11. DISTRIBUTED FE ANALYSIS OF LAMINATED COMPOSITES

11.1 OPENING REMARKS

In recent years, continuous fibre reinforced laminated composite plates have been increasingly used as structural elements in civil, military and aerospace fields because of their desirable properties such as high strength-to-weight ratio, high stiffness-to-weight ratio etc. In addition there exists the possibility of optimum structural design through the variation of fibre orientation, stacking sequence and choice of fibre and matrix materials. By using composites as a structural material, the weight of a structure can be reduced drastically by as much as 35% [127]. A multiphase or two material laminae consists of a stiff filament material embedded in a compatible matrix material. Examples of filaments are glass, boron, carbon, graphite, and steel whereas matrix materials have included polyesters, aluminum, and epoxies. Fibre reinforced composite have potential to become very useful as a structural material in lightweight construction except their analysis is complex. Unlike isotropic materials, composite materials exhibit low out-of-plane moduli relative to in-plane moduli. This may result in quite considerable transverse shear deformation and transverse normal stresses and strains, which can significantly influence the response and the failure mechanisms of such laminated anisotropic composite plates. A number of theories including the effects of transverse shear deformation and transverse normal stress and strains have been proposed but finite element formulation based on higher order displacement theories are more attractive.

Finite element solution of large size composite plates is highly computationally intensive for which present day uniprocessor computers may found to be slow. To improve the speed of computations use of supercomputers or cluster of workstations using message passing libraries can be done. But the relatively easier alternative for better computational efficiency is to use network of computers or LAN along with client-server approach for distributed computing. This chapter includes introduction of laminated composite, finite element formulation based on higher order shear deformation theory and its implementation over distributed computing environment.

11.2 LAMINATED COMPOSITES - TERMINOLOGY

Laminated composites consist of layers of at least two different materials that are bonded together. Lamination is used to combine the best aspects of the constituent layers in order to achieve a more useful material. The properties that can be emphasized by lamination are strength, stiffness, low weight, and corrosion-resistance; wear resistance, beauty or attractiveness, thermal insulation, acoustical insulation etc. Examples of laminated composites are bimetals, clad metals, laminated glass, plastic-based laminates, and laminated fibrous composites.

Lamina: A lamina is a flat arrangement of unidirectional fibers in matrix as shown in Fig.11.1. The fibers are the principal reinforcing or load-carrying agent. They are typically strong and stiff. The matrix can be organic, ceramic, or metallic. The function of matrix is to support and protect the fibers and to provide a means of distributing load among and transmitting load between the fibres.

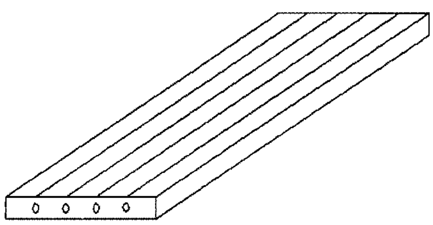

FIG. 11.1 LAMINA WITH UNIDIRECTIONAL FIBRES

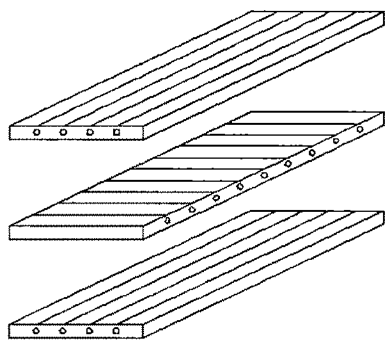

FIG. 11.2 THREE-PLY LAMINATE CONSTRUCTION

Laminate: A laminate is a stack of laminae with various orientations of principal material directions in the laminae as in Fig.11.2. The layers of a laminate are usually bounded together by the same matrix material that is used in the laminae. Laminate can be composed of plates of different materials or of laminae of the same material. A laminated circular cylindrical shell can be constructed by winding resin-coated fibers on a mandrel first with one orientation to the shell axis, then another, and so on until the desired thickness is built up.

A major purpose of lamination is to tailor the directional dependence of strength and stiffness of a material to match the loading environment of the structural element. Laminates are uniquely suited to this objective since the principal material directions of each layer can be oriented according to need.

As far as environmental resistance is concerned, composite materials are more efficient than traditional civil engineering materials such as steel, concrete, masonry, and plaster. Degradation in strength and stiffness for steel structures due to the corrosion problem requires frequent inspection, maintenance, and repair. Similarly, stress cracking due to the warm/cold weathering limits the service life of concrete structures. Timber is susceptible to moisture-swelling problems and paste attack.

Currently, composite materials are being used to retrofit and/or reinforce existing infrastructures. Flat composite laminates have been bonded to the exterior surface of reinforced concrete deck to increase its bending stiffness. Several pedestrian bridges have been built successfully. Composite materials are suitable for construction of highway bridges, power transmission towers, office/residential buildings, retaining walls, etc.

11.3 FINITE ELEMENT FORMULATION

In this section detailed finite element formulation for evaluating stiffness matrix and load vector of rectangular laminated plate element is discussed. The displacement model used accounts for membrane, bending and transverse shear deformations[128]. A eight nodded isoparametric element is used. A typical rectangular laminated plate with dimensions 'a' and 'b' and thickness 't' is composed of number of perfectly bonded orthotropic layers (laminae) which are placed one over another. In symmetric laminate these lamina are placed symmetrically with respect to the mid plane. A coordinate system is adopted such that the x-y plane coincides with the mid plane and the z-axis is perpendicular to the plane as shown in Fig. 11.3. The displacements in the x, y and z directions of the symmetrically laminated composite plates subjected to transverse load may be taken as follows. The displacement along the x, y and z directions are expressed in terms of higher order functions of thickness coordinates and mid plane variables [129].

$$u(x, y, z) = z\,\theta_x(x, y, 0) + z^3\,\theta_x^*(x, y, 0) = z\,\theta_x + z^3\,\theta_x^*$$
$$v(x, y, z) = z\,\theta_y(x, y, 0) + z^3\,\theta_y^*(x, y, 0) = z\,\theta_y + z^3\,\theta_y^*$$
$$w(x, y, z) = w(x, y, 0) + z^2\,w^*(x, y, 0) = w_0 + z^2\,w_0^* \qquad \ldots (11.1)$$

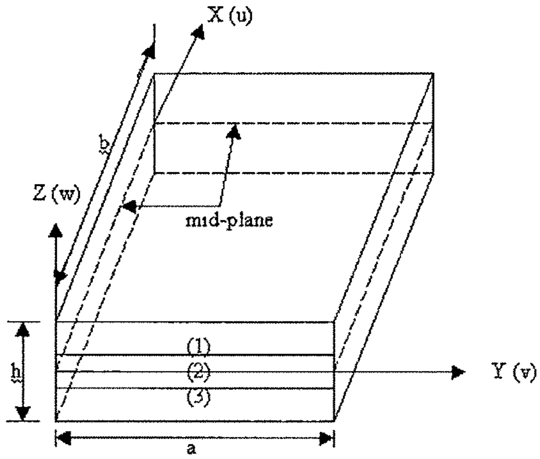

FIG. 11.3 GEOMETRY OF A RECTANGULAR LAMINATED COMPOSITE PLATE

where u, v, and w define the displacements of a point along x-, y- and z-directions respectively, θ_x and θ_y are the rotations of the normal to the mid plane at the same point, and w^*, θ_x^*, θ_y^* are the corresponding higher order terms. An advantage of the displacement model under consideration is that the assumed field variables w, θ_x, θ_y, w^*, θ_x^*, θ_y^* need only be of C^0 continuity. This model includes the effects of the transverse normal strain / stress also.

Strain expressions corresponding to model Eq. (11.1) are,

$$\varepsilon_x = \partial u / \partial x = z\,K_x + z^3\,K_x^*$$
$$\varepsilon_y = \partial v / \partial y = z\,K_y + z^3\,K_y^*$$
$$\varepsilon_z = \partial w / \partial z = z\,K_z$$
$$\gamma_{xy} = \partial u/\partial y + \partial v/\partial x = z\,K_{xy} + z^3\,K_{xy}^*$$
$$\gamma_{yz} = \partial v/\partial z + \partial w/\partial y = \phi_y + z^2\,\phi_y^*$$
$$\gamma_{xz} = \partial u/\partial z + \partial w/\partial x = \phi_x + z^2\,\phi_x^* \qquad \ldots (11.2)$$

Where the definitions of the various terms are as follows:

$K_x = \partial\theta_x/\partial x$, $\quad K_y = \partial\theta_y/\partial y$, $\quad K_{xy} = \partial\theta_x/\partial y + \partial\theta_y/\partial x$,

$K_x^* = \partial\theta_x^*/\partial x, \quad K_y^* = \partial\theta_y^*/\partial y, \quad K_{xy}^* = \partial\theta_x^*/\partial y + \partial\theta_y^*/\partial x,$

$\phi_x = \theta_x + \partial w_0/\partial x, \quad \phi_y = \theta_y + \partial w_0/\partial y,$

$\phi_x^* = 3\theta_x^* + \partial w_0^*/\partial x, \quad \phi_y^* = 3\theta_y^* + \partial w_0^*/\partial y,$

$K_z = 2\, w_0^*$ . . .(11.3)

The concise matrix form of Eq. 11.3 is,

$$\{\varepsilon_b^k\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix} = z \begin{Bmatrix} K_x \\ K_y \\ K_z \\ K_{xy} \end{Bmatrix} + z^3 \begin{Bmatrix} K_x^* \\ K_y^* \\ K_z^* \\ K_{xy}^* \end{Bmatrix} = z\,K + z^3\,K^* \quad ...(11.4a)$$

$$\{\varepsilon_s^k\} = \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = z \begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix} + z^2 \begin{Bmatrix} \phi_y^* \\ \phi_y^* \end{Bmatrix} = \phi + z^2\,\phi^* \quad ...(11.4b)$$

The above Eq. (11.4a) and (11.4b) are the expressions for the flexure and transverse shear strains respectively, at any point in the k^{th} layer of the laminate located at a distance z from the mid-plane. It should be noted that owing to the nature of Eq. (11.4b), the transverse shear strains vary parabolically through the plate thickness.

For an orthotropic lamina in a 3-D state, the strain-stress relationship at a point in each of the three orthogonal planes is given by,

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{13} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{13} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \\ \tau_{23} \\ \tau_{13} \end{Bmatrix}$$

Or $\{\varepsilon\} = [\,s\,]\{\sigma\}$ . . . (11.5)

The stress-strain constitutive relations can be obtained by inversion of strain-stress relations given by Eq. 11.5 and are written in following matrix form.

$\{\,\sigma\,\} = [\,c\,]\{\,\varepsilon\,\}$ . . . (11.6)

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{12}\\ \gamma_{23}\\ \gamma_{13}\end{Bmatrix}^k = \frac{1}{\Delta}\begin{bmatrix} E_1(1-\nu_{23}\nu_{32}) & E_1(\nu_{21}+\nu_{31}\nu_{23}) & E_1(\nu_{31}+\nu_{21}\nu_{32}) & 0 & 0 & 0\\ E_2(\nu_{12}+\nu_{13}\nu_{32}) & E_2(1-\nu_{13}\nu_{31}) & E_2(\nu_{32}+\nu_{12}\nu_{31}) & 0 & 0 & 0\\ E_3(\nu_{13}+\nu_{12}\nu_{23}) & E_3(\nu_{23}+\nu_{21}\nu_{13}) & E_3(1-\nu_{12}\nu_{21}) & 0 & 0 & 0\\ 0 & 0 & 0 & \Delta G_{12} & 0 & 0\\ 0 & 0 & 0 & 0 & \Delta G_{23} & 0\\ 0 & 0 & 0 & 0 & 0 & \Delta G_{13}\end{bmatrix}\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{12}\\ \tau_{23}\\ \tau_{13}\end{Bmatrix}^k$$

In which, $\Delta = (1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-2\nu_{12}\nu_{23}\nu_{31})$

In the stress-strain relation Eq. (11.6), the subscript k is introduced to designate k^{th} layer of the laminate. The relations given by Eq. (11.6) are the stress-strain constitutive relations with reference to lamina axes for a homogeneous orthotropic layer in a general 3-D state of stress and these are adopted here to develop a theory based on the displacement model given by Eq. (11.1).

As noted earlier, the relation given by Eq. (11.6) is the stress-strain constitutive relation for the orthotropic lamina referred to the lamina's principal axes (1,2,3). The principal material axes of a lamina may not coincide with the reference axes for the laminated plate. It is therefore necessary to transform the constitutive relation given by Eq. (11.6) from the lamina principal axes (1,2,3) to the reference axes of the laminate (x, y, z).

$$\sigma' = T\,\sigma \qquad \text{and} \qquad \varepsilon' = T\,\varepsilon \qquad \ldots (11.7)$$

The transformation matrix T is given by,

$$T = \begin{bmatrix} c^2 & s^2 & 0 & 2sc & 0 & 0\\ s^2 & c^2 & 0 & -2sc & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ -sc & sc & 0 & (c^2-s^2) & 0 & 0\\ 0 & 0 & 0 & 0 & c & -s\\ 0 & 0 & 0 & 0 & s & c\end{bmatrix} \qquad \ldots (11.8)$$

Where, $c = \cos\alpha$ and $s = \sin\alpha$ with α as angle between reference axes and principal axes of laminate

The relation between engineering and tensor strain vectors is given by,

$$\{ \varepsilon \} = [R] \{ \varepsilon_{ts} \}$$

$$\{ \varepsilon_{ts} \} = [R]^{-1} \{ \varepsilon \} \qquad \ldots (11.9)$$

R matrix is defined as,

$$[R] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \qquad \ldots (11.10)$$

The stress-strain constitutive relations with reference to laminate axes are obtained in the following form by making use of relations (11.6), (11.7) and (11.10),

$$\sigma = T^{-1} C R T R^{-1} \varepsilon \qquad \ldots (11.11)$$

It can easily be proved that,

$$R T R^{-1} = T^{-1t} \qquad \ldots (11.12)$$

Thus, the relation (11.11) can be rewritten as,

$$\sigma = Q \varepsilon \qquad \ldots (11.13)$$

Where,

$$Q = T^{-1} C T^{-1t}$$

In matrix form,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & Q_{24} & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & Q_{34} & 0 & 0 \\ Q_{14} & Q_{24} & Q_{34} & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \qquad \ldots (11.14)$$

The matrix coefficients Q are defined as,

$$Q_{11} = C_{11}c^4 + (2C_{12}+4C_{44})\, s^2c^2 + C_{22}\, s^4$$

$$Q_{12} = (s^4 + c^4)\, C_{12} + (C_{11}+ C_{22} - 4\, C_{44})\, s^2\, c^2$$

$$Q_{13} = c^2\, C_{13} + s^2\, C_{23}$$

$$Q_{14} = (C_{11} - C_{12} - 2C_{44})\, c^3 s + (C_{12} - C_{22} + 2C_{44})\, s^3 c$$
$$Q_{22} = C_{11}\, s^4 + C_{22}\, c^4 + (2C_{12} + 4C_{44})\, s^2 c^2$$
$$Q_{23} = c^2 C_{23} + s^2 C_{13}$$
$$Q_{24} = (C_{11} - C_{12} - 2C_{44})\, s^3 c + (C_{12} - C_{22} + 2C_{44})\, c^3 s$$
$$Q_{33} = C_{33}$$
$$Q_{34} = (C_{13} - C_{23})\, sc$$
$$Q_{44} = (C_{11} + C_{22} - 2C_{12} - 2C_{44})\, s^2 c^2 + (c^4 + s^4)\, C_{44}$$
$$Q_{55} = c^2 C_{55} + s^2 C_{66}$$
$$Q_{56} = (C_{66} - C_{55})\, sc$$
$$Q_{66} = s^2 C_{55} + c^2 C_{66} \qquad \ldots (11.15)$$

And the coefficients of C matrix in Eq. (11.15) are defined by Eq. (11.6).

The solution of the fundamental equations of the displacement model based on higher order shear deformation theory for laminates anisotropic plates, can conveniently be obtained by using the finite element displacement formulation. Element properties are derived by assuming a displacement function, which ensures completeness within the element and compatibility across the element boundaries. The finite element theory is developed in this section for application to linear equilibrium problems of isotropic, orthotropic and multiplayer anisotropic plates with various loading and boundary conditions. In present work, 8-noded isoparametric quadrilateral element (Fig 11.4) is used. The finite element formulation starts with writing the shape functions, followed by the derivation of the strain-displacement matrix [B], and calculation of element stiffness matrix.

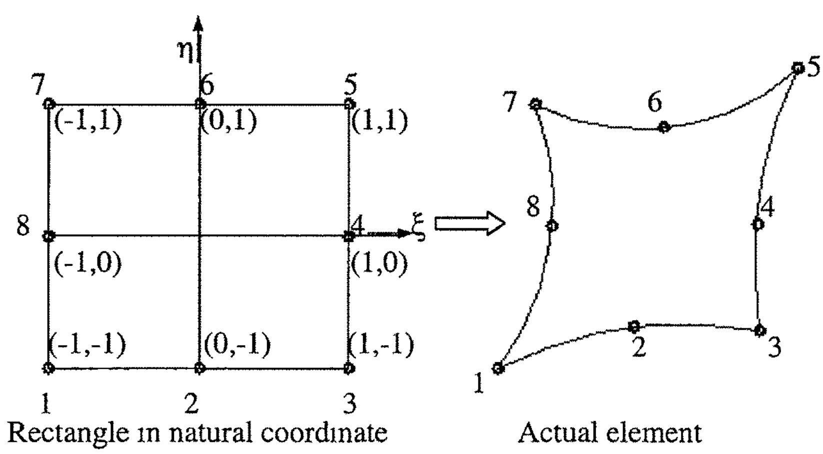

FIG. 11.4 EIGHT NODDED ISOPARAMETRIC ELEMENT

The vector,

$q = [w_{01}, \theta_{x1}, \theta_{y1}, w_0{}^*{}_1, \theta_x{}^*{}_1, \theta_y{}^*{}_1, w_{02}, \theta_{x2}, \theta_{y2}, w_0{}^*{}_2, \theta_x{}^*{}_2, \theta_y{}^*{}_2 \; . \; ..,\theta_y{}^*{}_8]$
denotes the element displacement vector. Thus the degrees of freedom at each node are:

w_0 = Transverse displacement at the geometrical mid-plane,

θ_x, θ_y = Rotations of the 'normal' to the geometrical mid-plane in x-z & y-z plane respectively,

w_0^* = Higher order term of transverse displacement w_0 at the geometrical mid-plane.

θ_x^*, θ_y^*= Higher order terms of rotations of the 'normal' to the geometrical mid-plane in x-z and y-z plane i.e. θ_x and θ_y respectively.

The shape functions for this element in terms of the non-dimensional coordinate system can be written as:

$$N_1 = \xi(\xi - 1)\eta(\eta-1)/4 \qquad N_2 = (1-\xi^2)\eta(\eta-1)/2$$
$$N_3 = \xi(\xi + 1)\eta(\eta-1)/4 \qquad N_4 = \xi(\xi + 1)(1-\eta^2)/2$$
$$N_5 = \xi(\xi + 1)\eta(\eta+1)/4 \qquad N_6 = (1-\xi^2)\eta(\eta+1)/2$$
$$N_7 = \xi(\xi - 1)\eta(\eta+1)/4 \qquad N_8 = \xi(\xi - 1)(1-\eta^2)/2 \qquad \text{. . . (11.16)}$$

Where, ξ and η are the non-dimensional coordinates (Fig. 11.4) of a given point on the element.

Now the displacement field is expressed in terms of the nodal values. Thus, if $d = [w_0, \theta_x, \theta_y, w_0^*, \theta_x^*, \theta_y^*]^T$ represents the displacement components of a point located at (ξ,η), and q is the element displacement vector, then

$$w_0 = N_1 w_{01} + N_2 w_{02} + \ldots + N_8 w_{08}$$
$$\theta_x = N_1\theta_{x1} + N_2\theta_{x2} + \ldots + N_8\theta_{x8}$$
$$\theta_y = N_1\theta_{y1} + N_2\theta_{y2} + \ldots + N_8\theta_{y8}$$
$$w_0^* = N_1 w_{01}^* + N_2 w_{02}^* + \ldots + N_8 w_{08}^*$$
$$\theta_x^* = N_1\theta_{x1}^* + N_2\theta_{x2}^* + \ldots + N_8\theta_{x8}^*$$
$$\theta_y^* = N_1\theta_{y1}^* + N_2\theta_{y2}^* + \ldots + N_8\theta_{y8}^* \qquad \text{. . . (11.17)}$$

Where,

$$\underset{(6x48)}{[N]} = \sum_{i=1}^{NN} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & 0 & N_i \end{bmatrix} \quad \ldots (11.18)$$

The strain-displacement matrix relating strain components to element nodal variables can be formed as:

$$[\varepsilon] = [B]\,[\delta] \quad \ldots (11.19)$$

Where each $[\delta]_i^T = [w_0, \theta_x, \theta_y, w_0^*, \theta_x^*, \theta_y^*]_i^T$ for $i = 1$ to 8.

Now, considering the flexure strain terms and shear strain terms separately and from Eq. (11.15), writing the strain-displacement relationship in terms of the bending curvature-displacement relation $[B_b]$ and shear rotation-displacement relation $[B_s]$.

The shear rotation – displacement relations are,

$$\phi = \begin{Bmatrix} \phi_x \\ \phi_y \\ \phi_x^* \\ \phi_y^* \end{Bmatrix} = \begin{Bmatrix} \theta_x + \partial w_0/\partial x \\ \theta_y + \partial w_0/\partial y \\ 3\theta_x^* + \partial w_0^*/\partial x \\ 3\theta_y^* + \partial w_0^*/\partial y \end{Bmatrix} \quad \ldots (11.20)$$

And the bending curvature-displacement relations are,

$$K = \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \\ K_x^* \\ K_y^* \\ K_{xy}^* \\ K_z \end{Bmatrix} = \begin{Bmatrix} \partial\theta_x/\partial x \\ \partial\theta_y/\partial y \\ \partial\theta_x/\partial y + \partial\theta_y/\partial x \\ \partial\theta_x^*/\partial x \\ \partial\theta_y^*/\partial y \\ \partial\theta_x^*/\partial y + \partial\theta_y^*/\partial x \\ 2\,w_0^* \end{Bmatrix} \quad \ldots (11.21)$$

So, B matrix for curvature and shear can be given as,

$$[B_b]_{(7\times48)} = \sum_{i=1}^{NN} \begin{bmatrix} 0 & \partial N_i/\partial x & 0 & 0 & 0 & 0 \\ 0 & 0 & \partial N_i/\partial y & 0 & 0 & 0 \\ 0 & \partial N_i/\partial y & \partial N_i/\partial x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \partial N_i/\partial x & 0 \\ 0 & 0 & 0 & 0 & 0 & \partial N_i/\partial y \\ 0 & 0 & 0 & 0 & \partial N_i/\partial y & \partial N_i/\partial x \\ 0 & 0 & 0 & 2N_i & 0 & 0 \end{bmatrix} \quad \ldots (11.22)$$

$$[B_s]_{(4\times48)} = \sum_{i=1}^{NN} \begin{bmatrix} \partial N_i/\partial x & N_i & 0 & 0 & 0 & 0 \\ \partial N_i/\partial y & 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & 0 & \partial N_i/\partial x & 3N_i & 0 \\ 0 & 0 & 0 & \partial N_i/\partial y & 0 & 3N_i \end{bmatrix} \quad \ldots (11.23)$$

The element stiffness matrix $[K^e]$ is calculated by,

$$[K^e]=\int_A \{[B_b(x,y)]^T[D_b][B_b(x,y)]+[B_s(x,y)]^T[D_s][B_s(x,y)]\}dA \quad \ldots (11.24)$$

Note that the matrices $[B_b]$ and $[B_s]$ are evaluated based on the shape functions given above. Upon evaluating matrices $[D_b]$, $[D_s]$, $[B_b]$ and $[B_s]$ the element stiffness matrix can be evaluated. However, since the shape functions and, thus, the matrices $[B_b]$ and $[B_s]$ are defined in terms of the non-dimensional coordinate system, the element stiffness matrix must be evaluated as follows:

$$[K^e] = \int_{-1}^{+1}\int_{-1}^{+1} \{[B_b(x,y)]^T[D_b][B_b(x,y)]+[B_s(x,y)]^T[D_s][B_s(x,y)]\} \, |J| \, d\xi \, d\eta \quad \ldots (11.25)$$

The Gauss-Quadrature integration technique is used to evaluate the integrals. In the present formulation a selective integration scheme is used to evaluate element stiffness matrix. For the bending stiffness terms 3 × 3 integration scheme and for the shear stiffness terms 2 × 2 integration scheme has been adopted. Thus the stiffness matrix has been evaluated as follows:

$$[K^e]= \sum_{a=1}^{NG} \sum_{b=1}^{NG} \{[B_b(x,y)]^T[D_b][B_b(x,y)]+[B_s(x,y)]^T[D_s][B_s(x,y)]\} |J| W_a W_b \quad \ldots (11.26)$$

where W_a and W_b are the weighting factors corresponding to Gauss sampling points and NG is the number of Gauss points selected for the integration schemes.

In the evaluation of the load vector the entire laminate is considered as a single layer of thickness t_l. The applied external forces may consist of independent or combination of the following load cases:

1) Gravity Load : The gravity loads, generally the self-weight of the element, always act in the global z-direction. Let 'ρ' be the uniform mass density of the element material and 'g' be the acceleration due to gravity in z-direction. The element load vector at node i is given by,

$$P_{gi} = \int_A \rho \, g \, t \, [N_i]^T \, dA \qquad \text{. . . (11.27)}$$

$$[P_g^e] = \sum_{a=1}^{NG} \sum_{b=1}^{NG} \rho \, g \, t \, [N_i]^T \, |J| \, W_a \, W_b \qquad \text{. . . (11.28)}$$

The above equation represents the element load vector for all the nodes.

2) Uniform normal surface pressure: To evaluate the nodal loads due to normal surface pressure Po, the displacement normal to the surface of the element is required. As here, there is only the transverse displacement, the transverse normal pressure acting either innermost or outermost surface is considered. The load vector at node i is given by,

$$[P_{pi}] = \int_A P_0 \, [N_i]^T \, dA \qquad \text{. . . (11.29)}$$

$$[P_p^e] = \sum_{a=1}^{NG} \sum_{b=1}^{NG} P_0 \, [N_i]^T \, |J| \, W_a \, W_b \qquad \text{. . . (11.30)}$$

The above equation represents the element load vector for all the nodes.

3) Sinusoidal normal surface pressure: The load vector at node i due to sinusoidal distributed normal pressure is obtained from Eq. (11.30) by replacing P_0 by,

$$P_0 \, \frac{\sin m\pi x}{a} \, \frac{\sin n\pi y}{b} \qquad \text{. . . (11.31)}$$

Where, P_0 is amplitude of loading in the z-direction and the element load vector is given by Eq. 11.31

4) Point load along the transverse direction: When the point of application is not coincident with nodal point and P_{pt} be the point load normal to the surface of the element, the load vector at node i is given by,

$$[P_{pti}] = P_{pt}\,[N_i]^T \qquad \ldots (11.32)$$

11.4 IMPLEMENTATION IN DISTRIBUTED COMPUTING ENVIRONMENT

To implement finite element analysis method over distributed processing environment the entire process is divided into various subtasks. Further these subtasks are distributed over number of computers and communication between computers is carried out by intermediate files. Two alternative approaches have been considered for implementation of distributed computing. In first alternative, derivation of stiffness matrix for each lamina is carried out on different computers concurrently and assembly of all stiffness matrices is carried out to get stiffness matrix of laminated plate. Load vector is calculated by considering single lamina. After applying boundary conditions displacements corresponding to free degrees of freedom are calculated. Subsequently the stresses in each lamina are calculated concurrently over different computers. The flow chart of the same process is shown in Fig. 11.5.

The simplicity of the implementation is the advantage of this approach. As the bandwidth of stiffness matrix for each lamina is minimum, its derivation needs not require much time, but the size of file to store stiffness matrix of each lamina increases with problem size. The communication between processes is carried out using FTP and so for larger file transfer more communication time is required. The mode of communication is using packet switching in which large sizes of data is divided into small packets and subsequently transfer of packets is carried out. It is observed that due to network congestion the packets are lost and so files cannot be transferred properly, which terminates the process uncompleted.

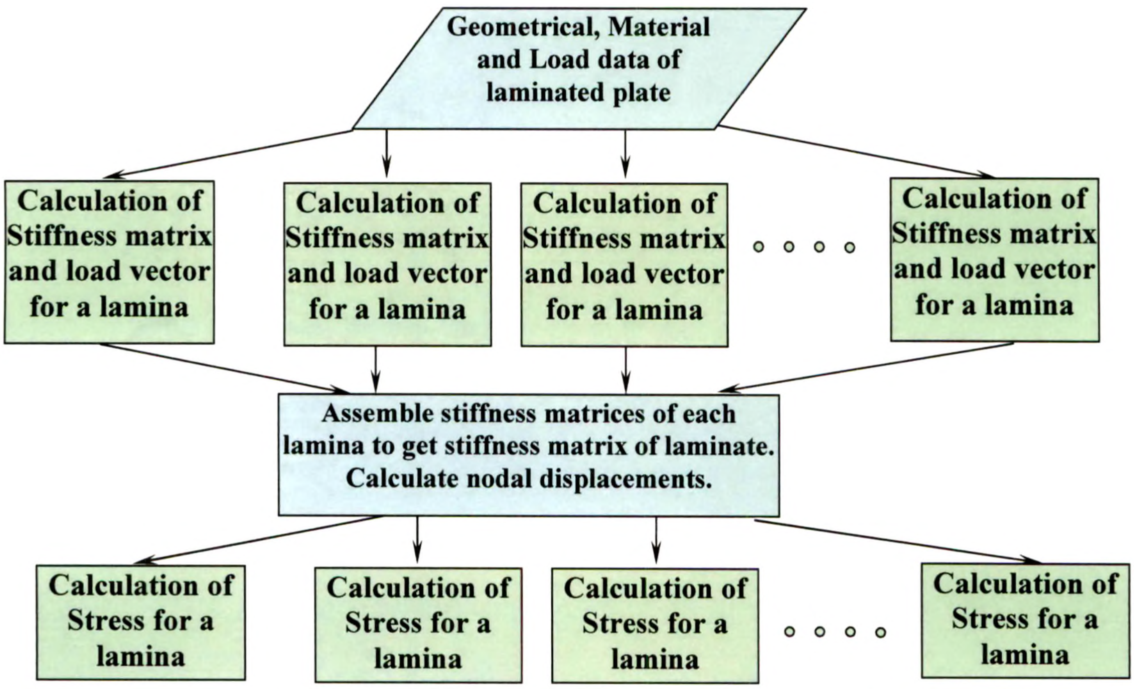

FIG. 11.5 FLOWCHART FOR DISTRIBUTED LAMINATED PLATE ANALYSIS

In the second alternative the constitutive matrix of different lamina is lumped together and stiffness matrix and load vector of each element is calculated using plate theory as discussed in previous section that includes effect of each lamina. To have advantage of distributed processing, substructure technique is used. Using this technique the finite element analysis of laminated composite plate is subdivided into five subtasks as shown in Table 11.1.

In second subtask to calculate stiffness matrix and load vector corresponding to boundary nodes, degrees of freedom corresponding to internal nodes are numbered first and degrees of freedom corresponding to boundary nodes are numbered last. This increases the bandwidth of the matrix, which in turn increases the time required for calculation of substructure stiffness matrix and load vector. With increasing number of substructure the bandwidth reduces and speed of computation increases. To reduce the communication time only upper part of banded substructure stiffness matrix is stored in the intermediate file. In third subtask also half band stiffness matrices of all substructures are assembled and subsequently after imposing boundary condition of actual structure displacements are calculated. In fourth subtask to calculate displacements of internal nodes from boundary nodes, stiffness matrix of substructure is to be formed, but in this process all nodes are numbered in sequence to keep the bandwidth minimum, which reduces the computational time. With increase in number of substructure time required for second and fourth subtask reduces while for third subtask increases.

TABLE 11.1 SUBTASKS FOR DISTRIBUTED ANALYSIS OF LAMINATED COMPOS.

No.	Subtask	Function of Subtask
1.	LAMANA1	Decompose entire finite element domain into small parts, known as substructures and distribute data of each substructure to different computers.
2.	LAMANA2	Calculates stiffness matrix and load vector corresponding to boundary / interface degrees of freedom, using static condensation, for each substructure in parallel on different computers and communicates the same for further assembly.
3.	LAMANA3	Collects stiffness matrix and load vector of each substructure and assemble to have stiffness matrix and load vector corresponding to interface degrees of freedom. After incorporating boundary degrees of freedom, calculates displacements of interface nodes and distribute the appropriate displacements of each substructure to different computers.
4.	LAMANA4	For each substructure, calculates displacements of internal nodes from displacements of boundary nodes and stresses in all laminae of each element and communicates the same for overall results.
5.	LAMANA5	After combining displacements and stresses in elements of each substructure form the results of entire finite element domain.

11.5 ANALYSIS PROBLEM AND RESULTS

A simply supported square laminated composite plate as shown in Fig 11.6 is selected for analysis. It consists of four layers symmetrically placed with respect to mid plane. It is subjected to sinusoidal transverse lading. The size of laminated plate is 100 cm × 100 cm. Plate consists of four laminates, each of 6.5 cm thick, so the total thickness of plate is 25 cm. Intensity of sinusoidal load is 10 kN/cm^2. Various material properties considered for laminated plate are as follows.

$E_1 / E_2 = 25$, $E_3 / E_2 = 1$, $\nu_{12} = \nu_{23} = \nu_{13} = 0.25$, $G_1 / E_2 = 0.2$, $G_2 / E_2 = 0.5$ and $G_3 / E_3 = 0.2$

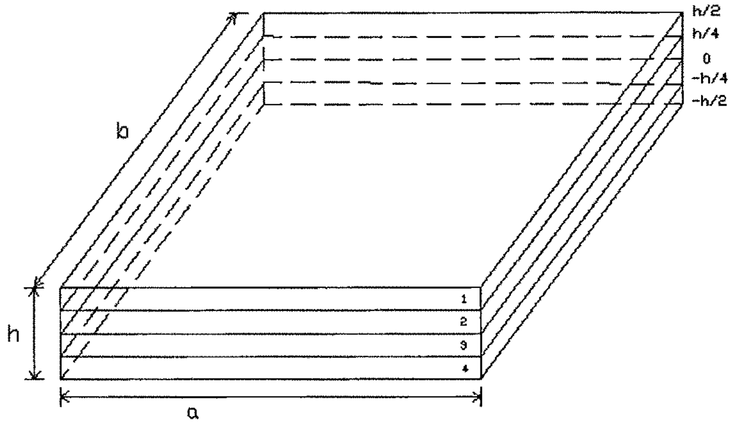

FIG. 11.6 LAMINATED PLATE HAVING FOUR LAMINA

For analysis of composite plate 8 nodded quadrilateral isoparametric element is used. Due to two way symmetry, only quarter plate is analyzed. Typical discretization of quarter plate in 8-noddeed isoparametric elements along with element and node numbering is shown in Fig 11.7.

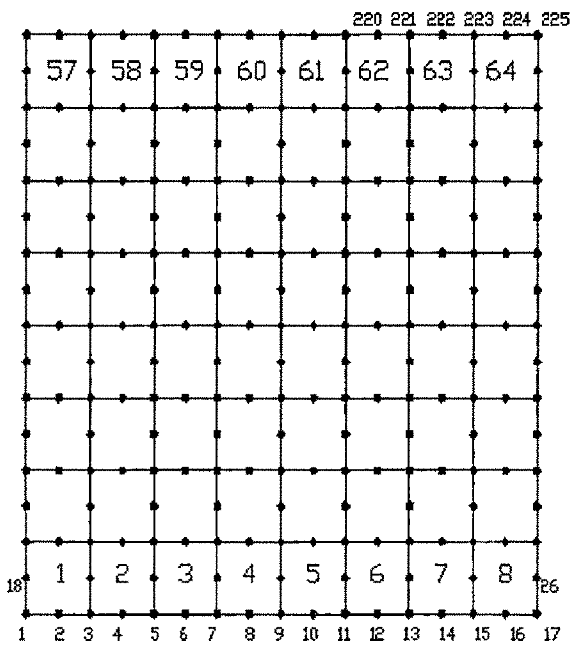

FIG. 11.7 DISCRETIZATION OF LAMINATED PLATE

Further to implement the analysis over distributed computing environment, the mesh is divided into various number of substructures. A typical substructuring system is shown in Fig. 11.8.

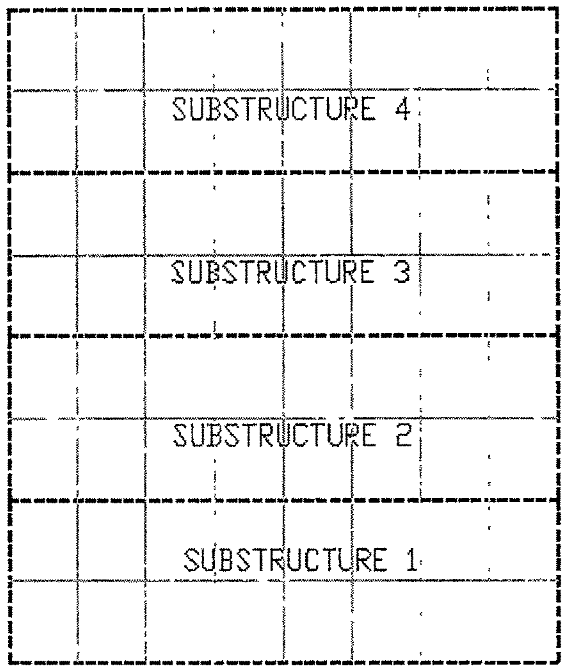

FIG. 11.8 DIVISION OF PLATE INTO FOUR SUBSTRUCTURES

A computer program is developed which discretize the plate into desired number of elements and further data for substructure is generated depending on number of substructures. For generating data of substructures number of elements are kept same to balance the computational load among various computer having identical configuration, which is known as static load balancing. As this is a coarse grain implementation, dynamic load balancing during runtime depending on completion of process on different computers is difficult and hence it is not considered here.

In the present work, the laminated plate is divided into 1600 elements and 4961 nodes. Each node is having six degrees of freedom as discussed in earlier section. So problem is having total 29766 degrees of freedom. The mesh is further subdivided into three, four and six substructures to use same number of computers to solve the problem. The configuration of application consists of allotting various processes to different computers of network and the communication between various computers is carried out by FTP. Typical screen shot of configuration of application using WebDedip, when distributed over four computers is shown in Fig. 11.9.

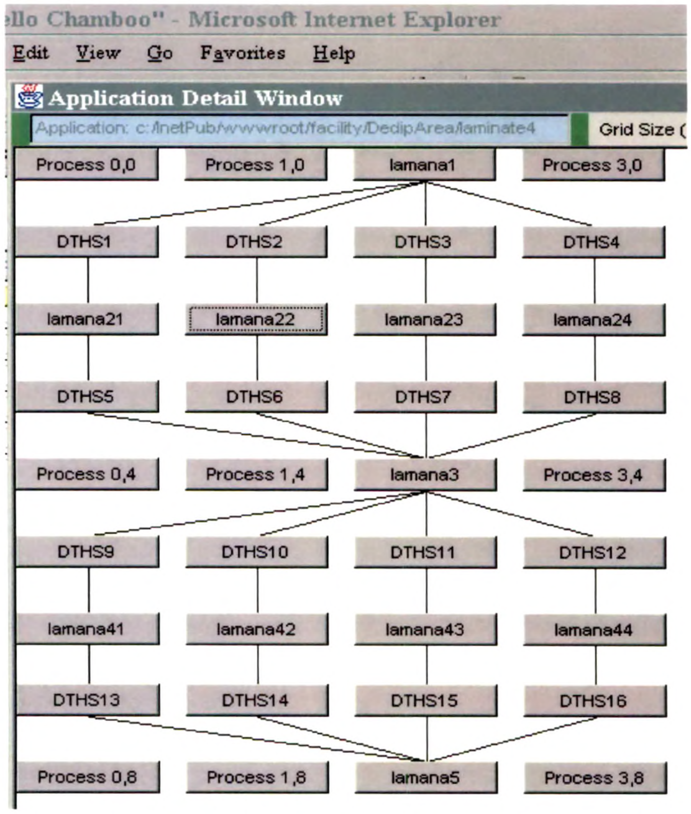

FIG. 11.9 CONFIGURATION OF APPLICATION ON 4 COMPUTERS

After building and successful completion of the application WebDedip gives the summary of application comprising the IP address of computer on which application was run along with start and completion time. A typical screen shot of summary of application as obtained from WebDedip is shown in Fig. 11.10.

Sr.	Process Name	Node No.	Start Time	Expected Time	End Time	Status
1	lamana1	222.222.8.51	23:11:08	23:11:18	23:11:14	NormalComplet...
2	lamana21	222.222.8.61	23:11:30	23:11:40	23:22:11	NormalComplet...
3	lamana22	222.222.8.67	23:11:33	23:11:43	23:21:48	NormalComplet...
4	lamana24	222.222.8.69	23:11:40	23:11:50	23:21:43	NormalComplet...
5	lamana3	222.222.8.51	23:22:34	23:22:44	23:23:25	NormalComplet...
6	lamana41	222.222.8.61	23:23:41	23:23:51	23:24:25	NormalComplet...
7	lamana42	222.222.8.67	23:23:44	23:23:54	23:24:26	NormalComplet...
8	lamana43	222.222.8.68	23:23:48	23:23:58	23:24:31	NormalComplet...
9	lamana44	222.222.8.69	23:23:51	23:24:01	23:24:33	NormalComplet...
10	lamana5	222.222.8.51	23:24:55	23:25:05	23:25:09	NormalComplet...
11	DTHS1	222.222.8.51	23:11:14	23:11:14	23:11:30	NormalComplet...
12	DTHS2	222.222.8.51	23:11:14	23:11:14	23:11:33	NormalComplet...
13	DTHS3	222.222.8.51	23:11:14	23:11:14	23:11:37	NormalComplet...
14	DTHS4	222.222.8.51	23:11:14	23:11:14	23:11:40	NormalComplet...
15	DTHS5	222.222.8.51	23:22:11	23:22:11	23:22:34	NormalComplet...
16	DTHS6	222.222.8.51	23:21:48	23:21:48	23:22:13	NormalComplet...
17	lamana23	222.222.8.68	23:11:37	23:11:47	23:21:21	NormalComplet...
18	DTHS7	222.222.8.51	23:21:21	23:21:21	23:21:45	NormalComplet...
19	DTHS8	222.222.8.51	23:21:43	23:21:43	23:22:06	NormalComplet...
20	DTHS9	222.222.8.51	23:23:25	23:23:25	23:23:41	NormalComplet...
21	DTHS10	222.222.8.51	23:23:25	23:23:25	23:23:44	NormalComplet...
22	DTHS11	222.222.8.51	23:23:25	23:23:25	23:23:48	NormalComplet...
23	DTHS12	222.222.8.51	23:23:25	23:23:25	23:23:51	NormalComplet...
24	DTHS13	222.222.8.51	23:24:25	23:24:25	23:24:41	NormalComplet...
25	DTHS14	222.222.8.51	23:24:26	23:24:26	23:24:46	NormalComplet...
26	DTHS15	222.222.8.51	23:24:31	23:24:31	23:24:51	NormalComplet...
27	DTHS16	222.222.8.51	23:24:33	23:24:33	23:24:55	NormalComplet...

FIG. 11.10 SUMMARY OF DISTRIBUTED APPLICATION

The results obtained by above procedure are found in good agreement with that given in literature. Comparison of various response quantities like transverse deflection and in-plane stresses with that of reference [129] is shown in Table 11.2. These results include thin to moderately thick plates. The numerical results are presented in non-dimensional form as follows:

Non dimensional displacement: $w' = 100\, E_2\, h^3\, w \,/\, q_0\, a^4$

Non dimensional stresses: $(\sigma_x' / \sigma_y' / \tau_{xy}') = h^2\,(\sigma_x / \sigma_y / \tau_{xy}) \,/\, q_0\, a^4$

Where, w = Transverse displacement, $\sigma_x / \sigma_y / \tau_{xy}$ = In-plane stresses, h = Thickness of composite laminate, a = Lateral dimension of plate and q_0 = Intensity of uniformly distributed load

TABLE 11.2 COMPARISON OF CALCULATED AND REFERENCE RESULTS

a/h	Method	w' at (a/2, b/2, 0)	σ_x' at (a/2, b/2, h/2)	σ_y' at (a/2, b/2, h/4)	τ_{xy}' at (0, 0, h/2)
4	**Present study**	**1.90**	**0.71**	**0.63**	**0.05**
	FSDT	1.71	0.41	0.58	0.03
	HSDT	1.89	0.67	0.63	0.04
	Elasticity	1.94	0.72	0.66	0.05
10	**Present study**	**0.72**	**0.56**	**0.39**	**0.03**
	FSDT	0.66	0.50	0.36	0.02
	HSDT	0.71	0.55	0.39	0.03
	Elasticity	0.74	0.56	0.40	0.03
20	**Present study**	**0.51**	**0.54**	**0.30**	**0.02**
	FSDT	0.49	0.53	0.50	0.02
	HSDT	0.51	0.54	0.30	0.02
	Elasticity	0.51	0.54	0.31	0.02
100	**Present study**	**0.43**	**0.53**	**0.26**	**0.02**
	FSDT	0.43	0.54	0.27	0.02
	HSDT	0.43	0.54	0.27	0.02
	Elasticity	0.43	0.54	0.27	0.02

The average time required for various processes, when entire structure is divided into different number of substructures, is tabulated in Table 11.3. Time required in computation and communication is also shown. From the time required by various processes, it is observed that second process which calculates substructure stiffness matrix and load vector consumes more time. As the size of substructure reduces, in case of more number of substructures, time for computation of substructure stiffness matrix and load vector reduces. As the ratio of number of internal nodes to boundary nodes increases time for substructure stiffness matrix and load vector computation increases.

TABLE 11.3 TIME REQUIRED FOR SEQUENTIAL AND PARALLEL PROCESSING

Processes	NB	NEQ	Average time (Sec)	Sequential time (Sec)	Parallel time		Speed up	Efficiency (%)
					Comp (Sec)	Comm (Sec)		
3 substructures								
lamana1	-	-	6	3471	1195	90	2.70	90.04
DTHS	-	-	23					
lamana2	9438	10206	1090					
DTHS	-	-	26					
lamana3	1248	2820	44					
DTHS	-	-	22					
lamana4	714	10209	48					
DTHS	-	-	19					
lamana5	-	-	7					
4 substructures								
lamana1	-	-	6	2687	725	99	3.26	81.52
DTHS	-	-	26					
lamana2	7110	7806	611					
DTHS	-	-	25					
lamana3	1200	3342	51					
DTHS	-	-	26					
lamana4	750	7806	43					
DTHS	-	-	22					
lamana5	-	-	14					
6 substructures								
lamana1	-	-	8	1294	279	115	3.28	54.67
DTHS	-	-	23					
lamana2	4734	5334	181					
DTHS	-	-	67					
lamana3	1080	4170	50					
DTHS	-	-	25					
lamana4	714	5334	22					
DTHS	-	-	20					
lamana5	-	-	18					

Based on the time required for various processes in sequential and parallel implementation, speedup is calculated. The comparison of ideal speedup and observed speedup is shown in Fig. 11.11. From calculated speedup and ideal speedup, efficiency is also calculated as a measure of performance. Comparison of communication and computation time for different substructures is shown in Fig. 11.12. These timings are observed when application is configured on network of Pentium IV computers running on WINDOWS-XP OS at 1.8 GHz with 256 MB RAM. The computers are connected through ethernet network with speed of 100 MBPS.

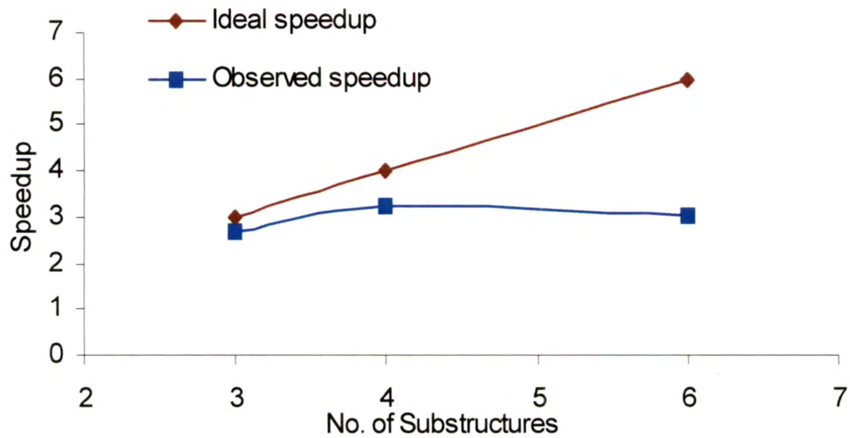

FIG. 11.11 COMPARISON OF IDEAL AND OBSERVED SPEEDUP

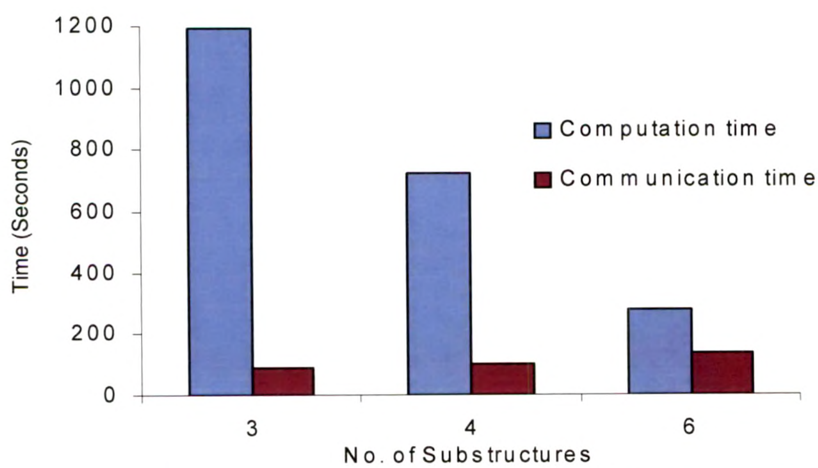

FIG. 11.12 COMPUTATION AND COMMUNICATION TIME

11.6 CLOSING REMARKS

Distributed implementation in WebDedip environment based on substructure concept of finite element analysis of laminated composite plate using higher order shear deformation theory was discussed in this chapter. Local Area Network was used for the implementation without any additional resources. This implementation has lead to following conclusions:

- Computation time decreases with more number of computers but communication time increases.

- Efficiency of computation increases with higher ratio of computation time to communication time. But the overall time to complete the solution of problem reduces with more number of computers.