Summary

Controllability of some nonlinear systems by using tools from analysis such as fixed point theory etc. Along with controllability results, we made attempt to obtain a computational procedures for the actual computation of steering control.

Linear controllability result for second order systems, often required in solving of real life problems, has been obtained by Skelton[42]. Here, we have investigated the controllability of the system governed by a matrix second order nonlinear(MSON) differential equation:

$$\left.\begin{array}{l} \frac{d^2x(t)}{dt^2} + A^2x(t) = Bu(t) + f(t, x(t)) \\ x(0) = x_0, \;\; x'(0) = y_0. \end{array}\right\} \tag{1}$$

where, the state $x(t) \in R^n$ and the control $u(t) \in R^m$, A^2 is a constant matrix of order $n \times n$ and B is a constant matrix of order $n \times m$ and $f : [0, T] \times R^n \to R^n$ is a nonlinear function satisfying Caratheodory conditions (refer Joshi and Bose [47]). The initial states x_0 and y_0 are in R^n. The corresponding Matrix Second Order Linear (MSOL) system is :

$$\left.\begin{array}{l} \frac{d^2x(t)}{dt^2} + A^2x(t) = Bu(t) \\ x(0) = x_0, \;\; x'(0) = y_0. \end{array}\right\} \tag{2}$$

We have proved another controllability result for the linear system(2). In this case, we could also provide a computational algorithm for the actual computation of controlled state and steering control. Furthermore, a controllability result for the nonlinear system(1) with a controlled linear part has been proved by using Banach's contraction principle. For deriving these results, we have not reduced the system into first order one and analysed the system in its original form itself. We have made use of Sine and Cosine matrices to express the solutions of (1) and (2). Sine and Cosine of the matrix A is computed using the Páde approximation. We have also gone a step further and generalized this result by introducing two special type of matrices Φ and Ψ, exhibiting simillar properties as Sine and Cosine matrices.

Impulsive dynamical system attracted the attention of many researchers (Lakshmikantham and Bainov[52], George, Nandakumaran and Arapostathis[32], Liu[56]). Anguraj and Arjunan[3] studied the existence and uniqueness of the solution of a nonlinear impulsive evolution equation. Using fixed point approach we have studied the controllability of the system described by the integro-differential equation:

$$\left.\begin{aligned} &x'(t) = Ax(t) + f(t, x(t), Tx(t), Sx(t)) + B(t)u(t),\ 0 < t < T,\ t \neq t_k \\ &x(0) = x_0, \\ &\Delta x(t_k) = I_k x(t_k),\ \ k = 1, 2, 3, ..., p \end{aligned}\right\} \tag{3}$$

in a Banach space X, where $f \in C([0,T] \times X \times X \times X, X)$, A is infinitesimal generator of C_0 semigroup with impulsive condition and $B(t)$ is a bounded linear operator from X to U and the control function $u(\cdot)$ is in $L^2([0,T]; U)$ and U is another Banach space.

$$Tx(t) = \int_0^t K(t,s)x(s)ds,\ \ K \in C[D, R^+]$$

$$Sx(t) = \int_0^T H(t,s)x(s)ds, \quad H \in C[D_0, R^+]$$

where $D = \{(t,s) \in R^2 : 0 \leq s \leq t \leq T\}$, $D_0 = \{(t,s) \in R^2 : 0 \leq t, s \leq T\}$ and $0 < t_1 < t_2 < t_3 < ... < t_p < T$

$$\Delta x(t_k) = x(t_k^+) - x(t_k^-)$$

where $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$ respectively.

As a part of this thesis work we have also derived algorithms for computing controllability results for both linear and nonlinear systems by using spectral analysis of Grammian matrix. We have considered the following nonlinear n-dimensional first order system: (elimination of f will convert it into equivalent linear system.)

$$\left.\begin{array}{l} \frac{dx(t)}{dt} = A(t)x(t) + B(t)u(t) + f(t, x(t)) \\ x(t_0) = x_0. \end{array}\right\} \tag{4}$$

where the state $x(t) \in R^n$, the control $u(t) \in R^m$, $A(t)$ and $B(t)$ are matrices of order $n \times n$ and $m \times n$ respectively. $x_0 \in R^n$ is the initial state and $f(t, x(t))$ is a nonlinear function. In this approach we have not computed the inverse of the controllability Grammian and thus made the computation of steering control easier.

At the end, we have considered a nonlinear Urysohn delay integral inclusion of Volterra type given by:

$$x(t) \in (Hx)(t) + \int_0^t g(t,s,x_s)F(s,x_s)ds + \int_0^t K(t,s)u(s)ds. \tag{5}$$

where, for each $t \in [0,T]$ the state x(t) is in R^n and the control $u(t) \in R^m$.

For any given real number $0 < r < T$ and for any function $x \in C([-r,T]; R^n)$ and

$s \in [0, T]$, we define an element $x_s \in C([-r, 0]; R^n)$ by $x_s(\theta) = x(s+\theta), \quad -r \le \theta \le 0$. The initial conditions are given by

$$x(\theta) = \phi(\theta), -r \le \theta \le 0, \tag{6}$$

for a fixed, $\phi \in C[-r, 0]$.

$H : L^{\infty}([-r, T]; R^n) \to C([0, T]; R^n)$ is the Urysohn operator defined by

$$(Hx)(t) = \phi(0) + \int_0^T h(t, s, x_s)ds$$

where, $h : [0, T] \times [0, T] \times L^{\infty}([-r, 0]; R^n) \to R^n$ is a nonlinear function, $g : [0, T] \times [0, T] \times L^{\infty}([-r, 0]; R^n) \to M_{n\times n}$ is also a nonlinear function, where $M_{n\times n}$ is a space of $n \times n$ matrices. For $(t, s) \in [0, T] \times [0, T]$, K(t,s) is $n \times n$ matrix, $F : [0, T] \times L^{\infty}([-r, 0]; R^n) \to 2^R$ is a set-valued mapping. The controllability problem was converted to a fixed point problem for set-valued mapping. We have proved the controllability result for the inclusion (5)-(6) by using Bohnenblust-Karlin extension of Kakutani's fixed point theorem for set-valued mappings. We have imposed sufficient conditions on the nonlinear functions g, h and F to guarantee the existence of a fixed point for a set-valued mapping.