Chapter 5

Exact Controllability of Nonlinear Impulsive System

Contents

In this chapter we study the exact controllability of a nonlinear impulsive control system governed by the integro-differential equation of the form

$$\begin{aligned} x'(t) &= Ax(t) + f(t, x(t), Tx(t), Sx(t)) + Bu(t), \quad 0 < t < T, \quad t \neq t_k \\ x(0) &= x_0, \\ \Delta x(t_k) &= I_k(x(t_k)) \end{aligned}$$

in a Banach space X, where $f \in C([0,T] \times X \times X \times X, X)$, $0 < t_1 < t_2 < ... < t_m < T$ and $I_k \in C(X, X)$, $k = 1, 2, ..., m$. Under Lipschitz condition on the nonlinear function f, we obtain the controllability results using fixed point theorem. In Section 5.1, we discuss the problem considered. Section 5.2 deals with the main results. Summary is given in Section 5.3.

5.1 Introduction

In the dynamics of many practical systems, there are an abrupt change in the states such as impulse or shock experienced in a short duration of time. Such systems are modeled in terms of impulsive differential equations ([52],[39]). The theory of impulsive differential equations is a new and important branch of differential equation theory, which has an extensive physical background and realistic mathematical model and hence has been emerging as an important area of investigation in recent years (refer, Lakshmikantham et. al[52]).

The existence of solution to impulsive integro-differential equations in Banach spaces has been studied by several authors([37],[40],[56] - [59],[72]). However, the controllability properties of such systems are yet to be investigated.

In [54], Leela studied the controllability aspect of a linear finite dimensional time invariant impulsive system. George et. al. [32] generalized the controllability result to nonlinear system with impulses. Recently Boukhamla and Mazouzi [13], obtained the controllability result for impulsive linear systems in infinite dimensional settings.

In this chapter, we investigate the controllability property of a impulsive control system governed by the nonlinear integro-differential equation:

$$\left.\begin{array}{l} x'(t) = Ax(t) + f(t, x(t), Tx(t), Sx(t)) + B(t)u(t), \;\; 0 < t < T, \;\; t \neq t_k \\ x(0) = x_0, \\ \Delta x(t_k) = I_k(x(t_k)), \;\; k = 1, 2, ..., m. \end{array}\right\} \quad (5.1.1)$$

in a Banach space X,

where, $f \in C([0,T] \times X \times X \times X, X)$, A is infinitesimal generator of a C_0 semigroup $\{G(t)|_{t\geq 0}\}$ with impulsive condition (refer Pazy, [64]) and $B(t)$ is a bounded linear operator from U to X and the control function $u(\cdot)$ is in $L^2([0,T]; U)$, U is another Banach space. T and S are operators defined by

$$Tx(t) = \int_0^t K(t,s)x(s)ds, \;\; K \in C[D, R^+]$$

$$Sx(t) = \int_0^T H(t,s)x(s)ds, \;\; H \in C[D_0, R^+]$$

where

$$D = \{(t,s) \in R^2 : 0 \le s \le t \le T\}$$

$$D_0 = \{(t,s) \in R^2 : 0 \le t, s \le T\}$$

$x_0 \in X$ is the initial condition. $0 < t_1 < t_2 < t_3 < ... < t_m < T, I_k : X \to X$ is an impulsive function, $k = 1, 2, ..., m$. $\Delta x(t_k)$ denotes the jump of $x(t)$ at $t = t_k$, that is,

$$\Delta x(t_k) = x(t_k^+) - x(t_k^-)$$

where $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$ respectively.

Anguraj and Arjunan [3] obtained existence and uniqueness of the solution of the impulsive evolution system(5.1.1) without control term. The purpose of this chapter is to investigate the controllability property of system(5.1.1) under suitable assumption on the nonlinear function f.

Let $[0,T]$ be the time interval and $t_1, t_2, ..., t_m$ be m time points on $[0,T]$. Let $PC([0,T],X) = \{x : [0,T] \to X$ such that $x(t)$ is continuous at $t \ne t_k$ and left continuous at $t = t_k$ and the right limit $x(t_k^+)$ exists for $k = 1, 2, ..., m\}$.
Evidently, $PC([0,T],X)$ is a Banach space with norm (refer Guo and Liu [40])

$$||x||_{PC} = \sup_{t\in[0,T]} ||x(t)||.$$

A function $x(\cdot) \in PC([0,T],X)$ is a mild solution of equations (5.1.1) if it satisfies

$$x(t) = G(t)x_0 + \int_0^t G(t-s)f(s, x(s), Tx(s), Sx(s))ds +$$

$$\sum_{0<t_k<t} G(t-t_k)I_k(x(t_k)) + \int_0^t G(t-s)B(s)u(s)ds, \quad 0 \le t \le T \tag{5.1.2}$$

Definition 5.1.1. ***Controllability***
We say that the system (5.1.1) is exactly controllable in an interval $[0,T]$, if for every initial state $x_0 \in X$ and final state $x_1 \in X$, there is a control $u(\cdot) \in L^2([0,T];U)$, for which the solution $x(\cdot)$ satisfies $x(0) = x_0$ and $x(T) = x_1$.

Now, we obtain the controllability result using fixed point theorem, under the following hypotheses:

(H1)A is a general unbounded operator, which generates a strongly continuous semigroup $G(\cdot)$.

(H2)For each $t \in [0,T]$ $B(t)$ is a bounded linear operator with

$$b = \sup_{t\in[0,1]} ||B(t)|| < \infty$$

(H3)$f : [0,T] \times X \times X \times X \to X$ and $I_k : X \to X, k = 1, 2, ..., m$ are continuous mapping and there exist constants $L_1, L_2, L_3 > 0,\ h_k > 0, k = 1, 2, 3, ..., m$ such that

$$||f(t, x_1, x_2, x_3) - f(t, y_1, y_2, y_3)|| \leq L_1||x_1 - y_1|| + L_2||x_2 - y_2|| + L_3||x_3 - y_3||$$

for $t \in [0,T],\ x_1, x_2, x_3, y_1, y_2, y_3 \in X$, and

$$||I_k(x) - I_k(y)|| \leq h_k||x - y||,\quad x, y \in X$$

$$Let\ M = \max_{t\in[0,T]} ||G(t)||_{B(X)} = \max_{t\in[0,T]} ||G^*(t)||_{B(X)},\quad L = max\{L_1, L_2, L_3\}$$

$$K^* = \sup_{t\in[0,T]} \int_0^t |K(s,t)|dt < \infty,\ H^* = \int_0^T |H(s,t)|dt < \infty$$

5.2 Main Result

For obtaining the controllability result of (5.1.1), we assume that the corresponding linear system:

$$\left.\begin{array}{l} x'(t) = Ax(t) + B(t)u(t),\ \ 0 < t < T,\ \ t \neq t_k \\ x(0) = x_0, \\ \Delta x(t_k) = I_k(x(t_k)) \end{array}\right\} \tag{5.2.1}$$

is controllable and hence the controllability Grammian $W(0,T)$ defined by:

$$W(0,T) = \int_0^T G(T-s)B(s)B(s)^*G^*(T-s)ds \tag{5.2.2}$$

is invertible.

The nonlinear impulsive system (5.1.1) is controllable on $[0, T]$ if and only if there exists a control u which steers a given initial state x_0 of the system to a desired final state x_1. That is, there exists control function u such that

$$x_1 = x(T) = G(T)x_0 + \int_0^T G(T-s)f(s, x(s), Tx(s), Sx(s))ds +$$

$$\sum_{0<t_k<T} G(T-t_k)I_k(x(t_k)) + \int_0^T G(T-s)B(s)u(s)ds$$

Let us define a control $u(t)$ by

$$u(t) = B^*(t)G^*(T-t)W^{-1}(0,T)[x_1 - G(T)x_0 -$$

$$\int_0^T G(T-s)f(s, x(s), Tx(s), Sx(s))ds - \sum_{0<t_k<T} G(T-t_k)I_k(x(t_k))] \tag{5.2.3}$$

where $x(\cdot)$ satisfies the nonlinear system (5.1.2). Now substituting the control $u(t)$ as defined in (5.2.3) into the nonlinear integral equation (5.1.2), we get

$$x(t) = G(t)x_0 + \int_0^t G(t-s)f(s, x(s), Tx(s), Sx(s))ds +$$

$$\sum_{0<t_k<t} G(t-t_k)I_k(x(t_k)) + \int_0^t G(t-s)B(s)B^*(s)G^*(T-s)W^{-1}(0,T)[x_1 - G(T)x_0 -$$

$$\int_0^T G(T-r)f(r, x(r), Tx(r), Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_k(x(t_k))]ds \tag{5.2.4}$$

If the equation (5.2.4) is solvable then $x(t)$ satisfies $x(0) = x_0$ and $x(T) = x_1$. This implies that the system (5.1.2) is controllable with control u as given by (5.2.3). Hence the controllability of the nonlinear impulsive system (5.1.1) is equivalent to the solvability of the equation (5.2.4).

We now obtain the controllability result by invoking Banach contraction principle to establish solvability of (5.2.4).

Theorem 5.2.1. *Suppose that*

(i) The assumptions $(H1) - (H3)$ are satisfied.

(ii) The linear system is exactly controllable and $w = ||W^{-1}(0,T)||$.

(iii) $M(1+M^2b^2wT)(LT(1+K^*+H^*)+\sum h_i)<1.$

Then the nonlinear impulsive system (5.1.1) *is controllable on* $[0,T]$. □

Proof. Defining an operator F on $PC([0,T];X)$ by

$$(Fx)(t) = G(t)x_0 + \int_0^t G(t-s)f(s,x(s),Tx(s),Sx(s))ds+$$

$$\sum_{0<t_k<t} G(t-t_k)I_k(x(t_k)) + \int_0^t G(t-s)B(s)B(s)^*G^*(T-s)W^{-1}(0,T)[x_1 - G(T)x_0 -$$

$$\int_0^T G(T-r)f(r,x(r),Tx(r),Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_k(x(t_k))]ds \quad (5.2.5)$$

It is clear that $F : PC([0,T];X) \to PC([0,T];X)$. Now we show that F is a contraction. For any $x, y \in PC([0,T];X)$

$$\begin{aligned}
&||(Fx)(t)-(Fy)(t)|| \\
\leq\ & \int_0^t ||G(t-s)||\,||f(t,x(s),Tx(s),Sx(s)) - f(t,y(s),Ty(s),Sy(s))||ds + \\
& \sum_{0<t_k<t} ||G(t-t_k)||\,||I_k(x(t_k)) - I_k(y(t_k))|| + \int_0^t ||G(t-s)||\,||B(s)||\,||B^*(s)|| \\
& ||G^*(T-s)||\,||W^{-1}(0,T)||[\int_0^T ||G(T-r)||\,||f(t,x(r),Tx(r),Sx(r)) - \\
& f(t,y(r),Ty(r),Sy(r))||dr + \sum_{0<t_k<T} ||G(T-t_k)||\,||I_k(x(t_k)) - I_k(y(t_k))||]ds \\
\leq\ & M\int_0^t (L_1||x-y|| + L_2||Tx-Ty|| + L_3||Sx-Sy||)ds + M||x-y||\sum h_k \\
& +M^2b^2w\int_0^t [\int_0^T M(L_1||x-y|| + L_2||Tx-Ty|| + L_3||Sx-Sy||)dr \\
& +M||x-y||\sum h_k]ds \\
\leq\ & MT(L_1 + L_2K^* + L_3H^*)||x-y|| + M||x-y||(\sum h_k + M^2b^2w) \\
& \int_0^t (MT(L_1 + L_2K^* + L_3H^*)||x-y|| + M||x-y||\sum h_k)ds
\end{aligned}$$

$$\begin{aligned} &\leq MT(L_1 + L_2K^* + L_3H^*)||x-y|| + M||x-y||(\sum h_k + M^3b^2) \\ &\quad WT^2(L_1 + L_2K^* + L_3H^*)||x-y|| + M^3b^2w||x-y||\sum h_k \\ &= M(1 + M^2b^2WT)\{LT(1 + K^* + H^*) + \sum h_k\}||x-y|| \end{aligned}$$

Now using the assumption (iii), we have

$$||(Fx)(t) - (Fy)(t)|| \leq \alpha||x-y||, \ \forall x, y \in PC([0,T]; X), \ \ where \ \ \alpha < 1$$

Therefore, F is a contraction on $PC([0,T], X)$. Hence by the Banach contraction principle, F has a unique fixed point in X. Let x be the fixed point in X. Therefore,

$$x(t) = G(t)x_0 + \int_0^t G(t-s)f(s, x(s), Tx(s), Sx(s))ds +$$

$$\sum_{0<t_k<t} G(t-t_k)I_k(x(t_k)) + \int_0^t G(t-s)B(s)B^*(s)G^*(T-s)W^{-1}(0,T)[x_1 - G(T)x_0 -$$

$$\int_0^T G(T-r)f(r, x(r), Tx(r), Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_k(x(t_k))]ds$$

Obviously, $x(0) = x_0$ and $x(T) = x_1$. Hence the impulsive system (5.1.1) is controllable. □

Corollary 5.2.1. *In case of $I_k's$ are constant, we have $h_k = 0, k = 1, 2, ..., m$. So the condition (iii) in the Theorem 3.1 becomes*

$$M(1 + M^2b^2wT)LT(1 + K^* + H^*) < 1$$

and thus the nonlinear impulsive system (5.1.1) is controllable.

Further, an algorithm for the computation of steering control and trajectory is given by the following corollary:

Corollary 5.2.2. *A computational algorithm for the computation of steering control and trajectory is given by:*

$$u_n(t) = B^*(t)G^*(T-t)W^{-1}(0,T)[x_1 - G(T)x_0 -$$

$$\int_0^T G(T-s)f(s, x_n(s), Tx_n(s), Sx_n(s))ds - \sum_{0<t_k<T} G(T-t_k)I_k(x_n(t_k))]$$

$$x_{n+1}(t) = G(t)x_0 + \int_0^t G(t-s)f(s, x_n(s), Tx_n(s), Sx_n(s))ds +$$

$$\sum_{0<t_k<t} G(t-t_k)I_k(x_n(t_k)) + \int_0^t G(t-s)B(s)B^*(s)G^*(T-s)W^{-1}(0,T)[x_1 - G(T)x_0 -$$

$$\int_0^T G(T-r)f(r, x_n(r), Tx_n(r), Sx_n(r))dr - \sum_{0<t_k<T} G(T-t_k)I_k x_n(t_k)]ds$$

for $n = 0, 1, 2, \ldots$ *with arbitrary* $x_0(t)$.

In case, the nonlinear function f is uniformly bounded, we can relax the inequality constraint (iii) in Theorem 5.2.1.

Theorem 5.2.2. *Let the nonlinear function f is Lipschitz and uniformly bounded. That is, $||f(t, x, y, z)|| \leq K \quad \forall\ x, y, z \in X$. Then the nonlinear impulsive system (5.1.1) is exactly controllable on X under the assumption $(H_1) - (H_3)$.* □

Proof. Let us consider the Banach space PC([0,T];X). Defining an operator $F : PC([0,T]; X) \to PC([0,T]; X)$ by

$$(Fx)(t) = G(t)x_0 + \int_0^t G(t-s)f(s, x(s), Tx(s), Sx(s))ds +$$

$$\sum_{0<t_k<t} G(t-t_k)I_k(x(t_k)) + \int_0^t G(t-s)B(s)B(s)^*G^*(T-s)W^{-1}(0,T)[x_1 - G(T)x_0 -$$

$$\int_0^T G(T-r)f(r, x(r), Tx(r), Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_k x(t_k)]ds \tag{5.2.6}$$

Since f is uniformly bounded, we have
$||(Fx)(t)||$

$$\begin{aligned}\leq\ & ||G(t)||\ ||x_0|| + \int_0^t ||G(t-s)||\ ||f(s, x(s), Tx(s), Sx(s))||ds + \\ & \sum_{0<t_k<T} ||G(t-t_k)||\ ||I_k(x(t_k))|| + \int_0^t ||G(t-s)||\ ||B(s)||\ ||B^*(s)|| \\ & ||G^*(T-s)||||W^{-1}(0,T)||\ [||x_1|| - ||G(T)||\ ||x_0|| - \int_0^T ||G(T-r)|| \\ & ||f(r, x(r), Tx(r), Sx(r))||dr - \sum_{0<t_k<T} ||G(T-t_k)||\ ||I_k||\ ||x(t_k)||]ds\end{aligned}$$

$$\begin{aligned}
&\leq M||x_0|| + MK\int_0^t ds + M\sum h_k + M^2b^2w\int_0^t (||x_1|| + Mx_0 + MK\int_0^T dr \\
&\quad + M\sum h_k)ds \\
&\leq M||x_0|| + MKT + M\sum h_k + M^2b^2w(||x_1|| + M||x_0|| + MKT + M\sum h_k)T \\
&= M(||x_0|| + KT + \sum h_k) + M^3b^2w(||x_0|| + KT + \sum h_k)T + M^2b^2w||x_1|| \\
&= (M + M^3b^2w)(||x_0|| + KT + \sum h_k) + M^2b^2w||x_1|| \\
&= \rho \ (say)
\end{aligned}$$

This implies F maps the whole space into the sphere of radius ρ. Let, $E = \{x \in PC([0,T];X);\ ||x|| < \rho\}$ and $Q = \{Fx : x \in E\}$. We observe that Q is uniformly bounded. Using the uniform continuity of $G(t)$, we have

$$\begin{aligned}
&||(Fx)(t_1) - (Fx)(t_2)|| \\
&\leq ||(G(t_1) - G(t_2))x_0|| + ||\int_0^{t_1} G(t_1 - s)f(s, x(s), Tx(s), Sx(s))ds \\
&\quad + \int_0^{t_2} G(t_2 - s)f(s, x(s), Tx(s), Sx(s))ds|| \\
&\quad + ||\sum_{0<t_k<t_1} G(t_1 - t_k)I_k(x(t_k)) - \sum_{0<t_k<t_2} G(t_2 - t_k)I_k(x(t_k))|| \\
&\quad + ||\int_0^{t_1} G(t_1 - s)B(s)B^*(s)G^*(T - s)W^{-1}(0,T)[x_1 - G(T)x_0 - \\
&\quad \int_0^T G(T - r)f(r, x(r), Tx(r), Sx(r))dr - \sum_{0<t_k<T} G(T - t_k)I_kx(t_k)]ds \\
&\quad - \int_0^{t_2} G(t_2 - s)B(s)B^*(s)G^*(T - s)W^{-1}(0,T)[x_1 - G(T)x_0 \\
&\quad - \int_0^T G(T - r)f(r, x(r), Tx(r), Sx(r))dr - \sum_{0<t_k<T} G(T - t_k)I_kx(t_k)]ds|| \\
&\leq ||G(t_1) - G(t_2)||\ ||x_0|| + \int_0^{t_1} ||G(t_1 - s) - G(t_2 - s)|| \\
&\quad ||f(s, x(s), Tx(s), Sx(s))||ds + \int_{t_1}^{t_2} ||G(t_2 - s)||\ ||f(s, x(s), Tx(s), Sx(s))||ds \\
&\quad + \sum_{0<t_k<t_1} ||G(t_1 - t_k) - G(t_2 - t_k)||\ ||I_k(x(t_k))|| + \sum_{t_1<t_k<t_2} ||G(t_2 - t_k)||\ ||I_k(x(t_k))|| \\
&\quad + \int_0^{t_1} ||G(t_1 - s) - G(t_2 - s)||\ ||B(s)||\ ||B^*(s)||\ ||G^*(T - s)||\ ||W^{-1}(0,T)||\ ||[x_1
\end{aligned}$$

$$
\begin{aligned}
&-G(T)x_0 - \int_0^T G(T-r)f(r,x(r),Tx(r),Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_kx(t_k)]||ds \\
&+ \int_{t_1}^{t_2} ||G(t_2-s)||\ ||B(s)||\ ||B^*(s)||\ ||G^*(T-s)||\ ||W^{-1}(0,T)||\ ||[x_1 - G(T)x_0 \\
&- \int_0^T G(T-r)f(r,x(r),Tx(r),Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_kx(t_k)]||ds \\
\leq\ & I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7
\end{aligned}
$$

where,

$$
\begin{aligned}
I_1 &= ||G(t_1) - G(t_2)||\ ||x_0|| \\
&\leq \frac{\epsilon}{7}, \quad for\ |t_1 - t_2| \leq \delta_1 \\
I_2 &= \int_0^{t_1} ||G(t_1-s) - G(t_2-s)||\ ||f(s,x(s),Tx(s),Sx(s))||ds \\
&\leq KT\frac{\epsilon}{7KT} = \frac{\epsilon}{7}, \quad for\ |t_1 - t_2| \leq \delta_2 \\
I_3 &= \int_{t_1}^{t_2} ||G(t_2-s)||\ ||f(s,x(s),Tx(s),Sx(s))||ds \\
&\leq MK(t_1-t_2), \quad for\ |t_1 - t_2| \leq \delta_3 = \frac{\epsilon}{7MK} \\
&\leq \frac{\epsilon}{7} \\
I_4 &= \sum_{0<t_k<t_1} ||G(t_1-t_k) - G(t_2-t_k)||\ ||I_k(x(t_k))|| \\
&\leq \frac{\epsilon}{7\sum h_k}\sum h_k, \quad for\ |t_1 - t_2| \leq \delta_4 \\
&= \frac{\epsilon}{7}, \quad for\ |t_1 - t_2| \leq \delta_4 \\
I_5 &= \sum_{t_1<t_k<t_2} ||G(t_2-t_k)||\ ||I_k(x(t_k))|| \\
&\leq M \sum_{t_1<t_k<t_2} h_k \\
&\leq \frac{\epsilon}{7}\ (By\ assuming \sum h_k\ is\ very\ small\ for\ |t_1 - t_2| \leq \delta_5)
\end{aligned}
$$

Now

$$
||[x_1 - G(T)x_0 - \int_0^T G(T-r)f(r,x(r),Tx(r),Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_kx(t_k)]||ds
$$

$$\begin{aligned}
&\leq [||x_1|| + ||G(T)||\,||x_0|| + \int_0^T ||G(T-r)||\,||f(r,x(r),Tx(r),Sx(r))||dr \\
&\quad + \sum_{0<t_k<T} ||G(T-t_k)||\,||I_k x(t_k)||ds] \\
&\leq ||x_1|| + M||x_0|| + MKT + M\sum h_k \\
&= D\ (say)
\end{aligned}$$

Therefore we have,

I_6

$$\begin{aligned}
&= \int_0^{t_1} ||G(t_1-s) - G(t_2-s)||\,||B(s)||\,||B^*(s)||\,||G^*(T-s)||\,||W^{-1}(0,T)||\,||[x_1 \\
&\quad -G(T)x_0 - \int_0^T G(T-r)f(r,x(r),Tx(r),Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_k x(t_k)]||ds \\
&\leq b^2 MwD\frac{\epsilon}{7b^2MwD} = \frac{\epsilon}{7}\ for\ |t_1-t_2| \leq \delta_6
\end{aligned}$$

Finally,

I_7

$$\begin{aligned}
&= \int_{t_1}^{t_2} ||G(t_2-s)||\,||B(s)||\,||B^*(s)||\,||G^*(T-s)||\,||W^{-1}(0,T)||\,||[x_1 - G(T)x_0 \\
&\quad -\int_0^T G(T-r)f(r,x(r),Tx(r),Sx(r))dr - \sum_{0<t_k<T} G(T-t_k)I_k x(t_k)]||ds \\
&\leq M^2b^2wD(t_2-t_1)\ \ for\ \ |t_1-t_2| \leq \delta_7 = \frac{\epsilon}{7M^2b^2wD} \\
&\leq \frac{\epsilon}{7}
\end{aligned}$$

Now taking $\delta = min\{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7\}$, we have

$$||(Fx)(t_1) - (Fx)(t_2)|| \leq \frac{\epsilon}{7} + \frac{\epsilon}{7} + \frac{\epsilon}{7} + \frac{\epsilon}{7} + \frac{\epsilon}{7} + \frac{\epsilon}{7} + \frac{\epsilon}{7}$$

That is,

$$||(Fx)(t_1) - (Fx)(t_2)|| \leq \epsilon$$

It implies Q is equicontinuous. Therefore by Arzola-Ascoli theorem Q is relatively compact. From Theorem 5.2.1 we have proved that F is Lipschitz continuous and hence F is continuous from E onto E. Thus applying Schauder's fixed point theorem

F has a fixed point and thus, the equation (5.2.6) is solvable. And hence the nonlinear impulsive system (5.1.1) is exactly controllable. □

5.3 Summary

In this chapter, we have discussed the exact controllability of nonlinear impulsive system. Here, we have proved the controllability result of a nonlinear impulsive evolution system by reducing the system into solvability problem. Fixed point theorem has been used for this purpose by imposing sufficient conditions on the nonlinear function f. Further, we have proved controllability results by using Schauder's fixed point theorem, in case of the nonlinear function f is uniformly bounded.