Chapter 1

Theoretical Developments in Particle Physics

One of the most challenging task in particle physics is to encompass the diversity and the complexity observed in the decay modes and fractional widths of particles. For example, there are twenty-two quantitative modes and total forty-nine decay modes of $K^{\pm}$, and ratio of highest to lowest of these fractions amounts to the order of 10^{11}. The spectroscopy and decay rates of various hadronic states are quite important to study due to availability of huge amount of high precession data acquired using large number of experimental facilities viz. BESIII at the Beijing Electron Positron Collider (BEPC), E835 at Fermilab and CLEO at the Cornell Electron Storage Ring (CESR), the B-meson factories, BaBar at PEP-II, Belle at KEKB, the CDF and D0 experiments at Fermilab, the Selex experiment at Fermilab, ZEUS and H1 at DESY, PHENIX and STAR at RHIC, NA60 and LHCb at CERN and new future facility $\overline{\mathsf{P}}$ANDA at FAIR, GSI. The plethora of observations from these facilities offer greater challenges and opportunities in theoretical high energy physics. The hadronic states are not only identified with their masses but also with their various decay rates. All the hadronic states along with experimentally identified decay channels are reported in Particle Data Group (PDG) [1].

1.1 Status of Experimental facilities

The study of Hadron Physics has created lot of interest because of many experimental facilities available world wide. They are collecting huge data in the heavy flavor sectors as well as open flavor sectors. These facilities are working on possible interpretation of data within Standard Model (SM) and beyond Standard Model (BSM). The BSM includes possible exotic states that are bound states with more than three quarks namely tetra-quark, penta-quark, hexa-quark or hybrid (consisting of quarks and gluons) states. The BSM also includes the rare decays, search for supersymmetry, leptoquarks phenomenology, lepton flavor violation decays and many more.

The experimental collider facilities world wide are divided mainly into two categories: fixed target method and particle colliders. The detailed Physics objectives of the experimental facilities are given in Tab. 1.1.

1.2 Status of Theoretical approaches

The experimental facilities tabulated in Tab. 1.1 are aimed at different areas and all of them are trying to understand the structure and dynamics of the basic building blocks of nature. Presently, the analysis and interpretation of huge data coming from experiments is the most crucial and challenging task. Theoretical methods are focused towards the direction of explaining these data and providing predictions for investigation by the upcoming experimental facilities. The theoretical approaches may be divided in three categories: (i) theories based on first principles such as lattice quantum chromodynamics (LQCD), (ii) QCD sum rules and (iii) theories based on effective field theories as well as phenomenological potential models.

1.2.1 Lattice Quantum Chromodynamics

The Lattice Quantum Chromodynamics (LQCD) is based on the first principles utilizing non-perturbative approach to calculate the hadronic spectrum and matrix element for any interaction explained using Feynman diagram. It is a non-perturbative lattice gauge theory formulated on grid of Euclidian space time allowing the construction of the correlation function between the hadronic state with the help of quark and gluon degrees of freedom. Here, the potential term comes from the interaction at the lattice point and therefore LQCD is considered to be the most realistic theory among all other theories and the results from LQCD calculations have been the closest to experimental data so far.

Table 1.1: Experimental facility and their objective

Collaboration	Country	Type	Objective
CMS	CERN, Switzerland	$p\bar{p}$ $Pb-Pb$	Search for Higgs bosons Look for Physics BSM eg. Supersymmetry Heavy ion Collision
ATLAS	CERN, Switzerland	$p\bar{p}$	Search for Higgs bosons Search for CP violation in B and D meson decays Search for supersymmetry
ALICE	CERN, Switzerland	$Pb-Pb$ $Pb-p$	Study of Quark gluon plasma Physics of strongly interacting matter
LHCb	CERN, Switzerland	$p\bar{p}$	B Physics Search for CP violation Search for FCNC decays
$BABAR$	SLAC, USA	e^+e^-	Search for CP violation in B meson CKM measurement Heavy quarkonium production
Belle	KEK, Japan	e^+e^-	Search for CP violation Search for rare decays in B mesons Search for exotic states
CLEO	CESR, USA	e^+e^-	Study of B Physics including Υ resonance Quarkonium production Study of Charm Physics
BESIII	BEPC, China	e^+e^-	Charm Physics Search for CP violation in $D_{(s)}$ decays Production and decays of light hadrons Search for Physics beyond SM
$D0$	Fermilab, USA	$p\bar{p}$	Search for Higgs bosons B Physics
CDF	Fermilab, USA	$p\bar{p}$	Search for Physics beyond SM Production and decay of top and b quark
SELEX	Fermilab, USA	Fixed Target	Search for charm meson and baryons Search for exotic states
FOCUS	Fermilab, USA	Fixed Target	Spectroscopy of charmed hadrons Search for rare and forbidden decays Search for doubly charmed baryons and pentaquark
CBM	FAIR, Germany	$A-A$	Explore QCD phase diagram at high baryon density
PANDA	FAIR, Germany	Fixed Target	Study of hadron structure and exotic hadrons
NUSTAR	FAIR, Germany		Study of nuclear structure Astrophysics
STAR	BNL, USA	$Au-Au$	Characteristics of the quark-gluon plasma Properties of QGP and equation os state

With the advancement in the computation facilities, many interesting results are available, particularly in the heavy quark sector. In the heavy flavor spectrum, the LQCD have provided most accurate results with a very small standard deviation. LQCD have successfully provided the mass spectrum of charmonia and bottomonia in the papers [2–12] and also successfully predicted the leptonic decay constants [13, 14]. However, the information regarding the excited states are yet to be reported. Also the LQCD have not yet provided the information regarding all decay properties of heavy quarkonia and mass spectra of B_c mesons. The reviews on LQCD calculations on heavy quarkonium physics is given in the Ref. [15, 16]. In the open flavor sector, the meson form factors are computed for the channel $D \to (\pi, K)\ell^+\nu_\ell$ channels [17–19]. Heavy to light meson form factors are also computed in the papers [20–22], where the authors of [22] have computed the form factors for $D_s \to \eta^{(\prime)}$ as a pilot study only. The $D_s \to \phi$ form factors are also computed for the first time by the HPQCD collaborations [21]. However, the branching fractions computation is not yet reported using LQCD. The mass spectrum for the heavy baryons is also reported by Refs. [23–28].

1.2.2 QCD sum rules

The QCD sum rules (QCDSR) also known as Shifman–Vainshtein–Zakharov sum rules [29] is the nonperturbative tool for hadronic phenomenology. In QCDSR, the hadrons are written in terms of interpolating quark currents and treated in the framework of gauge invariant operator product expansion. This calculation technique gives excellent agreement with the experimental data and also believed to the best theoretical approach after the LQCD.

1.2.3 Effective Field Theories

The computation of hadronic process from the first principles is not trivial and also requires lot of computational power. The alternative to the real scale processes is the Effective Field Theories (EFT). EFT is the fundamental framework to hadronic interaction with the quantum field theory. The EFTs are generally known as the nonrelativistic QCD (NRQCD) [30, 31] which takes into account the energy scale of the order $m_Q \gg m_Q v \gg m_Q v^2$. The remaining energy scale $m_Q \ll m_Q v$ is extracted using the potential NRQCD (pNRQCD) [32,33]. The NRQCD has been successfully employed for the spectroscopy of heavy quarkonia (charmonia, bottomonia and B_c mesons). Both pNRQCD and NRQCD have incorporated the correction in interacting potential of the order $1/m_Q^2$. It is important to note here that this potential is determined by LQCD simulations. They have successfully predicted the mass spectra of the heavy quarkonia. For computing the decay properties of the heavy quarkonia, the relativistic corrections are also employed to match the experimental data. In the heavy quarkonium spectroscopy, these theories have played very important role in development of the heavy quarkonium physics as the LQCD and QCDSR have not provided the detailed information for the excited states as well as for the decay properties.

EFTs are also developed for understanding the hadrons containing at least one heavy quark (c and/or b) whose masses are more than the QCD scale Λ_{QCD}, e.g. The heavy quark effective theory. This theory assumes that the heavy quark in hadron moves with the constant velocity and hence is considered to be the spectator in the rest frame of heavy hadrons. This theory basically falls in the category of low energy physics which is useful for studying the $D_{(s)}$ and $B_{(s)}$ mesons and understanding the flavor dynamics in these mesons. This can be helpful in studying the weak and strong interactions in the charm and bottom sectors. Their decay properties, especially the leptonic and semileptonic decays allow the direct measurement of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements via hadronic form factors. The CKM matrix is unitary matrix providing the information regarding quark mixing which basically takes place in the weak interactions. The flavor dynamics also gives the information regarding the origin of the CP violation. Currently, the CP violation is one of the main search for most of the experimental facilities.

1.2.4 Other Phenomenological Approaches

The very important problem in theoretical approaches in Particle Physics is quark confinement wherein the isolated color charged quark is not freely available and the quarks and gluons are permanently confined within the hadrons. In the theories based on first principles, the confinement evolves from the QCD computations. The confinement of quarks within hadrons is assumed in the phenomenological approaches. The earliest confinement model is the Bag model developed by Bogolioubov *et al.* that considers the quarks to be confined within the spherical volume and the force acting between the quarks to be the attractive force with strength of the attraction of the order of quark mass [34, and the references therein]. After the discovery of asymptotic freedom, the more advanced MIT Bag model was developed for the hadronic interactions.

The phenomenological approaches include both relativistic as well as nonrelativistic treatment of the quarks comprising the hadrons. The oldest but still effective approach is the potential models inspired from the QCD. In potential models, the interaction is chosen from the LQCD calculations. The next task is to solve the relativistic Dirac equation or the nonrelativistic Schrödinger equation to obtain the bound state mass of the hadrons. For the heavy quarkonium, heavy baryons and the exotic states involving the heavy quarks, the Schrödinger mechanism may be adopted because of the inclusion of heavy quark (c and/or b quark). This is justified to a great extent as the heavy quarks have Significantly low momentum compared to the bound state system constituting the basis for nonrelativistic treatment for heavy hadron spectroscopy. However, for open flavor meson spectroscopy, the nonrelativistic treatment is not valid because of the inclusion of light flavor quark.

In potential model calculations, the short distance behaviour is of the Coulomb type interaction and long distance coefficients are essentially confinement part and in the literature, there are various forms of potentials are available. For the hadronic interaction, relativistic as well as non relativistic potential models are reported. The oldest and still applicable potential is Cornell potential using the linear confinement. The potential equation is given by [35],

$$V(r) = Ar - \frac{4}{3}\frac{\alpha_s}{r} \tag{1.1}$$

This potential is also supported by LQCD calculations. Where A is the confinement strength analogous to the string tension and r is the inter-quark separation. For the heavy quarkonia, the Cornell potential is used for computing the mass spectra of ground state as well as excited states. This potential also takes care of the asymptotic freedom, at short distances the Coulomb term dominates where as at the large distances, the confinement term dominates. Here, α_s is the strong running coupling constant.

In Cornell potential Eq. (1.1), the confinement is the special case of the general Martin potential of the form [36]

$$V(r) = A + Br^{\nu}, \qquad n > 0 \tag{1.2}$$

with A and B to be the model constant parameters. This potential was employed for upsilon and charmonium spectra [36].

The logarithmic confinement is also used for computing the quarkonium mass spectra by Quigg-Rosner. The potential is of the type [37]

$$V(r) = CLog\left(\frac{r}{r_0}\right) \tag{1.3}$$

where C and r_0 are the constants to be determined from the experimental data.

The harmonic confinement also has been employed to compute the mass spectra and decay properties of mesons and baryons. Here, the relativistic Dirac equation is reduced nonrelativistically and the binding energy is obtained. The potential is given by [38]

$$V(r) = \frac{1}{2}(1+\gamma_0)A^2r^2 + B \tag{1.4}$$

Where A and B are the relativistic harmonic model (RHM) parameters. Initially RHM was applied to the light flavor sector only and later extended to the heavy flavor sectors (ERHM) [38,39] with inclusion of Colour Confinement Model [40–42].

The ψ and Υ spectroscopy are also computed in Buchmuller and Tye potential given by [43]

$$V(r) = -\frac{4}{3}\frac{1}{(2\pi)^3}\int d^3q \;\; e^{iqr}\left(\frac{4\pi\alpha_s}{q^2}\right) \tag{1.5}$$

There are many other potentials available in the literature. All the potentials have their different range of applicability. The potential models should reproduce the experimental ground state masses and also predict the excited states correctly. Also the potential models should correctly predict the decay properties. The computation of decay properties depend on the wave function chosen to solve the Schrödinger equation or Dirac equation. But not all potential models successfully predict the mass spectra as well as all decay properties. Till date, not a single potential model has successfully predict the mass spectra and decay properties. As a result, attempts towards the development of potential model to computed all properties of hadrons are still in progress. For example, in calculations of decay properties, different relativistic correction factors are also incorporated to match the experimental data.

1.3 Objectives of the present study and organisation of thesis

The heavy quarkonia ($c\bar{c}$, $b\bar{b}$ and $c\bar{b}$) are the most powerful systems for understanding the heavy quark antiquark interactions in QCD. For charmonia and bottomonia, there are more than 15 experimentally identified states for both the systems and for B_c mesons only pseudoscalar states for $1S$ and $2S$ states are available. As explained in the previous section, there are many ways in which these systems are studied. The oldest but still relevant approach is the potential model wherein the widely accepted potential for the interaction between constituent quarks is the Cornell potential given by Eq. (1.1). There are many individual studies for the spectroscopy and decay properties of the heavy quarkonia. Many of them provide good prediction for mass spectra but when it comes to predicting the decay properties such as weak decays, not all models provide successful computation of the decay properties [35, 36, 43–50]. In Chapter 2, we compute the mass spectra of charmonia, bottomonia and B_c mesons with the least number of model parameters. For computing the spectra, we employed the nonrelativistic approach for Cornell potential Eq. (1.1) and Schrödinger equation is solved numerically. For computing the masses of excited states, we add the spin dependent part of one gluon exchange potential perturbatively. With the help of model parameters and numerical wave function, we compute various decay properties such as leptonic decay constants, various annihilation widths (digamma, three gamma, digluon, three gluon, γgg, dilepton), electromagnetic transition widths. We also compute the weak decays of B_c mesons in a spectator model and also compute its life time. We compare our findings with the available experimental data, LQCD results and other theoretical and we observe that our results are in good agreement with them.

In chapter 3, we compute the spectroscopy of doubly heavy baryons in harmonic confinement scheme considering the potential of the type Lorentz scalar plus vector potential Eq. (1.4). Here, we employ the nonrelativistic reduction of the Dirac equation. For computing octet and decuplet masses, the spin dependent part of the one gluon exchange potential is employed perturbatively. Using the model parameters and spin flavor wave function, we compute the magnetic moment of the doubly heavy baryons. We also compute the radiative decay width for the transition $3/2^+ \to 1/2^+$ using the transition magnetic moments. We compare our findings with the available experimental data as well with the other theoretical approaches.

In last decade, with the advancement of the experimental facilities, lots of new and sometimes unexpected results have been reported. The first unexpected result came in 2003 when Belle collaboration reported the first new state $X(3872)$ in the channel $B \to K(\pi^+\pi^- J/\phi)$ [51]. The structure of this state was beyond the conventional quark model. It was observed that this state has a structure of 4 quark state. Later on, this state was also confirmed by BABAR collaboration in the same channel [52, 53]. Further this state was also confirmed by other experimental facilities such as CDF [54], and LHCb [55] collaborations. After 10 years of the discovery, LHCb collaboration has determined its quantum number to be $J^{PC} = 1^{++}$ [56]. Further, many different multi-quark states – so called exotic states were also identified. In PDG 2018, more than 20 tetra-quark states are reported along with several penta-quark and hexa-quark states. There are different ways in which these states are perceived theoretically. These include four quark, di-mesonic, hadro-quarkonium or composite molecular states. In chapter 4, we study exotic states considering them as di-meson molecules. For computing the bound state masses of these states, we solve the Schrödinger equation for the generalised Woods-Saxon potential. We also compute the strong two body decay widths using the interaction Lagrangian mechanism. We compare our results for masses as well as decay widths with the experimental data and other theoretical predictions.

In chapters 2, 3 and 4, we have successfully computed the mass spectra and decay properties of heavy quarkonia, doubly heavy baryons and exotic states considering the nonrelativistic treatment for heavy hadrons. But this nonrelativistic treatment is not applicable for the spectroscopy of open flavor mesons because of inclusion of light quark. In some crude approximation there are also some papers available in the literature where open flavor mesons are considered in the nonrelativistic approximation. But since their momenta are close to the bound state mass, it can not be treated nonrelativistically.

Next in chapter 5, we compute the decay properties of open flavor mesons, particularly charmed mesons. The open flavor mesons are important tool for understanding the dynamics of weak and strong interactions in the charm sectors. The semileptonic branching fractions of these mesons are proportional to the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, therefore these channels provide the direct determinations of $c \to q$ matrix element where $q = d, s$. We compute the decay properties of charmed and charmed-strange (D and D_s) mesons in a effective quantum field theory approach. We study the weak decay properties such as leptonic and semileptonic decay decays of $D_{(s)}$ mesons in the covariant confined quark model (CCQM) with built-in ifrared confinement developed by G. V. Efimov and M. A. Ivanov [57, 58]. The interaction Lagrangian is written in terms of the constituent quarks. In this model, the confinement of the quark can be introduced using the compositeness conditions [59,60]. One of the important key feature of the CCQM is computation of form factors in the entire physical range of momentum transfer. We study the leptonic branching fractions $D_{(s)} \to \ell^+\nu_\ell$ for $\ell = e, \mu$ and τ and semileptonic branching fractions for the channels $D_{(s)} \to (P, V)\ell^+\nu$ for $\ell = e$ and μ. Here P and V corresponds to the pseudoscalar and vector mesons. It is important to note that in semileptonic decays $D_{(s)}$ mesons the tau mode is kinematically forbidden. We also compare our findings of form factors and branching fractions with latest BESIII data along with other experimental data and other theoretical predictions.

Finally in chapter 6, we conclude the present study. We also discuss the future prospects of research in the area of weak decays using the covariant confined quark model. We also discuss the other possibilities of the application of potential models for heavy quarkonium spectroscopy.