Chapter 2

LEP Constraints on Unified Theories

2.1 Prelude

Since 1989 LEP has been producing electron positron collisions at various center of mass energies centered around the peak of the resonance production of the Z_0 bosons. The four experiments ALEPH, DELPHI, OPAL and L3 has till now collected about five million events. The analysis of this data on most varied subjects have been performed. The two most publicized results are the very precise measurements of the number of light neutrino and the absence of the standard neutral Higgs bosons in the energy range upto 63 GeV.

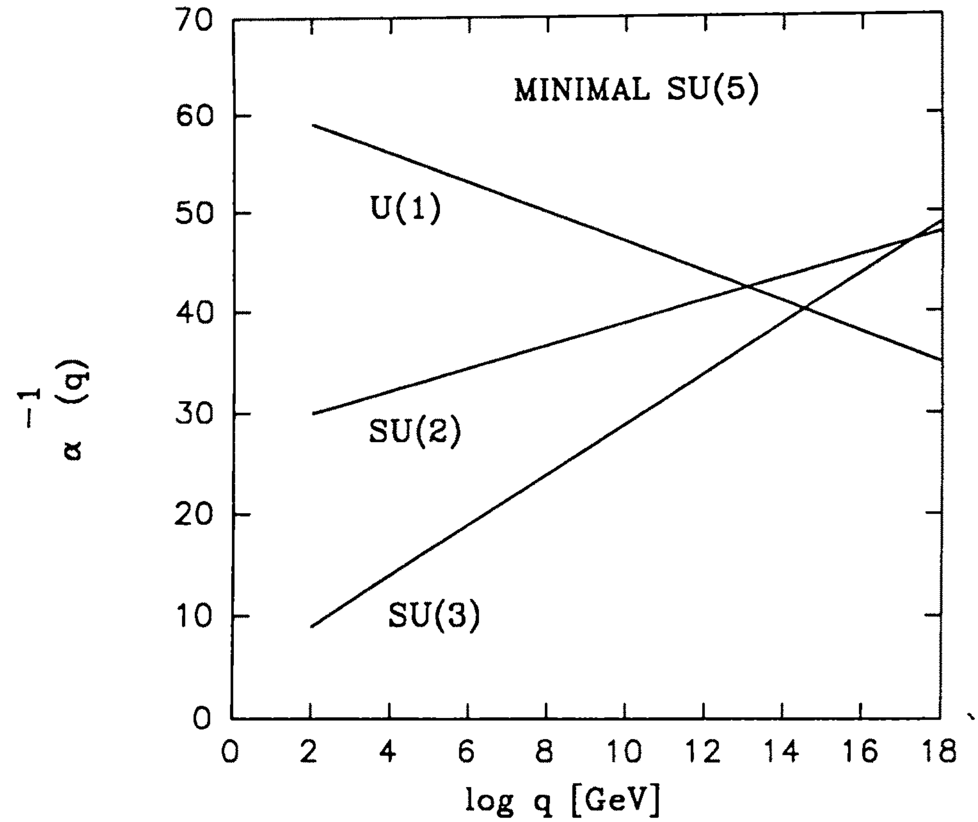
Recent interest in Grand Unified Theories has been motivated by the precision data that has emerged from LEP in the past years [1]. The measurements of the Z mass and width and also the jet cross sections and energy-energy correlations provide very accurate values of $\sin^2\theta_W$ and α_s at the scale M_Z. These precision values of $\sin^2\theta_W$ and α_s, when evolved using the renormalization group equations, can be used to put strong constraints on unified theories [2], [3], [4], [5].

In particular, in the analysis of Refs.[2],[4], it has been shown that using the recent experimental values

$$\begin{aligned}\sin^2\theta_W &= 0.2333 \pm 0.0008 \\ \alpha_s &= 0.113 \pm 0.005\end{aligned} \tag{2.1}$$

a unique intersection point of the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ couplings is not obtained in the standard model with one Higgs doublet (See Figure 1). Further in Ref.[2], it is shown that the criterion of unique intersection of the couplings at the unification scale is satisfied in the minimal supersymmetric extension of the standard model, and an unification scale of around 10^{16} GeV is obtained with the supersymmetry scale around 1 TeV (See Figure 2).

There are other ways of modifying the minimal standard model in order to get consistency with the data and the solutions that immediately suggest themselves are the inclusion of the effects of additional fermion generations or Higgs particles. However, the addition of new fermion generations changes the slopes of all the three couplings equally because the fermions contribute the same amount to the beta function coefficients of the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ groups. As for the Higgs, it has been shown [2] that, in the non-supersymmetric



MINIMAL SU(5)
U(1)
SU(2)
SU(3)
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
log q [GeV]
α⁻¹ (q)

FIGURE 1

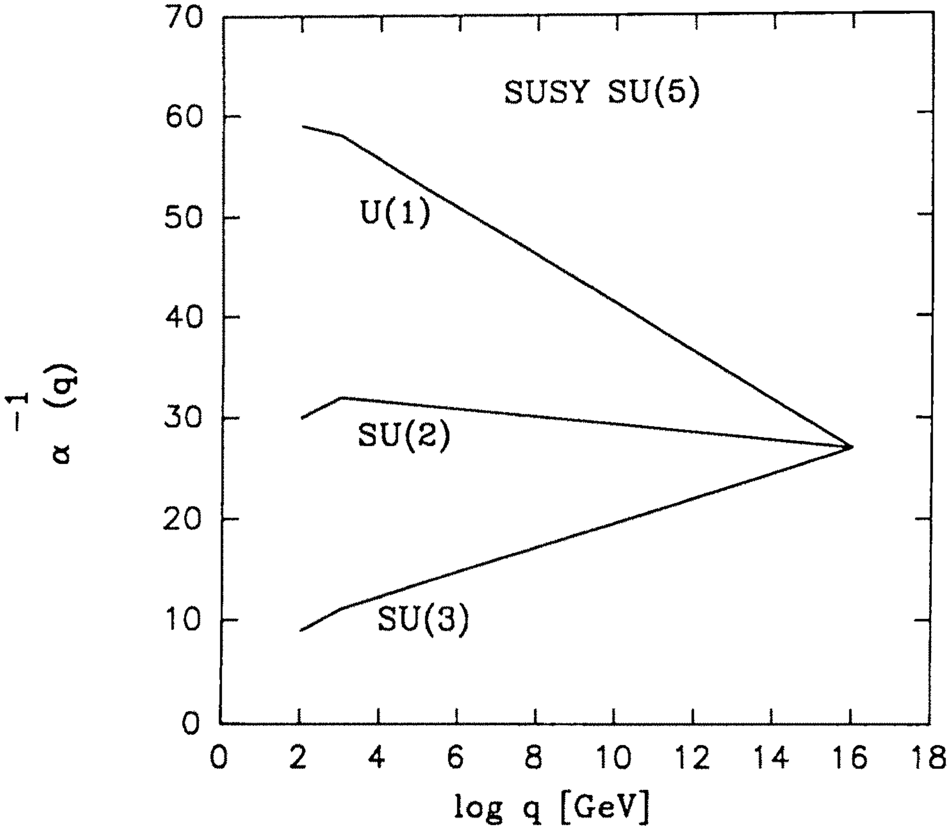

SUSY SU(5)
U(1)
SU(2)
SU(3)
α^{-1} (q)
log q [GeV]
0
10
20
30
40
50
60
70
0
2
4
6
8
10
12
14
16
18

FIGURE 2

case, with six or more Higgs doublets it is possible to obtain a unique intersection point for the couplings, but the value of the unification scale is too low and is ruled out by the measured value of the proton lifetime, $\tau_p = 5.5 \times 10^{32}$ years[6].

The failure of these attempts seems to indicate that the possibility of any unification group breaking in one step to the minimal non-supersymmetric standard model is, indeed, ruled out by present experimental data. Non-minimal (but non-supersymmetric) extensions of the standard model have also been studied [10] where it has been shown that either the introduction of a pair of leptoquarks at a scale of around 100 GeV or of a split **45** dimensional multiplet can also satisfy the unification constraints. The consensus seems to be that if we demand coupling constant unification, then there must be some new physics between the electroweak scale and the unification scale. The possibility of a desert between presently available energies and the unification scale seems to be ruled out. However the minimal left-right symmetric GUTs are consistent with the LEP results. We will see in this chapter that the LEP results put strong lower bounds on the right handed breaking scales of the left-right symmetric GUTS.

2.2 Effects of Higher Dimensional Operators

In this section, we show that this is not a necessary conclusion [1]. This we do by considering the presence of higher dimensional operators in the $SU(5)$ invariant Lagrangian. Such operators, scaled by powers of the Planck mass, arise due to quantum gravity effects [11] or due to spontaneous compactification of the extra spatial dimensions in Kaluza-Klein theories [12]. Such non-renormalizable terms involving fermion and Higgs fields have been used to show that the predictions of the minimal $SU(5)$ model for the fermion masses can be made consistent with observations [13]. Similar terms in the gauge part of the Lagrangian involving the gauge and Higgs scalar fields imply modifications in the gauge coupling constants at the unification scale [11], [14], [15]. We present, in the following, an analysis of the modification of the coupling constants at the unification scale due to the presence of five- and six-dimensional operators in the Lagrangian. We then check whether there is a consistent choice of couplings of the higher dimensional operators which yield $\sin^2\theta_W$, the unification scale, M_U, and the $SU(5)$ coupling constant α_G such that the experimental constraints from LEP and those coming from the measurement of the proton lifetime are simultaneously satisfied.

We start with a $SU(5)$ invariant Lagrangian which breaks at a scale M_U into $SU(3)_c \times SU(2)_L \times U(1)_Y$ due to a scalar Higgs field, ϕ, which transforms under the **24**-dimensional adjoint representation of $SU(5)$. This Lagrangian in the domain of energies $M_U \leq E \leq M_{Pl}$ (where M_{Pl} denotes the Planck mass) is given as a combination of the usual four dimensional terms and the new higher dimensional terms which have been induced by the physics beyond the Planck scale (or compactification scale). In principle, such non-renormalizable operators can be induced even due to the presence of a group G' which breaks to $SU(5)$ at a scale above the unification scale. We note here that the compactification scale can be even two orders of magnitude below the Planck scale in Kaluza-Klein theories [16]. The Lagrangian can be written as

$$L = L_0 + \sum_{n=1} L^{(n)} \tag{2.2}$$

[1] This section follows Ref [9]

where

$$L_0 = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \tag{2.3}$$

and the sum in Eq. 2.2 runs over all possible higher dimensional operators. We write down the five- and six-dimensional operators explicitly as

$$L^{(1)} = -\frac{1}{2}\frac{\eta^{(1)}}{M_{Pl}}\mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}) \tag{2.4}$$

$$\begin{aligned} L^{(2)} = & -\tfrac{1}{2}\tfrac{1}{M_{Pl}^2}\Big[\eta_a^{(2)}\{\mathrm{Tr}(F_{\mu\nu}\phi^2F^{\mu\nu}) + \mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}\phi)\} + \\ & \eta_b^{(2)}\mathrm{Tr}(\phi^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \eta_c^{(2)}\mathrm{Tr}(F^{\mu\nu}\phi)\mathrm{Tr}(F_{\mu\nu}\phi)\Big] \end{aligned} \tag{2.5}$$

where,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \tag{2.6}$$

$$A_\mu = A_\mu^i\frac{\lambda_i}{2} \tag{2.7}$$

with,

$$\mathrm{Tr}(\lambda_i\lambda_j) = \frac{1}{2}\delta_{ij} \tag{2.8}$$

In the above equations $\eta^{(n)}$ specify the couplings of the higher dimensional operators. Since

$$\mathrm{Tr}(F_{\mu\nu}\phi^2F^{\mu\nu}) = \mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}\phi) \tag{2.9}$$

we have used the same coupling $\eta_a^{(2)}$for both these operators in Eq. 2.5.

At the unification scale M_U, the Higgs field acquires the vacuum expectation value

$$\langle\phi\rangle = \frac{1}{\sqrt{15}}\phi_0\mathrm{diag}(1,1,1,-\frac{3}{2},-\frac{3}{2}) \tag{2.10}$$

The $SU(5)$ symmetry breaks at this scale because of the non-invariance of the Higgs vacuum expectation value under the $SU(5)$ symmetry. The magnitude of $\langle\phi\rangle$ is itself proportional to the unification scale, M_U, and, hence, one can replace the Higgs field appearing in Eq.2.5 by its vacuum expectation value (ignoring the small fluctuations of the Higgs field around $\langle\phi\rangle$). With this replacement one obtains the following $SU(3)\times SU(2)\times U(1)$ invariant Lagrangian:

$$\begin{aligned} -\tfrac{1}{2}(1+\epsilon_C)\mathrm{Tr}(F_{\mu\nu}^{(3)}F^{(3)\mu\nu}) - \tfrac{1}{2}(1+\epsilon_L)\mathrm{Tr}(F_{\mu\nu}^{(2)}F^{(2)\mu\nu}) \\ -\tfrac{1}{2}(1+\epsilon_Y)\mathrm{Tr}(F_{\mu\nu}^{(1)}F^{(1)\mu\nu}) \end{aligned} \tag{2.11}$$

Thus, even in the presence of the higher dimensional operators that we have considered above, we obtain the usual $SU(3)\times SU(2)\times U(1)$ invariant Lagrangian merely scaled by constant factors $(1+\epsilon_i)$ $i = C, L, Y$. Defining the physical gauge fields below the unification scale to be

$$A_i' = A_i(1+\epsilon_i)^{\frac{1}{2}} \tag{2.12}$$

we recover the usual $SU(3) \times SU(2) \times U(1)$ invariant Lagrangian with modified coupling constants

$$
\begin{aligned}
g_3^2(M_U) &= \bar{g}_3^2(M_U)(1+\epsilon_C)^{-1} \\
g_2^2(M_U) &= \bar{g}_2^2(M_U)(1+\epsilon_L)^{-1} \\
g_1^2(M_U) &= \bar{g}_1^2(M_U)(1+\epsilon_Y)^{-1}
\end{aligned}
\tag{2.13}
$$

The couplings $\bar{g}_i$ are the couplings that would have appeared in the absence of the higher dimensional operators, whereas the g_i are the physical couplings which are evolved down to lower scales.

It is expedient to introduce the parameter $\epsilon^{(n)}$ associated with a given operator of dimension $n+4$ in the following way:

$$
\epsilon^{(n)} = \left[\frac{1}{\sqrt{15}}\frac{\phi_0}{M_{Pl}}\right]^n \eta^{(n)} \tag{2.14}
$$

The vacuum expectation value ϕ_0 is related to M_U by

$$
\phi_0 = \left[\frac{6}{5\pi\alpha_G}\right]^{\frac{1}{2}} M_U \tag{2.15}
$$

where $\alpha_G = g_0^2/4\pi$ is the GUT coupling. We then have

$$
\epsilon^{(n)} = \left[\left\{\frac{2}{25\pi\alpha_G}\right\}^{\frac{1}{2}} \frac{M_U}{M_{Pl}}\right]^n \eta^{(n)} \tag{2.16}
$$

The change in the coupling constants are then related to the $\epsilon^{(n)}$s through the following equations.

$$
\begin{aligned}
\epsilon_C &= \epsilon^{(1)} + \epsilon_a^{(2)} + \frac{15}{2}\epsilon_b^{(2)} + \ldots \\
\epsilon_L &= -\frac{3}{2}\epsilon^{(1)} + \frac{9}{4}\epsilon_a^{(2)} + \frac{15}{2}\epsilon_b^{(2)} + \ldots \\
\epsilon_Y &= -\frac{1}{2}\epsilon^{(1)} + \frac{7}{4}\epsilon_a^{(2)} + \frac{15}{4}\epsilon_b^{(2)} + \frac{7}{8}\epsilon_c^{(2)} + \ldots
\end{aligned}
\tag{2.17}
$$

The ellipsis in the above equations denote the contribution of operators with dimension greater than six.

As shown above, the effect of the higher dimensional operators is to modify the gauge coupling constants The unification scale, M_U, is defined, as usual, through the boundary condition

$$
\bar{g}_3^2 = \bar{g}_2^2 = \bar{g}_1^2 = g_0^2 \tag{2.18}
$$

In the presence of the higher dimensional operators, the couplings $\bar{g}_i$ are not the physical couplings g_i but are related to them via the relations in Eq. 2.13. The result is that the following modified boundary condition is required to be satisfied at the unification scale

$$
g_3^2(1+\epsilon_C) = g_2^2(1+\epsilon_L) = g_1^2(1+\epsilon_Y) = g_0^2 \tag{2.19}
$$

The crucial point is that the physical couplings at the unification scale are g_3^2, g_2^2 and g_1^2 and these are the quantities that are evolved down to lower energy scales. The condition of equality in Eq.2.19 is, however, not on these physical couplings but on the "bare" couplings $g_i^2(1+\epsilon_i)$. The mismatch of the physical couplings at the unification scale can, therefore, be interpreted as due to the higher dimensional operators. With this in mind, one may use the standard one-loop renormalization group equations[7]

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M_U) + \frac{b_i}{2\pi}\ln(\frac{M_U}{M_Z}) \tag{2.20}$$

with the beta function coefficients given by

$$b_1 = \frac{41}{10}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7 \tag{2.21}$$

where we have taken $N_f = 3$ and $N_{\text{Higgs}} = 1$. Solving the RG equations yield

$$\ln(\frac{M_U}{M_Z}) = \frac{6}{67\alpha}\frac{1}{D}\left[\left\{1 - \frac{8}{3}\frac{\alpha}{\alpha_s}\right\} + \left\{\epsilon_C - \frac{5\epsilon_Y + 3\epsilon_L}{3}\frac{\alpha}{\alpha_s}\right\}\right] \tag{2.22}$$

$$\sin^2\theta_W = \frac{1}{D}\left[\sin^2\theta_W^{(5)} - \frac{19}{134}\epsilon_C + \frac{1}{67}\left\{21 + \frac{41}{2}\frac{\alpha}{\alpha_s}\right\}\epsilon_L + \frac{95}{402}\frac{\alpha}{\alpha_s}\epsilon_Y\right] \tag{2.23}$$

$$\frac{1}{\alpha_G} = \frac{3}{67}\frac{1}{D}\left[\frac{11}{3\alpha_s} + \frac{7}{\alpha}\right] \tag{2.24}$$

$$D = 1 + \frac{1}{67}(11\epsilon_C + 21\epsilon_L + 35\epsilon_Y) \tag{2.25}$$

where $\sin^2\theta_W^{(5)}$ is the usual minimal $SU(5)$ prediction

$$\sin^2\theta_W^{(5)} = \frac{23}{134} + \frac{109}{201}\frac{\alpha}{\alpha_s} \tag{2.26}$$

With the above equations at hand, we now consider whether it is possible to obtain a consistent choice of the parameters ϵ_C, ϵ_L and ϵ_Y such that we can satisfy the constraints on $\sin^2\theta_W$ and M_U from present experiments. First we restrict ourselves to five- dimensional operators only and try to see whether these operators alone can provide the required numerical values for $\sin^2\theta_W$ and M_U. The restriction to five-dimensional operators implies from Eq.2.17 the following relations:

$$\epsilon_C = \epsilon^{(1)}; \quad \epsilon_L = -\frac{3}{2}\epsilon^{(1)}; \quad \epsilon_Y = -\frac{1}{2}\epsilon^{(1)} \tag{2.27}$$

We use the values of $\sin^2\theta_W$ and α_s derived from LEP data, given in Eq.2.1, and $\alpha = 1/127.9$. Since ϵ_C, ϵ_L and ϵ_Y are all determined in terms of a single parameter $\epsilon^{(1)}$, specifying the value of $\sin^2\theta_W$ in Eq.2.23 at the scale M_Z fixes up these parameters uniquely. For the central value of $\sin^2\theta_W$ ($= 0.2333$), we obtain the solution $\epsilon^{(1)} = -0.0441$, which from Eq.2.22 gives $M_U = 3.8\times 10^{13}$ GeV. The corresponding value of $\alpha_G = 0.0245$. Using

$$\tau_p \approx \frac{1}{\alpha_G^2}\frac{M_U^4}{M_p^5} \tag{2.28}$$

(where M_p is the proton mass), we find that the value of M_U is too low to be consistent with the experimental limits on proton lifetime. We also find that by varying $\sin^2\theta_W$ over the allowed range, the values of M_U and α_G do not change appreciably. Thus, we find that it is not possible to obtain a consistent solution with five-dimensional operators alone.

We would now like to check whether it is possible to obtain a consistent solution if we admit both five- and six-dimensional operators. Then, from Eq.2.17, we see that ϵ_C, ϵ_L and ϵ_Y are now independent parameters. By feeding in the value of $\sin^2\theta_W$, we obtain one constraint on these three parameters. The other constraint that these parameters need to satisfy is, of course, the proton lifetime constraint. From Eq.2.28, we see that the proton lifetime is controlled by both M_U and α_G. There is a further constraint that we impose on the parameters. If we require that successive terms in the sum in Eq.2.17 be scaled by inverse powers of M_{Pl}, then this can be ensured by requiring that $|\eta^{(2)}_{a,b,c}| \leq |\eta^{(1)}|$ and with $\eta^{(1)}$ not too large. Restricting ourselves to the space of parameters that satisfy these constraints, we present some sample values of the parameters in Table 2.1.

ϵ_L	ϵ_C	ϵ_Y	$(M_U)_{\text{min}}$	α_G
-0.85	-0.86 - -0.845	-0.893 - -0.864	10^{17}	10^{-3}
-0.90	-0.913 - -0.905	-0.926 - -0.897	10^{17}	10^{-3}
-0.95	-0.956 - -0.953	-0.963 - -0.949	10^{16}	10^{-4}
-0.99	-0.9913 - -0.9906	-0.9926 - -0.9897	10^{16}	10^{-4}

Table 2.1: The ranges of the various parameters obtained with the central value of $\sin^2\theta_W = 0.2333$

We thus find that if we include both five- and six-dimensional operators, then there are a whole range of parameters that are consistent with the values of $\sin^2\theta_W$ and M_U that are required for agreement with experiment. In earlier papers [11], [14], where the effect of only five-dimensional operators was considered, the value of $\sin^2\theta_W$ obtained is too small to be in conformity with the latest values. The effect of six-dimensional operators was also included in Ref.[15]. Our work goes beyond the analysis presented in Ref.[15] in that we have included a more general set of six-dimensional operators. The effect of the extra operators that we have considered cannot *a priori* be neglected. Even if we restrict ourselves to the $\mathrm{Tr}(F_{\mu\nu}\phi^2F^{\mu\nu})$ operator, as in Ref.[15], we have checked that for the range of parameters chosen in Table. II of Ref.[15], the values of $\sin^2\theta_W$ obtained are not in conformity with the LEP results.

Our analysis thus shows that by including the effects of higher dimensional operators arising due to quantum gravity or spontaneous compactification of extra spatial dimensions in Kaluza-Klein theories, it is possible to show that the predictions of a minimal $SU(5)$ GUT is in conformity with the latest LEP values of $\sin^2\theta_W$ and α_s, and also with the experimental constraints on proton lifetime.

2.3 Constraints on Left-Right Symmetric GUTs

An important class of GUTs are the left-right symmetric models where the Standard Model comes from a larger group with a $SU(2)_L \times SU(2)_R$ symmetry [2]. One interesting possibility is that the left-right symmetry survives to relatively low energies, and would therefore have testable consequences at current experiments or in experiments planned in the near future.

Earlier phenomenological studies [17] [18] [19] have indicated that it is indeed possible to have a low value for the left-right symmetry breaking scale, M_R, in various grand unified

[2]This section follows Ref. [5]

theories with intermediate mass scales and in partially unified theories. In these analyses, however, values of $\sin^2\theta_W$ from as low as 0.21 to as high as 0.28 were considered.

Such a large variation in $\sin^2\theta_W$ was expected from the existence of a extra neutral gauge boson, Z', coming from either a $U(1)_R$ or $SU(2)_R$ symmetry. Due to the mixing of the Z with the Z' the usual ρ parameter, defined as

$$\rho \equiv \frac{M^2_{W^\pm}}{M^2_Z \cos^2\theta_W} \tag{2.29}$$

changes by a positive quantity, $\Delta\rho_M$, given as [20]

$$\Delta\rho_M = \sin^2\xi \left[\frac{M^2_{Z'}}{M^2_Z} - 1\right] \tag{2.30}$$

where ξ is the mixing angle. From the definitions of $\sin^2\theta_W$ and ρ one can see that

$$\delta(\sin^2\theta_W) \approx -\frac{\sin^2\theta_W \cos^2\theta_W}{\cos 2\theta_W}\Delta\rho_M \tag{2.31}$$

The quantity $\Delta\rho_M$ is determined entirely by the measured value of M_W/M_Z, as can be seen clearly from Eq. 2.30. The results of a recent fit [20] to the LEP data [1] yield the following bound:

$$\Delta\rho_M \leq 0.010 - 0.003\left[\frac{m_t(GeV)}{100}\right]^2 \tag{2.32}$$

where the dependence on the top mass, m_t, comes through the radiative corrections. Using this value of $\Delta\rho_M$ we get $\delta(\sin^2\theta_W) \approx 3.32 \times 10^{-3}$ which is well within the quoted errors on $\sin^2\theta_W$. It is with this very stringently bound value of $\sin^2\theta_W$ that we wish to study whether a low value for the left-right symmetry breaking scale, M_R, is allowed.

In this section, we will first assume the existence of precisely such a mass scale and study the evolution of the couplings *via* the renormalization group (RG) equations at the one-loop level. To keep matters simple at first, we consider a symmetry breaking scheme without any other intermediate mass scale other than M_R and neither do we specify the GUT group, G. We take $M_R \approx 1$ TeV and try to determine if there is any unification point below the Planck scale ($\approx 10^{19}$ GeV). The central values for the couplings at the scale M_Z(= 91.176 GeV), obtained from α_s, $\sin^2\theta_W$ and α [2] [3]

$$\alpha_1(M_Z) = 0.016887;\ \alpha_2(M_Z) = 0.03322;\ \alpha_3(M_Z) = 0.11 \tag{2.33}$$

are evolved to the scale M_R. Using the matching conditions of the coupling constants at M_R

$$\begin{aligned}
\alpha^{-1}_{1Y}(M_R) &= \frac{3}{5}\alpha^{-1}_{2R}(M_R) + \frac{2}{5}\alpha^{-1}_{1(B-L)}(M_R) \\
\alpha^{-1}_{2L}(M_R) &= \alpha^{-1}_{2R}(M_R)
\end{aligned} \tag{2.34}$$

the evolution equations become

$$\begin{aligned}
\alpha^{-1}_{1(B-L)}(q) &= \frac{5}{2}\alpha^{-1}_{1Y}(M_Z) - \frac{3}{2}\alpha^{-1}_{2L}(M_Z) - \\
&\quad 5b_{1Y}M_{RZ} + 3b_{2L}M_{RZ} - 2b_{1(B-L)}M_{qR} \\
\alpha^{-1}_{2L,R}(q) &= \alpha^{-1}_{2L}(M_Z) - 2b_{2L}M_{RZ} - 2b_2 M_{qR} \\
\alpha^{-1}_{3c}(q) &= \alpha^{-1}_{3c}(M_Z) - 2b_{3c}M_{RZ} - 2b_{3c}M_{qR}
\end{aligned} \tag{2.35}$$

where $M_{ij} \equiv 4\pi \ln(M_i/M_j)$ and the beta function coefficients, b_i's are given as:

$$b_1 = \frac{4}{(4\pi)^2};\ b_2 = -\frac{1}{(4\pi)^2}\frac{10}{3};\ b_3 = -\frac{7}{(4\pi)^2} \tag{2.36}$$

Here we have taken the number of fermion families, $n_f = 3$. In Fig. 3 we have plotted the gauge coupling constants $\alpha^{-1}_{1(B-L)}$, $\alpha^{-1}_{2L,R}$ and α^{-1}_{3c} as a function of energy, taking $M_R = 1$ TeV. We see from the Figure 3 that as a result of choosing a low value for M_R, there is no unification point even when we evolve upto a scale as high as 10^{19} GeV. The $SU(3)_c$ and $SU(2)_{L,R}$ do intersect at about 10^{17} GeV, but the $U(1)$ scale remains much too high for any possibility of unification to exist. We stress that the discrepancy is of such a large magnitude that the inclusion of the experimental errors on the input values of the coupling constants at M_Z will not redeem the situation and therefore this analysis using the central values should suffice to illustrate the point.

The above illustrative exercise certainly indicates that a low value of M_R is inconsistent with grand unification with no intermediate scales. To investigate this more thoroughly and to consider, in particular, the effect of introducing intermediate mass scales we will study various unification schemes with left-right symmetry, in detail. The traditional model of left-right symmetry is based on $SO(10)$, [17] E_6 [21] or $SU(16)$ [22] unification groups. Recently a very interesting proposal for unification starting from a $SU(15)$ group has been made [23]. Since baryon number is a local symmetry in this model, it is possible to suppress proton decay and allow unification at very low scales ($\approx 10^9$ GeV). This model is not left-right symmetric but a simple extension of this model which uses a $SU(16)$ unification group [24] is left-right symmetric. In this work, we consider the $SO(10)$, E_6 and $SU(16)$ based-models. Other than these we also consider partially unified models, which are left-right symmetric [17], where one starts from the semi-simple group $SU(4)_c \times SU(2)_L \times SU(2)_R$ instead of a simple group. These models are constrained by the value of $\sin^2\theta_W$ but not by the value of α_s and consequently there is more freedom in these models than in the grand unified models.

The detailed breaking chain that gives a intermediate left-right symmetry starting from a $SO(10)$ model is as follows [10]:

$$\begin{aligned} SO(10) \quad & \xrightarrow{M_U} SU(4) \times SU(2)_L \times SU(2)_R \\ & \xrightarrow{M_1} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \\ & \xrightarrow{M_R} SU(3)_c \times SU(2)_L \times U(1)_Y \\ & \xrightarrow{M_W} SU(3)_c \times U(1)_{em} \end{aligned}$$

The matching conditions at M_R are precisely the same as those given in Eq. 2.34. The weak breaking scale, M_W, is taken to be 250 GeV in our computations. The renormalization group equations for this breaking chain imply the following relations between the standard model coupling constants and the unification coupling constants:

$$\begin{aligned} \alpha^{-1}_{1Y}(M_W) &= \alpha^{-1}_{SO(10)}(M_U) + (\frac{6}{5}b_{2R} + \frac{4}{5}b_4)M_{U1} + \\ & \quad (\frac{6}{5}b_{2R} + \frac{4}{5}b_{1(B-L)})M_{1R} + 2b_{1Y}M_{RW} \\ \alpha^{-1}_{2L}(M_W) &= \alpha^{-1}_{SO(10)}(M_U) + 2b_{2L}M_{U1} + 2b_{2L}M_{1R} + 2b_{2L}M_{RW} \\ \alpha^{-1}_{3c}(M_W) &= \alpha^{-1}_{SO(10)}(M_U) + 2b_4 M_{U1} + 2b_{3c}M_{1R} + 2b_{3c}M_{RW} \end{aligned} \tag{2.37}$$

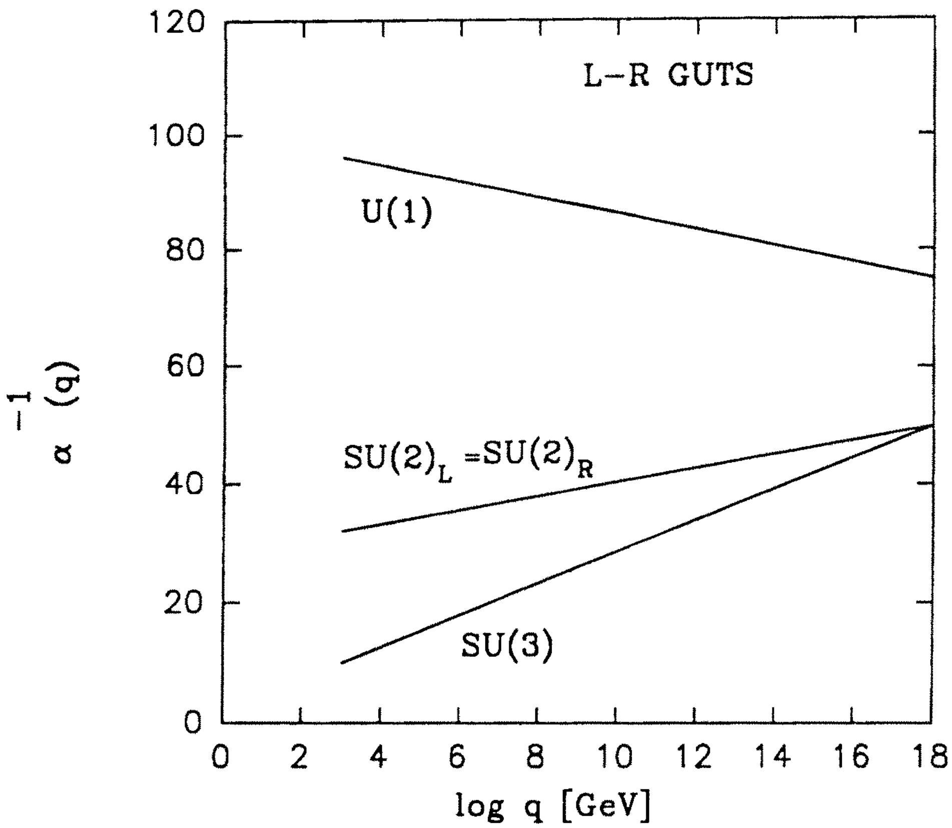

L−R GUTS
U(1)
SU(2)_L =SU(2)_R
SU(3)
120
100
80
60
40
20
0
0
2
4
6
8
10
12
14
16
18
log q [GeV]
α^−1 (q)

FIGURE 3

The analysis for the E_6-based theories is very similar to that of the $SO(10)$-based theories. In the E_6 case, however, there are two independent routes which lead to the Standard Model group. In one of these, the E_6 group goes to the Standard Model group *via* $SU(4)_c \times SU(2)_L \times SU(2)_R$ (exactly as in the $SO(10)$ case). The other possibility, which is of our interest, is

$$
\begin{aligned}
E_6 \overset{M_U}{\longrightarrow} & SU(3)_c \times SU(3)_L \times SU(3)_R \\
\overset{M_1}{\longrightarrow} & SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \\
\overset{M_R}{\longrightarrow} & SU(3)_c \times SU(2)_L \times U(1)_Y \\
\overset{M_W}{\longrightarrow} & SU(3)_c \times U(1)_{em}
\end{aligned}
$$

The renormalization group equations for this case are

$$
\begin{aligned}
\alpha_{1Y}^{-1}(M_W) &= \alpha_{E_6}^{-1}(M_U) + (\frac{8}{5}b_{3R} + \frac{2}{5}b_{3L})M_{U1} + \\
& \quad (\frac{6}{5}b_{2R} + \frac{4}{5}b_{1(B-L)})M_{1R} + 2b_{1Y}M_{RW} \\
\alpha_{2L}^{-1}(M_W) &= \alpha_{E_6}^{-1}(M_U) + 2b_{3L}M_{U1} + 2b_{2L}M_{1R} + 2b_{2L}M_{RW} \\
\alpha_{3c}^{-1}(M_W) &= \alpha_{E_6}^{-1}(M_U) + 2b_{3c}M_{U1} + 2b_{3c}M_{1R} + 2b_{3c}M_{RW}
\end{aligned} \tag{2.38}
$$

The M_{ij}'s are as defined earlier and the beta function coefficients are

$$
b_1 = 0;\ b_2 = -\frac{1}{(4\pi)^2}\frac{22}{3};\ b_3 = -\frac{1}{(4\pi)^2}11;\ b_4 = -\frac{1}{(4\pi)^2}\frac{44}{3} \tag{2.39}
$$

Note that the fermionic contribution to the beta functions have not been written down in the above equations since our intention is to use these in the equations for $\sin^2\theta_W$ and α_s, where the fermionic contributions cancel exactly. The linear combinations of the couplings that yield $\sin^2\theta_W$ and α_s are the following:

$$
\begin{aligned}
\sin^2\theta_W &= \frac{3}{8} - \frac{5}{8}\alpha(\alpha_{1Y}^{-1} - \alpha_{2L}^{-1}) \\
1 - \frac{8}{3}\frac{\alpha}{\alpha_s} &= \alpha(\alpha_{2L}^{-1} + \frac{5}{3}\alpha_{1Y}^{-1} - \frac{8}{3}\alpha_{3c}^{-1})
\end{aligned} \tag{2.40}
$$

Using the experimental numbers, $\sin^2\theta_W = 0.236$ and $\alpha_s = 0.11$ [2][3], the above relations reduce to the following:

$$
\alpha_{1Y}^{-1} - \alpha_{2L}^{-1} = 29.097 \tag{2.41}
$$

and

$$
\alpha_{2L}^{-1} + \frac{5}{3}\alpha_{1Y}^{-1} - \frac{8}{3}\alpha_{3c}^{-1} = 104.755 \tag{2.42}
$$

Substituting the expressions for the couplings from Eq. 2.37 and in Eq. 2.38 and solving we get the same solutions for both the $SO(10)$ and E_6 cases

$$
\begin{aligned}
m_{U1} &= -26.1741 + \frac{3}{2}m_{RW} \\
m_{12} &= 88.5009 - 4m_{RW}
\end{aligned} \tag{2.43}
$$

where $m_{ij} = M_{ij}/4\pi$. Using the fact that m_{U1} is positive, one can readily see that the minimum value of m_{RW} is 17.4494 i.e.

$$M_R|_{min} \equiv M_W exp(m_{RW}) \tag{2.44}$$

which means that M_R cannot be lower than 10^9 GeV, in both the $SO(10)$ and E_6 models.

Now consider left-right symmetry coming from $SU(16)$ as the Grand unification group. $SU(16)$ can break to the left-right symmetric group *via* a number of chains. But all the symmetry breaking chains which proceed *via* the $SU(4) \times SU(2)_L \times SU(2)_R$ group will give a lower bound on M_R, similar to what happens in the case of $SO(10)$ or E_6 models. We will, therefore, not discuss the chains which have an intermediate $SU(4) \times SU(2)_L \times SU(2)_R$ group.

It was noticed in the $SU(15)$ GUT that if $SU(3)_L \times SU(2)^q_L \times SU(3)_R \times U(1)_B \times SU(2)^l_L \times U(1)_l$ group breaks at the un-unification scale M_1, then the condition for low energy unification is that M_1 has also to be lowered. This idea can be extended to the $SU(16)$ GUT and it can be seen that lowering the un-unification scale one can achieve low energy unification. We shall now try to explore this scenario and see if we can have low-energy left-right symmetry in $SU(16)$ GUT with low energy unification. We shall assume that the Higgs structure is such that proton decay is suppressed. The detailed breaking chain we consider is the following:

$$\begin{aligned} SU(16) \quad & \xrightarrow{M_U} SU(3)_L \times SU(2)^q_L \times SU(3)_R \times SU(2)^q_R \times U(1)_B \times \\ & \qquad SU(2)^l_L \times SU(2)^l_R \times U(1)_{lep} \\ & \xrightarrow{M_1} SU(3)_c \times SU(2)^{q+l}_L \times SU(2)^{q+l}_R \times U(1)_{(B-L)} \\ & \xrightarrow{M_R} SU(3)_c \times SU(2)_L \times U(1)_Y \\ & \xrightarrow{M_W} SU(3)_c \times U(1)_{em} \end{aligned}$$

In the above breaking chain, $U(1)_B$ is proportional to the baryon number and $U(1)_{lep}$ is proportional to lepton number both properly normalized. The matching conditions at M_R are again those given in Eq. 2.34 with appropriate modifications to account for the quark-lepton un-unification group while those at the scale M_1 for the $U(1)$ groups can be easily seen to be

$$\alpha^{-1}_{1(B-L)}(M_1) = \frac{1}{4}\alpha^{-1}_{1B}(M_1) + \frac{3}{4}\alpha^{-1}_{1lep}(M_1) \tag{2.45}$$

From the fermion transformation properties at different levels, we can check that $SU(3)_L$ and $SU(3)_R$ are normalized to 1, $SU(2)^q_L$ and $SU(2)^q_R$ are normalized to 3/2, and $SU(2)^l_L$ and $SU(2)^l_R$ are normalized to 1/2. All other groups are normalized to 2. The beta function coefficients properly normalized are

$$\begin{aligned} & b_{1(B-L)} = 0;\ b_{1Y} = 0;\ b_{1B} = 0;\ b_{1lep} = 0 \\ & b^q_{2L} = b^q_{2R} = -\frac{1}{(4\pi)^2}\frac{88}{9};\ b^l_{2L} = b^l_{2R} = -\frac{1}{(4\pi)^2}\frac{88}{3};\ b_2 = -\frac{1}{(4\pi)^2}\frac{22}{3} \\ & b_3 = -\frac{1}{(4\pi)^2}11;\ b_{3L} = -\frac{1}{(4\pi)^2}22;\ b_{3R} = -\frac{1}{(4\pi)^2}22 \end{aligned} \tag{2.46}$$

The values of the beta function coefficients given above are, as before, without the fermionic contributions since these cancel exactly in the expressions for $\sin^2\theta_W$ and α_s. The evolution equations for the $SU(16)$ case are as follows:

$$\begin{aligned}
\alpha_{1Y}^{-1}(M_W) &= \alpha_{SU(16)}^{-1}(M_U) + (\frac{9}{10}b_{2R}^q + \frac{3}{10}b_{2R}^l + \frac{1}{5}b_{1B} + \frac{6}{10}b_{1lep}) \\
&\quad M_{U1} + (\frac{6}{5}b_2 + \frac{4}{5}b_{1(B-L)})M_{1R} + 2b_{1Y}M_{RW} \\
\alpha_{2L}^{-1}(M_W) &= \alpha_{SU(16)}^{-1}(M_U) + (\frac{3}{2}b_{2L}^q + \frac{1}{2}b_{2L}^l)M_{U1} + 2b_2M_{1R} \\
&\quad +2b_2M_{RW} \\
\alpha_{3c}^{-1}(M_W) &= \alpha_{SU(16)}^{-1}(M_U) + (b_{3L} + b_{3R})M_{U1} + 2b_3(M_{1R} + M_{RW})
\end{aligned} \tag{2.47}$$

As in the $SO(10)$ case, we use Eqs. 2 41 and 2.42 and solve for the ratios of the mass scales. The resulting equations are

$$\begin{aligned}
m_{RW} &= 17.4494 \\
m_{1R} &= 18.7031 - 2m_{U1}
\end{aligned} \tag{2.48}$$

In this case, therefore, we see that we get a fixed value of $m_{RW} = 17.4494$, which is precisely the value of the minimum of m_{RW} in the $SO(10)$ case. This implies a value of $\approx 10^9$ for the left-right symmetry breaking scale, M_R.

Finally, we investigate the possibility of left-right symmetry from a partially unified model [17]. The detailed breaking chain is as follows:

$$\begin{aligned}
&SU(4) \times SU(2)_L \times SU(2)_R \overset{M_1}{\longrightarrow} \\
&SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \\
&\overset{M_R}{\longrightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y \\
&\overset{M_W}{\longrightarrow} SU(3)_c \times U(1)_{em}
\end{aligned}$$

The matching condition at M_R is the same as in the preceding examples. The evolution equations in this case read as follows:

$$\begin{aligned}
\alpha_{1Y}^{-1}(M_W) &= \frac{3}{5}\alpha_2^{-1}(M_1) + \frac{2}{5}\alpha_4^{-1}(M_1) + \\
&\quad (\frac{6}{5}b_{2R} + \frac{4}{5}b_{1(B-L)})M_{1R} + 2b_{1Y}M_{RW} \\
\alpha_{2L}^{-1}(M_W) &= \alpha_2^{-1}(M_1) + 2b_{2L}M_{1R} + 2b_{2L}M_{RW} \\
\alpha_{3c}^{-1}(M_W) &= \alpha_4^{-1}(M_1)2b_{3c}M_{1R} + 2b_{3c}M_{RW}
\end{aligned} \tag{2.49}$$

In writing the above equations we have used the fact that, due to the left-right symmetry beyond the scale M_R, $\alpha_{2L} = \alpha_{2R} = \alpha_2$. The beta function coefficients remain the same as before. Note that the important difference in this case is that the single coupling at the unification scale which appeared in the unified models, is now replaced by the disparate coupling strengths at the scale of partial unification. To express the couplings at the scale M_W in terms of $\sin^2\theta_W$ and α_s an appropriate linear combination needs to be constructed such that the different couplings at the partial unification scale are exactly canceled. The resulting equation is

$$\frac{1}{4\pi}\left[\frac{(1-2\sin^2\theta_W)}{\alpha}-\frac{2}{3\alpha_s}\right] = 2(b_{2R}-b_{2L}+\frac{2}{3}b_{1(B-L)}-\frac{2}{3}b_3)M_{1R} +2(\frac{5}{3}b_{1Y}-b_{2L}-\frac{2}{3}b_3)M_{RW} \tag{2.50}$$

Using the experimental values of $\sin^2\theta_W$ and α_s, we get

$$53.74-2m_{RW}=m_{2R} \tag{2.51}$$

To get the minimum of M_R in this case, we also use the constraint that M_2 should be less than (or equal to!) the Planck scale, 10^{19} GeV i.e.

$$m_{RW}+m_{2R}\leq 38.23 \tag{2.52}$$

From these equations, we get $m_{RW}\geq 15.51$ which implies a lower bound on M_R equal to $\approx 10^9$ GeV.

We now consider the effect of including the Higgs contribution in the renormalization group equations. The specific Higgs representations under different symmetry groups are given in Table (2.2).

$SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{(B-L)}$	$SU(3)_c\times SU(2)_L\times U(1)_Y$
$1,1,3,+\sqrt{\frac{3}{2}}$	$1,1,0$
$1,2,2,0$	$1,2,-\frac{1}{2}\sqrt{\frac{3}{5}}$
$1,2,2,0$	$1,2,+\frac{1}{2}\sqrt{\frac{3}{5}}$
$1,3,1,-\sqrt{\frac{3}{2}}$	$1,3,-\sqrt{\frac{3}{5}}$

Table 2.2: The explicit Higgs representations under the left-right symmetric group and the electroweak group.

Using these representations, the beta function coefficients are given as

$$b_{1(B-L)}=\frac{1}{(4\pi)^2}\frac{3}{2};\ b_{1Y}=\frac{1}{(4\pi)^2}\frac{2}{5};\ b_{2L}=b_{2R}=-\frac{1}{(4\pi)^2}\frac{20}{3};$$
$$b_{2L}^{ew}=-\frac{1}{(4\pi)^2}\frac{41}{6};\ b_3=-\frac{1}{(4\pi)^2}11 \tag{2.53}$$

Solving as before, we find that with the Higgs contribution the lower bound of M_R increases to $\approx 10^{11}$ GeV.

We have also studied the effect on M_R of including supersymmetry. The essential difference in the analysis is that the beta function coefficients are modified. Taking into account these modifications, we find that in the supersymmetric case the lower bound on M_R is $\approx 10^{11}$ GeV.

In conclusion, the most recent experimental data provide very strong constraints on left-right symmetric models. We have shown that if a left-right symmetric group coming from either a grand unified or partially unified group breaks at an intermediate mass scale, M_R, then the tightly constrained values of $\sin^2\theta_W$ and α_s can be used to put a lower bound on the value of M_R. This lower bound is $\approx 10^9$ GeV, irrespective of the unification group. Grand unified theories and partially unified theories, therefore, completely rule out the possibility of seeing the right handed partners of $W^{\pm}$ at the energies available in current experiments or those planned in the near future. Conversely, the discovery of these particles at such energies can be used to refute unification models. It is of importance to note, however, that our analysis puts no constraints whatsoever on the existence of extra Z at low energies, as an extra $U(1)_R$ can survive down to electroweak breaking scales. The inclusion of the Higgs or supersymmetry increases the lower bound on M_R.

2.4 Contraints on Non-Perturbative Unification

The compatibility of the simple supersymmetric GUT with no intermediate breaking scales and the couplings determined from LEP is remarkable. Nonetheless, it is important to study other models, which are alternatives to grand unification, and see whether they are viable in the light of the available experimental information on couplings [3].

An interesting alternative to GUTs was proposed by Maiani, Parisi and Petronzio [26] several years ago. In this scheme, the couplings enter a non-perturbative phase at a high energy scale, i.e. the theory is asymptotically divergent. Starting from the renormalization group equation for a coupling α,

$$\frac{d\alpha}{dt} = \beta(\alpha), \tag{2.54}$$

where $\beta(\alpha)$ is the beta function and $t = \ln(Q^2/\mu^2)$, μ being some reference scale, we obtain

$$t = \int_{\alpha(\mu)}^{\alpha(Q^2)} \frac{d\alpha}{\beta(\alpha)}. \tag{2.55}$$

For $\beta(\alpha) > 0$ (asymptotically divergent theory) there is a value of t, given by

$$t = \int_{\alpha(\mu)}^{\infty} \frac{d\alpha}{\beta(\alpha)} < \infty, \tag{2.56}$$

for which $\alpha \to \infty$. If perturbation theory is to be valid at all energy scales, we require $\alpha(\mu) = 0$, so that $t_c = \infty$, $\alpha(\mu) = 0$ is the infra-red fixed point. But if $\alpha(\mu) \neq 0$ but small, i.e. it is sufficiently close to the infra-red fixed point, then there is a finite cut-off in energy beyond which the theory is non-perturbative.

In Ref. [26], it was assumed that the standard SU(3)×SU(2)×U(1) theory, due to new fermion generations that get switched on around the weak scale $\Lambda_F = 250$ GeV, is asymptotically divergent beyond Λ_F. The couplings $\alpha_{1,2,3}$ are sufficiently close to zero at Λ_F but not quite zero. As a consequence, the theory is cut off at a scale Λ. At this scale, the most interesting situation is that not just one but all three couplings are large, i.e. of O(1). In fact, it has been shown [27] that such a non-perturbative scenario exhibits a "trapping" mechanism, whereby if one of the couplings grows large, the other couplings will also increase. This effect, by means of which all three couplings are large and of the same order of magnitude at Λ, leads to what is called non-perturbative unification. In Ref. [26] the cut-off scale Λ was assumed

[3] This section follows Ref [25]

to be the Planck scale, however, in subsequent studies [28, 29], Λ was determined to be of the order of 10^{15} -10^{17} GeV. Since the low-energy couplings are close to the infra-red fixed point, they are insensitive to the values of the couplings at the scale Λ.

One natural extension of the above scenario is the inclusion of supersymmetry. This was first considered in Ref. [28], and was later discussed in Refs. [29, 30]. Other than solving the hierarchy problem, the inclusion of supersymmetry is attractive because it provides a framework for the existence of new particles needed to make the theory asymptotically divergent. In the case of the simplest $N = 1$ supersymmetric extension of the scenario, it suffices to consider $n_f = 5$, where n_f is the number of fermion generations.

In this section, we use the recent LEP values to check whether any strong constraints on the non-perturbative unification scenario can be obtained. The values of $\sin^2\theta_W$ and α_s from LEP are very precise compared to that available from older experiments. One strong constraint is on the number of extra chiral generations. The present limit on the oblique parameters S, T and U allows only three chiral fermion generations, while the vectorial generations are not constrained. Thus in addition to the three chiral fermion generations we are allowed to have only an even number of generations.

We shall first specify the supersymmetric non-perturbative unification scenario in detail. While discussing the results we shall also comment on the results of the non-supersymmetric case. We consider an SU(3)×SU(2)×U(1) supersymmetric gauge theory with the assumption that an $N = 1$ supersymmetry holds above the scale Λ_s. We assume $n_f = 5$ supersymmetric generations and two Higgs supermultiplets. In the discussion of the non-supersymmetric case we shall consider one Higgs scalar and $n_f = 8$ and 9. From the requirement that the Yukawa couplings do not become arbitrarily large, a bound on the fermion masses can be obtained [31, 32]. This bound is that fermion masses are, in general, smaller than 200–250 GeV. We assume that the extra fermion generations, which are required for the theory to be asymptotically divergent, are of the order of 250 GeV in mass.

Having specified the theory we can now address the question of the evolution of the three couplings. The two-loop renormalization group equations for the couplings are given by the following coupled differential equations:

$$\mu\frac{d\alpha_i(\mu)}{d\mu} = \frac{1}{2\pi}\left[a_i + \frac{b_{ij}}{4\pi}\alpha_j(\mu) + \frac{b_{ik}}{4\pi}\alpha_k(\mu)\right]\alpha_i^2(\mu) + \frac{2b_{ij}}{(4\pi)^2}\alpha_i^3(\mu), \tag{2.57}$$

where i, j, $k = 1$, 2, 3 and $i \neq j \neq k$, and a_i and b_{ij} are the one- and two-loop beta function coefficients. In the range of energies between M_Z and the supersymmetric threshold, M_s, we use the non-supersymmetric beta functions to evolve the couplings, whereas from M_s onward the supersymmetric beta functions are effective. We retrieve the result for the non-supersymmetric scenario by taking $M_s = \Lambda_{MPP}$ and large n_f.

In the non-supersymmetric case the one-loop beta function coefficients are [33]

$$b_j = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + n_f \begin{pmatrix} \frac{20}{9} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix} \tag{2.58}$$

while the two-loop beta functions are

$$a_{ij} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{pmatrix} + n_f \begin{pmatrix} \frac{95}{27} & 1 & \frac{44}{9} \\ \frac{1}{3} & \frac{49}{3} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} & \frac{13}{6} & 0 \\ \frac{1}{2} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.59}$$

In the supersymmetric case the one-loop beta functions take the form [33]

$$b_j = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + n_f \begin{pmatrix} \frac{10}{3} \\ 2 \\ 2 \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \tag{2.60}$$

while the two-loop beta functions are

$$a_{ij} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 54 \end{pmatrix} + n_f \begin{pmatrix} \frac{190}{27} & 2 & \frac{88}{9} \\ \frac{2}{3} & 14 & 8 \\ \frac{11}{9} & 3 & \frac{68}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.61}$$

In all these equations, n_f and n_h denote the number of fermion generations and the number of Higgs doublets respectively.

We integrate the coupled differential equations in Eq. (2.57) numerically, with the initial values of the three couplings $\alpha_{1,2,3}$ taken to be of O(1) at the unification scale Λ. What we do in practice is to evolve downwards using the renormalization group equations for several values of Λ, and check what the predicted values of the couplings at the scale M_Z are. The extra fermion generations are assumed to contribute to the beta functions for all energies greater than 250 GeV.

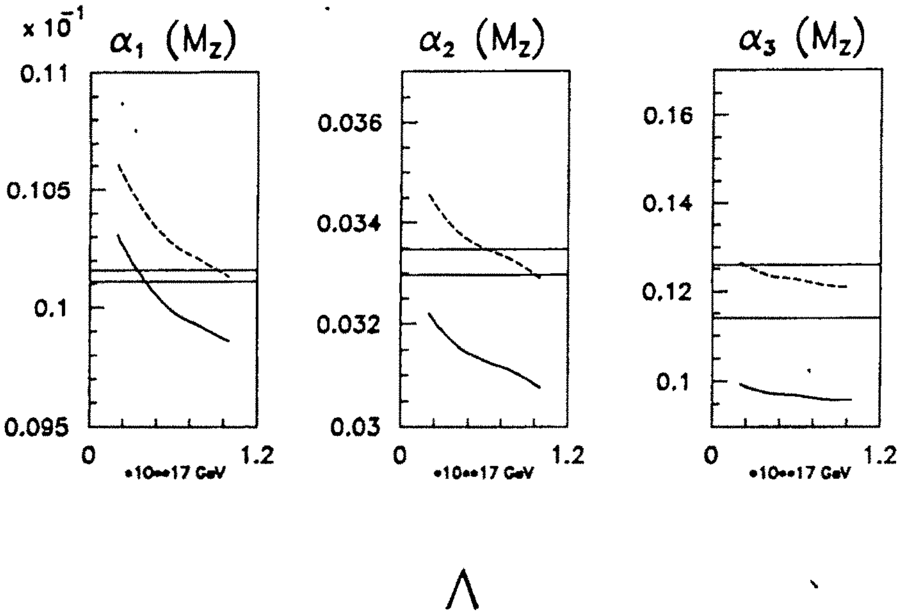
We shall first comment on the non-supersymmetric scenario and then present our main result, namely the supersymmetric extension. In this case we find that for $n_f \leq 8$, $\alpha_2(M_z)$ remains too small, and that $\alpha_{1,2}(M_z)$ falls within the experimental bound for $n_f \geq 9$. But for $n_f \geq 9$ the strong coupling constant evolves extremely fast and $\alpha_3(M_z)$ becomes too large. Thus the precision LEP data rule out the non-supersymmetric scenario completely.

The results of the computation for the supersymmetric version are shown in Figure 4, where $\alpha_{1,2,3}(M_Z)$ are shown as a function of Λ. The solid, dashed and dotted curves are for $M_s =$ 250 GeV, 1.2 TeV and 5 TeV, respectively. The horizontal lines in the Figure 4 show the upper and lower bounds on the couplings at M_Z as determined by the LEP experiment. These are as follows [34]:

$$\begin{aligned} \alpha_1 &= 0.0101322 \pm 0.000024 \\ \alpha_2 &= 0.03322 \pm 0.00025 \\ \alpha_3 &= 0.120 \pm 0.006. \end{aligned} \tag{2.62}$$

It is clear from the Figure 4 that the non-perturbative unification scheme is certainly viable if we have $M_s = 1.2$ TeV and Λ close to 0.78×10^{17} GeV. We have checked that the range of values allowed is $M_s = 1.2 \pm 0.2$ TeV and $\Lambda = (0.7\text{–}0.8) \times 10^{17}$ GeV. We have also checked that the couplings at M_Z are not sensitive to the choice of the couplings at Λ. We have checked this by varying these from 0.75 to 10.

Let us now summarize our results of this section. We have studied the non-perturbative unification scenario first proposed by Maiani, Parisi and Petronzio. We point out that the non-supersymmetric version of this scenario is ruled out by LEP data. However, the supersymmetric extension of this scenario remains a viable alternative to conventional grand unified theories and is capable of predicting the precision values of couplings determined from LEP. Our numerical results show that the non-perturbative scale, Λ, at which all couplings are large, is around $0.7\text{–}0.8 \times 10^{17}$ GeV, with the supersymmetric threshold M_s around 1.0–1.4 TeV. If the scale M_s gets either larger or smaller it is then not possible to reproduce the



x 10^-1
α1 (MZ)
0.11
0.105
0.1
0.095
0
1.2
*10**17 GeV
α2 (MZ)
0.036
0.034
0.032
0.03
0
1.2
*10**17 GeV
α3 (MZ)
0.16
0.14
0.12
0.1
0
1.2
*10**17 GeV
Λ

FIGURE 4

values of the couplings at M_Z. We should note that the agreement with the data is obtained only for a constrained range of parameters of this scenario. In principle, the effect of higher-order corrections could be large and this may ruin the agreement. It is also likely that more accurate measurements of the strong coupling α_3 at low energies may be sufficient to either put strong constraints or completely rule out this scenario. It is nevertheless interesting that this scenario, at the two-loop level, is a possible alternative to conventional grand unification.

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