Chapter 2

Physico-Mathematical Background

Contents

This chapter deals with the Physico-Mathematical background which is necessary to understand the subsequent chapters. The materials are taken from various sources like [3, 4, 12, 19, 27, 28, 33, 35, 43, 51, 60, 63, 65, 73, 104, 106].

2.1 Basic Definitions

Definition 2.1.1 ***Fluid*** Fluid can be defined as a substance which is capable of flowing and which deforms continuously as long as the shearing force is applied. This continuous deformation under the action of forces compels the fluid to flow and this tendency of fluid is called ***fluidity***.

Definition 2.1.2 ***Surface Tension*** It is defined as the tensile force acting on the surface of a liquid when it in contact with a gas, or on the surface between two immiscible liquids. In this case the contact surface behaves like a membrane. The unit of surface tension is N/m^2.

Definition 2.1.3 ***Viscosity*** Viscosity is that property of a fluid by virtue of which it offers resistance to the movement of a one layer of a fluid over an adjacent layer. The unit of viscosity is Ns/m^2.

Definition 2.1.4 ***Density*** The density of a fluid is defined as mass per unit volume. It is generally denoted by ρ. The unit of density is kg/m^3.

Definition 2.1.5 ***Compressible Fluid*** Compressible fluid is that type of fluid in which the density of the fluid changes from point to point or in other words the density is not constant for the fluid.

Definition 2.1.6 ***Incompressible Fluid*** Incompressible fluid is that type of fluid in which the density of the fluid is constant.

Definition 2.1.7 ***Ideal Fluid*** **(*Perfect Fluid*)** Ideal fluids are those fluids which have no viscosity, surface tension and are incompressible. In nature, ideal fluid does not exit.

Definition 2.1.8 ***Real Fluid*** **(*Practical Fluid*)** These fluids are actually available in nature. The fluids possesses the property like viscosity, surface tension and compressibility.

Definition 2.1.9 ***Newtonian Fluid*** The Newtonian fluid is the fluid in which the shear stress is directly proportional to the rate of shear strain or velocity gradient. For example, Glycerin, light-hydrocarbon oils, silicone oils, air, gases, etc.

Definition 2.1.10 ***Non-Newtonian Fluid*** The non-Newtonian fluid is the fluid in which the shear stress is not directly proportional to the rate of shear strain or velocity gradient. For example, slurries, tooth paste, gel, etc.

Definition 2.1.11 ***Porous Medium*** It is referred as a solid body with pores (void spaces) in it.

Definition 2.1.12 ***Permeability*** Permeability is that property of a porous material which characterizes the ease with a fluid may be made to flow through the material by an applied pressure gradient. The value of the permeability is determined by the structure of the porous material. The unit of permeability is m^2.

Definition 2.1.13 ***Porosity*** Porosity is measure of the void spaces in a material, and is defined as fraction of the volume of void spaces over the total volume of the material.

Definition 2.1.14 ***Magnetic Dipole*** The magnetic dipoles, generally known as north and south poles, commonly exist in magnetic materials. The magnetic dipoles are not separate poles unlike an electric dipole.

Definition 2.1.15 ***Magnetic Field Strength*** The magnetic field strength H at any point in a magnetic field is the force experienced by unit north pole placed at that point. Its unit is A/m.

Definition 2.1.16 ***Magnetic Induction* (*Flux Density*)** The magnetic induction (flux density) B in any material is defined as number of lines of force through a unit area of cross-section perpendicularly. Its unit is kg/s^2A.

Definition 2.1.17 ***Magnetic Dipole Moment*** The magnetic dipole moment is equal to the product of the magnetic pole strength and the length of the magnet.

Definition 2.1.18 ***Magnetization* (*Intensity of Magnetization*)** Magnetization M is the measure of magnetism of magnetic materials and is defined as the magnetic moment per unit volume. Its unit is A/m.

Definition 2.1.19 ***Magnetic Susceptibility*** Magnetic susceptibility is used to explain the magnetization of material. It is defined as the ratio of magnetization to the magnetic field strength. That is,

$$\text{magnetic susceptibility } \bar{\mu} = \frac{M}{H}.$$

Definition 2.1.20 ***Magnetic Permeability*** The magnetic permeability μ is defined as the ratio of amount of magnetic density *B* to the applied magnetic field intensity *H*. It is used to measure the magnetic lines of forces penetrating through a material. That is,

$$\text{magnetic permeability } \mu = \frac{B}{H}.$$

2.2 Definitions of Various Flows

Definition 2.2.1 ***Uniform Flow*** If the flow velocity having the same magnitude and direction at every point in the fluid flow, then the flow is said to be uniform.

Definition 2.2.2 ***Non-uniform Flow*** If at a given instant, the velocity is not the same at every point in the fluid flow, then the flow is said to be non-uniform.

Definition 2.2.3 ***Steady Flow*** A steady flow is that flow in which the characteristics like velocity, pressure, etc. are independent of time.

Definition 2.2.4 ***Unsteady Flow*** An unsteady flow is that flow in which the characteristics like velocity, pressure, etc. are dependent on time.

Definition 2.2.5 ***Rotational Flow*** The flow in which the fluid particles rotate about their own axis is called rotational flow.

Definition 2.2.6 ***Irrotational Flow*** The flow in which the fluid particles does not rotate about its own axis is called irrotational flow.

Definition 2.2.7 ***Laminar Flow*** A flow in which every fluid particle traces out a definite curve and the curves traced out by any two different particles never intersect is called laminar flow. It is a layer wise flow.

Definition 2.2.8 ***Turbulent Flow*** A flow in which every fluid particle does not trace out a definite curve and the curves traced out by the particles intersect is called turbulent flow.

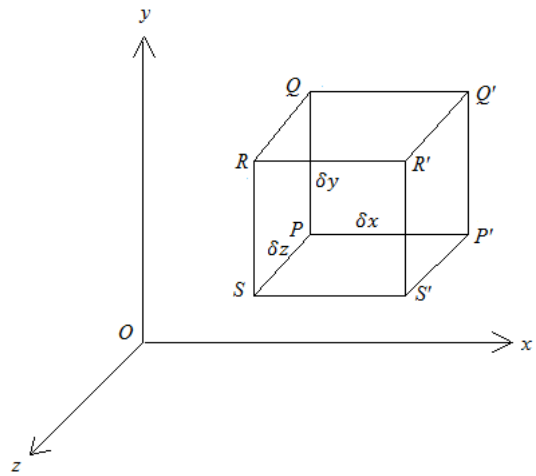
2.3 Brief Introduction of Fluid Mechanics

Fluid mechanics is that branch of science which deals with the behaviour of the fluid (liquids or gases) at rest as well as in motion. It deals with static, kinematic and dynamic aspects of fluids. The study of fluids at rest is called ***fluid static***. The study of fluids in motion where pressure forces are not considered, is called ***fluid kinematics*** and if the pressure forces are considered, then it is called ***fluid dynamics***. Thus, fluid dynamics is the sub-discipline of fluid mechanics that deals with the fluid flow. Fluid dynamics deals with fluid behaviour in motion under the influence of body as well as surface forces. Hydrodynamics is a subject dealing with the motion of incompressible fluids. Hydrodynamics is originated in the middle of 18^{th} century when Euler discovered the equation of motion for an inviscid fluid. Later, Navier and Stokes gave the equation of motion for viscous fluids. The equation for turbulent motion was discovered by Reynolds and the well known boundary layer theory was introduced by Prandtl. Besides, many other scientists and mathematicians including Benard, Kelvin, Taylor, Karman etc. gave their excellent contributions to this subject. On the basis of various fields and fluid interactions, hydrodynamics may be classified as

1. Electrohydrodynamics (EHD)
2. Magnetohydrodynamics (MHD)
3. Ferrohydrodynamics (FHD)

Electrohydrodynamics (EHD) is the branch of science dealing with the motion of fluids with electric force effects. Magnetohydrodynamics (MHD) deals with the motion of electrically conducting fluids in the presence of magnetic field. Ferrohydrodynamics (FHD) came into existence in the mid of sixties after the invention of ferrofluid.

- **Equation of Continuity in Cartesian Coordinates** This equation is based on the principle of conservation of mass of fluids. Consider a point P in a flowing fluid at which fluid velocity is given by $\mathbf{q}(x, y, z, t) = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$, and density $\rho = \rho(x, y, z, t)$, where **i**, **j**, **k** are unit vectors along the coordinate directions of Cartesian axes *Oxyz*, and t is time.



Construct a rectangular parallelepiped $PQRSP'Q'R'S'$, with edges parallel to the coordinate axes and of lengths δx, δy, δz respectively.

If $\delta x \delta y \delta z$ are infinitestimals, the rate at which fluid enters the parallelepiped through the face *PQRS* is given by

$$\text{area of } PQRS \times \text{velocity in } x\text{- direction} \times \text{density} = \delta y \delta z \, u\rho.$$

The rate at which fluid leaves the parallelepiped through the face $P'Q'R'S'$ is given by

$$\delta y \delta z \, u\rho + \frac{\partial}{\partial x}(\delta y \delta z \, u\rho)\delta x.$$

The rate at which fluid accumulates due to flow crosses the faces *PQRS* and $P'Q'R'S'$ is given by

$$\delta y \delta z \, u\rho - \left\{\delta y \delta z \, u\rho + \frac{\partial}{\partial x}(\delta y \delta z \, u\rho)\delta x\right\} = -\frac{\partial}{\partial x}(\delta y \delta z \, u\rho)\delta x$$

$$= -\frac{\partial}{\partial x}(u\rho)\delta x \delta y \delta z.$$

Similarly, the net inflow rate through the face $PP'S'S$ and $QQ'R'R$ is given by

$$-\frac{\partial}{\partial y}(\, v\rho)\delta x \delta y \delta z,$$

and from the other two faces, the net inflow is

$$-\frac{\partial}{\partial z}(w\rho)\delta x\delta y\delta z.$$

Therefore, the rate at which fluid is flowing into the parallelepiped through the six faces is

$$-\left\{\frac{\partial}{\partial x}(u\rho)+\frac{\partial}{\partial y}(v\rho)+\frac{\partial}{\partial z}(w\rho)\right\}\delta x\delta y\delta z.$$

… (2.1)

The amount of fluid within the parallelepiped is $\rho\delta x\delta y\delta z$. The rate of increase of amount of fluid within the parallelepiped per unit time is

$$\frac{\partial}{\partial t}(\rho\delta x\delta y\delta z).$$

… (2.2)

Equating equation (2.1) and equation (2.2), we get

$$\left\{\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(u\rho)+\frac{\partial}{\partial y}(v\rho)+\frac{\partial}{\partial z}(w\rho)\right\}\delta x\delta y\delta z=0.$$

Dividing by $\delta x\delta y\delta z$, in the limit as $\delta x\delta y\delta z\to 0$, we get the equation of continuity at the point P as

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(u\rho)+\frac{\partial}{\partial y}(v\rho)+\frac{\partial}{\partial z}(w\rho)=0.$$

… (2.3)

If the fluid flow is steady, the equation of continuity becomes

$$\frac{\partial}{\partial x}(u\rho)+\frac{\partial}{\partial y}(v\rho)+\frac{\partial}{\partial z}(w\rho)=0.$$

… (2.4)

If the fluid is incompressible, so that the mass density ρ does not changes with x, y, z and t. Therefore, the equation of continuity becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0.$$

… (2.5)

The vector form of equation (2.3) can be written as

$$\frac{\partial \rho}{\partial t}+\boldsymbol{\nabla}\cdot(\rho\mathbf{q})=0.$$

… (2.6)

Equation (2.3) is most general form of equation of continuity in Cartesian coordinates which is applicable for steady as well as unsteady flow, uniform as well as non-uniform flow, and compressible as well as incompressible fluids.

The equation of continuity for an incompressible flow presented in cylindrical form as

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{\partial w}{\partial z}=0.$$

… (2.7)

where r, θ, z are cylindrical polar coordinates.

➢ **Momentum Equation (Navier-Stokes Equation)** It is based on the principle of conservation of momentum. The following Navier-Stokes equations can be derived by considering forces acting on small fluid element having the shape of a parallelepiped. By considering forces acting on the element as pressure forces, gravity force or body force and shear forces due to viscosity, then Navier-Stokes equations in Cartesian coordinates are given as

$$g_x-\frac{1}{\rho}\frac{\partial p}{\partial x}=\frac{du}{dt}-\frac{\eta}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right),$$

$$g_y-\frac{1}{\rho}\frac{\partial p}{\partial y}=\frac{dv}{dt}-\frac{\eta}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right),$$

$$g_z - \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{dw}{dt} - \frac{\eta}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right).$$

On addition, we get the resultant equation in Cartesian coordinates as

$$g_x + g_y + g_z - \frac{1}{\rho}\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} + \frac{\partial p}{\partial z}\right)$$
$$= \frac{d(u+v+w)}{dt} - \frac{\eta}{\rho}\left[\frac{\partial^2 (u+v+w)}{\partial x^2} + \frac{\partial^2 (u+v+w)}{\partial y^2} + \frac{\partial^2 (u+v+w)}{\partial z^2}\right],$$

… (2.8)

where p is the pressure force, g_x, g_y and g_z are components of gravitational force or body force in the x, y and z- directions, respectively.

The vector form of (2.8) can be written as

$$\rho\left[\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}\cdot\nabla)\mathbf{q}\right] = \mathbf{F} - \nabla p + \eta\,\nabla^2\mathbf{q},$$

that is,

$$\rho\frac{D\mathbf{q}}{Dt} = \mathbf{F} - \nabla p + \eta\,\nabla^2\mathbf{q},$$

… (2.9)

where $\mathbf{F}$ is resultant of body force or gravitational force of fluid mass and $\frac{D}{Dt}$ is the substantial derivative (convective or material). To be precise, the convention $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{q}\cdot\nabla$ is adopted.

In cylindrical polar coordinates, Navier-Stokes equation for an incompressible fluid can be written as

$$\frac{\partial \mathbf{q}}{\partial t} - (\mathbf{q}\times curl\mathbf{q}) = \mathbf{F} - \nabla\left(\frac{p}{\rho} + \frac{q^2}{2}\right) - \frac{\eta}{\rho}\nabla\times curl\mathbf{q},$$

… (2.10)

which is one of the base of our study. Where q is magnitude of fluid velocity $\mathbf{q}$.

➢ **Darcy's Law** The governing equation for fluid motion in a vertical porous column were first introduced by Darcy in 1856. Accordingly, the law is given by

$$V = -\frac{\varphi}{\eta}\nabla P,$$

where V is the space averaged velocity (Darcian velocity), φ is the permeability of the porous region, η is the coefficient of viscosity and P is the pressure in the porous region.

2.4 Various Types of Lubrication

A lubricant is any substance that will form a film between the rubbing surfaces and thereby preventing the actual contact of the surfaces. The lubricants keep the machine elements apart and allow them to move relatively with minimum efforts. It will minimize friction and wear between rubbing surfaces. It will carry away the heat generated due to frictional work converted. It will provide protection against corrosion. Such a system where lubricant is used is called ***lubricated system*** and the process of minimizing the friction and wear using lubricant is called ***lubrication***. This means the purpose of lubrication is to reduce friction, wear and heating of machine parts moving relative to each other.

The following are the classification of lubrication.

1. Hydrodynamic lubrication
2. Hydrostatic lubrication
3. Elastohydrodynamic lubrication
4. Boundary lubrication
5. Solid film lubrication

1. ***Hydrodynamic (or Fluid-film) Lubrication*** In this type, the surface of bearing are separated by a relatively thick film of lubricant to prevent metal to metal contact. The film pressure is created by the moving surface forcing the lubricant into a wedge shaped zone, therefore creating a pressure that separates the sliding surfaces.
2. ***Hydrostatic Lubrication*** In this type, the lubrication is forced into the bearing at a pressure high enough to separate the surfaces. This type of lubrication is helpful during the starts and

stops of machine when the relative motion of the surfaces, is not fast enough to generate a required pressure.

3. ***Elastohydrodynamic Lubrication*** Generally in a rolling element bearing, under the load, due to an elastic deformation, the line or point contact becomes a contact area. Elastohydrodynamic lubrication is concerned with the formation of a thin fluid film at the contact area of a rolling element. Under the favourable conditions of speed, load and fluid viscosity, the elastohydrodynamic fluid film can be of sufficient thickness to separate the rolling surfaces. Because of this there may have significant reduction of wear also.
4. ***Boundary Lubrication*** Boundary lubrication is a special case of hydrodynamic lubrication where a very thin film of lubrication is adequate. This type of lubrication can be consider whenever the load on a bearing is light and the shaft speed is low and wear is not a critical problem. Moreover, porous bearing material with self lubrication properties can be used for boundary lubrication. Sometimes in practice, in spite of full fluid film lubrication there may arise situation like boundary lubrication because of vibrations, disturbances and occasional fluid starvation.
5. ***Solid Film Lubrication*** This type of lubrication is used in a situation where a fluid lubrication is not desirable because there are certain industries like food and pharma where the risk of contamination of product by fluid lubrication is catastrophic. In such situation, solid or powder form of lubrication such as graphite, Teflon etc. are being used as lubricant.

2.5 Basic Assumptions of Lubrication Theory

The following are the basic assumptions of lubrication in our present research work.

1. The fluid is Newtonian and incompressible.
2. The fluid flow is laminar.
3. The gravity and inertia forces acting on the fluid can be ignored compared with the viscous force.
4. The film thickness is very small compared with the bearing geometry.
5. The viscosity of fluid is constant through the film thickness.
6. The bearing surfaces are assumed to be perfectly rigid so that elastic deformations of the bearing surfaces may be disregarded.

7. Bearing surfaces are assumed to be perfectly smooth.
8. Porous matrix of the bearing surface is assumed to be homogeneous.
9. Validity of the Darcy's law is assumed.
10. Temperature changes in the lubricant are neglected.

etc.

2.6 Different Type of Bearings

It can be said that the bearing is a system of machine elements whose function is to support a generated load due to machine operation by reducing friction between the relatively moving surfaces. The load may be radial, axial or combination of these. The bearing can be categorize according to the direction of the applied load. A radial or journal bearing supports a radial load. A thrust bearing, on the other hand, supports a thrust or an axial load whereas conical bearings support both radial and axial load.

A substance called lubricant is introduced in the clearance space of a bearing for its efficient functioning. Any substance which has some amount of viscosity is known as lubricant. Use of lubricants reduces the loss of energy caused by friction. In addition, heat generated by friction is also carried away by the lubricant and thus it works as a cooling agent.

Commonly used bearings are rolling element bearing, fluid film bearings, slider bearing. Rolling element bearings have much wider use in industries since rolling friction is lower than the sliding friction.

During machine operating conditions, if two mating surfaces are completely separated by lubricant film, such a type of lubrication is called ***fluid film lubrication***. Bearings operating under this kind of lubrication are fluid film bearings. As metal to metal contact is completely avoided by this system of lubrication, it is sometime known as perfect lubrication. Fluid film bearings are lubricated by the hydrodynamic flow of lubricant which is generated by relative surface motion or external pressurization. In a hydrostatic bearing also known as 'externally pressurized' bearing, the load is supported due to pressure in the fluid which is supplied from an external source. In addition to these two types, there is another class of fluid film bearing known as 'squeeze film' bearing, which supports a load because of oscillating relative normal motion.

Journal or shaft bearings which supports radial load may be divided into full journal bearings, where the contact angle of the bushing with the journal is 360^o, and the partial journal bearings, in which the contact angle is either 180^o or less. Full journal bearings are widely used in bearings industrial machinery. These bearings can take up rotating load. The partial journal bearings have limited applications and are used when the direction of radial load does not change.

To design a hydrodynamic bearing, a few important characteristics, like load carrying capacity, flow requirement and power loss due to viscous friction are to be predicated accurately. These parameters can be determined if the pressure within the bearing is known. To study and find this type of phenomena, one has to solve a particular form of the Navier-Stokes equations along with the mass continuity equation after making some basic assumptions. The governing equation thus formed is called the generalized ***Reynolds equation***. The basic momentum equation can also be set up from balance of the element of fluid in this lubricating film. The generalized Reynolds equation contains viscosity, density, film thickness, surface motion etc. as parameters.

- **Porous Bearings** These are special type bearings in which the bearing material possesses the property of self lubrication and requires no additional lubricant. Particularly, with the recent advancements in powder metallurgy such bearings have found all-round applicability and are practically indispensable for various applications in air craft and automotive accessories, domestic appliances, farm and construction machineries, printing textile and baking industries, etc. During operation lubricant is stored in these pores and feeds through interconnecting pores to the bearing surfaces. The lubricant which is forced from the loaded zone of the bearing is reabsorbed by capillary action. Since these bearings can operate for long periods of time without additional supply of lubricant, they are used in inaccessible or inconvenient places where relubrication or frequent maintenance is difficult. High porosity with a maximum amount of lubricant is used for high speed, light load applications such as fractional horse motor bearings. A low lubricant content low porosity material is satisfactory for oscillating and reciprocating motion. Bronze is used as most common porous bearing material. These bearings are wear resistant, ductile, and corrosion resistant with good embedability which gives them a wide range of applications from home appliances to domestic machinery. Copper-iron porous metal bearings are useful in applications involving shocks and heavy loads. Phenolics, acetals and nylons are widely used plastic materials. Phenolic have been replaced by wood bearings and metals in such applications as propellers, electrical switch gears, rolling mills and water turbine bearings. Acetals are used for non-expensive bearings in wide variety of automotive appliances and industrial applications. Synovial joints are the biological bearings provided by nature in the human body, play an important role during the body locomotion of a bone past another, supporting considerable load involved and providing low friction. The biological bearing uses a poroelastic bearing material known as articular cartilage.

➢ **Bearing Design Characteristics** The restrictions and environmental conditions imposed by the bearing system such as choice of lubricant, bearing material specifications, bearing life, cost, bearing alignment, positioning precision, direction and magnitude of loads, bearing ambient pressure, supply pressure, flow rate available from the system, heat flow etc. establish various requirements for the bearing design. Definitions of both, the range of imposed bearing requirements and bearing performance limitations, are required to assure the compatibility of the bearing and its design. To prepare a bearing that will reliably meet performance requirements with the specific maintenance is really a difficult task. Following parameters are required to design and analyze the bearings.

1. Lubricant flow in the bearing
2. Lubricant side leakage from the bearing
3. Pressure distribution in the film region
4. Load carrying capacity
5. Centre of pressure (or attitude angle in case of journal bearing)
6. Friction force (or coefficient of friction)
7. Film stiffness
8. Squeeze film versus film thickness relationship (in case of squeeze film bearing)
9. Range for stability of the bearing both for initial velocity disturbances and initial position disturbances.

etc

2.7 Brief History of Development of Magnetic Fluid

The first documented attempt to produce Magnetic Fluid (MF) or Ferrofluid (FF) was made by Gown knight in 1779. He tried to suspend fine iron fillings in water. But this attempt was unsuccessful. The next noteworthy attempt was made by Bitter in 1940. He prepared a magnetic colloid which was stable under gravity, but the colloid losses its stability in a magnetic field. Bitter exploited this magnetic field induced instability to observe the domain boundaries on the surface of ferromagnetic material. Subsequently, Elmore refined the Bitter colloid and studied its physical properties. In such colloids number concentration of magnetic particles were small. So, maximum magnetization that can be achieved was also very low. During this period magnetic clutch 'slurry' was also developed. Such slurry, composed of micron size iron particles dispersed in an oil, solidifies in the presence of an applied magnetic field. In short, the slurry looses its flowability in magnetic field and becomes highly viscous. Ideas for a magnetic fluid seal were also mooted at this time. The requirement of magnetic fluids whose viscosity were not affected by magnetic fields limited the progress.

A real break through was achieved when Solman S. Papell in 1963 whilst working at 'NASA' (National Aeronautics and Space Administration) devised a method of stabilizing MFs against aggregation. His main interest was to mix this MF with rocket fuel so that it can be pumped in zero gravitational field by means of externally applied magnetic field. The MF synthesized by Papell exhibited a large saturation magnetization (i.e. 100 Gauss) and its viscosity remained low even in presence of magnetic field. Papell's method of preparation consisted of ball milling magnetite in a liquid in the presence of stabilizing surfactant until the magnetite was in colloid state. His fluid consisted of finely divided particles of magnetite in kerosene. To keep the particles from clumping together, Papell used oleic acid as dispersing agent. At the same time R. E. Rosensweig and his associates were also working on synthesis of magnetic fluids. Whilst working at AVCO-USA, they succeeded in making magnetic fluids that were about ten times as strong as Papell's original magnetic fluid. The particle size in their fluid is of the order of 100 $\mathring{A}$ and it did not congeal even when subjected to a strong magnetic field. It would become magnetized but it remained a fluid. Rosensweig and Neuringer were the first two, who proposed a mathematical model of this fluid, consequently a new branch of fluid mechanics now known as ferrohydrodynamics is established.

Ferrofluid (FF) industry was also first established by Rosensweig and his associate. Moskowitz founded in 1969 ferrofluidics corporation, U.S.A., is still at present a major source of MFs and devices based on magnetic fluids. It has its branches in France, Japan and Germany. Apart from this industry many other industries also manufacture several types of magnetic fluids and ferro fluidic devices. Some of these are Servoflow; U.S.A., Georgia Pacific; U.S.A., Matsumoto; Japan, Toiho industries; Japan and U.S.S.R.

In India too, an increasing trend of research interest in magnetic fluid has been evident through the pioneering contribution by Mehta and Coworkers [47]. Recent contributions from diversified viewpoints are due to Malik *et. al.* [69] and Vaidyanathan *et. al.* [101].

- **Brief Idea About Magnetic Fluids** A MF is a colloid consisting of submicron sized single domain ferromagnetic particles dispersed in a liquid called carrier liquid with a suitable surfactant added so as to prevent agglomeration and sedimentation. Thus it is a multicomponent (solid magnetite particles, surfactant and carrier liquid) and two phase (solid-liquid) system.

It is noted that most of the applications in various branches of engineering and science requires the stable MF against gravitational and gradient magnetic fields. Also it should be stable against agglomeration which arise from magnetic and London type van der Waals forces. It is observed that by making a small size of the magnetite particle and choosing a suitable surfactant, it is possible to prepare a stable magnetic fluid. Of course, for a specific application the criteria for selecting the various parameters such as carrier liquids, concentration, size and composition of particles, particles coating etc. are different.

Thus, magnetic fluid is a multi-component and multiphase system having essential magnetic properties and fluidity.

- **About Magnetic Fluid Effect** FFs or MFs [74] are stable colloidal suspensions containing fine ferromagnetic particles dispersing in a liquid, called carrier liquid, in which a surfactant is added to generate a coating layer preventing the flocculation of the particles. When an external magnetic field $\mathbf{H}$ is applied, FFs experience a magnetic body forces $(\mathbf{M} \cdot \nabla)\mathbf{H}$ which depends upon the magnetization vector $\mathbf{M}$ of ferromagnetic particles. Owing to these features FFs are useful in many applications like in sensors, sealing devices, filtering apparatus, elastic damper, bearings, etc. [5, 23, 42, 45, 64]. The application of FF as lubricant is also one of them.

- **Fluid Dynamics Involving Magnetic Fluid** In (classical) ordinary fluid dynamics there are only three forces viz. (a) the pressure gradient (b) gravity force (c) the viscous force which have been taken into account an accordingly the equation of motion is stated as the sum of the gradients of all those forces remains equal to rate of change of velocity multiplied by the density.

Whereas in case of ferrohydrodynamics the magnetic body force also acts along with four interparticle forces viz. magnetic attraction, van der Waals attraction, steric repulsion and electric repulsion. Among which we have considered only two namely magnetic body force and magnetic attraction, since magnetic attraction is a long range potential and varies very slowly with distance.

The magnetic body force originates from the interaction of magnetic field H with the magnetization of the fluid and is of great importance regarding all further investigation [46].

2.8 Basic Flow Equations

The basic flow equations used in our work based on R. E. Rosensweig model are given by [76]

$$\rho\left[\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}\cdot\nabla)\mathbf{q}\right] = -\nabla p + \eta\,\nabla^2\mathbf{q} + \mu_0(\mathbf{M}\cdot\nabla)\mathbf{H}$$

… (2.11)

$$\nabla\cdot\mathbf{q} = 0$$

… (2.12)

$$\nabla\times\mathbf{H} = 0$$

… (2.13)

$$\mathbf{M} = \bar{\mu}\,\mathbf{H}$$

… (2.14)

$$\nabla\cdot(\mathbf{H}+\mathbf{M}) = 0$$

… (2.15)

where ρ is fluid density, p fluid film pressure, η is fluid viscosity, $\mathbf{q}$ is fluid velocity in film region, μ_0 is free space permeability, $\mathbf{M}$ is the magnetization vector, $\mathbf{H}$ is magnetic field vector, $\bar{\mu}$ is magnetic susceptibility and t is time.