Chapter 3

Two Porous Layers Slider Bearing with a Convex Pad Upper Surface Considering Slip and Squeeze Velocity

Contents

3.1 Introduction

Wu [1] in an innovative analysis, dealt with the case of squeeze film behaviour for porous annular disks in which he showed that owing to the fact that fluid can flow through the porous material as well as through the space between the bounding surfaces, the performance of a porous walled squeeze film can differ substantially from that of a solid walled squeeze film. Later [2] extended the above analysis of [1] by introducing the effect of velocity slip to porous walled squeeze film with porous matrix attached to the upper plate. They found that the load carrying capacity decreases due to the effect of porosity and slip. Prakash and Vij [3] investigated a porous inclined slider bearing without the effect of magnetic fluid (MF) and found that porosity caused decrease in the load carrying capacity and friction, while it increases the coefficient of friction. Gupta and Bhat [4] found that the load carrying capacity and friction could be increased by using a transverse magnetic field using conducting lubricant.

With the advent of ferrofluid (FF), Agrawal [5] studied its effects on a porous inclined slider bearing and found that the magnetization of the magnetic particles in the lubricant increases load capacity without affecting the friction on the moving slider. In [6] Verma studied squeeze film bearing with MF as lubricant using three porous layers attached to the lower plate and showed that load carrying capacity increases due to the effect of MF lubricant as compared to conventional viscous fluid as lubricant. Shah *et.al.* in [7, 8] studied respectively convex pad porous surface slider bearing with slip velocity and axially undefined journal bearing with anisotropic permeability, slip and squeeze velocity. In both the papers lubricant used was FF and showed that the performance of the bearing is better. Other references [9 -15] have also analyzed effects of FFs in their study from different viewpoints.

In all above investigations, none of the authors in their study considered the effects of two porous layers attached to the lower plate (slider) for a slider bearing having convex pad stator with slip and squeeze velocity using FF as a lubricant. The porous layer in the bearing is considered because of its advantageous property of self-lubrication. With this motivation the present Chapter proposes the study of performance of a slider bearing having convex pad stator with two porous layers attached to the lower plate with a FF lubricant under a magnetic field oblique to the lower surface. Here, the effects of slip velocity at the film and porous interface, as well as squeeze velocity when the upper plate approaches to lower one are also included for study.

A mathematical model of the above problem in the form of Reynolds equation is derived. Fixed size of porous matrix is considered for computation of dimensionless load carrying capacity for same as well as different values of the permeabilities k_1 and k_2 of the upper and lower porous matrixes respectively as shown in Figure 3.1. The results are also obtained when squeeze velocity $\dot{h} = 0$ and $\dot{h} \neq 0$.

3.2 Formulation of the Mathematical Model

The configuration of the slider bearing having convex pad stator with squeeze velocity $\dot{h}$ is displayed in Figure 3.1. The lower surface (slider) is of having length A in x -direction and breadth B in y -direction and moving with uniform velocity U in the x-direction. The upper convex pad surface is a stator with central thickness H_c. The film thickness is h and given by expression (refer [7])

$$h = H_c \left\{ 4\left(\frac{x}{A} - \frac{1}{2} \right)^2 - 1 \right\} + h_1 \left\{ a - \frac{a}{A}x + \frac{x}{A} \right\};\ a = \frac{h_2}{h_1},\ 0 \le x \le A,$$

… (3.1)

where h_2 and h_1 are maximum and minimum film thicknesses respectively.

The slider has attached with porous matrix of thickness d_2 first and then d_1 as shown in Figure 3.1. The stator moves normally towards the slider with a uniform velocity

$$\dot{h} = dh / dt,$$

where t is time.

Also,

$$\mathbf{q} = u\,\mathbf{i} + v\,\mathbf{j} + w\mathbf{k},$$

… (3.2)

where u, v, w are components of film fluid velocity in *x, y* and *z* -directions respectively.

The magnetic field considered here is oblique to the lower surface and is defined as

$$H^2 = K\,x(A - x),$$

… (3.3)

where *K* is chosen to suit the dimensions of both sides.

By combining basic flow equations (2.18) to (2.22) and using equation (3.2) under the usual assumption of lubrication, neglecting inertia terms and that the derivatives of velocities across the film predominate, equation governing the lubricant flow in the film region in x-direction yields

$$\frac{\partial^2 u}{\partial z^2}=\frac{1}{\eta}\frac{\partial}{\partial x}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right), \qquad \dots (3.4)$$

where p is the pressure in the film region, μ_0 is the free space permeability, $\bar{\mu}$ is the magnetic susceptibility and η is the fluid viscosity.

Using slip boundary conditions [16]

$$u=\frac{1}{s}\frac{\partial u}{\partial z}+U\ ;\quad \frac{1}{s}=\frac{\sqrt{k_1}}{\alpha}\ \text{ when }\ z=0, \qquad \dots (3.5)$$

and

$$u=0\ \text{ when }\ z=h, \qquad \dots (3.6)$$

equation (3.4) becomes

$$u = \frac{(h-z)s}{(1+sh)}U + \left\{ \frac{(z+h+shz)(z-h)}{2\mu(1+sh)} \right\} \frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right), \qquad \dots (3.7)$$

where s represents slip parameter, α represents slip coefficient and k_1 represents permeability of the upper porous layer. All are dependent on the structure of the porous material.

Continuity equation for the film region is given by

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0. \qquad \dots (3.8)$$

Integrating continuity equation (3.8) for the film thickness h; that is, from 0 to h, and making use of equation (3.7), yields

$$\frac{\partial}{\partial x}\left\{ \frac{sh^2}{2(1+sh)}U - \frac{h^3(4+sh)}{12\mu(1+sh)} \frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right) \right\} = w_0 - V, \qquad \dots (3.9)$$

as $w|_{z=h} = w_h = V = -\dot{h}$, which represents squeeze velocity in the downward z -direction.

Also,

$$w|_{z=0} = w_0 .$$

The velocity components in the porous region are given by

$$\bar{u}_i = -\frac{k_i}{\eta}\frac{\partial P_i}{\partial x} + \frac{\mu_0 \bar{\mu} k_i}{2\eta}\frac{\partial H^2}{\partial x} \, (x \text{ -direction}),$$

… (3.10)

$$\bar{w}_i = -\frac{k_i}{\eta}\frac{\partial P_i}{\partial z} + \frac{\mu_0 \bar{\mu} k_i}{2\eta}\frac{\partial H^2}{\partial z} \, (z \text{ -direction}),$$

… (3.11)

where $i = 1, 2$ represents index of velocity components in the porous matrix of thickness d_1 and d_2 respectively; P_1, P_2 are the pressures in the porous layers and k_1, k_2 are permeabilities there (Refer Figure 3.1).

Considering continuity of the flow between two porous layers d_1 and d_2 with respect to z, one obtains

$$\left[-\frac{k_1}{\eta}\frac{\partial P_1}{\partial z} + \frac{\mu_0 \bar{\mu} k_1}{2\eta}\frac{\partial H^2}{\partial z}\right]_{z=-d_1} = \left[-\frac{k_2}{\eta}\frac{\partial P_2}{\partial z} + \frac{\mu_0 \bar{\mu} k_2}{2\eta}\frac{\partial H^2}{\partial z}\right]_{z=-d_1}.$$

… (3.12)

Also, at the surface of the impermeable lower plate

$$\left[-\frac{k_2}{\eta}\frac{\partial P_2}{\partial z} + \frac{\mu_0 \bar{\mu} k_2}{2\eta}\frac{\partial H^2}{\partial z}\right]_{z=-(d_1+d_2)} = 0.$$

… (3.13)

Using equations (3.10) and (3.11) in the continuity equation for porous region

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z} = 0,$$

yields

$$\frac{\partial^2}{\partial x^2}\left(P_i - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right) + \frac{\partial^2}{\partial z^2}\left(P_i - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right) = 0,$$

… (3.14)

where $i = 1, 2$.

Integrating equation (3.14) with respect to z over the porous layer of the thickness d_1, yields

$$\frac{\partial}{\partial z}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right)\bigg|_{z=0} - \frac{\partial}{\partial z}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right)\bigg|_{z=-d_1} = -\int_{-d_1}^{0} \frac{\partial^2}{\partial x^2}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right) dz.$$

… (3.15)

Again, integrating equation (3.14) with respect to z over the porous layer of the thickness d_2, yields

$$\frac{\partial}{\partial z}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right)\bigg|_{z=-d_1} - \frac{\partial}{\partial z}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right)\bigg|_{z=-(d_1+d_2)} = -\int_{-(d_1+d_2)}^{-d_1} \frac{\partial^2}{\partial x^2}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right) dz.$$

… (3.16)

Using condition (3.12), equation (3.15) becomes

$$\frac{\partial}{\partial z}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\Bigg|_{z=0} = -\int_{-d_1}^{0} \frac{\partial^2}{\partial x^2}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)dz + \frac{k_2}{k_1}\frac{\partial}{\partial z}\left(P_2 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\Bigg|_{z=-d_1} .$$

… (3.17)

Using equations (3.13) and (3.16), equation (3.17) becomes

$$\left[\frac{\partial}{\partial z}\left(P_1 - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\right]_{z=0} = -\left(d_1 + \frac{k_2}{k_1} d_2\right)\frac{d^2}{d x^2}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right),$$

… (3.18)

using Morgan-Cameron approximation [16].

As the normal component of velocity across the film-porous interface are continuous, therefore

$$w\big|_{z=0} = \bar{w}\big|_{z=0} .$$

… (3.19)

Using equation (3.11) at z = 0, equations (3.18), (3.19) and the fact that $\partial H^2 / \partial z = 0$, equation (3.9) becomes

$$\frac{d}{dx}\left[\left\{12 k_1\left(d_1 + \frac{k_2}{k_1} d_2\right) + \frac{h^3 (4 + sh)}{(1 + sh)}\right\}\frac{d}{dx}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\right] = 12\mu V + 6\mu U \frac{d}{dx}\left(\frac{s h^2}{1 + sh}\right),$$

… (3.20)

which is known as Reynolds equation of the considered phenomena.

Defining dimensionless quantities

$$X = \frac{x}{A},\ \ \bar{h} = \frac{h}{h_1},\ \ \bar{s} = s h_1,\ \ a = \frac{h_2}{h_1},\ \ \bar{p} = \frac{h_1^{\,2}\, p}{¦\ AU},\ \ «* = \frac{«_0 \bar{«}\, K\, A\, h_1^{\,2}}{¦\ U},$$

$$\cdot\ = \frac{k_1 d_1 + k_2 d_2}{h_1^{\,3}},\ \ S = \frac{-2\ VA}{U h_1},$$

… (3.21)

the magnetic field H defined in equation (3.3) becomes

$$H^2 = K\, A^2 X (1 - X),$$

… (3.22)

and the equation (3.20) becomes

$$\frac{d}{dX}\left[G \frac{d}{dX}\left\{ \bar{p} - \frac{1}{2} «* X (1 - X) \right\} \right] = \frac{dE}{dX},$$

… (3.23)

where

$$G = 12 \cdot\ + \frac{\bar{h}^3 (4 + \bar{s}\,\bar{h})}{(1 + \bar{s}\,\bar{h})},\ \ E = \frac{6\,\bar{s}\,\bar{h}^2}{(1 + \bar{s}\bar{h})} - 6 S\, X.$$

… (3.24)

Equation (3.23) is known as Reynolds equation in dimensionless form.

3.3 Solution

Solving equation (3.23) under the boundary conditions

$$\overline{p} = 0 \text{ when } X = 0, 1,$$

… (3.25)

yields

$$\overline{p} = \frac{1}{2} \text{«} * X(1 - X) + \int_0^X \frac{E - Q}{G}\, dX \;,$$

… (3.26)

where

$$Q = \frac{\int_0^1 \frac{E}{G}\, dX}{\int_0^1 \frac{1}{G}\, dX} \;.$$

The load carrying capacity W of the bearing can be expressed in dimensionless form as

$$\overline{W} = \int_0^1 \overline{p}\, dX \;,$$

where

$$\overline{W} = \frac{W h_1^{\,2}}{A^2 B \text{¦} U} \;.$$

Using equation (3.26),

$$\bar{W} = \frac{W\, h_1^{\;2}}{A^2 B ¦\, U} = \frac{« *}{12} - \int_0^1 \frac{E - Q}{G}\; X\, dX \; .$$

... (3.27)

3.4 Results and Discussion

The values of the dimensionless load carrying capacity $\bar{W}$ has been calculated for the following value of the parameters using Simpson’s 1/3 rule with step size 0.1.

$$h_1 = 0.05\,(m),\;\; h_2 = 0.10\,(m),\; \bar{«} = 0.05,\; U = 1\;(m/s),$$

$$A = 0.15\;(m),\; ¦ = 0.012\;(N\,s/m^2),\; «_0 = 4® \times 10^{-7}\,(N/A^2),\; H_c = 0.3(m),$$

$$\dot{h} = 0.005\;(m/s),\; d_1 = 0.01(m),\; d_2 = 0.01(m),\; \sim\; = 0.1,\; K = 10^9.$$

The FF used here is water based. The magnetic field considered here is oblique to the lower plate and its strength is in between $O(10^2)$ – $O(10^3)$ in order to get maximum magnetic field at $x = A / 2$.

The calculation of magnetic field strength is shown below [8]:

From equation (3.3),

$$H^2 = K\, x(A - x),$$

$$\text{Max. } H^2 = 10^{-4}\; K,$$

$$\text{For } H = O(10^3),\; K = O(10^{10}).$$

According to [17] the maximum magnetic field strength one can take is of $O(10^5)$.

The calculated values of $\bar{W}$ presented by Tables 3.1 and 3.2.

Table 3.1 presents the values of $\bar{W}$ by interchanging the values of k_1 and k_2 for two different cases of $\dot{h}=0$ and $\dot{h}\neq 0$. It is observed from the table that when $k_1 > k_2$, dimensionless load carrying capacity increases about 6.35 % in both the cases; that is, for $\dot{h}=0$ and $\dot{h}\neq 0$ as compared to $k_1 < k_2$.

Table 3.2 presents the values of $\bar{W}$ by considering two same values of k_1 and k_2 for two different cases of $\dot{h}=0$ and $\dot{h}\neq 0$. It is observed from the table that the dimensionless load carrying capacity increases about 113% for $k_1 = k_2 = 0.0001$ in both the cases; that is, for $\dot{h}=0$ and $\dot{h}\neq 0$ as compared to $k_1 = k_2 = 0.01$.

3.5 Conclusions

The problem on slider bearing having convex pad stator with two porous layers attached to the lower plate is discussed here for it optimum performance with a FF lubricant under a magnetic field oblique to the lower surface. The effects of slip velocity at the film and porous interface, as well as squeeze velocity when the upper plate approaches to lower one, is also considered for study. The FF flow model considered here is due to R. E. Rosensweig and the FF is considered to be of water based with magnetic field strength considered of order between $10^2 - 10^3$ in order to get maximum magnetic field at $x = A / 2$. From the results and discussion it is concluded that better dimensionless load carrying capacity can be obtained for smaller values of k_1 and k_2 and for $k_1 > k_2$.

Also, it should be noted from equation (3.4) that, a constant magnetic field does not enhance load carrying capacity in Rosensweig's FF flow model.

3.6 Figure

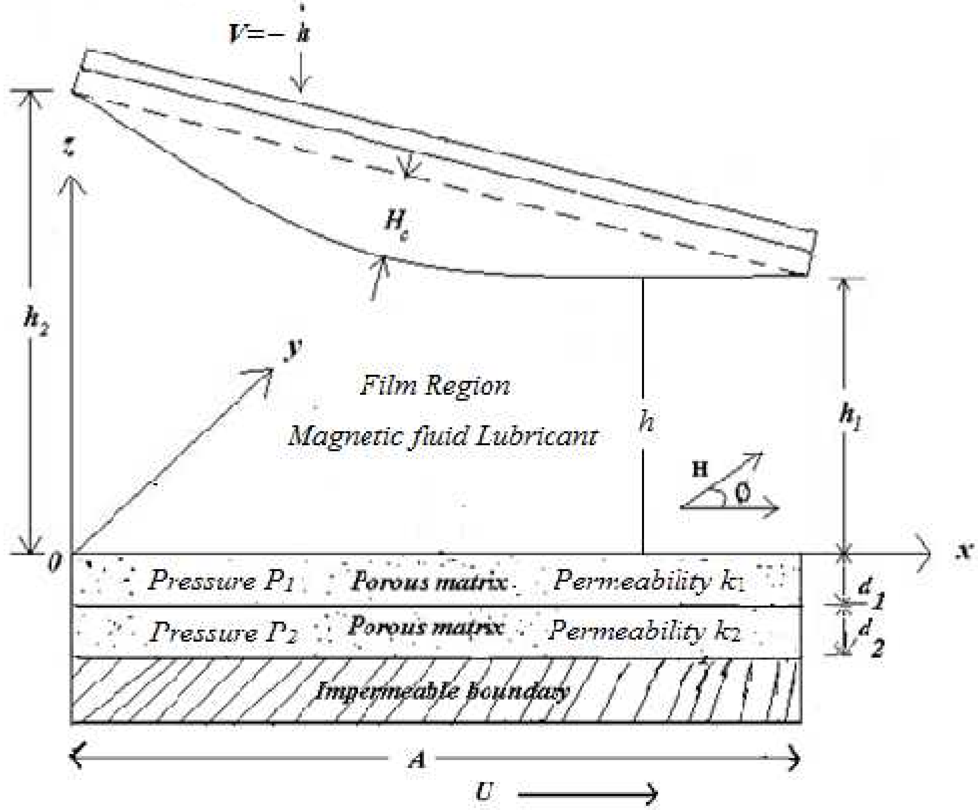

Figure 3.1 Convex pad surface slider bearing.

3.7 Tables

$\overline{W}$			
k_1	k_2	$\dot{h} = 0$	$\dot{h} \neq 0$
0.1	0.0001	0.1620510	0.1621569
0.0001	0.1	0.1523362	0.1524624
% increase in $\overline{W}$		6.38	6.35

Table 3.1 Values of $\overline{W}$ for interchanging values of k_1 and k_2 considering $\dot{h} = 0$ and $\dot{h} \neq 0$.

$\overline{W}$			
k_1	k_2	$\dot{h} = 0$	$\dot{h} \neq 0$
0.0001	0.0001	0.3386675	0.3385819
0.01	0.01	0.1588051	0.1592045
% increase in $\overline{W}$		113.26	112.67

Table 3.2 Values of $\overline{W}$ for same values of k_1 and k_2 considering $\dot{h} = 0$ and $\dot{h} \neq 0$.

3.8 References

[1] Wu H, Squeeze - film behavior for porous annular disks, *Journal of Lubrication Technology*, *Transactions of ASME*, Series F, 92(4), 593-596 (1970).

[2] Sparrow E M, Beavers G S and Hwang I T, Effect of velocity slip on porous walled squeeze films, *Journal of Lubrication Technology*, 94, 260-265 (1972).

[3] Prakash J and Vij S K, Hydrodynamic lubrication of a porous slider, *Journal of Mechanical Engineering Science*, 15, 232-234 (1973).

[4] Gupta J L and Bhat M V, An inclined porous slider bearing with a transverse magnetic field, *Wear*, 55, 359-367 (1979).

[5] Agrawal V K, Magnetic fluid based porous inclined slider bearing, *Wear*, 107, 133-139 (1986).

[6] Verma P D S, Magnetic fluid-based squeeze film, *International Journal of Engineering Science*, 24(3), 395-401 (1986).

[7] Shah R C and Bhat M V, Ferrofluid lubrication of a porous slider bearing with a convex pad surface considering slip velocity, *International Journal of Applied Electromagnetic and Mechanics*, 20, 1-9 (2004).

[8] Shah R C and Bhat M V, Anisotropic permeable porous facing and slip velocity on squeeze film in an axially undefined journal bearing with ferrofluid lubricant, *Journal of Magnetism and Magnetic Materials*, 279, 224-230 (2004).

[9] Chi C Q, Wang Z S and Zhao P Z, Research on a new type of ferrofluid – lubricated

journal bearing, *Journal of Magnetism and Magnetic Materials*, 85, 257-260 (1990).

[10] Sinha P, Chandra P and Kumar D, Ferrofluid lubrication of cylindrical rollers with cavitation, *Acta Mechanica*, 98, 27-38 (1993).

[11] Shah R C and Bhat M V, Ferrofluid squeeze film between curved annular plates including rotation of magnetic particles, *Journal of Engineering Mathematics*, 51, 317-324 (2005).

[12] Ahmad N and Singh J P, Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity, *Journal of Engineering Tribology*, 221, 609-613 (2007).

[13] Liu J, Analysis of a porous elastic sheet damper with a magnetic fluid, *Journal of Tribology*, 131, 0218011-15 (2009).

[14] Shah R C and Patel D B, Mathematical Modelling of newly designed Ferrofluid Based Slider Bearing Including Effects of Porosity, Anisotropic Permeability, Slip Velocity at Both the Ends, and Squeeze Velocity, *Applied Mathematics*, 2 (5), 176-183 (2012).

[15] Shah R C and Parsania M M, Comparative Study of Parallel Plate Slider Bearing with Other Slider Bearings Using Magnetic Fluid as Lubricant, *American journal of Mathematics and Statistics*, 3(4), 179-189 (2013).

[16] Shah R C and Bhat M V, Ferrofluid lubrication equation for porous bearing considering anisotropic permeability and slip velocity, *Indian Journal of Engineering & Materials Sciences*, 10, 277-281 (2003).

[17] Tipei N, Theory of Lubrication with Ferrofluids: Application to short bearings, *Transactions of the ASME*, 104, 510-515 (1982).