CHAPTER - VII : STATISTICAL ANALYSIS OF THE DATA

As pointed out in the previous chapter, before the inventory scores can be comprehended, interpreted and be made meaningful, it is always necessary to subject them to rigorous statistical treatment for standardising the scores. The standardisation of a test is a statistical procedure which gives meaning to the test as a whole. The standardisation procedure involves :

(1) Statistical analysis of the data
(2) Establishment of the validity of the test
(3) Determining the reliability
(4) Fixing the norms.

Statistical analysis of the test results

The maximum score that a testee can have in this inventory is 200 as there are 100 items, each carrying a maximum of two marks. On summarisation, it was noted that the range of the scores was (162 - 54) + 1 = 109. The range within which all the scores were distributed was divided into 12 class-intervals, each interval being of 10 units.

Table 29 shows the frequency distribution of all scores, subjected to further statistical analysis. A number of useful statistics for later use and understanding have been computed and shown herewith.

Table 29 - Frequency distribution of inventory scores; calculation of the mean and standard deviation

Step intervals	Mid point	Frequency	x'	fx'	fx'^2
160-169	164.5	1	5	5	25
150-159	154.5	10	4	40	160
140-149	144.5	26	3	78	234
130-139	134.5	62	2	124	248
120-129	124.5	85	1	85	85
110-119	114.5	99	0	0	0
110-109	104.5	91	-1	-91	91
90-99	94.5	60	-2	-120	240
80-89	84.5	36	-3	-108	324
70-79	74.5	16	-4	- 64	256
60-69	64.5	9	-5	- 45	225
50-59	54.5	5	-6	- 30	180
Total :	N = 500	500		-126	2068

Mean

Assumed mean = 114.5

Correction = $\frac{\sum fx'}{N} = \frac{-126}{500} = -.252$ ~~$i = 10$~~

Interval = 10 $\therefore$ Ci = -2.52

True mean = AM + Ci = 114.5 − 2.52 = 111.98

or 112.0

Median

N = 500 $\therefore$ $\frac{N}{2} = 250$

$$\text{Median} = l + \left(\frac{N/2 - F}{fm}\right) i$$

Where l = lower limit of the class interval upon which the median lies ;

$\frac{N}{2}$ = one half the total number of scores ;

fm = frequency within the interval upon which the median falls ;

F = sum of the scores on all intervals below l ;

i = length of the class interval.

$$\therefore \text{Median} = 109.5 + \frac{250 - 217}{99} \times 10 = 112.8$$

On the same lines the following values were obtained for other percentile points :-

Q_3 or P_{75} = 126.5 ; Q_1 or P_{25} = 99.3 ; P_{90} = 137.4

and P_{10} = 85.0

Measure of variability

'The standard deviation ' :

$$SD \text{ or } \sigma = \sqrt{\frac{fx'^2}{N} - c^2} \times i$$

$$\therefore \; SD = \sqrt{\frac{2068}{500} - .062} \times 10$$

$$= 2.018 \times 10 = 20.18 \text{ or } 20.2$$

Reliability of the mean, median and the standard deviation

The above results showing the mean, the median and the standard deviation have been obtained on a sample of 500 teachers. It is necessary to test the reliability of these statistics. Would we get the same results if the test were administered to the whole population, i.e. to all the teachers of the primary schools in Mysore State ? To what extent would the obtained results diverge from the true theoretical values ? To answer this question, other statistics showing reliability are needed. The reliability of the above statistics could be determined by calculating the stadard error (SE) of each of them.

Reliability of the mean

$$SE \text{ of mean or } \sigma_M = \frac{\sigma}{\sqrt{N}} = \frac{20.2}{\sqrt{500}} = 0.90$$

∴ The true mean would lie between the following limits :

Mean + (2.58 x 0.9) and mean - (2.58 x 0.9) i.e. 112.0 + 2.3 and 112.0 - 2.3 at 0.01 level of confidence.

Reliability of the median

Standard error of the median = 1.2533 x σ_M

= 1.2533 x .9 = 1.128.

∴ The true median will be between the limits 112.8 + (2.58 x 1.115) and 112.8 - (2.58 x 1.128) i.e. 112.8 + 2.9 and 112.8 - 2.9 at 0.01 level of confidence.

Reliability of the standard deviation

Standard error of the standard deviation

$$= \frac{\sigma}{\sqrt{2n}} = \frac{20.2}{\sqrt{1000}} = \frac{20.2}{31.62} = 0.63$$

∴ The true σ will be between the limits 20.2 + (2.58 x 0.63) and 20.2 - (2.58 x 0.63) i.e. 20.2 + 1.62 and 20.2 - 1.62 at 0.01 level of confidence.

Nature of the distribution of the inventory scores

The values of mean, median and the standard deviation do give us some idea of the way the scores on the inventory are distributed. But it is only a rough estimate. The important thing in the construction of any inventory is to find out the actual distribution of the scores of the inventory and use statistical methods to verify the nature of the distribution of the raw scores.

The following statistical procedures were adopted for testing whether the scores on the inventory were distributed normally or not.

It has been found that usually the scores obtained on intelligence ~~tests~~, achievement and other such tests distribute according to the normal probability curve. But here in this inventory it is teacher-efficiency that is being measured. It would not strike strange if this distribution of teacher-efficiency scores is not normal. However, as the graph and X^2 test show, it is found to the credit of the vast labour of the investigator that the scores are distributed normally, implying that the random, representative sample tested belonged to a normal population as far as the trait of teacher-efficiency was concerned. Deviations from normality could be understood also from the measures of their skewness and kurtosis.

(1) Calculation of skewness and Kurtosis

Skewness : Skewness of the distribution is calculated as under :

$$SK = \frac{P_{90} + P_{10}}{2} - P_{50}$$

$$= \frac{137.3 + 85.0}{2} - 112.8$$

$$= \frac{222.3}{2} - 112.8 = 111.15 - 112.8 = -1.65$$

This shows that there is negative skewness, i.e. the scores are a little massed at the right end of the scale. This is evident from the average also which is 112.0 instead of 100.

Another formula used to calculate the skewness of the distribution is :

$$SK = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

$$= \frac{3(112.0 - 112.8)}{20.2} = \frac{3(-.8)}{20.2}$$

$$= \frac{-2.4}{20.2} = -0.118.$$

By this formula also the curve is negatively skewed. But these results are numerically different from one another. This is because 'the two measures of skewness are computed from different reference values in the distribution[1]. Anyhow the two formulae agree in indicating some negative skewness for the distribution of 500 scores.

The important question of how much skewness a distribution must exhibit before it may be said to be significantly

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1. Garrett, H.E.: Statistics in Psychology and Education, p 121.

skewed cannot be answered until the 'standard error' of the skewness is calculated.

Standard error of the skewness

$$SK = \frac{0.5185\ D}{\sqrt{N}} \quad \text{in which } D = P90 - P10$$

$$SK = \frac{(137.3 - 85.0)}{\sqrt{500}}\ 0.5185$$

$$= \frac{52.3}{\sqrt{500}} \times 0.5185 = \frac{27.12}{22.36}$$

$$= 1.21$$

$$\text{Critical ratio } t = \frac{SK}{\sigma_{Sk}} = \frac{-1.65}{1.21}$$

Thus, dividing -1.65 by 1.21, we get t = 1.40. Assuming the distribution of 't's to be normal, it is clear from the table that for n - 1 df 't' falls far short of the 0.05 level. This shows that the distribution is not significantly skewed.

Kurtosis

The term Kurtosis refers to the 'peakedness' or flatness of a frequency distribution as compared with the normal distribution.

Formula for measuring Kurtosis is :

$$Ku = \frac{Q}{P90 - P10}$$

$$Q = \frac{Q3 - Q1}{2} = \frac{126.5 - 99.3}{2} = \frac{27.2}{2} = 13.6$$

$$Ku = \frac{13.6}{52.3} = 0.260$$

For a normal curve, Ku = .263. If the obtained Ku is greater than .263, the distribution is platykurtic, if less than .263, the distribution is leptokurtic. Ku of the distribution of the scores is very slightly less than .263. Hence it is very slightly leptokurtic.

Standard error of the kurtosis

The Kurtosis of the distribution deviates - .003 from the Ku of the normal distribution which is 0.263. The direction of the deviation indicates that the distribution is leptokurtic. The significance of this deviation from normal Ku may be estimated by calculating σ Ku using the following formula :

$$\sigma_{Ku} = \frac{.27779}{\sqrt{N}}$$

$$\sigma_{Ku} = \frac{.27779}{\sqrt{500}} = 0.012$$

$$t \text{ or } Ku_d/Ku = \frac{.003}{.012} = 0.25$$

Assuming a normal sampling distribution for t, 0.25 is far below the .05 level and hence the deviation (peakedness) of this frequency distribution from the normal distribution is not significant.

The Chi-Square Test

The Chi-Square test represents a useful method of comparing the experimentally obtained results with those to be expected theoretically, on some hypothesis. The hypothesis in the present case is that the distribution of the scores on the present inventory follows the normal curve and that any deviation of the obtained data from the theoretically expected data is insignificant and attributable to chance factor only. If the distribution of the test scores were theoretically normal, the value of chi-square would be zero. But in this case the distribution of the scores show some skewness. The chi-square test is therefore applied to test the hypothesis of normal distribution.

$$\text{(Chi-Square) } x^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

where f_o = obtained frequency ;

f_e = expected frequency (on the hypothesis of normal distribution in this case).

Thus, the obtained $X^2 = 11.33$ as shown in table 30. For df = 9 P = 0.3. This value of P shows the normal nature of the distribution.

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Table 30 - Checking normality of the distribution of the inventory scores of teachers by the X^2 test

	Class intervals	Mid points	Observed frequency	x - M	$X = \frac{x - M}{\sigma}$	Area P(x)	AP(x)	Expected frequency N x AP(x)	$f_o - f_e$	$(f_o - f_e)^2$	$\left(\frac{f_o - f_e}{f_e}\right)^2$
1	160-169	164.5	1	48	2.37	.9911	0.0089	5	-4	16	3.20
2	150-159	154.5	10	38	1.88	.9699	0.0212	10	-	-	-
3	140-149	144.5	26	28	1.38	.9162	0.0537	27	-1	1	0.04
4	130-139	134.5	62	18	0.90	.8159	0.1003	50	12	144	2.88
5	120-129	124.5	85	8	0.40	.6554	0.1603	80	5	25	0.31
6	110-119	114.5	99	-2	-0.099	.4602	0.1952	98	1	1	0.01
7	100-109	104.5	91	-12	-0.60	.2743	0.1859	93	-2	4	0.04
8	90-99	94.5	60	-22	-1.09	.1379	0.1364	68	-8	64	0.94
9	80-89	84.5	36	-32	-1.59	.0539	0.0820	41	-5	25	0.61
10	70-79	74.5	16	-42	-2.09	.0183	0.0373	19	-3	9	0.47
11	60-69	64.5	9	-5[illegible]	[illegible].57	.0051	0.0132	6	3	9	1.50
12	50-59	54.5	5	-62	-[illegible].07	.0011	0.0054	3	2	4	1.33
									X^2	=	11.33

Mean = 112.0 ; SD = 20.2 ; Taking df = 9, P = 0.3 ; ∴ The observed difference is not significant.

∴ The obtained distribution of the score is normal

$$\sum \frac{(f_o - f_e)^2}{f_e} = 11.33 = x^2$$

Graphical representation of the test results

The normal nature of the distribution of the data has been proved satisfactorily. The ordinary frequency distribution conveys picture of the situation only in numbers. However, for quick easy grasp, a pictorial presentation would not be out of place.

There are 3 common methods of representing a distribution of scores graphically viz. the histogram, the frequency polygon and the smoothed curve.

The smooth curve

In order to smooth the frequency polygon, it is necessary to find out the smoothed or 'adjusted' frequencies for the different step intervals. Table 31 gives the original frequencies and the 'smoothed' frequencies for the various step intervals.

Table 31

Scores	Original frequencies	Smoothed frequencies
40-49	0	1.66
50-59	5	4.66
60-69	9	10.00
70-79	16	20.33
80-89	36	37.33
90-99	60	62.33
100-109	91	83.33
110-119	99	91.66
120-129	85	82.00
130-139	62	57.66

Table 31 (contd.)

140–149	26	32.66
150–159	10	12.33
160–169	1	3.66
170–179	0	0.33

The Histogram and the Smoothed frequency Polygan are shown in Graph II.

Superimposition of an ideal normal curve on the curve obtained from the inventory data :

The equation of the normal probability curve is :

$$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

in which x = scores (expressed as deviations from the mean) laid off along the baseline of x-axis ;

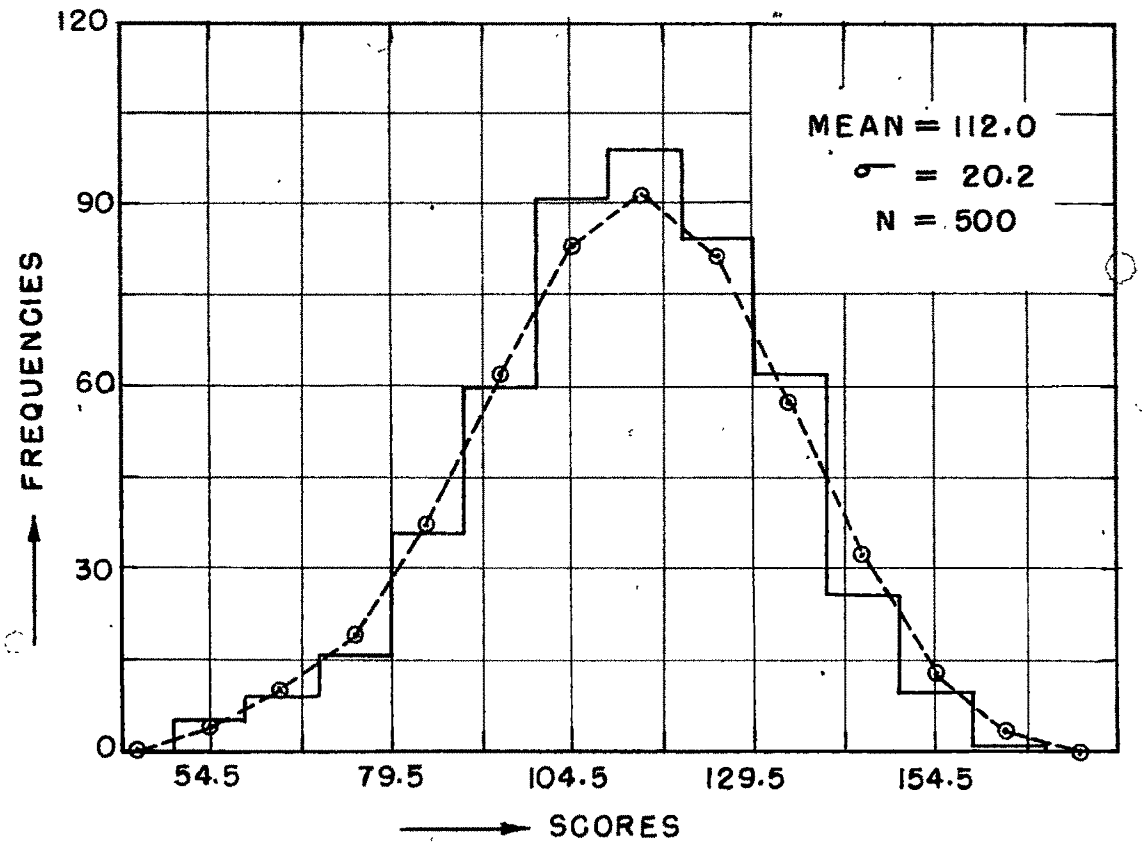
y = the height of the curve above the x-axis, i.e. the frequency of a given x-value or the number achieving a certain score ;

n = number of cases ;

σ = standard deviation of the distribution ;

π = 3.1416 and e = 2.7183 (base of the Napierian system of logarithms).

To plot a normal curve, the height of a maximum ordinate



GRAPH II HISTOGRAM AND THE SMOOTHED FREQUENCY POLYGON SHOWING THE DISTRIBUTION OF SCORES OF 500 EXAMINEES IN THE FINAL RUN OF THE INVENTORY.

(yo) is to be computed first. This is the frequency at the middle of the distribution. The maximum ordinate (yo) can be determined from the above equation.

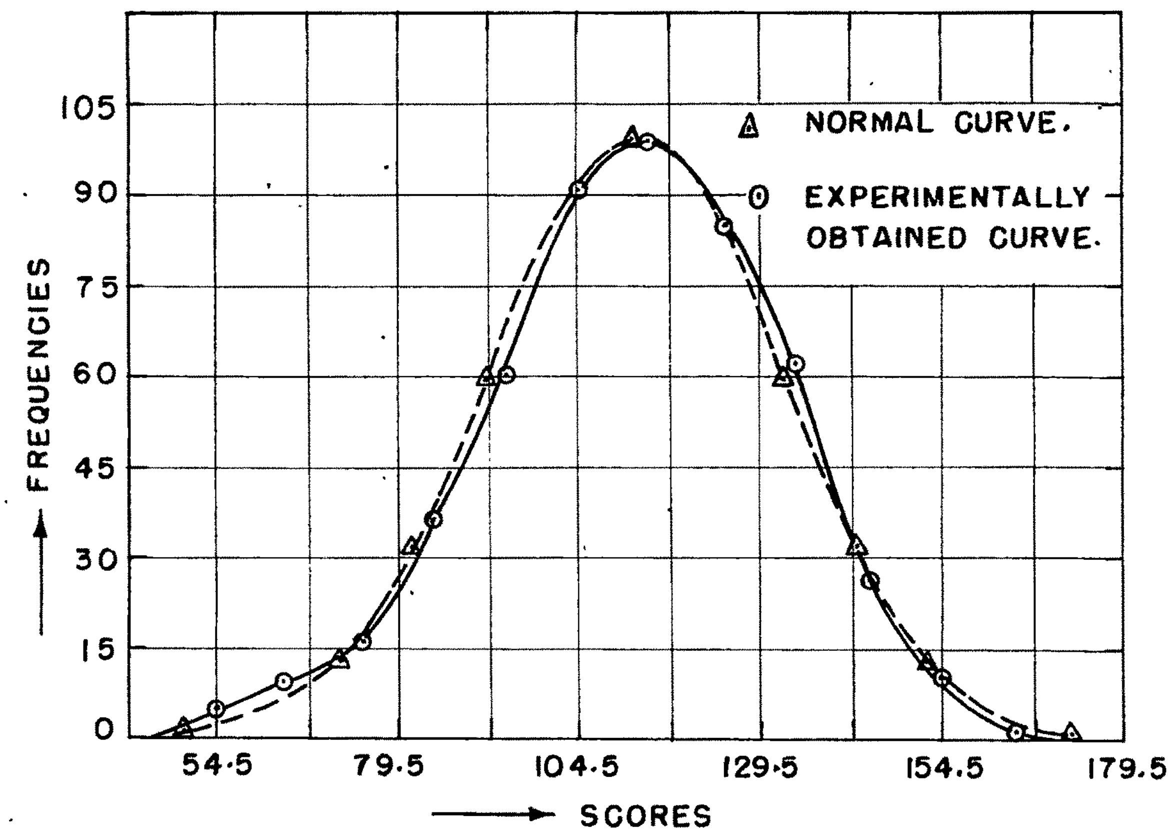
When x = 0 in the equation (x at the mean of the normal curve is 0) the term $e^{\frac{-x^2}{2\sigma^2}}$ equals 1 and yo = $\frac{N}{\sigma\sqrt{2\pi}}$. Yo is the frequency at the median point in the score distribution.

From the tables, we can find out the values of y at different points on the base line.

Below are given the different values of y corresponding to different points on the x-axis.

Points on the x-axis		Normal curve ordinates
At mean yo	112 (mean)	9.86
yo $\pm$ 1σ	(91.8, 132.2)	5.98
yo $\pm$ 1.5σ	(81.7, 142.3)	3.20
yo $\pm$ 2σ	(71.6, 152.4)	1.33
yo $\pm$ 3σ	(51.4, 172.6)	0.11

Using the above results the ideal normal curve has been superimposed on the curve obtained from the test data. Graph III gives a visual comparison between the two graphical



GRAPH III SUPERIMPOSITION OF THE NORMAL CURVE ON THE EXPERIMENTALLY OBTAINED CURVE.

representations - one theoretical and the other experimentally obtained. The visual presentation shows that the distribution of the obtained curve is very much similar to an ideal theoretical normal distribution.

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