Masses and radiative leptonic decay properties of Bc meson

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Introduction

We employ non-relativistic treatment with the help of Schrödinger equation in order to study the Bc spectroscopy. The Schrödinger equation for the bound state of Bc system is solved using numerical integration together with convexity arguments and nodal theorem for wave function [1]. The pure-leptonic decays of heavy mesons are very interesting from theoretical as well as experimental point of view [2,3]. In the present paper we have studied the decay constants and the leptonic decay width using the non-relativistic treatment.

Methodology

We solve the Schrödinger Equation numerically with the quark-antiquark potential of the form [4-6],

$$V(r) = -\frac{k\alpha_s}{r} + Ar^{\upsilon} + V_{SD} \tag{1}$$

Where A is the potential parameter, υ is a general potential index corresponding to the confining part of the potential. For present computation of masses and decay properties, we have taken as $\upsilon = 1$. α_s is the strong running coupling coefficient which can be determined from

$$\alpha_s(M^2) = \frac{4\pi}{\left(11-\frac{2}{3}n_f\right)\ln\frac{M^2+M_b^2}{\Lambda^2}} \tag{2}$$

Where the scale is taken as M=$2m_Q m_{\bar{q}}/(m_Q+m_{\bar{q}})$, M_b=0.95 GeV, Λ=413 MeV. We fit the values of *k* and *A* for ground state of $b\bar{b}$ using experimental value of *b* quark mass and then determine the *c* quark mass by fitting $c\bar{c}$ ground state mass [7]. We choose the scale for the *Bc* system as α_s = 0.255. The obtained values for k = 1.173, A = 0.17, m_b = 4.66 GeV and m_c = 1.275 GeV are employed for further computation. V_{SD} is the spin dependent part of the potential [8].

$$V_{SD} = V_{SS(r)}\left[S(S+1)-\frac{3}{2}\right] + V_{LS}(r)(\vec{L}.\vec{S}) + V_T(r)\left[S(S+1)-\frac{3(\vec{S}.\vec{r})(\vec{S}.\vec{r})}{r^2}\right] \tag{3}$$

The spin-orbit term containing $V_{LS}(r)$ and the tensor term $V_T(r)$ describe the fine structure of the meson state, while the spin-spin term containing $V_{SS}(r)$ proportional to $2(\overrightarrow{S_q}.\overrightarrow{S_{\bar{q}}})=S(S+1)-3/2$. The coefficient of these spin-dependent terms of Eq.3 can be written in terms of the vector and scalar parts of the static potential as [8]

$$V_{LS}(r) = \frac{1}{2m_1m_2r}\left(3\frac{dV_V}{dr}-\frac{dV_S}{dr}\right)$$

$$V_T(r) = \frac{1}{6\,m_1m_2r}\left(3\frac{d^2V_V}{dr^2}-\frac{1}{r}\frac{dV_S}{dr}\right) \tag{5}$$

$$V_{SS}(r) = \frac{1}{3m_1m_2}\nabla^2V_V$$

The Bc mass spectroscopy is computed with these parameters and the result is given in Table 1.

TABLE I: Masses of Bc Meson (GeV)

State $n\,^{2S+1}L_J$	Present	[9]	[10]	[11]
$1\,^1S_0$	6.293	6.270	6.349	6.264
$1\,^3S_1$	6.317	6.332	6.373	6.337
$2\,^1S_0$	6.777	6.835	6.821	6.856
$2\,^3S_1$	6.811	6.881	6.855	6.899
$3\,^1S_0$	7.152	7.193	7.125	7.244
$3\,^3S_1$	7.187	7.235	7.210	7.280

Decay Constants

The pseudoscalar and vector decay constants are computed using the Van Royen Weisskopf formula for color are zero separation between the constituent quarks in ground state [12]

$$f_p = \sqrt{\frac{3}{\pi M_p}}\,R_{1S}(0)\;;\; f_v = \sqrt{\frac{3}{\pi M_v}}\,R_{1S}(0) \tag{5}$$

M_p and M_v are the masses of the pseudoscalar and vector meson respectively. The values of f_p and f_V are given in the Table II with the charge radii of the S-wave *Bc* mesons.

TABLE II: pseudoscalar and vector decay constants

State	Decay constant (MeV)				
	Present	[14]	[16]	[17]	[18]
$1\,^1S_0$	412	350	360	456	607
$1\,^3S_1$	411	--	--	--	604

Radiative Leptonic Decay Width

In this section, we compute the radiative decay width using the relation [13]

$$\Gamma(Bc \to l\overline{\gamma v}) = \frac{G_F^2|V_{cb}|^2}{8\pi^2}\, f_{Bc}^2 m_{Bc}^3 \,\frac{m_l^2}{m_{Bc}^2}\,\left(1-\frac{m_l^2}{m_{Bc}^2}\right)^2$$

As the mass of the lepton is very low compared to the *Bc* meson, the decays of pseudoscalar mesons into light lepton pairs are helicity suppressed, i.e. their decay widths are suppressed m_l^2/m_{Bc}^2, therefore the above formula becomes

$$\Gamma(Bc \to \gamma l\bar{v}) = \frac{\alpha G_F^2|V_{cb}|^2}{2592\pi^2}\, f_{Bc}^2 m_{Bc}^3[x_b + x_c] \quad (6)$$

Where $\alpha = 1/137$ is the electromagnetic coupling constant, G_F Fermi coupling constant = 1.66×10^{-5}, $|V_{cb}|$=0.044 [13] is the CKM matrix element. x_b and x_c is given by:

$$x_b = \left(3-\frac{m_{Bc}}{m_b}\right)^2 \text{ and } \quad x_c = \left(3-2\,\frac{m_{Bc}}{m_b}\right)^2 \qquad (7)$$

The computed radiative leptonic decay width for S-wave *Bc* mesons is 1.59×10^{-17} GeV which is comparable in order with 6.44×10^{-17} GeV as obtained by C. Cheng et al. [16].

Summary

The results from Table (I) suggests that our results for the *Bc* meson S-wave masses are in good agreement with that by D. Ebert et al [9] with small deviation from other references. It is also evident from Table II that the decay constants are in tune with other theoretical models too. As the experimental results for the same are not available, we compare the outcome of the present work with existing phenomenological models. It is found that the present non-relativistic computation can provide good framework to study *Bc* meson as both the quarks can be treated non-relativistically. Further study on the decay properties using fine-tuned parameters is underway.

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Semi-leptonic and pionic decays of Doubly Strange baryons

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Introduction

The doubly strange b-baryon Ω_b^- was reported by $D0$ [1] and CDF [2] Collaboration through the channel $\Omega_b^- \to J/\psi\Omega^-$ at $\sqrt{1.96}$ TeV. Doubly strange c-baryon Ω_c^0 was observed by $E687$ [3] significantly in the channel of $\Omega_c^0 \to \Sigma^+ K^- K^- \pi^+$ and later was confirmed by other groups [4]. We employ the extended harmonic confinement model in order to understand semi leptonic and pionic decay modes of these states to compute their masses and decay widths.

Methodology

The mass of baryon in the N energy eigenstate and J spin state can be computed as[5]

$$M_N^J = \sum_{i=1}^{3} \epsilon_N(q_i)_{conf} + \sum_{i<j=1}^{3} \epsilon(q_i, q_j)_{coul} + \sum_{i<j=1}^{3} \epsilon_N^J(q_i, q_j)_{S.D.} \quad (1)$$

where the first term is the confinement part, second term is due to the Coulomb interaction between the constituent quarks and the third term corresponds to the spin-dependent interactions.
The confinement energy of the baryonic system is given by [6],

$$\epsilon(q)_{conf} = \sqrt{(2N+3)\Omega_N(q) + M_q^2 - \frac{3M_q}{\sum_{i=1}^{3} M_{q_i}}}$$

where the size parameter, $\Omega_N(q)$ of RHM radial wave function is energy dependent and is given by

$$\Omega_N(q) = A\sqrt{E_N + M_q} \quad (2)$$

M_q is the constituent quark mass. The Coulomb of eq. 1 can be computed as

$$\epsilon(q_1, q_2)_{coul} = \left\langle NS \left| \frac{k\alpha_s^{eff}}{r} \right| NS \right\rangle \quad (3)$$

where α_s^{eff} is the strong running coupling coefficient. The spin-spin interaction is computed using the spin hyperfine interaction of the residual confined one gluon exchange potential [5–9]

$$V_{\sigma_i \cdot \sigma_j} = \frac{\alpha_s(\mu) N_i^2 N_j^2}{4} \frac{\lambda_i \lambda_j}{[E_i + m_i][E_j + m_j]} \times \left[4\pi\delta^3(r) - C^4 r^2 D_1(r)\right] \left(-\frac{2}{3}\sigma_i \sigma_j\right)$$

where $N_{i/j}$ is the normalization constant, C is the confinement strength of the gluon, r is the inter-quark distance, $\lambda_i\lambda_j$ is the spin factor, $D_1(r)$ is the confined gluon propagator and can be fitted to $\sim \frac{k_1}{r} exp(-C^2 r^2/2)$ [7, 8].

$$\epsilon_N^J(q_i, q_j)_{S.D.} = \langle NS|V_{SD}|NS\rangle \quad (4)$$

Here we have used $m_b = 4829$ MeV, $m_c = 1479$ MeV and $m_s = 410$ MeV. The potential parameters k, k_1 and C are fine tuned to obtain the experimental mass of Ω^-. The parameters used in this computation are $k = 0.006$, $k_1 = 21.36$ and $C = 100$ MeV.

Decay of Doubly strange baryons

In this section we compute the decay of Ω_c^0 and Ω_B^- baryon. The general definition for the semi-electronic decay width is given by [10]

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$$\frac{d\Gamma}{dw} = \frac{G_f^2 M^5}{192\pi^3} |V_{CKM}|^2 \sqrt{w^2 - 1} P(w) \quad (5)$$

where $P(w)$ contains the hadronic and leptonic tensor. After evaluating the integration over $w = 1$ in the hadronic form factors one will get the following relation for the decay width for electronic $(1/2)^+ \to (1/2)^+$ transition [10]

$$\Gamma_{\Omega^{0/-}_{c/b} \to \Xi^{+/0}_{c/b} e^- \bar{\nu}} = \frac{G_f^2 |V_{CKM}|^2}{15\pi^3} (M - m)^5 \quad (6)$$

where G_f is the Fermi coupling constant. The pionic decay width using the transition amplitude is computed using [10]

$$\Gamma_{\Omega_b^- \to \Xi_b^0 \pi^-} = \frac{(\Delta M)^{\frac{3}{2}}}{192\pi M^7} |A((ss)_1 \to (us)_0 \pi^-)|^2$$

here $\Delta M = [M^2 - (m - m_\pi)^2][M^2 - (m + m_\pi)^2]$ and the weak di-quark decay amplitude can be approximated with $|a_{weak}| \sim (1...2) \times 10^{-6}$ as [10]

$$A((ss)_1 \to (us)_0 \pi^-) \sim 2 M V_{us} V_{ud}^* a_{weak}$$

Where V_{us} and V_{ud} are the CKM matrices. We compute the semi-leptonic and pionic decay widths of Ω_c^0 and Ω_b^- without any additional parameters and the results are given in the table II.

Conclusion

The ground state masses of Ω_c^0 and Ω_b^- are computed using the methodology explained in the first section and compared with the experimental data. We also compute the semi-leptonic and pionic decay widths of Ω_c^0 and Ω_b^-. It is observed from table II that our results are well within the range as proposed by [10].

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TABLE I: ground state masses in MeV

State	quark content	present	[11]
Ω_c^0	css	2694.63	2695.2 ± 1.7
Ω_b^-	bss	6049.58	6048.8 ± 3.2

TABLE II: baryonic decay widths in GeV

mode of decay	present	[10]
$\Omega_c^0 \to \Xi_c^+ e^- \bar{\nu}$	9.05×10^{-18}	2.6×10^{-18}
$\Omega_c^0 \to \Xi_c'^+ e^- \bar{\nu}$	3.65×10^{-19}	3.63×10^{-19}
$\Omega_b^0 \to \Xi_b^0 e^- \bar{\nu}$	16.17×10^{-18}	4.05×10^{-18}
$\Omega_b^- \to \Xi_b^0 \pi^-$	0.93×10^{-18}	$(0.7...2.6) \times 10^{-18}$
$\Omega_b^- \to \Xi_b^- \pi^0$	0.91×10^{-18}	$(0.3...1.3) \times 10^{-18}$

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Masses and magnetic moment of doubly heavy baryons

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Introduction

Doubly heavy baryons are composed of two heavy quarks (b and/or c) and one light quark (u, d or s). There have been many theoretical attempts to compute masses of these states [1]. Experimental observation of such heavy resonances are expected from the facilities such as LHCb and Bell II. We employ the extended relativistic harmonic confinement model (ERHM) to compute the masses of doubly heavy baryons. The magnetic moments of heavy flavour baryons are also computed using the spin-flavour wave functions of the constituent quarks and their effective masses within the baryon.

Theoretical framework

In the relativistic harmonic confinement model (RHM) with scalar plus vector potential for the quark confinement, coloured quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic oscillator type of potential. The RHM has been extended to accommodate multi-quark states from lighter to heavier flavour sectors with unequal quark masses [2]. The mass of baryon in the N energy eigenstate and J spin state can be computed as [2, 3]

$$M_N^J = \sum_{i=1}^{3} \epsilon_N(q_i)_{conf} + \sum_{i<j=1}^{3} \epsilon(q_i, q_j)_{coul} + \sum_{i<j=1}^{3} \epsilon_N^J(q_i, q_j)_{S.D.} \quad (1)$$

First term is confinement energy of the constituent quarks inside the baryon; second term is the residual colour coulomb interaction between confined quarks and the third term corresponds to the spin-dependent interactions. The colour coulomb interaction energy is computed using the residual coulomb potential $V_{coul}(q_i q_j) = \frac{k\alpha_s(\mu)}{\omega_n r}$. Where ω_n is the state dependent colour dielectric coefficient [2]. It is also the measure of confinement strength through the non- perturbative contributions to the confinement scale at the respective threshold energy of the quark- antiquark excitations.

The wave function for the baryons are constructed through the single particle wave function but with the three particle size parameters [2, 3]. The spin averaged mass of the doubly heavy baryons are obtained using the model parameters $k = 0.37$, confinement parameter $A = 2166$ MeV$^{3/2}$, quark masses $m_u = 240$ MeV, $m_d = 243$ MeV, $m_s = 450$ MeV, $m_c = 1275$ MeV, $m_b = 4660$ MeV. The octet and decuplet masses are computed by considering the residual two body chromomagnetic interaction through the spin dependent term of confined one gluon exchange potential perturbatively.

Magnetic Moment

Considering the mass of bound quark inside the baryon as effective mass, the magnetic moment is computed using [4]

$$m_i^{eff} = m_i \left(1 + \frac{\langle H \rangle}{\sum_i m_i}\right) \quad (2)$$

such that the mass of the bayron is

$$M_B = \sum_i^3 m_i^{eff} \quad (3)$$

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TABLE I: Masses of doubly heavy baryons in MeV

Baryon	quark content	present	[4]	[5]	[6]	[7]	[8]
Ξ_{cc}^{++}	ccu	3542	3439	3612	3620	3532	
Ξ_{cc}^{*++}	ccu	3677	3516	3706	3727	3623	
Ξ_{cc}^{+}	ccd	3544	3440	3605	3620	3537	3520
Ξ_{cc}^{*+}	ccd	3677	3518	3685	3727	3629	
Ω_{cc}^{+}	ccs	3644	3479	3702	3778	3667	
Ω_{cc}^{*+}	ccs	3717	3559	3783	3872	3758	
Ξ_{bc}^{+}	bcu	6928	6834	6919	6933	6988	
Ξ_{bc}^{*+}	bcu	6990	6865	6986	6980	7083	
Ξ_{bc}^{0}	bcd	6929	6838	6820	6933	–	
Ξ_{bc}^{*0}	bcd	6990	6870	—-	6980	–	
Ω_{bc}^{0}	bcs	7012	6893	6986	7088	7103	
Ω_{bc}^{*0}	bcs	7045	6936	7046	7130	7200	
Ξ_{bb}^{0}	bbu	10257	10114	10197	10202	10344	
Ξ_{bb}^{*0}	bbu	10289	10165	10236	10237	10431	
Ξ_{bb}^{-}	bbd	10257	10117	10197	10202	–	
Ξ_{bb}^{*-}	bbd	10289	10170	10236	10237	–	
Ω_{bb}^{-}	bbs	10333	10164	10260	10359	10397	
Ω_{bb}^{*-}	bbs	10350	10236	10297	10389	10495	

$*$ indicates $J^P = \frac{3}{2}^+$ state

TABLE II: Magnetic moments in μ_N

Baryon	present	[4]	[5]	RQM [11]	NRQM [11]	[9]
Ξ_{cc}^{++}	-0.169	-0.137	-0.208	-0.130	-0.010	
Ξ_{cc}^{*++}	2.72	2.749	2.670	–	–	2.59
Ξ_{cc}^{+}	0.853	0.859	0.785	0.720	0.740	
Ξ_{cc}^{*+}	-0.23	-0.168	-0.311	–	–	-0.20
Ω_{cc}^{+}	0.74	0.783	0.635	0.670	0.670	
Ω_{cc}^{*+}	0.16	0.121	0.139	–	–	0.12
Ξ_{bc}^{+}	-0.52	-0.400	-0.475	-0.120	-0.290	-0.387
Ξ_{bc}^{*+}	2.68	2.052	2.27	–	–	2.011
Ξ_{bc}^{0}	0.63	0.476	0.518	0.420	0.460	0.499
Ξ_{bc}^{*0}	-0.76	-0.567	-0.712	–	–	-0.551
Ω_{bc}^{0}	0.49	0.396	0.368	0.450	0.390	0.399
Ω_{bc}^{*0}	-0.32	-0.316	-0.261	–	–	-0.279
Ξ_{bb}^{0}	-0.89	-0.656	-0.742	-0.530	-0.580	-0.665
Ξ_{bb}^{*0}	2.30	1.576	1.870	–	–	1.596
Ξ_{bb}^{-}	0.32	0.190	0.251	0.180	0.189	0.208
Ξ_{bb}^{*-}	-1.32	-0.951	-1.11	–	–	-0.984
Ω_{bb}^{-}	0.16	0.109	0.101	0.040	0.100	0.111
Ω_{bb}^{*-}	-0.86	-0.711	-0.662	–	–	-0.703

$*$ indicates $J^P = \frac{3}{2}^+$ state

Here the magnetic moment is obtained in terms of its constituent quarks as

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_i \vec{\sigma_i} | \phi_{sf} \rangle \tag{4}$$

where $\mu_i = e_i/2m_i^{eff}$. e_i and σ_i shows the charge and spin of the quark constituting the baryonic state and $|\phi_{sf}\rangle$ represents the spin flavor wave function of the respective baryonic state. [10].

Results and Discussion

We have employed ERHM to compute masses of baryons double heavy quarks. The computed mass spectra is found to be matching with available results from other theoretical approaches and are listed with them in table I. The magnetic moments are computed without introducing any extra parameters or correction to the wave function and are found in agreement with other theoretical calculations. It is observed that presence of b quark in the baryons raises the magnitude of the magnetic moments by a factor. This suggests that inclusion of some relativistic corrections and use of other suggested approaches for computation of magnetic moments may improve the results.

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Masses and radiative decay of Ω_{cc}^{+} baryon

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Introduction

The fact that the energy scales of doubly heavy baryons are much larger in comparison to strong interaction scale Λ_{QCD}, makes study of their spectroscopy an important tool for testing quantum chromodynamics [1, 2]. We employ extended relativistic harmonic model for computing the masses of Ω_{cc}^{+} baryon. Though this state is yet to be observed experimentally, many theoretical models have computed their mass spectra and decay modes. We compute masses and radiative decay widths using the model parameters along with spin-flavor wave functions and compare the results with the available theoretical predictions.

Methodology

For computation of bound state masses of baryon, we use the relativistic harmonic confinement model in which the quarks are confined through the Lorentz scalar plus vector potential of the form

$$V_{conf} = \frac{1}{2}(1+\gamma_0)A^2r^2 \tag{1}$$

Where A is the confinement strength mean field parameter and γ_0 is the Dirac matrix. The non relativistic reduction of the Dirac equation is performed for the potential Eq. (1) and the energy eigen values (ϵ_{conf}) are obtained. We perturbatively add the Columb potential along with state dependent colour dielectric coefficient (ω_n) given by [3]

$$V_{coul} = \frac{k\alpha_s(\mu)}{\omega_n r} \tag{2}$$

Where $\alpha_s(\mu)$ is the strong runing coupling constant. The mass of baryon in the different $n^{2S+1}L_J$ state acording to different J^{PC} can be written as [3]

$$M_N^J = \sum_{i=1}^{3}\epsilon_N(q_i)_{conf} + \sum_{i<j=1}^{3}\epsilon(q_i,q_j)_{coul} + \sum_{i<j=1}^{3}\epsilon_N^J(q_i,q_j)_{S.D.} \tag{3}$$

where the last term corresponds to the expectation value of spin dependent part of confined one gluon exchange potential.

The potential parameters used in computation of octet and decuplet masses of Ω_{cc}^{+} are as follows: $k = 0.37$, confinement mean field parameter $A = 2166$ MeV$^{3/2}$, quark masses: $m_c = 1315$ MeV and $m_s = 470$ MeV.

TABLE I: Masses of Ω_{cc}^{+} baryon in MeV

State	Present	[4]	[5]	[6]	[7]	[8]
Ω_{cc}^{+}	3769.91	3770	3747	3713	3738	3650
Ω_{cc}^{*+}	3835.3	3824	3819	3785	3822	3810

$*$ indicates $J^P = \frac{3}{2}^+$ state

Radiative decays

The radiative decay width can be expressed in terms of transition magnetic moment (in nuclear magneton μ_N) as [9]

$$\Gamma_{B^*\to B\gamma} = \frac{\omega^3}{4\pi}\frac{2}{2J+1}\frac{e^2}{m_p^2}\mu^2_{B^*\to B\gamma} \tag{4}$$

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TABLE II: Radiative transition magnetic moment in μ_N and radiative decay width in keV

	Present	[10]	[11]	[4]	[12]	[13]
$\mu_{\Omega_{cc}^{*+}\to\Omega_{cc}^{+}\gamma}$	-0.877	1.54	0.789	–	–	-0.89
$\Gamma_{\Omega_{cc}^{*+}\to\Omega_{cc}^{+}\gamma}$	0.89	9.45	0.949	8.61	6.93	–

where, m_p is the mass of proton, μ is the transition magnetic moment that can be written in terms of magnetic moment of constituent quark of final and initial state of baryons as $\mu_{B^*\to B\gamma} = \langle B|\hat{\mu}_{B^*z}|B^*\rangle$.

Result and discussion

We employed the extended relativistic harmonic model (ERHM) for computing the masses of doubly heavy Ω_{cc}^{+} baryon and our results are tabulated in Table I. Since no experimental results are available for these state we compare our results with theoretical predictions such as relativistic quark model [4], hypercentral model [8] as well with lattice QCD [5–7]. Our results deviate by less than 2 % from those obtained using lattice QCD as well as relativistic quark model.

Next we compute the radiative transition width of Ω_{cc}^{+} baryons. The decay rate is expressed in terms of transition magnetic moment. Considering the masses of confined quarks as the effective mass, the magnetic moments are obtained using the spin flavor structure of constituent quarks. The computed results are tabulated in Table II and compared with the other theoretical predictions. Our results for transition magnetic moments are matching well with modified Bag model [11] and chiral constituent quark model [13]. We also compare our result for radiative decay width with modified Bag model [11], chiral constituent quark model [13] along with recent papers on chiral quark model [10] and relativistic quark model [4]. We observe that our results match very well with modified Bag model.

Conclusion

Employing extended relativistic harmonic confinement model, we compute the mass of doubly heavy Ω_{cc}^{+} baryon which are matching well with lattice QCD results. The computed radiative transition magneic moment and decay width are lower than the recent theoretical predictions. However, due to the fact that the results from lattice calculations and experimental results are still awaited, the present results might be of interest as they are in tune with chiral constituent quark model and modified Bag model. Further study on the radiative decay properties of doubly heavy baryons is underway.

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Chapter 174
Mass and Hadronic Decay Widths of Z States as Di-meson Molecule

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After the discovery of Z_c^+ and Z_b^+ hadronic states by BESIII [1] and BELLE [2] collaborations respectively, there have been many attempts to describe these states either as tetra quark states or as hadronic molecules. The charged states and the masses of these exotic states are nearer to the threshold of $D^+\bar{D}^*$ and $B\bar{B}^*$ and respectively suggesting them to be di-mesonic molecular states. We treat them as hadronic molecules of $D^+\bar{D}^*$ mesons and $B\bar{B}^*$ mesons. We consider the interaction between the constituent mesons of the type modified Woods-Saxon plus Coulomb repulsive terms of the form [3]

$$V(r) = \frac{V_0}{1 + Exp\left[\frac{r-R_0}{a}\right]} + \frac{C Exp\left[\frac{r-R_0}{a}\right]}{\left(1 + Exp\left[\frac{r-R_0}{a}\right]\right)^2} - \frac{b}{r} \tag{174.1}$$

where the potential parameters are the strength of the potential $V_0 = 15\ MeV$, $b = 0.08$, size of the hadron $R_0 = 1.75$ fm, diffuseness of the surface of the molecule $a = –0.51$ fm. C determines the depth of the potential ($0 < C < 150$ MeV) [3]. For computation of binding energy, we solve the Schrödinger equation numerically for the interaction potential defined in Eq. 174.1. Binding energy and thus the di-mesonic molecular masses are given in Table 174.1.

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Table 174.1 Masses of $Z_c^+(D^+\bar{D}^*)$ and $Z_b^+(B\bar{B}^*)$ molecular states (in MeV) with the variation in potential depth C (in MeV)

C	$D^+\bar{D}^*$		$B\bar{B}^*$	
	Binding energy	Mass	Binding energy	Mass
0	11.82	3864.74	5.58	10598.9
50	11.96	3864.61	7.05	10597.4
100	12.07	3864.5	8.04	10596.4
150	12.15	3864.42	8.72	10595.7
PDG [5]		3883.9±4.5		10607.2±2.0

Table 174.2 Decay widths of Z_c^+ and Z_b^+ molecular states (in MeV)

Decay mode	Decay width					
	C = 0	C = 50	C = 100	C = 50	Exp [6]	[4]
$Z_c^+ \to \psi(1s) + \pi$	11.7202	11.7553	11.7849	11.8064	–	10.43–23.89
$Z_c^+ \to \psi(2s) + \pi$	2.1166	2.1146	2.1127	2.1114	–	1.28–2.94
$Z_b^+ \to \Upsilon(1s) + \pi$	22.8443	22.9280	22.9998	23.0567	22.9±7.3	13.3–30.8
$Z_b^+ \to \Upsilon(2s) + \pi$	26.9257	26.9858	27.0443	27.0930	21.1±4.0	15.4–35.7

We employ the method of Phenomenological Lagrangian mechanism developed by Y. Dong et al. [4] to compute hadronic decay widths. The corresponding two body decay widths for Z_c^+ and Z_b^+ can be written as [4]

$$\Gamma_{Z_c^+ \to \Psi(ns)\pi^+} \simeq \frac{g^2_{Z_c\Psi(ns)\pi}}{96\pi M^3_{Z_c}} \lambda^{3/2}(M^2_{Z_c}, M^2_{\psi(ns)}, M^2_\pi)\left(1 + \frac{M^2_{\psi(ns)}}{2M^2_{Z_c}}\right) \tag{174.2}$$

$$\Gamma_{Z_b^+ \to \Upsilon(ns)\pi} \simeq \frac{g^2_{Z_b\Upsilon(ns)\pi}}{16\pi M_{Z_b}} \lambda^{1/2}(M^2_{Z_b}, M^2_{\Upsilon(ns)}, M^2_\pi) \tag{174.3}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the Källen function, $g's$ are the coupling constant corresponds to the coupling between hadron to its constituent mesons. The computed decay widths are given in Table 174.2.

Result and Conclusion

The masses of Z_c^+ and Z_b^+ considering them as molecular states of $D^+\bar{D}^*$ and $B\bar{B}^*$ respectively are found to be below the mass of their resonance. We have analysed the nature of potential with the parameters such as diffuseness and depth of the potential. We have also computed the hadronic decay widths of these states in formalism of interaction Lagrangian mechanism and compare with the experiments. Our predictions of decay widths are in good agreement with the data from experiments.

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Decay properties of Ξ_{cc}^{++} baryon

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Introduction

The doubly heavy baryons are among the best tools to understand the quantum chromodynamics and heavy quark effective theory. Using weak decays of doubly heavy baryons, one can determine the elements of CKM matrix that help to understand quark mixing angle. The first doubly heavy baryon Ξ_{cc}^{+} was discovered by SLEX collaboration [1, 2] and recently LHCb have discovered the doubly charmed Ξ_{cc}^{++} in the $\Lambda_c^+ K^- \pi^+ \pi^-$ mass spectrum at $\sqrt{s} = 13$ TeV [3]. There are many theoretical approaches in literature to compute the mass spectra and decay properties. The models based on lattice QCD [7], QCD sum rules [8] relativistic quark model (RQM) [9], hypercentral constituent quark model [10] and many more.

In this article we compute the mass of doubly heavy Ξ_{cc}^{++} in the extended version of relativistic harmonic confinement model (ERHM). The spin dependent part of the confined one gluon exchange interaction is employed to compute the mass of excited state. Using the potential parameters and spin flavor wave-functions, we compute the transition magnetic moments between $3/2^+ \to 1/2^+$ states. We also compare our findings with the available experimental data and other theoretical predictions.

Formulation

We employ the extended relativistic harmonic confinement model (ERHM) to compute the masses of Ξ_{cc}^{++} baryon. In ERHM, the quarks are confined through in the Lorentz scalar with vector harmonic oscillator potential of the form[4, 5].

$$V_{conf} = \frac{1}{2}(1+\gamma_0)A^2 r^2 \tag{1}$$

We employ the nonrelativistic reduction of Dirac equation to compute the bound state masses of the doubly heavy baryons for the potential Eq. (1). In above equation A is the confinement mean field parameter and γ_0 is the Dirac matrix. Using the wave function, we incorporate the Coulomb potential with color dielectric coefficient perturbatively. The Coulomb potential is given by

$$V_{coul} = \frac{k\alpha_s}{\omega r} \tag{2}$$

Here, in this equation ω is the state dependent color dielectric coefficient and α_s is the strong running coupling constant. We also include the spin dependent part of confined one gluon exchange potential perturbatively to compute the mass of excited state. We assume here that the light quark interacts with both the heavy quarks separately (three body description) and not with a heavy diquark as proposed by other theoretical approach [6] as that causes increase in the baryon mass as a consequence. The mass of baryonic system in the

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TABLE I: Masses of Ξ_{cc}^{++} baryon in MeV

State	Present	[9]	[10]	[7]
Ξ_{cc}^{++}	3620.75	3620	3511	3610 (23) (22)
Ξ_{cc}^{*++}	3752.28	3727	3687	3692 (28) (21)

$*$ indicates $J^P = \frac{3}{2}^+$ state

different $n^{2S+1}L_J$ state can be written as

$$M_N^J = \sum_{i=1}^{3} \epsilon_N(q_i)_{conf} + \sum_{i<j=1}^{3} \epsilon(q_i, q_j)_{coul} + \sum_{i<j=1}^{3} \epsilon_N^J(q_i, q_j)_{S.D.} \qquad (3)$$

The potential parameters are: Coulomb interaction strength $k = 0.37$, the mean field parameter $A = 2166$ MeV$^{3/2}$ and quark masses $m_c = 1315$ MeV and $m_u = 240$ MeV. The computed masses of Ξ_{cc}^{++} is tabulated in Tab. I

Transition Magnetic Moment

The radiative transition magnetic moment in terms of nuclear magneton (μ_N) is computed using

$$\mu_{B^* \to B\gamma} = \langle B | \hat{\mu}_{B^*z} | B^* \rangle \qquad (4)$$

where B and B' represents the constituent quarks of parent and daughter baryon respectively. We obtain the following result

$$\mu_{\Xi_{cc}^{*++} \to \Xi_{cc}^{++}} = 1.564 \ \mu_N$$

Our results are in good agreement with the other theoretical approaches such as χCQM [11] and the model based on effective mass scheme [12].

Conclusion

We have computed the masses of doubly heavy baryons employing the extended version of relativistic harmonic confinement model and the results are tabulated in Tab I in comparison with results from LQCD [7], relativistic quark model [9] and hypercentral quark model [10]. Our results are in good agreement with LQCD and RQM. We have also computed the weak transition magnetic moment and it is in compliance with the other theoretical predictions. We notice that the three body description of the double heavy baryons provide better mass spectra without addition of correction terms. The study on computation of weak decay properties and lifetimes of differently charged states of doubly heavy baryons are underway.

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