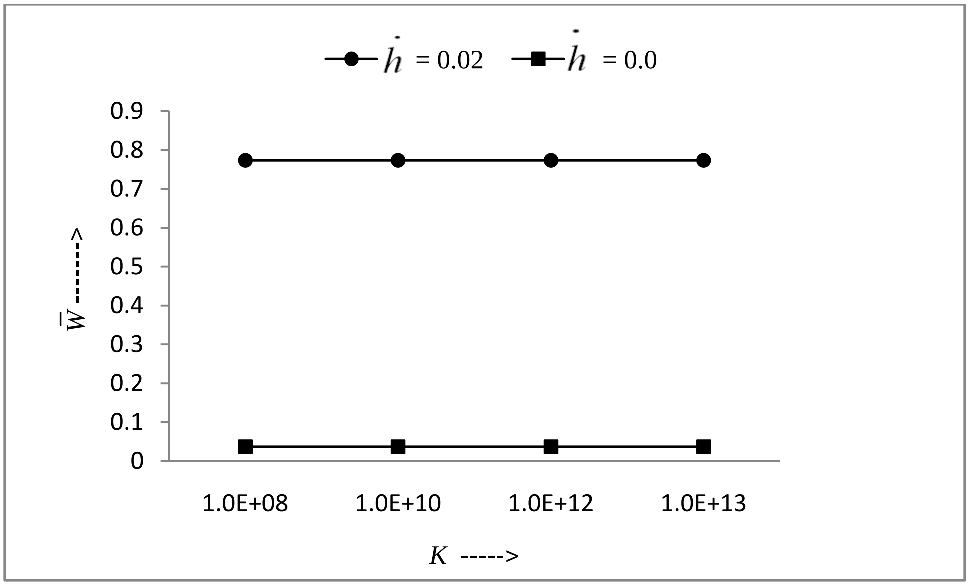

Figure 7.3 Values of $\bar{W}$ for various values of K when $\alpha^2 = 0.0$, $\varphi = 10^{-12}$ and $U = 6.28$

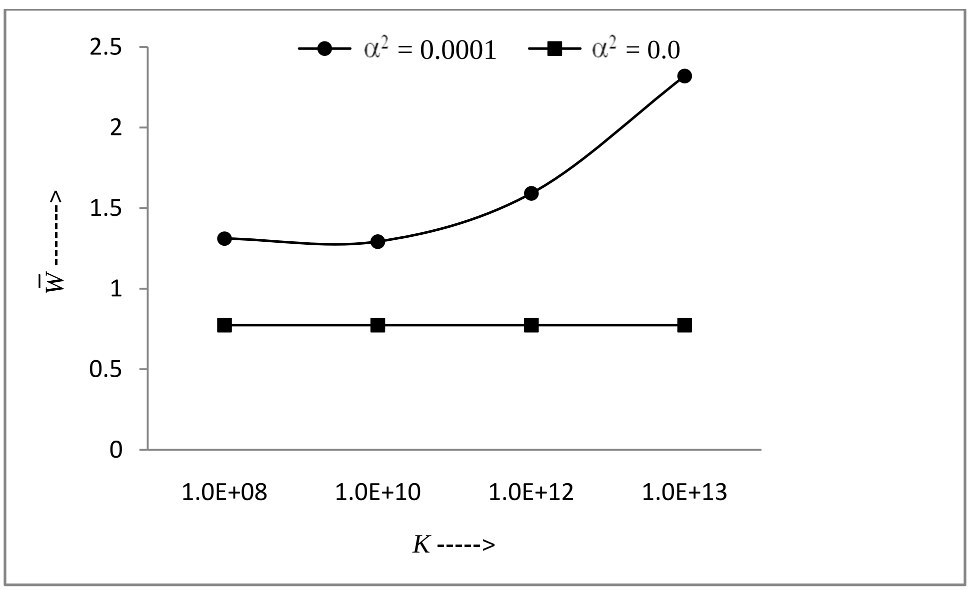

Figure 7.4 Values of $\bar{W}$ for various values of K when $\dot{h} = 0.02$, $\varphi = 10^{-12}$ and $U = 6.28$

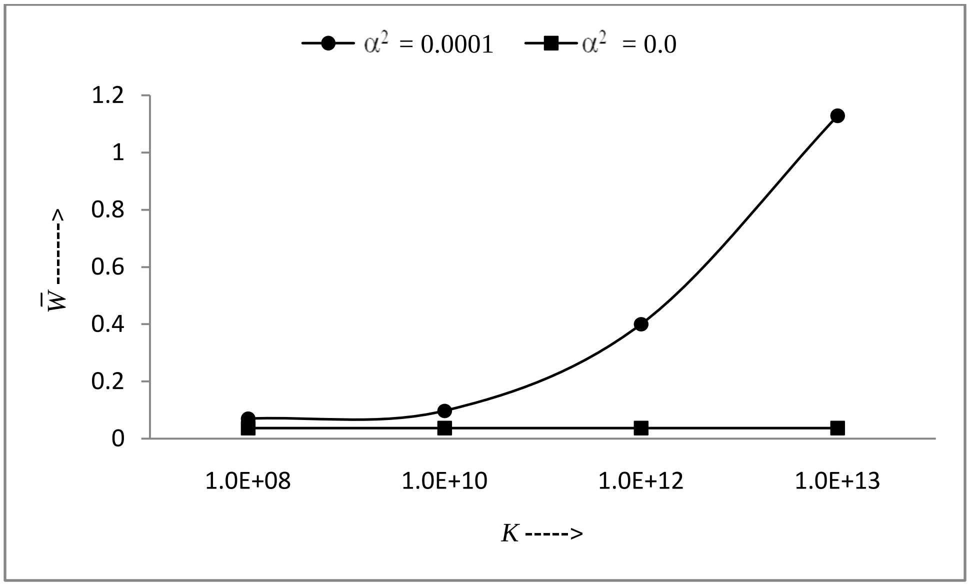

Figure 7.5 Values of $\bar{W}$ for various values of K when $\dot{h} = 0.0$, $\varphi = 10^{-12}$ and $U = 6.28$

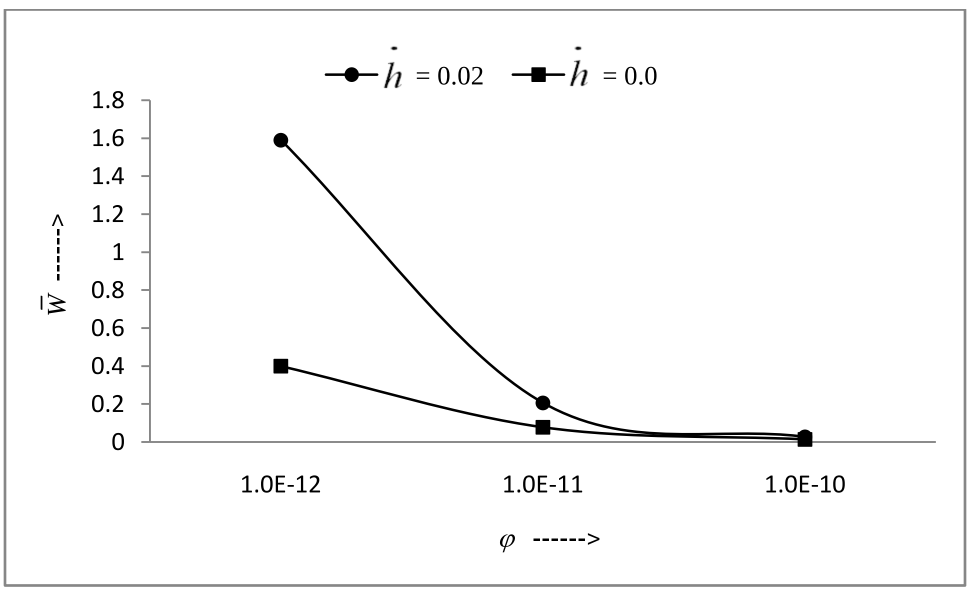

Figure 7.6 Values of $\bar{W}$ for various values of φ when $\alpha^2 = 0.0001$, $K = 10^{12}$ and $U = 6.28$

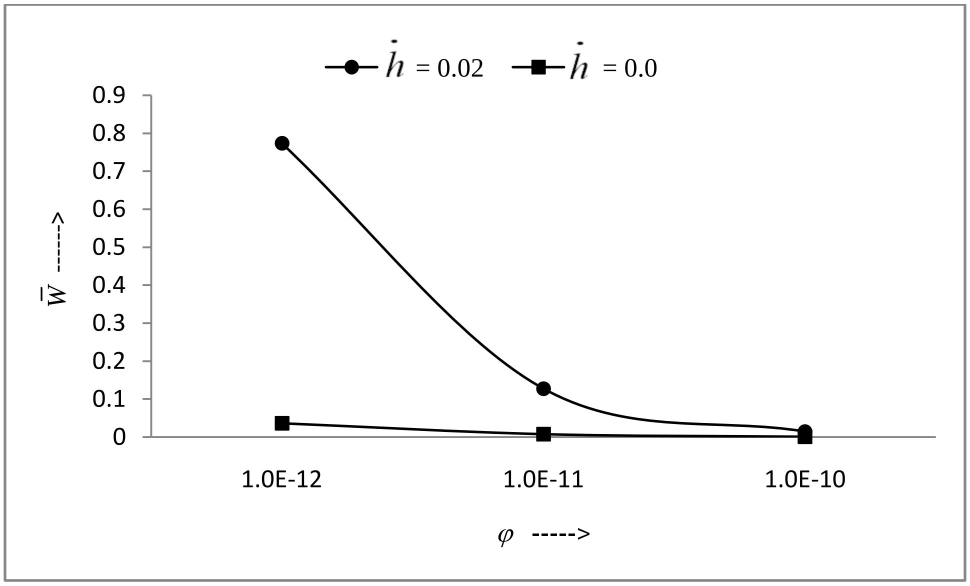

Figure 7.7 Values of $\overline{W}$ for various values of φ when $\alpha^2 = 0.0$, $K = 10^{12}$ and $U = 6.28$

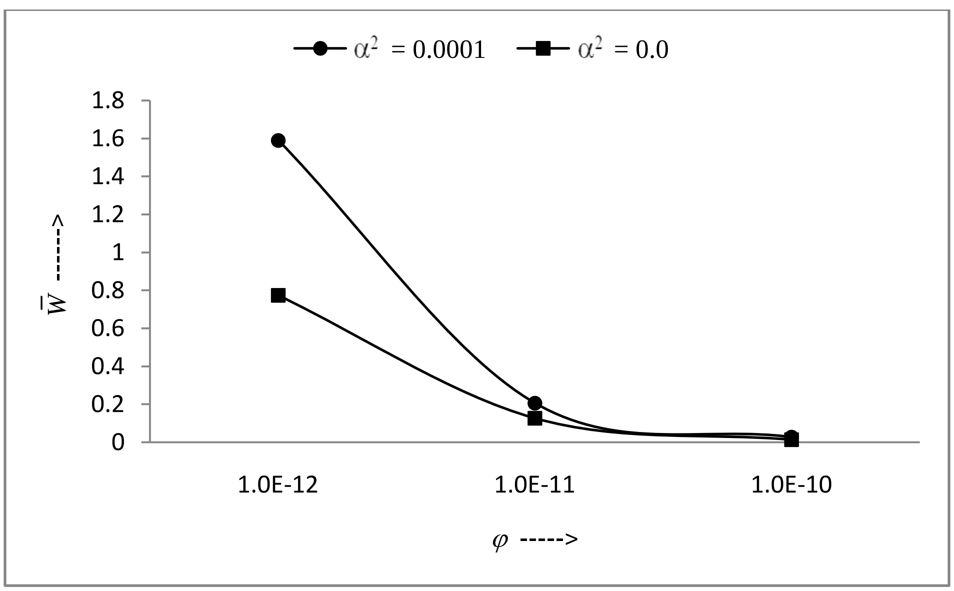

Figure 7.8 Values of $\overline{W}$ for various values of φ when $\dot{h} = 0.02$, $K = 10^{12}$ and $U = 6.28$

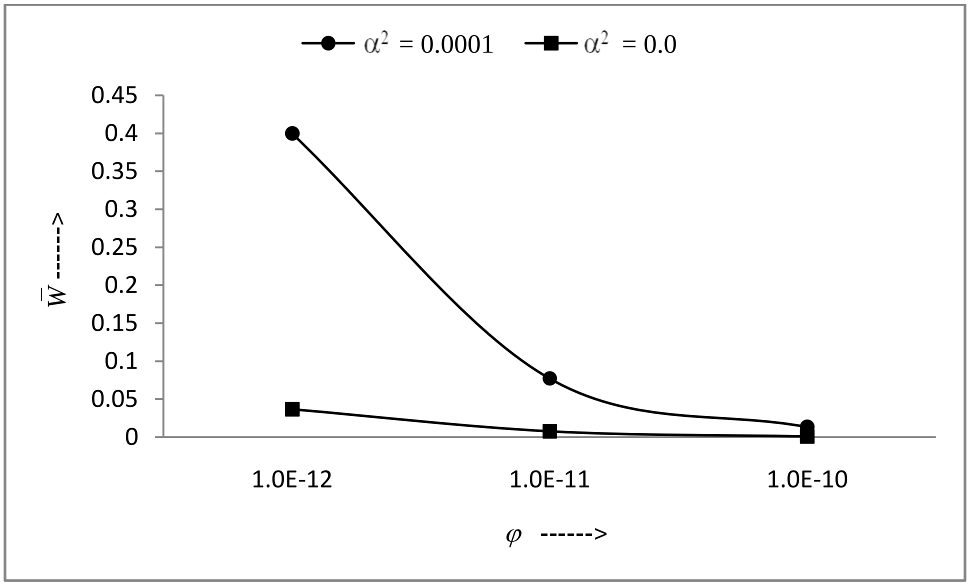

Figure 7.9 Values of $\bar{W}$ for various values of φ when $\dot{h} = 0.0$, $K = 10^{12}$ and $U = 6.28$

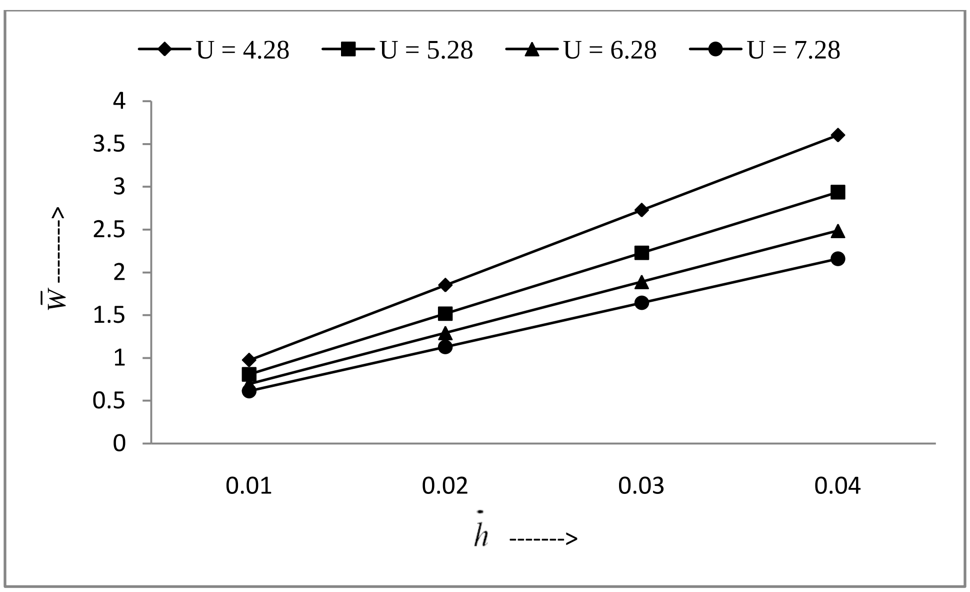

Figure 7.10 Values of $\bar{W}$ for various values of $\dot{h}$ and U when $K = 10^{10}$, $\varphi = 10^{-12}$ and $\alpha^2 = 0.0001$