

7.1 GENERAL REMARKS

The rapid growth of urban population and limited space has increased the number of high-rise buildings. In the design of high-rise buildings it is very important to consider the effects of lateral loads developed mainly due to earthquake and wind. To develop efficient structural system, it is necessary to understand the behavior of structure under these forces, which are dynamic in nature. The natural frequency of a structure is important property of structure that governs the behavior of structures under dynamic forces. Also the calculation of earthquake forces require time period of building. The time period or natural frequency can be calculated by considering motion of structure under no external force. The solution of eigen problem which involves stiffness and mass matrices gives natural frequency of structure.

For large size structure calculation of natural frequencies, through eigen value analysis using mass and stiffness property of structure, consumes more computational time. To reduce the computational time use of parallel or distributed computing can be done using supercomputers, transputers or network of high performance workstations. Recent advances in networking and communication technology has shifted the trend towards distributed computing using network of personal computers and in that one simpler way is client server approach in which the entire process is divided in to small tasks and communication between tasks can be done through file transfer, which has a benefit of not requiring any message passing functions in the program.

Substructure technique can be used for calculating natural frequency of structure. Substructure technique involves dividing entire structure in to number of smaller parts, which are known as substructures. The stiffness and mass matrices are calculated independently, corresponding to boundary nodes, for each substructure. Subsequently stiffness and mass matrices corresponding to boundary nodes of all substructures are assembled. Finally eigen value analysis of reduced equations is carried out to get the natural frequency of whole structure. In distributed implementation stiffness and mass matrices of

substructures are calculated on different computers in parallel using static condensation technique and are subsequently assembled for eigen value analysis using inverse iteration method. In substructure technique, stiffness and mass matrices corresponding to boundary nodes can be calculated by static or dynamic condensation method [117]. As the static condensation method is one-step method and dynamic condensation is iterative method, it is easier to implement static condensation method on distributed computing environment.

In this chapter dynamic analysis of multistorey building frame is implemented on distributed computing environment. Substructure technique is used for calculating natural frequency of plane frame. The data corresponding to each substructure are sent from master to number of slave computers where stiffness and mass matrices corresponding to boundary nodes are calculated concurrently. The stiffness and mass matrices are assembled on master computer and eigen value analysis of reduced equations is carried out to calculate natural frequency of entire structure. For mass matrix calculation lumped mass approach is used.

For distributed computing WebDedip is used where distributed application is implemented over Local Area Network. For illustration of use of this distributed data processing (DDP) environment in distributed dynamic analysis, an example of plane frame with 300 storey and 10 bays is selected.

7.2 DYNAMIC ANALYSIS FORMULATION

The analysis of structure in free motion i.e. the structure is not subjected to any external excitation and its motion is governed by initial conditions only, provides the most important dynamic properties of the structure which are the natural frequencies and the corresponding modal shapes. The equation of motion under free vibration can be written as,

$$[M]\{\ddot{y}\} + [K]\{y\} = \{0\} \qquad \dots (7.1)$$

The solution will be in the form of

$$y_i = a_i \sin(\omega t - \alpha), \quad i = 1, 2, \dots, n \qquad \dots (7.2)$$

where a_i is the amplitude of motion of the i^{th} coordinate and n is the number of degrees of freedom.

The substitution of Eq. (7.2) in (7.1) and rearranging terms gives,

$$[[K] - \omega^2[M]] \{a\} = \{0\} \quad \dots (7.3)$$

which is a set of n homogeneous algebraic system of linear equations with n unknown displacements a_i and an unknown parameter ω^2 . The solution for which not all $a_i = 0$, requires determinant of the matrix factor of $\{a\}$ be equal to zero.

$$| [K] - \omega^2[M] | = 0 \quad \dots (7.4)$$

This results in a polynomial equation of degree n in ω^2 which should be satisfied by n values of ω^2 , which is known as eigen value. From eigen value natural frequency and time period can be calculated as,

$$f = \omega/2\pi \text{ and } T = 2\pi/\omega \quad \dots (7.5)$$

To derive stiffness and mass matrices corresponding to boundary nodes of substructure, equation of motion is to be written in terms of degrees of freedom corresponding to boundary node $\{y_b\}$ and internal node $\{y_i\}$ as

$$\left(\begin{array}{c|c} [M_{ii}] & [M_{ib}] \\ \hline [M_{bi}] & [M_{bb}] \end{array} \right) \begin{Bmatrix} \{\dot{y}_i\} \\ \{\dot{y}_b\} \end{Bmatrix} + \left(\begin{array}{c|c} [K_{ii}] & [K_{ib}] \\ \hline [K_{bi}] & [K_{bb}] \end{array} \right) \begin{Bmatrix} \{y_i\} \\ \{y_b\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \quad \dots (7.6)$$

The substitution of $\{y\} = \{Y\} \sin \omega_j t$ results in the generalized eigen problem,

$$\left(\begin{array}{c|c} [K_{ii}] - \omega_j^2 [M_{ii}] & [K_{ib}] - \omega_j^2 [M_{ib}] \\ \hline [K_{bi}] - \omega_j^2 [M_{bi}] & [K_{bb}] - \omega_j^2 [M_{bb}] \end{array} \right) \begin{Bmatrix} \{y_i\} \\ \{y_b\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \quad \dots (7.7)$$

where ω_j^2 is the approximation of the j^{th} eigen value.

The approximate value of ω_j^2 is taken as zero initially so equation will become

$$\left(\begin{array}{c|c} [K_{ii}] & [K_{ib}] \\ \hline [K_{bi}] & [K_{bb}] \end{array} \right) \begin{Bmatrix} \{y_i\} \\ \{y_b\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \quad \dots (7.8)$$

By applying Gauss Jordan method and eliminating degrees of freedom corresponding to internal nodes $\{y_i\}$ gives

$$\left[\begin{array}{c|c} [I] & -[T_j] \\ \hline [0] & [D_j] \end{array} \right] \begin{Bmatrix} \{y_i\} \\ \{y_b\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \quad \dots (7.9)$$

where $[I]$ is identity matrix, $[0]$ is zero matrix, $[T_j]$ is transfer matrix and $[D_j]$ is reduced dynamic matrix.

From Eq. (7.9),

$$\{y_i\} - [T_j] \{y_b\} = \{0\} \text{ OR } \{y_i\} = [T_j] \{y_b\} \quad \dots (7.10)$$

Consequently $\{Y\}$ can be expressed as $\{Y\} = [T_j'] \{y_b\} \quad \dots (7.11)$

$$\text{where } [T_j'] = \begin{Bmatrix} -[T_j] \\ [I] \end{Bmatrix} \text{ and } \{Y\} = \begin{Bmatrix} \{y_i\} \\ \{y_b\} \end{Bmatrix} \quad \dots (7.12)$$

The reduced mass and stiffness matrices are calculated as

$$[M_j'] = [T_j']^T [M_j] [T_j'] \quad \text{and} \quad [K_j'] = [D_j] - \omega_j^2 [M_j'] \quad \dots (7.13)$$

where the transformation matrix $[T_j']$ is given by Eq. (7.12) and reduced dynamic matrix $[D_j]$ is obtained from Eq. (7.9). The reduced problem is in following form.

$$[[K_j'] - \omega_j^2 [M_j']] \{y_b\} = 0 \quad \dots (7.14)$$

The eigen solution of Eq. (7.14) gives frequency ω_j^2 and corresponding mode shape coefficient vector $\{y_b\}_j$ for reduced system. From $\{y_b\}_j$, mode shape coefficients for entire system can be determined by,

$$\{Y\}_j = [T_j'] \{y_b\}_j \quad \dots (7.15)$$

In static condensation method value of ω_j^2 is assumed as zero and reduced stiffness and mass matrices are calculated based on Eqs. (7.9), (7.12) and

(7.13). Solution of reduced equation gives natural frequency and mode shape coefficients.

The dynamic condensation procedure is iterative in nature in which value of ω_j^2 obtained from above procedure is substituted again in Eq. (7.7). Subsequently revised transformation matrix is calculated by Eq. (7.12) and revised reduced stiffness and mass matrices are calculated by Eq. (7.13) to get new value ω_{j+1}^2 . This three-step process is applied iteratively. From experience it is observed that within one or two iteration virtually exact eigen solution is obtained.

Substructure technique for dynamic analysis is implemented for plane frame problems. Dynamic analysis of plane frames using stiffness method requires calculation of stiffness and mass matrices. The stiffness matrix of plane frame member with two translational degrees of freedom and one rotational degree of freedom can be determined easily by direct stiffness approach [103] and can be written as

$$[k] = \frac{EI}{L^3} \begin{pmatrix} AL^2/I & 0 & 0 & -AL^2/I & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -AL^2/I & 0 & 0 & AL^2/I & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{pmatrix} \quad \dots (7.16)$$

where E is modulus of elasticity of material, A is cross sectional area, L is length of member and I is moment of inertia of member.

The lumped mass matrix $[M]_l$ and consistent mass matrix $[M]_c$ along member axis are given by [117],

$$[M]_l = \frac{mL}{2} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad [M]_c = \frac{mL}{420} \begin{pmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{pmatrix} \quad \dots (7.17)$$

where m is the mass per unit length of the member.

The stiffness and mass matrices can be transformed to structural axis by using rotation transformation matrix as,

$$[k]_s = [R]^T [k] [R] \quad \text{and} \quad [M]_s = [R]^T [M] [R] \quad \dots (7.18)$$

where $[R]$ is rotation transformation matrix as given by

$$[R] = \begin{pmatrix} c_x & c_y & 0 & 0 & 0 & 0 \\ -c_y & c_x & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_x & 0 & 0 \\ 0 & 0 & 0 & -c_y & c_x & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \dots (7.19)$$

Where $c_x = (x_k - x_j) / L$, $c_y = (y_k - y_j) / L$, $L = \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2}$, (x_k, y_k) and (x_j, y_j) are coordinates of k and j end of members respectively.

7.3 SUBSTRUCTURE TECHNIQUE IN DYNAMIC ANALYSIS

Steps for implementing substructure technique for dynamic analysis of plane frame are as follows:

1. Divide the entire structure in to various substructures such that degrees of freedom in each substructure is more or less same.
2. Form the stiffness and mass matrices corresponding to boundary nodes. For this assemble the stiffness and mass matrices corresponding to each member. Then eliminate the degrees of freedom corresponding to internal nodes by Gauss elimination method from stiffness matrix and obtain stiffness matrix corresponding to boundary nodes and transformation matrix as described above. This transformation matrix is used to find mass matrix corresponding to boundary nodes.
3. Assemble the stiffness and mass matrices of all substructures to find overall stiffness and mass matrices.
4. Obtain eigen values and eigen vectors by applying inverse iteration method from reduced stiffness and mass matrices.

The above procedure is implemented to regular two-dimensional rigid jointed frames. The data for various examples is described in Table 7.1 and Fig. 7.1. The spacing between plane frames in each example is assumed as 3.0 m and thickness of slab is assumed as 120 mm. So, each beam is subjected to (0.12 m

$\times 25 \text{ kN/m}^3 \times 3\text{m}) / 9.81\text{m}^2/\text{sec} \approx 1 \text{ kN-sec/m}^2$ mass per meter length in addition to mass due to self-weight. As, it is observed that the frequency calculated by lumped mass approach and consistent mass approach are almost same, lumped mass approach is used in the calculations.

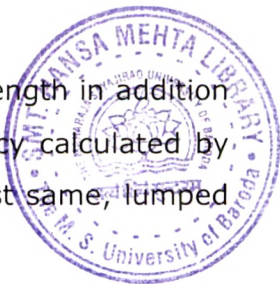


TABLE 7.1 DATA FOR VARIOUS EXAMPLES

Ex. No.	No. of storey	Height of storey (m)	No. of Bays	Width of bay (m)	Size of beam (m × m)	Size of column (m × m)	Additional mass at floor level (kN-sec/m ²)
1	10	4	5	5	0.3 × 0.5	0.3 × 0.6	1.0
2	20	4	5	5	0.3 × 0.5	0.3 × 0.8	1.0
3	30	4	5	5	0.3 × 0.5	0.5 × 0.9	1.0
4	40	4	5	5	0.3 × 0.5	0.6 × 1.0	1.0
5	50	4	5	5	0.3 × 0.5	0.6 × 1.2	1.0

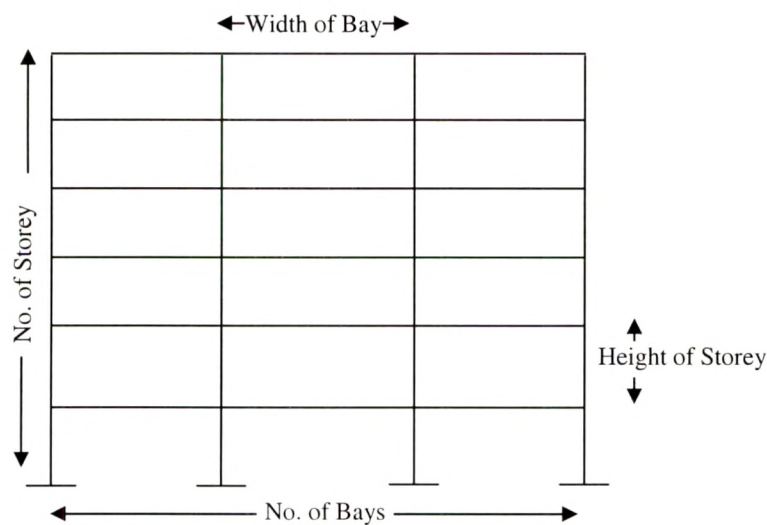


FIG. 7.1 WHOLE STRUCTURE

The building frame of each example is subdivided into various numbers of substructures. Typical detail of substructures is shown in Fig. 7.2. The stiffness and mass matrices of all substructures are assembled and eigen values and corresponding natural frequencies for first three modes are calculated. For regular structures the contribution of mass in first mode is maximum and it is recommended by IS 1893 [118] to consider number of modes when about 90% mass is participating. So, here only first three modes have been considered.

These natural frequencies are compared with natural frequencies calculated by considering full structure. The comparison of results obtained using method of static condensation is given in Table 7.2. Where values shown in bracket indicates % error.

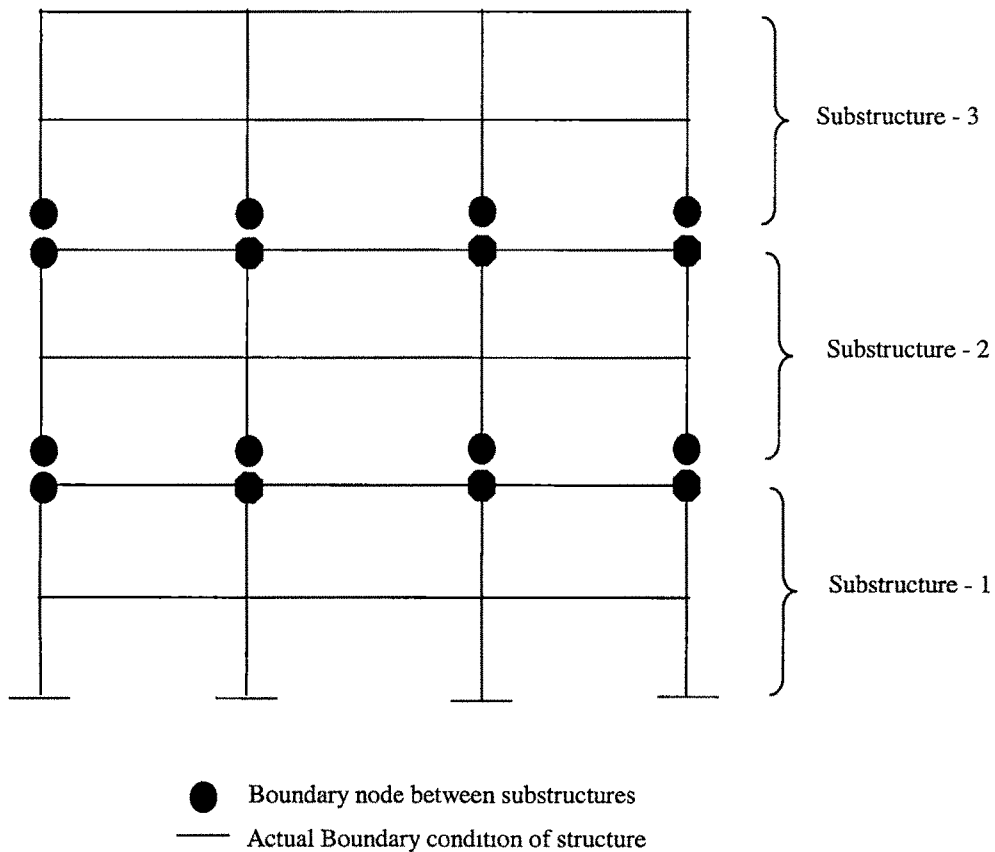


FIG. 7.2 PLANE FRAME DIVIDED INTO SUBSTRUCTURES

From Table 7.2 it is observed that first frequency matches closely with the exact value for all examples when different numbers of substructures are considered. For second mode and third mode the variation in frequency is larger compared to exact value when entire structure is divided into only two substructures. But as number of substructures increases, the difference goes on reducing. It is also observed from Table 7.2, that when structure is divided into five substructures, the frequency matches closely with the exact value. In the present study, dynamic condensation technique is also applied when % error is very large i.e. frequency of third mode when entire structure is divided into only two substructures. Within two iterations, dynamic condensation gives almost exact

value. But as shown in Fig. 7.3, when higher number of substructures is considered, % error is very small and so only static condensation technique is also sufficient.

TABLE 7.2 COMPARISON OF NATURAL FREQUENCY OF FIRST 3 MODES

Ex. No.	Mode No.	Frequency of whole structure (hz)	Frequency of structure (Hz) considering number of substructures			
			2	3	4	5
1	1	0.7501	0.7767 (3.55)	0.7545 (0.59)	0.754 (0.52)	0.7506 (0.07)
	2	2.306	2.531 (9.76)	2.37 (2.78)	2.338 (1.39)	2.322 (0.69)
	3	4.038	7.659 (89.67)	4.246 (5.15)	4.174 (2.70)	4.120 (2.03)
2	1	0.384	0.3894 (1.41)	0.3865 (0.65)	0.3848 (0.21)	0.3844 (0.10)
	2	1.181	1.286 (8.89)	1.223 (3.56)	1.203 (1.86)	1.192 (0.93)
	3	2.084	4.128 (98.08)	2.209 (6.00)	2.193 (5.23)	2.143 (2.83)
3	1	0.2372	0.2407 (1.48)	0.2382 (0.42)	0.2377 (0.21)	0.2372 (0.00)
	2	0.7329	0.7998 (9.13)	0.7639 (4.23)	0.7477 (2.02)	0.7399 (0.96)
	3	1.303	2.557 (96.24)	1.405 (7.83)	1.358 (4.22)	1.34 (2.84)
4	1	0.1656	0.1678 (1.33)	0.1664 (0.48)	0.166 (0.24)	0.1655 (-0.06)
	2	0.5128	0.560 (9.20)	0.5345 (4.23)	0.523 (1.99)	0.518 (1.01)
	3	0.9190	1.829 (99.02)	0.9855 (7.24)	0.9709 (5.65)	0.9471 (3.06)
5	1	0.1248	0.1264 (1.28)	0.1252 (0.32)	0.1249 (0.08)	0.1247 (-0.08)
	2	0.3899	0.424 (8.75)	0.405 (3.87)	0.3973 (1.90)	0.3935 (0.92)
	3	0.7057	1.38 (95.55)	0.754 (6.84)	0.7412 (5.03)	0.7282 (3.19)

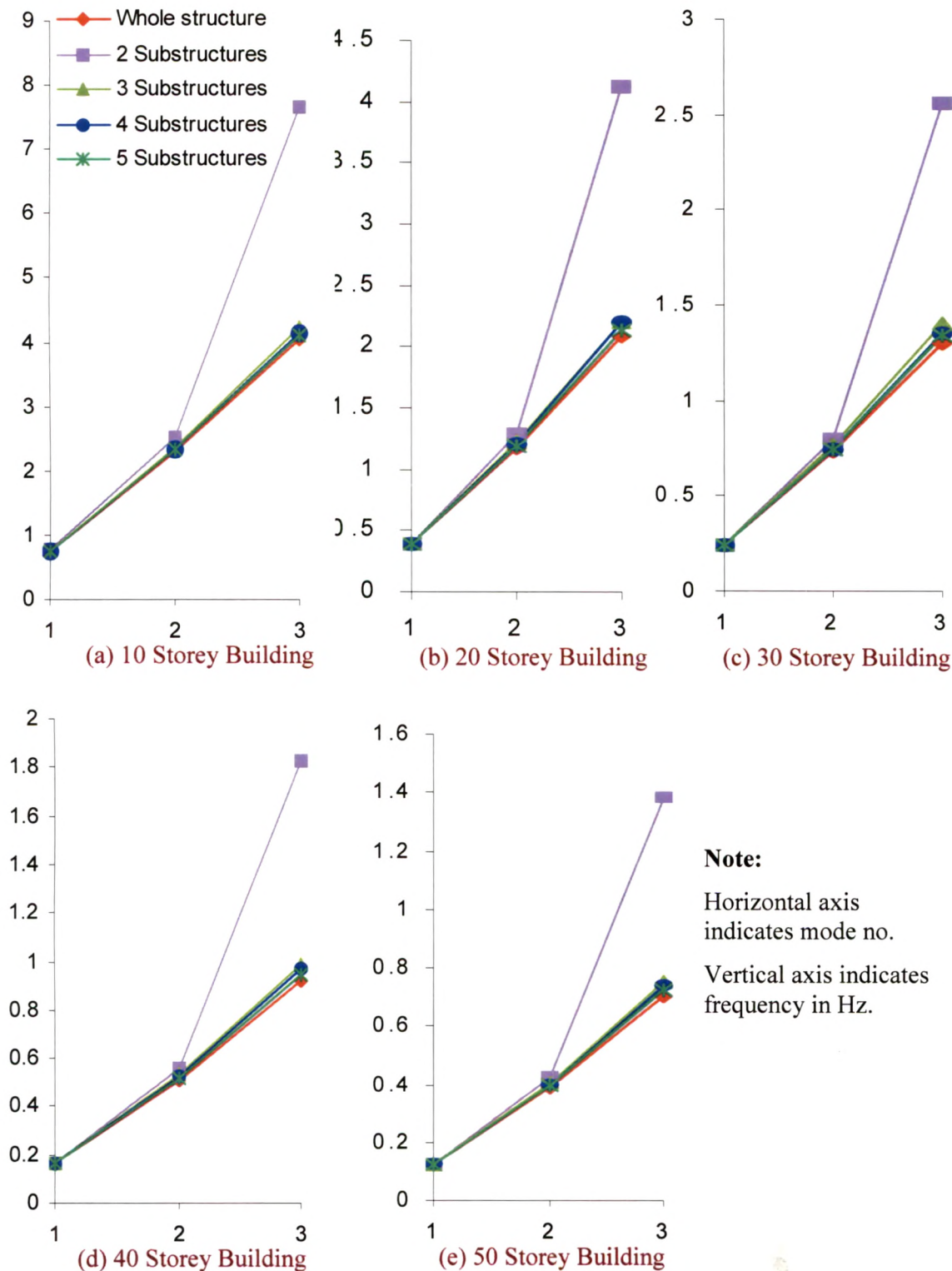


FIG. 7.3 COMPARISON OF FIRST 3 MODE FREQUENCY WITH DIFFERENT SUBSTRUCTURES

7.4 IMPLEMENTATION ON DDP ENVIRONMENT

To implement dynamic analysis of plane frames on WebDedip / DDP environment, the entire process is divided into number of tasks, which can run on different computers, and communication between them is carried out by intermediated files using FTP. Distributed dynamic analysis is divided into following three tasks.

The first task DPFY1 reads the structural data and depending on number computers to be used, decomposes the entire structure into number of substructures and prepares the data file for each substructure. This task is carried out on master computer where the data of entire structure is entered. The data files corresponding to each substructure are communicated to different computers for next task.

The second task DPFY2 evaluates the substructure stiffness and mass matrices using static condensation procedure as described in earlier section. This task is done on various slave computers concurrently. For calculation of mass matrix lumped mass approach is used. However in program provision is made to consider consistent or lumped mass approach. After calculation of stiffness matrix and mass matrices corresponding to boundary nodes of substructure they are written in intermediate file which can be transferred to master computer for next task. As for static condensation procedure internal nodes are numbered first and then the boundary nodes. The stiffness and mass matrices becomes populated and bandwidth increases, and therefore for calculation purpose stiffness and mass matrices are stored in square matrix form. But in intermediate file only upper triangular part is written to keep size of file smaller for transferring data.

The third task DPFY3 receives substructure stiffness and mass matrices from slave computers and assembles them depending on degrees of freedom of boundary nodes. This task assembles matrices in banded form to have low memory requirement and process can be made faster. Subsequently applying inverse iteration method, it evaluates eigen values and eigen vectors. Finally eigen values are written to output file. Fig. 7.4 shows implementation of dynamic analysis on distributed computing environment.

WebDedip / DDP environment gives facility to configure application over different computers. Configuration consists information about i.e. IP address of computer on which it will run and name of process on which it is depending and name of dependent processes. Also information about communication i.e. DTHS (Data Transfer from Host to Slave) in form of source process and destination process are to be entered.

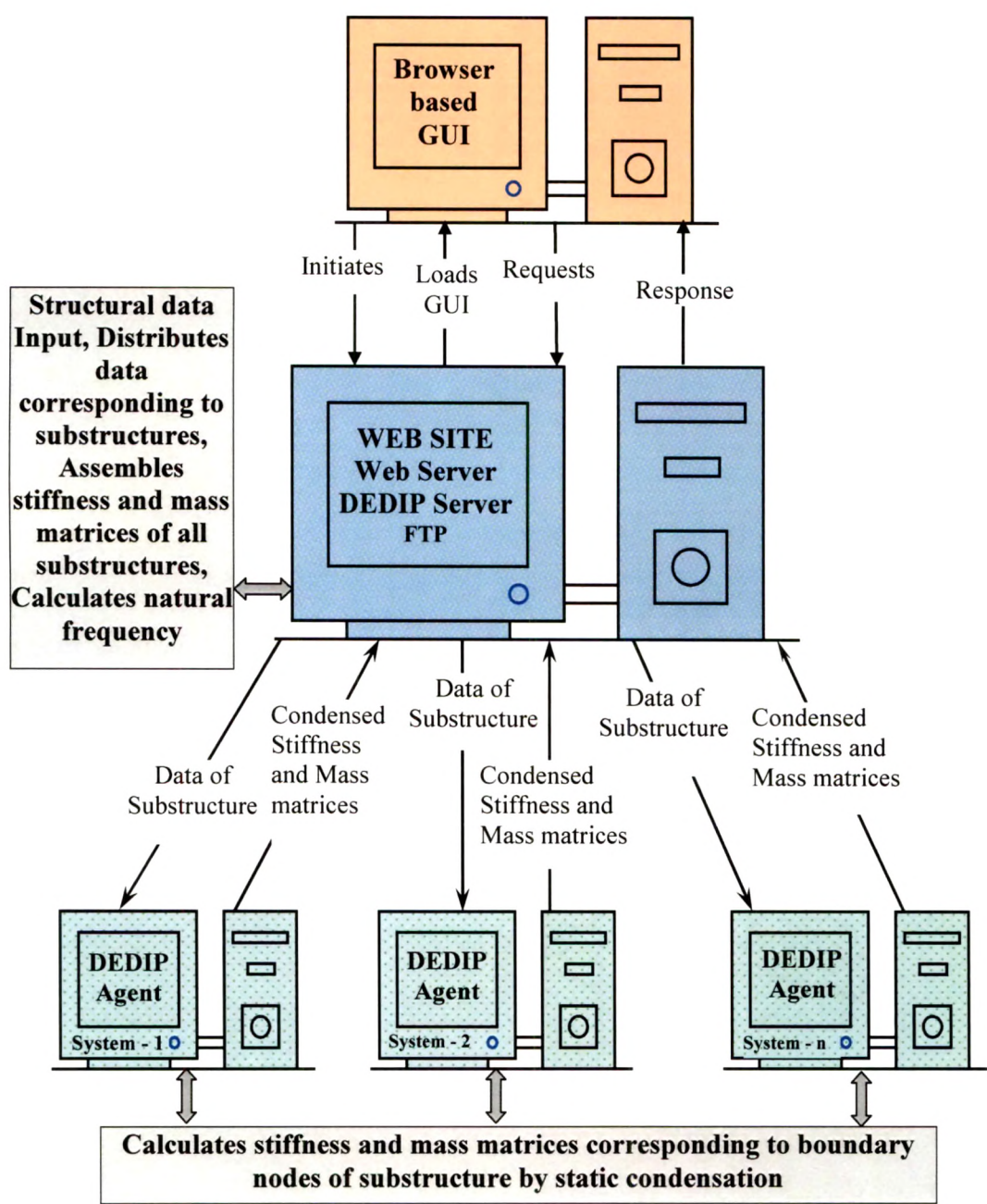


FIG. 7.4 IMPLEMENTATION OF DISTRIBUTED DYNAMIC ANALYSIS ON DDP

The various screen shots for configuring distributed dynamic analysis application using five computers are reproduced in Fig. 7.5.

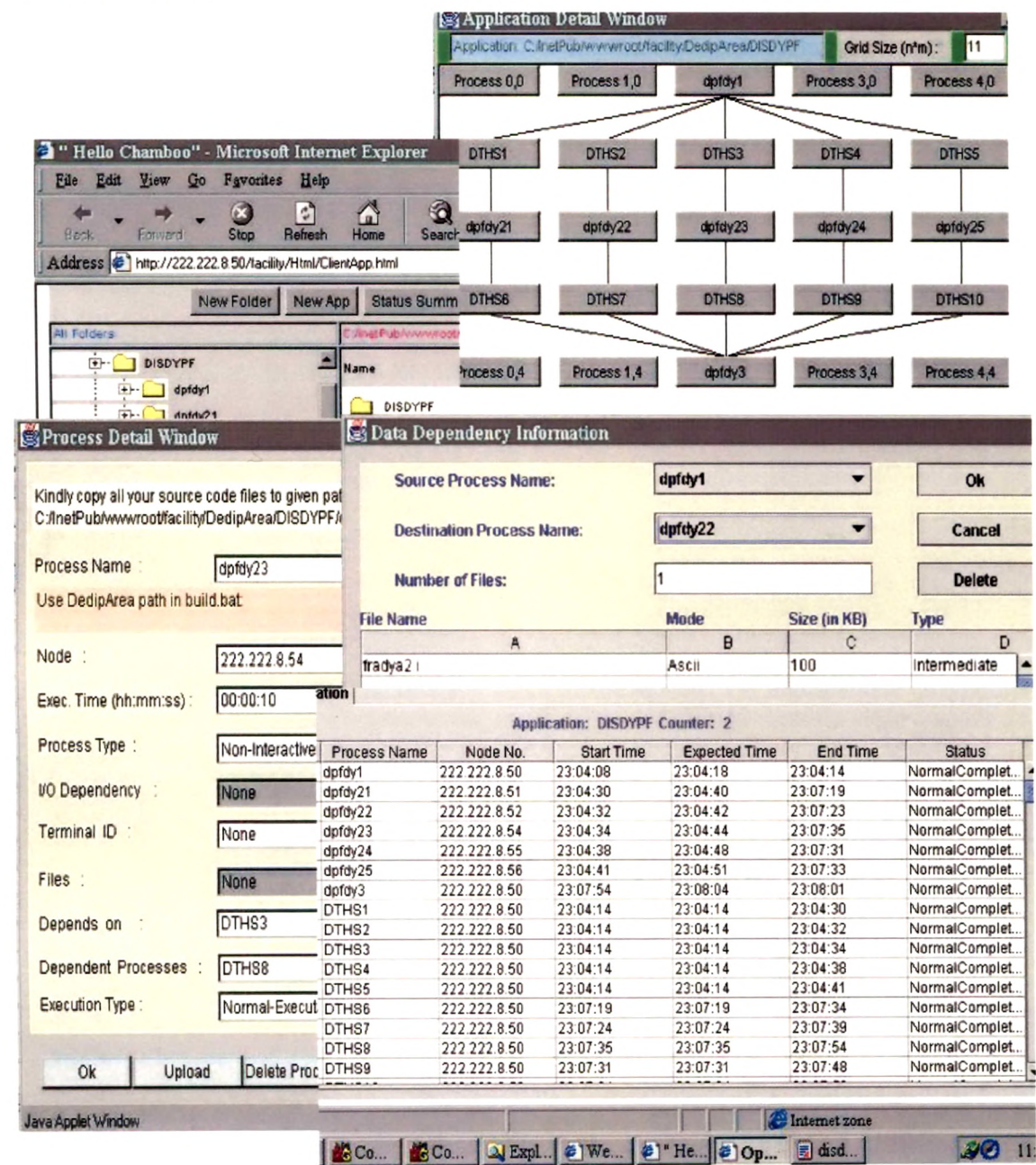


FIG. 7.5 SCREEN SHOTS TO CONFIGURE APPLICATION ON FIVE COMPUTERS

7.5 A 300 STORIED FRAME EXAMPLE

To illustrate the implementation, a plane frame having 300 stories with storey height 4.0 m and 10 bays with width of each bay 5.0 m has been taken as shown in Fig. 7.6. The size of columns is 1.0 m \times 2.0 m and that of beams is 0.3 m \times

0.6 m. The spacing between plane frames is assumed as 3.0 m and thickness of slab is assumed as 120 mm. So, each beam is subjected to $(0.12 \text{ m} \times 25 \text{ kN/m}^3 \times 3\text{m}) / 9.81\text{m/sec}^2 \approx 1 \text{ kN-sec/m}^2$ mass per meter length in addition to mass due to self-weight. Lumped mass approach is used in the calculations. The problem consists of 6300 members and 3311 nodes. As in rigid jointed plane frame structure, three degrees of freedom (two translational and one rotational) are considered, the problem is having thus a total of 9933 degrees of freedom.

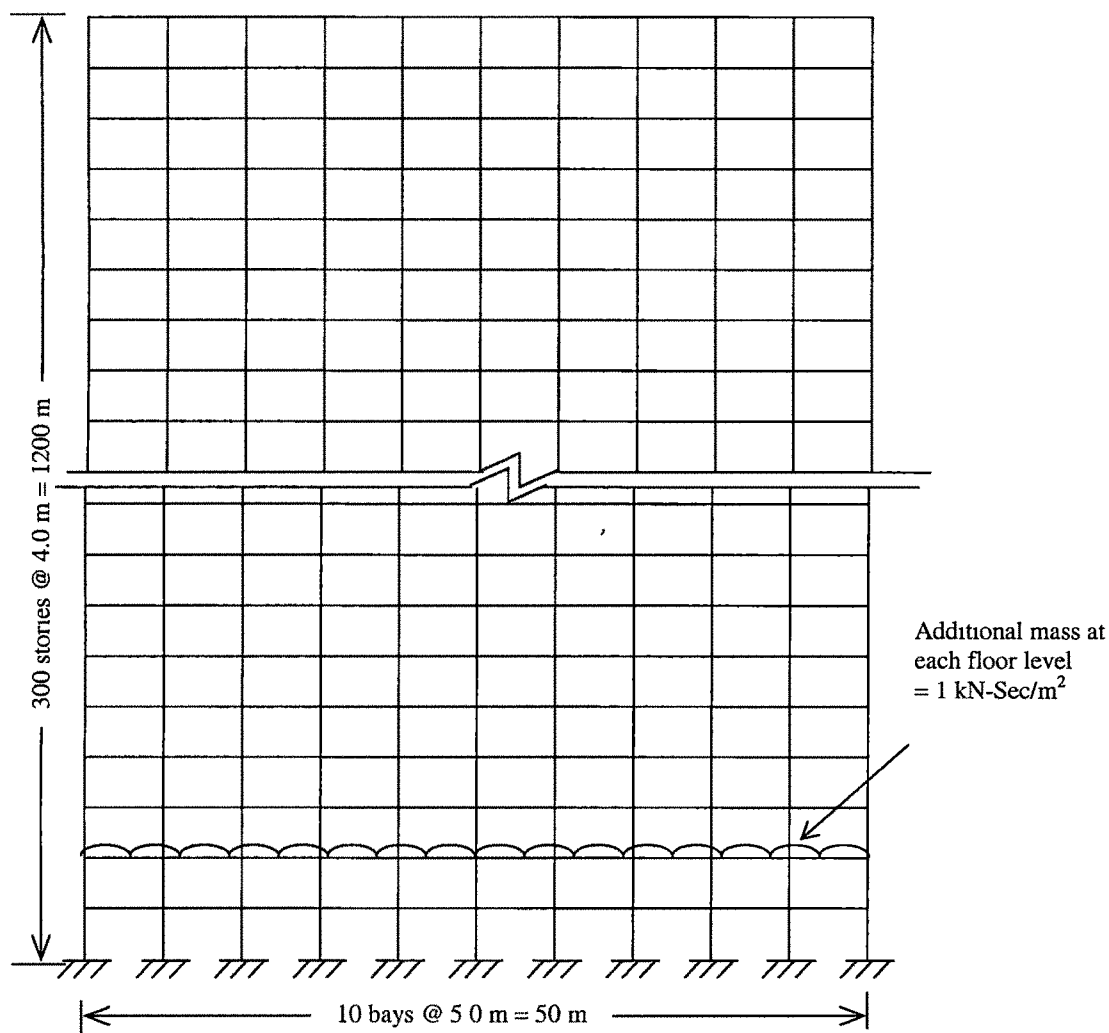


FIG. 7.6 A 300 STORIES FRAME EXAMPLE

The entire frame is divided in to various numbers of substructures for calculation of natural frequencies. As the building is regular, frequencies of first five modes are considered. The comparison of natural frequency when distributed over various numbers of computers is shown in Table 7.3. From the table it can be observed that, frequency of first mode closely matches when number of substructures are considered with that of entire structure. The frequency of

entire structure is also verified with commercial packages like SAP2000 and STAAD/Pro. For higher modes the frequency of whole structure matches with increasing number of substructures.

TABLE 7.3 COMPARISON OF FREQUENCY WITH DIFFERENT SUBSTRUCTURES

Mode no.	Frequency of whole structure (Hz)	Frequency of structure (Hz) considering number of substructures			
		2	3	4	5
1	0.0134	0.0140	0.0135	0.0132	0.0133
2	0.0487	0.0525	0.0506	0.0498	0.0493
3	0.0975	0.2273	0.1080	0.1056	0.1026
4	0.1437	0.5812	0.3070	0.1593	0.1581
5	0.1911	0.5903	0.4640	0.3770	0.2103

The DDP has facility, which gives the summary of application indicating the status of various processes i.e. node number (IP address), start time and end time. If any error occurs in process or if process could not be completed, the same can be checked through Operator Console of DDP. The time required for computation and communication can be obtained from this summary when process is distributed over number of computers. In this implementation number of slave computers is kept same as number of substructures.

The time spent in various processes and in communication and its comparison with sequential processing is reported in Table 7.4. The sequential time is obtained by processing all substructures on single computer in sequence. In this implementation data files corresponding to substructures and substructure stiffness and mass matrices need not required to be prepared. Also there are no different processes like that for preparing substructure data, calculating substructure stiffness and mass matrices and assembly as in case of distributed implementation. So the time taken by sequential process is quiet less than sum of time taken by all processes in distributed implementation. When substructures are distributed on different computers, time for calculating substructure stiffness and mass matrices for same size of substructure is different on different computers, even though computers are of identical configuration. This is because of local services running on computers connected through network.

TABLE 7.4 COMPARISON OF TIME REQUIRED WITH DIFFERENT COMPUTERS

Number of Substructures and Computers			2	3	4	5
Time (Sec) for Sequential Processing			5250	1872	1054	725
Time (Sec) for Distributed Processing	Computation Time	DPFDY1	6	6	6	6
		DPFDY2	2997	738	329	181
		DPFDY3	7	7	7	7
	Communication Time		40	43	44	46
	Total Time		3050	794	386	240
Observed Speedup			1.72	2.36	2.73	3.02
Ideal Speedup			2	3	4	5

The timings are observed when application is implemented over Local Area Network of Pentium IV computers with 256 MB RAM and running at 1.8 GHz speed. The communication time increases marginally with increase in number of substructures or computers. This indicates that communication time is not directly proportional to size of file transferred. The reason for the same can be use of FTP for communication. In FTP for communication between computers first authentication of user is done and subsequently file is transferred through socket. So actual file transfer time in 100 Mbps Ethernet network is very less but authentication time is more.

From the observation of time, the speed-up is calculated. Speed-up is the ratio of time required for sequential processing done on single computer to time required by distributed processing using number of computers. Ideal speed-up is equal to number of computers used. But due to time involved in communication of data between the computers, observed speed-up is lower than ideal speed-up. The speed-up observed is shown in Fig. 7.7.

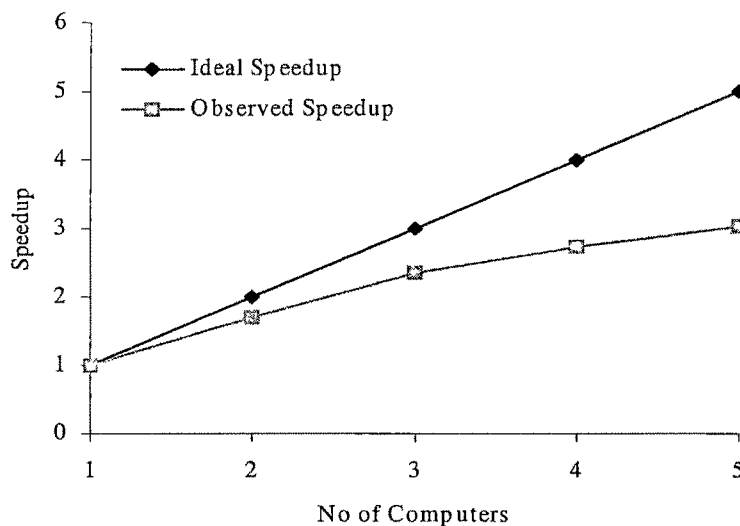


FIG. 7.7 COMPARISON OF TIME SPEEDUP

7.6 CLOSING REMARKS

In this chapter application of distributed processing was explored in the field of dynamic analysis. A dynamic analysis of plane frames, for calculation of natural frequency, was implemented over LAN using WebDedip / DDP environment. Substructure technique was used for distributed implementation. For deriving substructure stiffness and mass matrices, static and dynamic condensation techniques can be used. But through examples it was observed that with more number of substructures static condensation gives almost same results compared to that obtained by considering whole structure. In this coarse grain implementation, good computational efficiency of 60-80% was observed for a moderate size of problem.