3. SIZE EFFECT

3.1 SIZE-EFFECT

The size effect parameter is related to scaling on the problem, which is applied to every physical theory. The size effect in solid mechanics is understood as the Effect of the characteristic structure size (dimensions) D on the Nominal Strength σ_n of structure when Geometrically similar structures are compared.

According to the Classical Theory Nominal strength is given by $\sigma_n=\frac{P}{bd}$ where P is the Ultimate load (or load parameter) and b specimen width and specimen depth. This Nominal strength is independent on structural size. If we are taking Geometrical similar structure having a same material. Here Nominal Shear strength (σ_n) is constant. Any deviation in property is known as size effect. Deviation in terms of the length, width and Depth of the section of the beam. According to strength of material concept larger depth of beam and smaller depth of beam is failing at same stress but due to size effect larger depth of beam fails at lower stress.

3.2 IMPORTANCE OF SIZE EFFECT

One of the reasons to use various specimens of different size and shape used in various countries. The characteristics of material properties and its behavior under loading condition can be evaluated by using model analysis. Models were tested to know the Ultimate load carrying capacity and its failure patterns. Size effect is also used in standards and codes to give nonconservative predictions of prototype (and design) strength. Thus, by varying size effect parameter, strength of the structure member can be predicted under applied load condition.

3.3 CLASSICAL HISTORY OF SIZE EFFECT

A question of size effect was first discussed by Leonardo da Vinci (1500s), who stated that "Among cords of equal thickness the longest is the least strong". He also wrote that a cord "is so much stronger ... as it is shorter".

This rule implied inversely proportional to the nominal strength to the length of a cord, which is of course a strong exaggeration of the actual size effect.

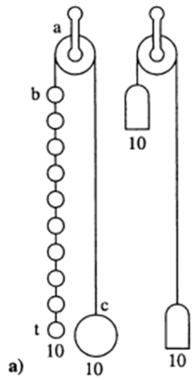

Fig.3-1

Illustrating the Size Effect Discussions by Leonardo Da Vinci In the Early 1500s

More than a century later, the exaggerated rule of Leonardo was rejected by Galileo (1638) in his famous book in which he founded Mechanics of Materials. Galileo argued that cutting a long cord at various points (F, D and E in Fig.3-2) should not make the remaining part stronger. He pointed out, that a size effect is manifested in the fact that large animals have relatively bulkier bones than small ones, which he called the "weakness of giants" (Fig.3-2).

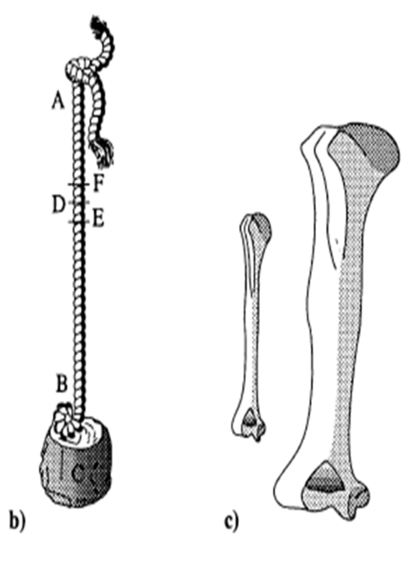

Fig.3-2
Illustrating the size effect discussions by Gallileo Galilei in 1638


DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI;
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con una Appendice del centro di grauità d'alcuni Solidi.
IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

Fig.3-3
Title page of the famous book of Galileo (1638), which founded Mechanics of Materials

After Midcentury, a major advance was made by Mariotte (1686). He experimented with ropes, paper and tin and made the observation, from today's viewpoint revolutionary, that "a long rope and a short one always supports the same weight unless that in a long rope there may happen to be some faulty place in which it will break sooner than in a shorter". He had proposed that this is a consequence of the principle of "the Inequality of the matter whose absolute resistance is less in one Place than another". In qualitative terms, he initiated the statistical theory of size effect, two and half centuries before Weibull. At that time, however, the Theory of Probability was at its birth and was not yet ready to handle the problem.

Marriotte's conclusions were later rejected by Thomas Young (1807). He took a strictly deterministic viewpoint and stated that "A wire of 2 inches in diameter is exactly 4 times as strong as a wire 1 inch in diameter", and other statement "the length has no effect either in increasing or diminishing the cohesive strength". This was a setback; he had not taken random scatter of material strength. Later from numerous extensive experiments clearly demonstrated that the presence of size effect for different materials.

The second major advance was the famous work of Griffith (1921). While founding the Theory of Fracture Mechanics, he also introduced fracture mechanics into the study of Size Effect. He concluded that "the weakness of isotropic solids, is due to the presence of discontinuities or flaws". The effective strength of technical materials could be increased 10 or 20 times at least if these flaws could be eliminated".

In Griffith's view, however, the flaws or cracks triggering failure were only microscopic, which was not characteristic of quasibrittleness materials. The sizes and random distribution of these flaws determined the local macroscopic strength of the material but did not affect the global scaling. Thus, Griffith's work represented a physical basis of Mariotte's statistical concept of size effect, rather than a discovery of a new type of Size Effect.

Fisher and Tippett (1925) and Frechet (1927) published fundamental papers and formulated the weakest-link model for a chain to explain size effect. Their work was early supplemented by the studies of Tippett (1925), and Peirce (1926), and later refined by von Mises (1936) and others.

The capstone on the edifice of statistical size effect initiated by Mariotte was laid by Weibull (1939) in Sweden. Weibull (1939) reached a crucial conclusion: The tail distribution of extremely small strength values with extremely small probabilities cannot be adequately described by any of the known distributions. He proposed for the tail of the extreme value distribution of local strength of a small material element, a power law with a threshold. The distribution of the strength of a chain based on this power law came to be known as the Wei bull distribution. With Weibull's work, the basic framework of the statistical theory of size effect became complete. Most subsequent studies until the 1980s dealt basically with refinements, justifications and applications of Weibull's theory. The researcher contribution in research work is as follows

Dugdale(1960), Barenblatt(1959,1962), Knauss (1973,1974),Wnuk(1974),Zaitsev and Wittmann (1974),Bazant(1976),Zech and Wittmann(1977),Mihashi and Izumi (1977),Kfouri and Rice(1977),Bazant and Cedolin(1979,1980),MihashiandZaitsev(1981),Petersson(1981),Mihashi(1983), Mandelbrot(1984),Bazant,Belytschkoand Chang (1984),Bazant (1984b), Brown(1987),Mecholsky and Mackin (1988), Carpinteri(1986 1989),Pijaudier-Cabot and Bazant(1987), Bazant and Pijaudier-Cabot(1988),Bazant and Lin (1988),Cahn (1989),Chen and Runt (1989),Kittl and Diaz (1988, 1989, 1990),Hornbogen(1989), Peng and Tian (1990), Saouma et al (1990), Bouchaud et al. (1990), Chelidze and Gueguen (1990),Bazant and Cedolin (1991),Long et al.(1991), Issa et al. (1992),Bazant (1992),Malcy et al.(1992), Mosolov and Borodich(1992), Borodich(1992), Lange et al. (1993),Planas and Elices (1988, 1989a, 1989b, 1993),Xie (1987,1989,1993) Xie et al.(1994, 1995), Saouma and Barton (1994),Mihashi et al., eds.(1994) , Feng et al. (1995), Carpinteri and Chiaia (1995),Wittmann, ed.(1995), Mihashi and Rokugo (1998), Bazant and Rajapakse (1999).

3.4 FACTORS INFLUENCING SIZE EFFECTS

A number of factors influence the strength properties and its material behavior systems. The strength properties include compressive, tensile strength, shear bond strength, fatigue strength, and creep of various dimensional changes. Along with these properties, the nature of the material and the geometric configuration of specimens are also important. The materials range from naturally occurring timber and rocks to manufactured materials, such as concrete, steel, plastics, etc.

Some of the properties of the above materials are affected more by changes in size than by other variables. Some properties may not influence the final interpretation of a model investigation to the same degree as the size effect because of their minor influence on the behavior of a structure. In the case of reinforced concrete, the change in compressive strength is not an important as the yield strength of the reinforcement in an under reinforced or lightly reinforced beam. On the other hand, in the investigation of shear strength of a slab or heavily reinforced beam, the compressive strength (as related to its tensile strength) plays a direct and important role. If It is not considered properly, the observed difference in the strength of specimens of two scales might be attributed incorrectly to the size effect.

Generally, theoretical studies treat the behavior of material systems and their physical results on a statistical basis. The basic philosophy being that the failure in heterogeneous materials is a statistical phenomenon. Thus, the larger the volume, the greater the chances for failure which will result in lower strength.

In general, variations in strength of similar shape but different specimen size of concrete are caused by the following factors:

1. Differential curing rates of the various size specimens.

2. Differences in the quality (density) of the material cast into the various size molds.

3. Change of quality of the cast material as a result of the water gain of the top layers and water leakage through the forms.

4. Differential drying of the various size specimens during testing.

5. Difference in induced stress conditions because of variation of quality of end capping of different size compressive specimens.

6. Statistical variations in strength as a result of volume effects.

7. Loading rate on specimens.

8. Strain gradient effects in flexural specimens.

3.5 SIZE EFFECT IN REINFORCED CONCRETE

Overall size effects in reinforced concrete models are important since the behavior of the model is to be extrapolated for predicting prototype behavior. Three types of behavior are important in comparing model and prototype structures made of reinforced concrete.

1. Bond characteristics

2. Cracking similitude (service conditions)

3. Ultimate strength and deformation

3.5.1 Bond Characteristics

Investigation into the bond characteristics is complicated by limited knowledge of the bond phenomenon in the prototype concrete. The bond strength of prototype deformed bars is mainly due to the mechanical wedge action and eventual cracking of concrete stressed by the deformations. This action reduces the size effect on bond to a certain degree, if bar deformations are reproduced in the model reinforcement.

A limited number of pullout tests by Aroni (1959) on smooth and square-twisted bars, indicated the existence of scale effects in bond strength of different-size bars tested with the bars in their usual condition. However, when the surface was polished, this scale effect disappeared, suggesting that it was related to the surface condition associated with a given size, rather than the size itself. Alami and Ferguson (1963) concluded from beam tests that models fail to predict the behavior of reinforced concrete prototype as the result of inadequate bond. which is the primary reason of failure, thus casting doubt on the use of models in the cases where the bond may be the expected cause of failure. For investigations concerned with pre-cracking behavior, or if the flexural or shear resistance is required, it is not necessary to satisfy all the requirements of bond similitude. It is sufficient to ensure that there is sufficient bond resistance so that premature cracking or bond failure does not occur. This can be achieved by providing sufficient embedment length to develop the yield strength of the bar.

The results of tests by Harris (1966), Mina (1967), and many others indicate that certain phenomena involving the bond as the primary cause for failure can be modeled with reasonable reliability. If these models are constructed carefully to eliminate any variation and also if the variables such as the concrete strength, steel yield strength, and mechanical deformations are controlled accurately. Clark (1971) showed that the bond between concrete and reinforcement has a significant effect on service load behavior. This is so particularly in small-scale models, where the reinforcement may take a variety of forms, such as threaded rods or deformed wires, which would exhibit entirely different bond characteristics.

From these various experiments, it may be concluded that certain deformed model bars exhibit bond characteristics that are comparable to those of the prototype reinforcing bars.

3.5.2 Cracks in Service Conditions

Inelastic load-deflection response of a reinforced concrete structure is often strongly dependent upon the degree and manner of cracking. Cracking modes can also influence behavior under reversed or repeated loading, moment and force redistribution in indeterminate systems, and service load conditions. The existence of size effects in cracking is defined as follows: crack width should vary with the size of model, and the number of cracks will be reduced with decreased model size. Initiation of cracking is a function of the tensile strength of concrete. As it is proved that tensile strength increases as the size of specimen is reduced. Therefore, it may be stated that on reducing the size of a structure the load level at which the first crack forms will be somewhat higher. Crack spacing and width will both be dependent on the bond between the two materials. If the bond properties are inadequate, there will be a reduced number of cracks with relatively fewer and wider cracks. Variation of strain gradient can also affect cracking, but very little research has been done on this parameter.

Many tests conducted on small-scale reinforced concrete beam specimens which reveal that the total number of major visible cracks decreases with decreasing beam size. However, the overall cracking patterns are found to be similar, and load-deflection behavior is properly modeled. These tests also indicate that only a small size effect is associated with cracking in scaled models, provided that the other conditions of similitude (mainly the properties of materials and bond strength) are satisfied.

3.5.3 Ultimate Strength and Deformation

In many model studies, the objective is to obtain the ultimate strength as well as the load-deflection behavior of a scaled model.

Tests of over reinforced beams have been reported by Sabnis (1969) with models at scale factors of 1/10 and 1/6 to compare the prediction of ultimate loads, moment-rotation behavior, and effectiveness of helical binders (for confinement) in the compression zone. The cross sections of specimens were 15 mm x 28 mm and 25 mm x 47 mm to match the scale of the available reinforcement. Comparable-size cylinders were tested to determine the various properties of the model concrete. It was found from moment-rotation curves that for prototype and two different sizes of models there was no size effect in these beams. The predictions of other related behavior were within ±10% for models of both scales. In punching shear investigations of slabs, Sahnis and Roll (1971) used a scale factor of 2.5 for model slabs as well as for the control cylinders. Since this type of failure is directly related to the tensile strength of concrete, great care in determining tensile strength is required in such tests. Predictions of the parameter $P_u/bd\sqrt{f_c}$ the punching shear strength of slabs were excellent, with no scale effect observed.

From tests on specimens tested under pure torsion and combined bending and torsion. Syarnal (1969) reported excellent deformation similitude to cracking load values of about 40 to 45% of the ultimate load. However, post cracking torque-twist and the predicted values of angles of twist showed considerable variation. He concluded that it is possible to obtain reasonably good deformation similitude (deflections, twists. etc.), for the entire loading range, between the prototype and its small-scale models (minimum dimensions= 50 mm) reinforced with deformed steel wires without excessive size effects.

Chowdhury (1974) tested model beam-column joints at the scale factor of 10. His tests were successful in predicting the complete behavior of reinforced concrete joints subjected to fully reversed loads. These included flexural and shear behavior and load-deflection response. Cracking patterns obtained were also very similar to those in the prototype.

3.6 INFLUENCE OF SIZE EFFECT ON STRUCTURAL STRENGTH

There are six different size effects that may cause the nominal strength to depend on structure size:

3.6.1 Boundary Layer Effect

It is also known as the wall effect. This effect is due to size of aggregate. Cement mortar adjacent to wall has smaller effect compare to larger size aggregate. Therefore, the surface layer, whose thickness is independent of the structure size and is of the same order of magnitude as the maximum aggregate size, has different properties. The size effect is due to the fact that in a smaller member, the surface layer occupies a large portion of the cross-section, while in a large member, it occupies a small part of the cross-section. In most situations, this type of size effect does not seem to be very strong. A second type of boundary layer effect arises because, under Normal Stress parallel to the surface, the mismatch between the elastic properties of aggregate and mortar matrix causes transverse stresses in the interior, while at the surface these stresses are zero. A third type of boundary layer size effect arises from the Poisson effect (lateral expansion) causing the surface layer to nearly be in plane stress, while the interior is nearly in plane strain.

This causes the singular stress field at the termination of the crack front edge at the surface to be different from that at the interior points of the crack front edge (Bazant and Estenssoro 1979). A direct consequence of this, easily observe in fatigue crack growth in metals, is that the termination of the front edge of a propagating crack cannot be orthogonal to the surface. The second and third types exist even if the composition of the boundary layer and the interior is the same.

3.6.2 Diffusion Phenomenon

Size effect is due to the fact that the diffusion half-times (i.e. half-times of cooling, heating, drying etc.) are proportional to the square of the size of the structure. At the same time, the diffusion process changes the material properties and produces residual stresses which in turn produce inelastic strains and cracking. For example, drying may produce tensile cracking in the surface layer of the concrete member. Due to different drying times and different stored energies, the extent and density of cracking may be rather different in small and large members. This develops different response in the members. For long-time failures, it is important that drying causes a change in concrete creep properties, that creep relaxes these stresses and in thick members the drying happens much slower than in thin members.

3.6.3 Hydration Heat

Hydration heat or other phenomena associated with chemical reactions. This effect is related to the previous one in that the half-time of dissipation of the hydration heat produced in a concrete member is proportional to the square or the thickness (size) of the member. Therefore, thicker members heat to higher temperatures, a well-known problem in concrete construction. Again, the non-uniform temperature rise may cause cracking, induce drying, and significantly alter the material properties.

3.6.4 Statistical Site Effect

Statistical size effect is caused by the randomness of material strength. Size effect has been explained traditionally in concrete structures. The theory of the size effect, originated by Weibull (1939), is based on the Model of a chain. The failure load of a chain is determined by the minimum value of the strength of the links in the chain, and the statistical size effect is due to the fact that the longer the chain, the smaller is the strength value that is likely to be encountered in the chain.

3.6.5 Fracture Mechanics Size Effect

This type of size effect is due to the release of stored energy of the structure into the fracture front. This is important source of size-effect.

3.6.6 Fractal Nature Of Crack Surfaces

If fractality played a significant role in the process of formation of new crack surface, it would modify the fracture mechanics size effect. Probably this size effect is only hypothetical conjecture.

3.7 SIZE EFFECT IN SHEAR STRENGTH OF CONCRETE

In 1955, the Wilkins Air Force Depot warehouse (Fig No. 3-4) in Shelby, Ohio, collapsed due to the shear failure of 36 in. (914 mm) deep beams which did not contain any stirrups at the location of failure. These beams had a longitudinal steel ratio of only 0.45%. They failed at a shear stress of only about 0.5 MPa whereas the ACI Building Code of that time (ACl Committee 318, 1951) permitted an allowable working stress of 0.62 MPa for the 20 MPa concrete used in the beams.

Figure.3-4 Wilkins Air Force Depot in Shelby, Ohio, 1955

Kani (1966 and 1967) was amongst the first to investigate the effect of absolute member size on concrete shear strength after the dramatic warehouse shear failures in 1955. His work consisted of beams without web reinforcement with varying member depths ‘d’, longitudinal steel percentages ‘ρ’ and shear span-to-depth ratios ‘a/d’. He determined that member depth and steel percentage had a great effect on shear strength and that there is a transition point at a/d≈2.5 at which beams fails in shear mode critical (i.e. the value of the bending moment at failure was minimum).

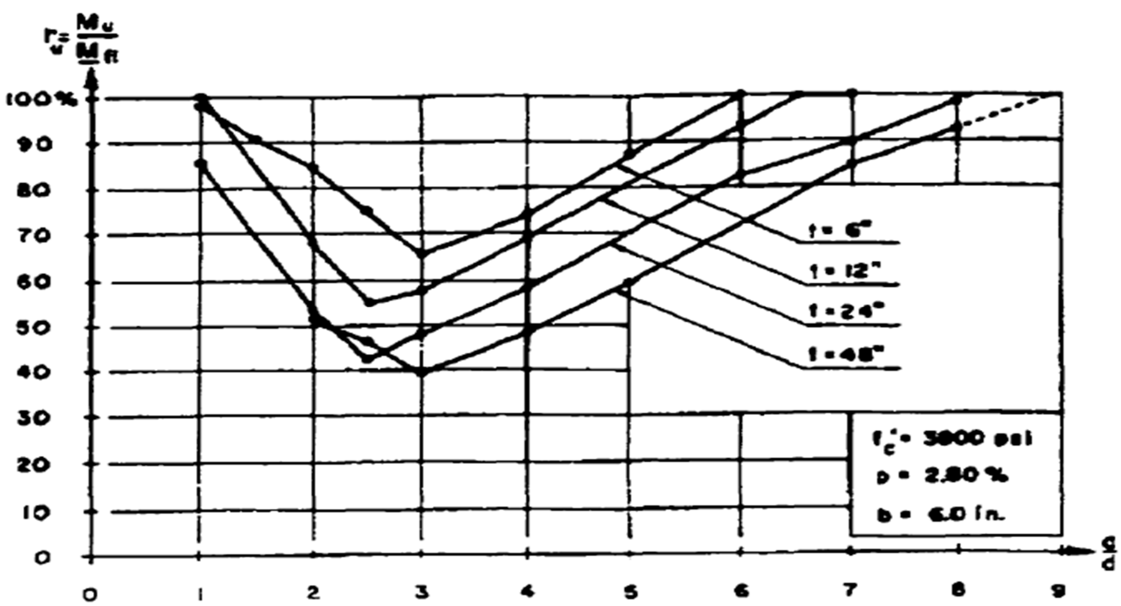

Figure.3-5 Relative strength (Ultimate Moment/Flexural Moment) vs. a/d ratio (Kani 1967)

Kani found this value of ‘a/d’ to be the transition point between failure modes and is the same for different member sizes and steel ratios. When ‘a/d’ value is lower than 2.5 the test beams developed arch action and had a considerable reserve of strength beyond the first cracking point. For ‘a/d’ values greater than 2.5 failure was sudden, brittle and in diagonal tension soon after the first diagonal cracks appeared. Kani’s work was summarized in the textbook "Kani on Shear in Reinforced Concrete" (Kani et al.1979).

Bazant and Kim (1984) derived a shear strength equation based on the theory of fracture mechanics. This equation accounts for the size effect phenomenon as well as the longitudinal steel ratio and incorporates the effect of aggregate size. This equation was calibrated using 296 previous tests obtained from the literature and was compared with the ACI Code equations. It was noted after the comparison that the practice used in the ACI Code of designing for diagonal shear crack initiation rather than ultimate strength does not yield a uniform safety margin when different beam sizes are considered. It was also found, according to the new equation, for larger specimen depths the factor of safety in the ACI Code almost disappears.

Mphonde and Frantz (1984) tested concrete beams without shear reinforcement with varying ‘a/d’ ratios from 1.5 to 3 and concrete strengths ranging from 21 to 103 MPa. They concluded that the effect of concrete strength becomes more significant with smaller ‘a/d’ ratios and that failures became more sudden and explosive with greater concrete strength. It was also found that there is a greater scatter in the results of specimens with small ‘a/d’ ratios due to the possible variations in the failure modes.

Bazant and Kazemi (1991) performed tests on geometrically similar beams with a size range of 1:16 and having a constant ‘a/d’ ratio of 3.0 and a constant longitudinal steel ratio. Beams tested varied in depth from 1 inch (25 mm) to 16 inches (406 mm). The study of size effect parameter required to verify the previously published formula.

Kim and Park (1994) performed tests on beams with a higher-than-Normal concrete strength (53.7 MPa). Test variables were longitudinal steel ratio, shear span-to-depth ratio ‘a/d’ and effective depth ‘d’. Beam heights varied from 170 mm to 1000 mm while the longitudinal steel ratio varied from 0.01 to 0.049 and ‘a/d’ varied from 1.5 to 6.0. Their findings were similar to Kani's from which it was concluded that the behavior of the higher strength concrete is similar to that of Normal-strength concrete.

Shioya (1989) conducted a number of tests on large-scale beams in which the influence of member depth and aggregate size on shear strength was investigated. In this study, lightly reinforced concrete beams containing no transverse reinforcement were tested under a uniformly distributed load. The beam depths in this experimental program ranged from 100 mm to 3000 mm. Shioya found that the shear stress at failure decreased as the member size increased and as the aggregate size decreased.

Stanik (1998) performed tests on a wide range of beam specimens at the University of Toronto. The specimens tested had varying depths ‘d’, ranging from 125 mm to 1000 mm, varying amounts of longitudinal steel (0.76% to 1.31%) as well as varying concrete strength, ranging from 37 MPa to 99 MPa. The longitudinal reinforcement was distributed in some specimens along the sides and some specimens contained the minimum amount of transverse reinforcement recommended by the CSA Standard (CSA 1994). In the series with longitudinal bars along the sides, a set of wider beams was also tested. The purpose was to evaluate the influence of the amount, as well as the distribution of the longitudinal steel on the shear strength. Stanik found that the size effect is very pronounced in lightly reinforced deep members. Members containing the minimum amount of transverse reinforcement or side distributed steel performed better than their counterparts with only bottom longitudinal reinforcing bars.

Deep members with side distributed reinforcement performed nearly as well as the shallow members containing only bottom longitudinal reinforcement. As well, the wider members containing side distributed steel were weaker than the narrower ones with similar side distributed steel. Stanik found very little gain in shear strength with the use of higher concrete strengths. Stanik used the modified compression field theory proposed by the CSA Standard (CSA 1994) to predict the response of his test beams. He found good agreement between his experimental results and these predictions. Stanik also performed a comparison between his experimental results and the ACI Code (ACI committee 318-1995) expressions. He found that the ACI expressions substantially overestimate the shear contribution of concrete, notably in the deeper members.

3.8 SIZE EFFECT PARAMETERS CONSIDERED FOR SHEAR STRENGTH AND CRACK WIDTH FOR MODERATE DEEP BEAM

Many researchers have considered various size effect parameters for Strength and flexure crack in RCC and Pre-stressed members. While a very little work had been done for Shear strength of moderate deep beam using size effect parameter incorporating fibers in it. List of size effect parameters are given below:

- Longitudinal reinforcement ratio (ρt)
- Shear reinforcement ratio (ρw)
- Steel rebar arrangement
- Grade of steel (fy)
- **Grade of concrete (Fck)**
- Steel stress (fs)
- Strain in rebar (εs)
- Strain in concrete at level of reinforcement (εc)
- Concrete cover (Cc)
- Bond factor (bond between steel and concrete at level of reinforcement)
- Duration of load application
- Modular ratio (m)
- **Diameter of bar to Percentage of Reinforcement (Φ/ρ)**
- Strain distribution factor
- **Shear span to depth ratio (a/d)**
- **Effective length to overall depth ratio (Leff/D)**
- Modulus of Elasticity of steel (Es)
- **Fiber aspect ratio (diameter/length)**

- **% of fiber reinforcement**
- Depth of neutral axis (xu)
- Maximum bar spacing (s).