Chapter 2

Physico-Mathematical Background

Contents

This Chapter deals with the Physico-Mathematical background necessary to understand the subsequent Chapters. The topics included are basic definitions, description of different flows, types of different lubrication with lubrication assumptions, bearing design systems and about ferrofluids (FFs) with basic equations. The materials are taken from various sources like [1-29].

2.1 Basic Definitions from Fluid Mechanics, Porous Medium and Physics

In this article various definitions which are required for the subsequent study are discussed.

2.1.1 Definitions from Fluid Mechanics

Definition 1 (Fluid)

It is a substance which is capable of flowing and which deforms continuously as long as the shearing force is applied. This continuous deformation under the action of forces compels the fluid to flow. Fluids can be liquids or gases.

Definition 2 (Surface Tension)

It is a tensile force acting on the surface of a liquid when it in contact with a gas, or on the surface between two immiscible liquids. In this case the contact surface behaves like a membrane.

The unit of surface tension is N/m.

Definition 3 (Viscosity)

It is that property of a fluid by virtue of which it offers resistance to the movement of a one layer of a fluid over an adjacent layer.

The unit of viscosity is $N\,s / m^2$.

Definition 4 (Density)

It is defined as mass per unit volume. It is generally denoted by ° .

The unit of density is $N\,s^2 / m^4$.

Definition 5 (Compressible Fluids)

These are the fluids in which density changes from point to point or in other words the density is not constant.

Definition 6 (Incompressible Fluids)

These are the fluids in which density remains constant.

Definition 7 (Inviscid or Ideal Fluids)

These are those fluids which have no viscosity, surface tension and are incompressible. In nature, ideal fluid does not exist, these are imaginary fluids, are also called perfect fluids or frictionless fluids.

Definition 8 (Viscous or Real Fluids)

These fluids are actually available in nature. The fluids possess the property like viscosity, surface tension and compressibility. These are also called Practical Fluids.

Definition 9 (Newtonian Fluids)

Fluids obeying Newton's law of viscosity ² $= « \frac{du}{dy}$ and for which viscosity « has constant value are called Newtonian fluids, that is for such fluids shear stress is directly proportional to the rate of shear strain or velocity gradient.

For example, water, glycerin, light-hydrocarbon oils, silicone oils, air, gases, etc.

Definition 10 (Non-Newtonian Fluids)

These are the fluids in which the shear stress is not directly proportional to the rate of shear strain or velocity gradient.

For example, slurries, tooth paste, gel, etc.

Definition 11 (Uniform Flow)

Flow is said to be uniform when the flow velocity having the same magnitude and direction at every point in the fluid flow.

Definition 12 (Non-uniform Flow)

The flow is said to be non-uniform when at a given instant, the velocity is not the same at every point in the fluid flow.

Definition 13 (Steady Flow)

The flow is said to be steady when fluid characteristics like density, velocity, pressure, etc. are independent of time.

Definition 14 (Unsteady Flow)

The flow is said to be unsteady when fluid characteristics like velocity, pressure, density, etc. are dependent on time.

Definition 15 (Rotational Flow)

The flow in which the fluid particles rotate about their own axis is called rotational flow.

Definition 16 (Irrotational Flow)

The flow in which the fluid particles do not rotate about its own axis is called irrotational flow.

Definition 17 (Laminar Flow)

A laminar flow is one in which fluid particles move in layers or lamina with one sliding over the other. The laminar flow is also called viscous flow or stream line. This type of flow occurs when viscosity is high and velocity is low.

Definition 18 (Turbulent Flow)

A flow in which fluid particles move in a zigzag way is called turbulent flow. All particles are disturbed and mix with each other; there is continuous change in momentum of the fluid particles.

2.1.2 Definitions from Porous Medium and Darcy's Law

Definition 1 (Porous Medium)

It is referred as a solid body with pores (void spaces) in it.

Definition 2 (Permeability)

It is that property of a porous material which characterizes the ease with a fluid may be made to flow through the material by an applied pressure gradient. The value of the permeability is determined by the structure of the porous material.

The unit of permeability is m^2.

Definition 3 (Porosity)

It is a measure of the void spaces in a material, and is defined as fraction of the volume of void spaces over the total volume of the material.

➢ **Darcy's Law**

The governing equation for fluid motion in a vertical porous column was first introduced by Darcy in 1856. It describes the flow of a fluid through the porous medium. In mathematical form, law is by

$$V = -\frac{k}{¦}\nabla P\,,$$

where V is the space averaged velocity (Darcian velocity), k is the permeability of the porous region, $¦$ is the coefficient of viscosity and P is the pressure in the porous region.

2.1.3 Definitions from Physics

Definition 1 (Magnetic Dipole)

The magnetic dipoles, generally known as north and south poles, commonly exist in magnetic materials. The magnetic dipoles are not separate poles unlike an electric dipole.

Definition 2 (Magnetic Field Strength)

The magnetic field strength H at any point in a magnetic field is the force experienced by unit north Pole placed at that point.

The unit of magnetic field strength is A/m.

Definition 3 Magnetic Induction (Flux Density)

The magnetic induction B in any material is defined as number of lines of force through a unit area of cross-section perpendicularly.

The unit of magnetic induction is kg/s^2A.

Definition 4 (Magnetic Dipole Moment)

The magnetic dipole moment is equal to the product of the magnetic pole strength and the length of the magnet.

Definition 5 Magnetization (Intensity of Magnetization)

Magnetization M is the measure of magnetism of magnetic materials and is defined as the magnetic moment per unit volume.

The unit of magnetization is A/m.

Definition 6 (Magnetic Susceptibility)

Magnetic susceptibility is used to explain the magnetization of material. It is defined as the ratio of magnetization M to the magnetic field strength H. That is,

$$\text{magnetic susceptibility } \overline{«} = \frac{M}{H}.$$

Definition 7 (Magnetic Permeability)

The magnetic permeability « is defined as the ratio of amount of magnetic density *B* to the applied magnetic field intensity *H*. It is used to measure the magnetic lines of forces penetrating through a material. That is,

$$\text{magnetic permeability } « = \frac{B}{H}.$$

2.2 Relationship between Pressure Gradient and Shear Stress

When the fluid is a rest then there is no shear stress but when the fluid is in motion then shear stresses are developed. If the fluid particles have relative motion so fluid particles have different velocities then the shape does not remain same. There is no relative motion between fluid particles, if the velocity of fluid is same at all point. Hence no shear stress will be produced.

Assuming experimental result that the shear stress is proportional to velocity gradient (shear strain), then

$$² \sim \frac{du}{dy} \Rightarrow ² = \text{constant} \times \frac{du}{dy}$$

The constant of proportionality is known as the dynamic viscosity « of the fluid

$$² = « \frac{du}{dy}$$

which is Newton’s law of viscosity. The value of « depends upon the types of fluid.

The viscous behavior of a non-Newtonian fluid may be given by the power law equation,

$$2 = k\left(\frac{du}{dy}\right)^n$$

where k is a consistency index and n is a flow behaviour index.

Relation between shear stress and the rate of shear strain for fluids is shown in the following Figure 2.1.

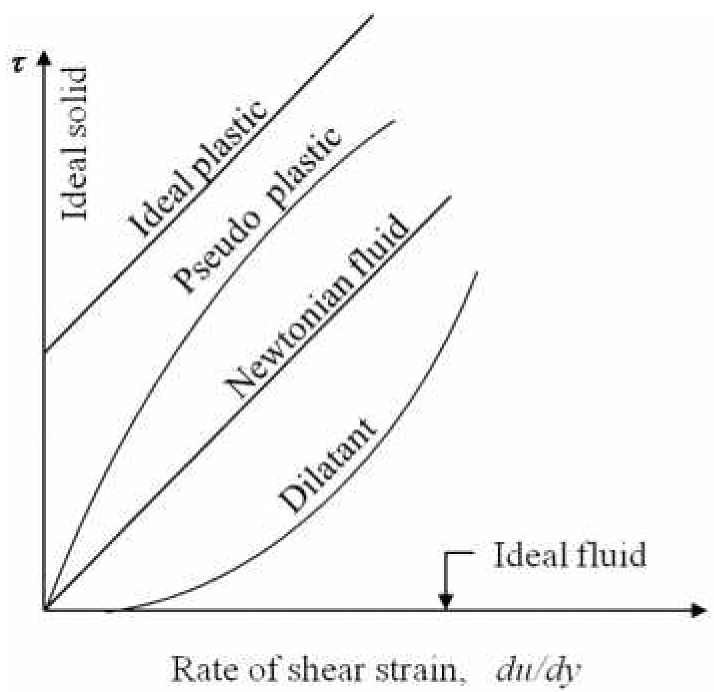

Figure 2.1

Definition 1 (Ideal plastic)

This substance indicates no deformation when stressed up to a certain point (yield stress) and beyond that it behaves like a Newtonian fluid.

Definition 2 (Pseudo-plastic)

Fluid for which the flow behaviour index *n* is less than unity is called pseudo-plastic.

For example, colloidal solution, milk, cement

Definition 3 (Dilatant substances)

Fluid for which the index *n* is greater than unity is called dilatant substances. Dynamic viscosity increase as the rate of shear increases for this substance.

For example, concentrated solution of sugar

Definition 4 (Ideal fluid)

A fluid in which shear stress is zero even if there is velocity gradient is called the ideal fluid. Ideal fluids have no viscosity.

Definition 5 (Thixotropic substances)

Fluids which show an apparent decrease in viscosity with time are known as thixotropic substances. For example, Jelly paints

Definition 6 (Rheopectic substances)

Fluids which show an apparent increase in viscosity with time are known as rheopectic substances.

2.3 Brief Review of Fluid Mechanics with Basic Equations

Fluid mechanics deals with the study of all fluids (liquids or gases) under static and dynamics situations. Fluid mechanics is a branch of continuous mechanics which deals with a relationship among forces, motions, pressure, etc. in a continuous material. The study of fluids at static *i.e.* at rest is called **fluid static.** The study of fluids in motion where pressure forces are not considered is called **fluid kinematics** and if the pressure forces are considered, then it is called **fluid dynamics**. Thus, fluid dynamics is that branch of fluid mechanics which deals with the study of the motion of fluids, the forces that are responsible for this motion and the interaction of fluid with solids. Fluid dynamics deals with fluid behaviour in motion under the influence of body as well as surface forces. Fluid dynamics plays an important role in industry, transportation, manufacturing, environment, energy and biology, etc.

Hydrodynamics, the study of fluid flow, which was also developed in the middle of 18^{th} century when Euler discovered the equation of motion for an inviscid fluid. Later, Navier and Stokes gave the equation of motion for viscous fluids. The equation for turbulent motion was discovered by Reynolds and the well known boundary layer theory was introduced by Prandtl. Besides, many other scientists and mathematicians including Benard, Kelvin, Taylor, Karman etc. gave their excellent contributions to this subject.

On the basis of various fields and fluid interactions, hydrodynamics may be classified as

1) Electrohydrodynamics (EHD)
2) Magnetohydrodynamics (MHD)
3) Ferrohydrodynamics (FHD)

- **Electrohydrodynamics (EHD)** is the branch of science dealing with the motion of fluids with electric force effects.

- **Magnetohydrodynamics (MHD)** deals with the motion of electrically conducting fluids in the presence of magnetic field.

- **Ferrohydrodynamics (FHD)** came into existence in the mid of sixties after the invention of FF.

2.3.1 Equation of Continuity

According to the principle of mass conservation 'matter can be neither created nor destroyed'. This principle can be applied to a flowing flow.

Consider the flow of the fluid through a control volume shown in the Figure 2.2 having length *dx*, *dy* and *dz in x*, *y* and *z* -directions respectively. A point *P* in a flowing fluid at which fluid velocity is given by $\mathbf{q}(x, y, z, t) = u\,\mathbf{i} + v\,\mathbf{j} + w\mathbf{k}$ and density of fluid = ° , where **i**, **j**, **k** are unit vectors along the coordinate directions of Cartesian axes and t is time.

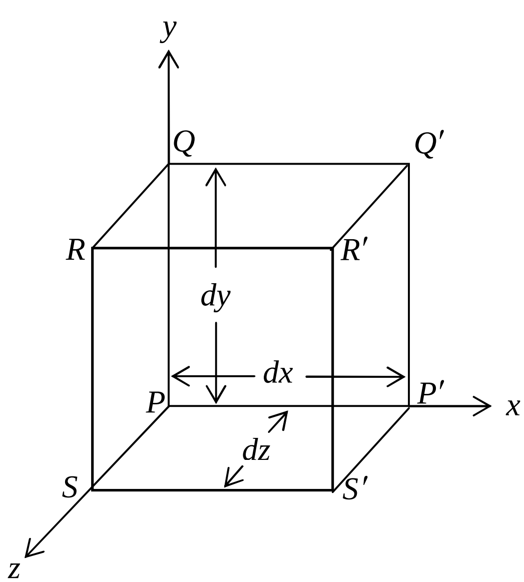

Figure 2.2

Construct a rectangular parallelepiped $P\,Q\,R\,S\,P'\,Q'\,R'\,S'$, with edges parallel to the coordinate axes and of lengths *dx*, *dy* and *dz* respectively.

Mass of the fluid entering through the face *PQRS* is given by

$$\text{density} \times \text{velocity in } x\text{-direction} \times \text{ area of } PQRS = \circ\, u\, dy\, dz.$$

Then the mass of the fluid leaving the face $P'\,Q'\,R'\,S'$ is given by

$$\circ\, u\, dy\, dz + \frac{\partial}{\partial x}(\circ\, u\, dy\, dz)\, dx.$$

Therefore, rate at which fluid accumulates due to flow in x -direction through the faces *PQRS* and $P'\,Q'\,R'\,S'$ is given by

$$\text{mass through the face } PQRS - \text{mass through the face } P'\,Q'\,R'\,S'$$

$$= \circ\, u\, dy\, dz - \circ\, u\, dy\, dz - \frac{\partial}{\partial x}(\circ\, u\, dy\, dz)\, dx$$

$$= -\frac{\partial}{\partial x}(\circ\, u\, dy\, dz)\, dx$$

$$= -\frac{\partial}{\partial x}(\circ\, u)\; dx\, dy\, dz\ .$$

Similarly, the net inflow in y -direction is given by

$$-\frac{\partial}{\partial y}(\circ\, v)\; dx\, dy\, dz,$$

And mass gain in z -direction is given by

$$-\frac{\partial}{\partial z}(\circ\ w)\ dx\,dy\,dz.$$

Thus, the net gain in the fluid flowing into the parallelepiped through the six faces is

$$-\left[\frac{\partial}{\partial x}(\circ\ u)\ +\frac{\partial}{\partial y}(\circ\ v)+\frac{\partial}{\partial z}(\circ\ w)\ \right]dx\,dy\,dz.$$

… (2.1)

Also, rate of increase of the mass within the control volume per unit time is

$$\frac{\partial}{\partial t}(\circ\times\text{volume})=\frac{\partial\circ}{\partial t}dx\,dy\,dz.$$

… (2.2)

According to principle of conservation of mass, equating equation (2.1) and equation (2.2), we get

$$-\left[\frac{\partial}{\partial x}(\circ\ u)\ +\frac{\partial}{\partial y}(\circ\ v)+\frac{\partial}{\partial z}(\circ\ w)\ \right]dx\,dy\,dz=\frac{\partial\circ}{\partial t}dx\,dy\,dz.$$

Dividing by $dx\,dy\,dz$, we get the equation of continuity in the Cartesian coordinates at the point P as

$$\frac{\partial\circ}{\partial t}+\frac{\partial}{\partial x}(\circ\ u)\ +\frac{\partial}{\partial y}(\circ\ v)+\frac{\partial}{\partial z}(\circ\ w)=0.$$

… (2.3)

Equation (2.3) is the most general form of **Equation of Continuity in Cartesian coordinates** which is applicable for steady as well as unsteady flow, uniform as well as non-uniform flow, and compressible as well as incompressible fluids.

- For the steady fluid flow $\frac{\partial \rho}{\partial t} = 0,$ hence the equation (2.3) of continuity becomes

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0.$$

… (2.4)

- If the fluid is incompressible, so that the mass density ρ, does not changes with *x*, *y*, *z* and t. Therefore, the equation (2.3) of continuity becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

… (2.5)

- **Vector Form of Continuity Equation**

The vector form of equation (2.3) can be written as

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{q}) = 0.$$

… (2.6)

- **Cylindrical Form of Continuity Equation**

The equation of continuity for an incompressible steady flow (2.5) presented in cylindrical form as

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u)+\frac{1}{r}\frac{\partial v}{\partial \overline{\ }}+\frac{\partial w}{\partial z}=0.$$

… (2.7)

Where $r, \overline{\ }, z$ are cylindrical polar coordinates.

2.3.2 Navier-Stokes Equation (Momentum Equation)

It is based on the principle of conservation of momentum. Considering an infinitely small mass of the fluid enclosed in an elementary parallelepiped shown in the Figure 2.3 of the sides *dx*, *dy* and *dz*.

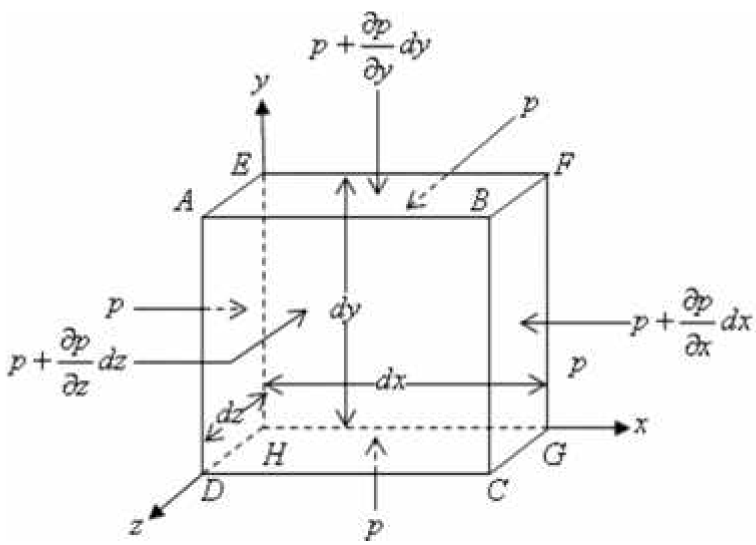

Figure 2.3

The motion of the fluid element is influenced by the following forces:

(i) Normal forces due to pressure :

The net pressure force in the x -direction is given by

$$pdydz - \left(p + \frac{\partial p}{\partial x} dx \right) dydz$$

$$= -\frac{\partial p}{\partial x} dxdydz.$$

(ii) Body or gravity force :

Let g_x, g_y and g_z are components of gravitational force or body force **g** in the x, y and z-directions, respectively. Then body force per unit mass of the fluid in the x -direction is

$$= m\, g_x = g_x \circ\ dxdydz.$$

(iii) Inertia forces :

Inertia force on the fluid mass along x -direction is given by

$$\text{mass} \times \text{acceleration}$$

$$= \circ\ dxdydz\ \frac{du}{dt}.$$

(iv) Shear forces :

The components of shear force per unit mass by the viscous forces be S_x, S_y and S_z in the x, y and z -directions, respectively.

Thus, shear force acting on the parallelepiped along x -direction is given by

$$S_x \circ\ dxdydz.$$

As per Newton's second law of the motion, the sum of all forces acting in the fluid element in any direction equals the resulting inertia forces in that direction, therefore along x -direction

$$-\frac{\partial p}{\partial x}\, dxdydz + g_x\, \rho\, dxdydz \; + S_x\, \rho\, dxdydz = \rho\, dxdydz\, \frac{du}{dt}$$

$$g_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{du}{dt} - S_x\,.$$

… (2.8)

Similarly,

$$g_y - \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{dv}{dt} - S_y\,,$$

… (2.9)

and

$$g_z - \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{dw}{dt} - S_z.$$

… (2.10)

Now, we find the values of S_x , S_y and S_z :

The shear force acting on the face $ADHE$ is given by

$$-\mu\, \frac{\partial u}{\partial x}\, dydz$$

The shear force acting on the face *BCGF* is given by

$$¦ \frac{\partial}{\partial x}\left(\mathrm{u} + \frac{\partial u}{\partial x} dx\right) dydz$$

$$= ¦ \left(\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} dx\right) dydz$$

The resultant force acting along the x -direction is given by

$$-¦ \frac{\partial u}{\partial x} dydz + ¦ \left(\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} dx\right) dydz$$

$$= ¦ \frac{\partial^2 u}{\partial x^2} dx\, dydz$$

… (2.11)

Similarly, the x-component of resultant shear force acting on the faces *DCGH* and *ABFE* is equal to

$$¦ \frac{\partial^2 u}{\partial y^2} dx\, dydz$$

… (2.12)

and, the x-component of resultant shear force acting on the faces *EFGH* and *ABCD* is given by

$$¦ \frac{\partial^2 u}{\partial z^2} dx\, dydz$$

… (2.13)

Total force, parallel to x-axis, on all the six faces of the parallelepiped is given by the sum of forces defined in equations (2.11), (2.12) and (2.13) and is given by

$$\mu \frac{\partial^2 u}{\partial x^2} dx\, dydz + \mu \frac{\partial^2 u}{\partial y^2} dx\, dydz + \mu \frac{\partial^2 u}{\partial z^2} dx\, dydz$$

$$= \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) dx\, dy\, dz$$

The shear (resistance) per unit mass is obtained by dividing above quantity by $\rho\, dx\, dy\, dz$,

we get

$$S_x = \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right).$$

Similarly,

$$S_y = \frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

and

$$S_z = \frac{\mu}{\rho} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right).$$

Putting these values of S_x, S_y and S_z in equations (2.8), (2.9) and (2.10), we get

$$g_x - \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{du}{dt} - \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),$$

… (2.14)

$$g_y - \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{dv}{dt} - \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right),$$

… (2.15)

and

$$g_z - \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{dw}{dt} - \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right).$$

… (2.16)

These equations (2.14), (2.15) and (2.16) are called **Navier-Stokes Equations** for viscous flow.

- **Vector Form of the Navier-Stokes Equations**:

$$\rho \frac{D\mathbf{q}}{Dt} = -\nabla p + \rho\,\mathbf{g} + \mu\,\nabla^2\mathbf{q},$$

… (2.17)

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \nabla \bullet \mathbf{q} \text{ and } \mathbf{q} = (u, v, w).$$

2.4 Various Types of Lubrication

In order to minimize friction and wear among the moving machine elements a substance is introduced and thereby preventing the actual contact of the surfaces, such a substance is called **lubricant**. Lubricants keep the machine elements apart and allow them to move relatively with minimum efforts. It will minimize friction and wear between rubbing surfaces. It will carry away the heat generated due to frictional work converted. It will provide protection against corrosion. Such a system where lubricant is used is called **lubricated system**.

The process of minimizing the friction and wear using lubricant is called **lubrication**. Hence the purpose of lubrication is to reduce friction, wear and heating of machine moving elements.

The following are the classification of the lubrication.

1) Hydrodynamic lubrication
2) Hydrostatic lubrication
3) Elastohydrodynamic lubrication
4) Boundary lubrication
5) Solid film lubrication

1) Hydrodynamic (or Fluid-film) Lubrication

Lubrication in which the surfaces of bearing are separated by a relatively thick film of lubricant to prevent metal to metal contact is called hydrodynamic lubrication. The film pressure is generated by the moving surfaces forcing the lubricant into a wedge shaped zone, therefore creating a pressure that separates the sliding surfaces.

2) Hydrostatic Lubrication

Lubrication in which external pressure is applied to the lubricant in the bearing system to maintain the fluid lubricant film where it would. This type of lubrication is helpful during the starts and stops of machine when the relative motion of the surfaces, is not fast enough to generate a required pressure.

The hydrostatic lubrication is suitable for extremely high load to be levitated at very low speed or at highly controlled precision position.

3) Elastohydrodynamic Lubrication

Elastohydrodynamic lubrication (EHL) is an approach of fluid-film lubrication in which hydrodynamic action is significantly enhanced by surface elastic deformation and due to high pressure lubricant viscosity increases. In a rolling element bearing, under the load, due to an elastic deformation, the line or point contact becomes a contact area. Elastohydrodynamic lubrication is concerned with the formation of a thin fluid film at the contact area of a rolling element. Sometimes, the elastohydrodynamic fluid film can be of sufficient thickness to separate the rolling surfaces, hence there may have significant reduction of wear also.

4) Boundary Lubrication

This type of lubrication can be considered whenever the load on a bearing is light and the shaft speed is low and wear is not a critical problem. Moreover, porous bearing material with self lubrication properties can be used for boundary lubrication.

Boundary lubrication is a special case of hydrodynamic lubrication where a very thin film of lubrication is sufficient.

5) Solid Film Lubrication

This type of lubrication is used in a situation where a fluid lubrication is not desirable because there are certain industries like food and pharma where the risk of contamination of product by fluid lubrication is disastrous. In such situation, solid or powder form of lubrication such as graphite, Teflon etc. are being used as lubricant.

2.4.1 Basic Assumptions of Lubrication Theory

The following are the basic assumptions of lubrication in our present research work.

1) The fluid flow is laminar.

2) The fluid is Newtonian and incompressible.

3) The gravity and inertia forces acting on the fluid are ignored.

4) The film thickness is very small compared with the bearing geometry.

5) The viscosity of fluid is constant through the film thickness.

6) The bearing surfaces are assumed to be perfectly rigid so that elastic deformations of the bearing surfaces may be disregarded.

7) Bearing surfaces are assumed to be perfectly smooth.

8) Porous matrix of the bearing surface is assumed to be homogeneous.

9) Validity of the Darcy's law is assumed.

10) Temperature changes in the lubricant are neglected.

etc.

2.5 Bearing types

Bearing is a system of machine elements whose function is to support a generated load due to machine operation by reducing friction and wear between the relatively moving surfaces. The load may be radial, axial or combination of these. The bearing can be categorized according to the direction of the applied load. A radial or journal bearing supports a radial load. A thrust bearing, on the other hand, supports a thrust or an axial load whereas conical bearings support both radial and axial load.

Lubricant is introduced in the clearance space of a bearing for its efficient functioning. Any substance which has some amount of viscosity is known as lubricant. Use of lubricants reduces the loss of energy caused by friction. In addition, heat generated by friction is also carried away by the lubricant and thus it works as a cooling agent.

Bearings which are commonly used: rolling element bearing, fluid film bearings, slider bearing. Rolling element bearings have much wider use in industries since rolling friction is lower than the sliding friction.

During machine operating conditions, if two mating surfaces are completely separated by lubricant film, such a type of lubrication is called **fluid film lubrication**. Bearings operating under this kind of lubrication are fluid film bearings. As metal to metal contact is completely avoided by this system of lubrication, it is sometime known as perfect lubrication. Fluid film bearings are lubricated by the hydrodynamic flow of lubricant which is generated by relative surface motion or external pressurization. In a hydrostatic bearing also known as 'externally pressurized' bearing, the load is supported due to pressure in the fluid which is supplied from an external source. In addition to these two types, there is another class of fluid film bearing known as 'squeeze film' bearing, which supports a load because of oscillating relative normal motion.

Shaft or Journal bearings which supports radial load may be divided into full journal bearings, where the contact angle of the bushing with the journal is 360^{o}, and the partial journal bearings, in which the contact angle is either 180^{o} or less. Full journal bearings are widely used in bearings industrial machinery. These bearings can take up rotating load. The partial journal bearings have limited applications and are used when the direction of radial load does not change.

To design a hydrodynamic bearing, a few important characteristics, like load carrying capacity, flow requirement and power loss due to viscous friction are to be predicated accurately. These parameters can be determined if the pressure within the bearing is known. To study and find this type of phenomena, one has to solve a particular form of the Navier-Stokes equations along with the mass continuity equation after making some basic assumptions. The governing equation thus formed is called the generalized **Reynolds equation**. The basic momentum equation can also be set up from balance of the element of fluid in this lubricating film. The generalized Reynolds equation contains viscosity, density, film thickness, surface motion etc. as parameters.

- **Porous Bearings**

Porous metal bearings were introduced in the 1920s. Plastics and composites involving polymers compounded with a wide variety of solid filler materials have found wide use. These are special type bearings in which the bearing material possesses the property of self lubrication and requires no additional lubricant. Particularly, with the recent advancements in powder metallurgy such bearings have found all-round applicability and are practically indispensable for various applications in air craft engines and automotive accessories, domestic appliances, farm and construction machineries, printing textile and baking industries, etc. During operation lubricant is stored in these pores and feeds through interconnecting pores to the bearing surfaces. The lubricant which is forced from the loaded zone of the bearing is reabsorbed by capillary action. Since these bearings can operate for long periods of time without additional supply of lubricant, they are used in inaccessible or inconvenient places where relubrication or frequent maintenance is difficult. High porosity with a maximum amount of lubricant is used for high speed, light load applications such as fractional horse motor bearings. A low lubricant content low porosity material is satisfactory for oscillating and reciprocating motion. Bronze is used as most common porous bearing material. These bearings are wear resistant, ductile, and corrosion resistant with good embedability which gives them a wide range of applications from home appliances to domestic machinery. Copper-iron porous metal bearings are useful in applications involving shocks and heavy loads. Phenolics, acetals and nylons are widely used plastic materials. Phenolic have been replaced by wood bearings and metals in such applications as propellers, electrical switch gears, rolling mills and water turbine bearings. Acetals are used for non-expensive bearings in wide variety of automotive appliances and industrial applications. Synovial joints are the biological bearings provided by nature in the human body, play an important role during the body locomotion of a bone past another, supporting considerable load involved and providing low friction. The biological bearing uses a poroelastic bearing material known as articular cartilage.

- **Bearing Design Characteristics**

The restrictions and environmental conditions imposed by the bearing system such as choice of lubricant, bearing material specifications, bearing life, cost, bearing alignment, positioning precision, direction and magnitude of loads, bearing ambient pressure, supply pressure, flow rate available from the system, heat flow etc. establish various requirements for the bearing design. Definitions of both, the range of imposed bearing requirements and bearing performance limitations, are required to assure the compatibility of the bearing and its design. To prepare a bearing that will reliably meet performance requirements with the specific maintenance is really a difficult task. Following parameters are required to design and analyze the bearings.

1) Lubricant flow in the bearing
2) Lubricant side leakage from the bearing
3) Pressure distribution in the film region
4) Load carrying capacity
5) Centre of pressure (or attitude angle in case of journal bearing)
6) Friction force (or coefficient of friction)

7) Film stiffness

8) Squeeze film versus film thickness relationship (in case of squeeze film bearing)

9) Range for stability of the bearing both for initial velocity disturbances and initial position disturbances.

etc.

2.6 Brief History about Development of Ferrofluids

The first documented attempt to produce ferrofluid (FF) or magnetic fluid (MF) was made by Gown knight in 1779. He tried to suspend fine iron fillings in water. But this attempt was unsuccessful. The next noteworthy attempt was made by Bitter in 1940. He prepared a magnetic colloid which was stable under gravity, but the colloid losses its stability in a magnetic field. Bitter exploited this magnetic field induced instability to observe the domain boundaries on the surface of ferromagnetic material. Subsequently, Elmore refined the Bitter colloid and studied its physical properties. In such colloids number concentration of magnetic particles were small. So, maximum magnetization that can be achieved was also very low. During this period magnetic clutch 'slurry' was also developed. Such slurry, composed of micron size iron particles dispersed in oil, solidifies in the presence of an applied magnetic field. In short, the slurry loses its flow ability in magnetic field and becomes highly viscous. Ideas for a FF seal were also mooted at this time. The requirement of FFs whose viscosity were not affected by magnetic fields limited the progress.

A real breakthrough was achieved when Solman S. Papell in 1963 whilst working at 'NASA' (National Aeronautics and Space Administration) devised a method of stabilizing MFs against aggregation. His main interest was to mix this MF with rocket fuel so that it can be pumped in zero gravitational fields by means of externally applied magnetic field. The MF synthesized by

Papell exhibited a large saturation magnetization (i.e. 100 Gauss) and its viscosity remained low even in presence of magnetic field. Papell's method of preparation consisted of ball milling magnetite in a liquid in the presence of stabilizing surfactant until the magnetite was in colloid state. His fluid consisted of finely divided particles of magnetite in kerosene. To keep the particles from clumping together, Papell used oleic acid as dispersing agent. At the same time R. E. Rosensweig and his associates were also working on synthesis of FFs. Whilst working at AVCO-USA, they succeeded in making FFs that were about ten times as strong as Papell's original FF. The particle size in their fluid is of the order of 100 A° and it did not congeal even when subjected to a strong magnetic field. It would become magnetized but it remained a fluid. Rosensweig and Neuringer were the first two, who proposed a mathematical model of this fluid, consequently a new branch of fluid mechanics now known as ferrohydrodynamics is established.

FF industry was also first established by Rosensweig and his associate. Moskowitz founded in 1969 ferrofluidics corporation, U.S.A., is still at present a major source of MFs and devices based on FFs. It has its branches in France, Japan and Germany. Apart from this industry many other industries also manufacture several types of FFs and ferro fluidic devices. Some of these are Servoflow; U.S.A., Georgia Pacific; U.S.A., Matsumoto; Japan, Toiho industries; Japan and U.S.S.R.

In India too, an increasing trend of research interest in FF has been evident through the pioneering contribution by Mehta *et.al.* [17]. Recent contributions from diversified viewpoints are due to Malik *et. al.* [18] and Vaidyanathan *et. al.* [19].

- ## Brief Idea About Ferrofluids

A MF is a colloid consisting of submicron sized single domain ferromagnetic particles dispersed in a liquid called carrier liquid with a suitable surfactant added so as to prevent agglomeration and sedimentation. Thus it is a multi-component (solid magnetite particles, surfactant and carrier liquid) and two phase (solid-liquid) system.

It is noted that most of the applications in various branches of engineering and science requires the stable MF against gravitational and gradient magnetic fields. Also it should be stable against agglomeration which arise from magnetic and London type van der Waals forces. It is observed that by making a small size of the magnetite particle and choosing a suitable surfactant, it is possible to prepare a stable FF. Of course, for a specific application the criteria for selecting the various parameters such as carrier liquids, concentration, size and composition of particles, particles coating etc. are different.

Thus, FF is a multi-component and multiphase system having essential magnetic properties and fluidity.

- ## About Ferrofluid Effect

FFs or MFs [20] are stable colloidal suspensions containing fine ferromagnetic particles dispersing in a liquid, called carrier liquid, in which a surfactant is added to generate a coating layer preventing the flocculation of the particles. When an external magnetic field $\mathbf{H}$ is applied, FFs experience a magnetic body forces $(\mathbf{M} \bullet \nabla)\mathbf{H}$ which depends upon the magnetization vector $\mathbf{M}$ of ferromagnetic particles. Owing to these features FFs are useful in many applications like in sensors, sealing devices, filtering apparatus, elastic damper, bearings, etc. [21-25]. The application of FF as lubricant is also one of them.

- **Fluid Dynamics Involving Ferrofluid**

In (classical) ordinary fluid dynamics there are only three forces viz. (i) the pressure gradient (ii) gravity force (iii) the viscous force which have been taken into account an accordingly the equation of motion is stated as the sum of the gradients of all those forces remains equal to rate of change of velocity multiplied by the density.

Whereas in case of ferrohydrodynamics the magnetic body force also acts along with four interparticle forces viz. magnetic attraction, van der Waals attraction, steric repulsion and electric repulsion. Among which we have considered only two namely magnetic body force and magnetic attraction, since magnetic attraction is a long range potential and varies very slowly with distance.

The magnetic body force originates from the interaction of magnetic field *H* with the magnetization of the fluid and is of great importance regarding all further investigation [26].

2.7 Basic Flow Equations for the Present Study

The basic flow equations used in our work based on R. E. Rosensweig model are given by [27]

Equation of Motion

$$\rho \left[\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \bullet \nabla)\,\mathbf{q} \right] = -\nabla p + \eta \nabla^2 \mathbf{q} + \mu_0 (\mathbf{M} \bullet \nabla)\mathbf{H}$$

… (2.18)

Equation of Continuity

$$\nabla \bullet \mathbf{q} = 0$$

… (2.19)

Equations of Electromagnetic field

$$\nabla \times \mathbf{H} = 0$$

… (2.20)

$$\mathbf{M} = \bar{«}\,\mathbf{H}$$

… (2.21)

$$\nabla \bullet (\mathbf{H} + \mathbf{M}) = 0$$

… (2.22)

where ° is fluid density, p fluid film pressure, η is fluid viscosity, $\mathbf{q}$ is fluid velocity vector in film region, $«_0$ is free space permeability, $\mathbf{M}$ is the magnetization vector, $\mathbf{H}$ is magnetic field vector, $\bar{«}$ is magnetic susceptibility and t is time.

In the equation (2.18), the term $«_0 (\mathbf{M} \bullet \nabla)\mathbf{H}$ is due to the effect of ferrofluid. Due to addition of this term, the increase of magnetic pressure in our problems is observed.

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