Part I

Background

Chapter 1

Introduction

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1.1 Introduction

Controllability is one of the basic properties of control systems appearing in various engineering disciplines. A system is said to be controllable if we can find a controller, which will steer the system from any initial state to a desired final state in a given finite time interval (refer Russel[68]). Kalman (refer [49]) introduced the concept of controllability of finite dimensional linear system in 1960's and subsequently this concept was extended to nonlinear systems (refer Mirza and Womack[61], Vidyasagar[77], Balachandran et.al. [8], [9], Joshi and George[46], Klamka[50], Shiv Prasad and Mukherjee[69]). The classical theory of controllability in finite dimensional space was extended for linear abstract systems defined on infinite dimensional spaces by Triggianni (refer [75]). Further, Quinn and Carmichael [65], Louis and Wexler[60], George[31], Zuazua[83] and many other authors obtained controllability results for nonlinear systems in infinite dimensional spaces.

Various notions of controllability such as exact controllability(refer Zuazua [83]), approximate controllability (refer Zhou[81], Geroge[31], Sukavanam[71]) , partial controllability(Nandakumaran and George[62],[63]), stochastic controllability (Arapostathis, George and Ghosh[5]) etc. were introduced in the literature.

The controllability theory of linear systems is almost saturated in the literature. Though there has been many results available for the nonlinear systems, many problems are still open for nonlinear systems. Furthermore, the computational algorithm for the steering control is important for engineering systems, which is not easily available in the literature. Development of powerful tools in differential equations, linear algebra and functional analysis resulted in the enrichment of control theory considerably.

In the present thesis, we investigate controllability of nonlinear systems by using some tools from analysis such as fixed point theory, spectral theory etc. Along with controllability results, we made an attempt to obtain a computational procedure for the actual computation of steering control. Second order systems often come in applications for which a linear controllability result has been obtained by Hughes and Skelton[42]. We study controllability of n-dimensional second order linear systems, and provide a controllability result for nonlinear second order systems using Banach's contraction principle. Our results here are computational in nature. In recent years, the field of impulsive dynamical systems attracted the attention of many researchers (Lakshmikantham and Bainov[52], George, Nandakumaran and Arapostathis[32], Liu[56]). Boukhamla and Mazouzi [13] obtained the controllability results for linear type system in Hilbert space settings. Anguraj and Arjunan[3] studied the existence and uniqueness of the solution of nonlinear impulsive evolution equations. We study the controllability of the same systems by employing fixed point theory. We also obtain computational controllability results for both linear and nonlinear systems by using spectral analysis of Grammian matrix. Finally, we take up a system whose dynamics is described by an integral inclusion. The controllability result here is established by using Bohnenblust-Karlin extension of Kakutani's fixed point theorem for set-valued mappings. The following section briefly discusses the problems dealt with and the results obtained as a part of this research work.

I. Controllability of Second Order Systems: Trigonometric Matrix Approach

Here we investigate the controllability of the system governed by a matrix second order nonlinear (MSON) differential equation:

$$\left.\begin{array}{l}\frac{d^2x(t)}{dt^2} + A^2x(t) = Bu(t) + f(t, x(t)) \\ x(0) = x_0, \ \ x'(0) = y_0.\end{array}\right\} \tag{1.1.1}$$

where, the state $x(t) \in R^n$ and the control $u(t) \in R^m$, A^2 is a constant matrix of order $n \times n$ and B is a constant matrix of order $n \times m$ and $f : [0, T] \times R^n \to R^n$ is a nonlinear function satisfying Caratheodory conditions (refer Joshi and Bose [47]). The initial states x_0 and y_0 are in R^n. The corresponding Matrix Second Order Linear (MSOL) system is :

$$\left.\begin{array}{l}\frac{d^2x(t)}{dt^2} + A^2x(t) = Bu(t) \\ x(0) = x_0, \ \ x'(0) = y_0.\end{array}\right\} \tag{1.1.2}$$

The system (1.1.2) has been studied by many researchers (refer Diwakar and Yadavalli [24], Hughes and Skelton [42], Wu and Duan [79]). This type of equations can model the dynamics of many natural phenomena to a significantly large extent(refer Hughes and Skelton [42], Fitzgibbon [27]). For example, Fitzgibbon [27] used the second order abstract differential equations for establishing the boundedness of the solutions of the equation governing the transverse motion of an extensible beam. A necessary and sufficient condition for the controllability of the matrix second order linear (MSOL) system has been proved in Hughes and Skelton [42]. They converted the second order system into first order system and obtained controllability result. However, no computational scheme for the steering control was proposed. Here, we prove another equivalent controllability result for the linear system(1.1.2) which also provides a computational algorithm for the actual computation of controlled state and steering control.. Furthermore, we prove a controllability result for the nonlinear system(1.1.1) with a controllable linear part. To prove the controllability result, we assume that the nonlinearity f is Lipschitz continuous. We do not reduce the system into first order and analyse the original form itself. Since, in many cases, it is advantageous to treat the second-order differential equations directly than converting them to first order systems. In our analysis, we invoke the tools of nonlinear functional analysis like fixed point theorem to obtain the controllability result for the nonlinear system.

We make use of matrix Sine and Cosine operators to express the solutions of (1.1.1) and (1.1.2). We employ an algorithm proposed by Hargreaves and Higham [41] for computing the Cosine and Sine of the matrix $A \in R^{n\times n}$. The algorithm uses the Páde approximations of Sine and Cosine of the matrix A.

We provide numerical examples for the computation of steering control for both linear and nonlinear systems.

II. Controllability of Second Order Systems: A General Operator Approach

Here we consider the control system described by the matrix second order nonlinear differential equation

$$\left.\begin{array}{l}\frac{d^2x(t)}{dt^2} + Ax(t) = B(t)u(t) + f(t, x(t)) \\ x(0) = x_0, \;\; x'(0) = y_0.\end{array}\right\} \tag{1.1.3}$$

where, the state $x(t) \in R^n$,the control $u(t) \in R^m$, A is matrix of order $n \times n$, B is a matrix of order $n \times m$ and $f : [0, T] \times R^n \to R^n$ is a nonlinear function. The initial states $x_0,\ y_0$ are in R^n.

We prove controllability result of (1.1.3) by introducing two special types of matrices Φ and Ψ instead of Sine and Cosine matrices.

III. Exact Controllability of Impulsive Systems

In the dynamics of many practical systems, there is an abrupt change in the state such as impulse or shock experienced in a short duration of time. Such systems are modeled in terms of impulsive differential equations (refer Lakshmikantham and Bainov[52], Nandakumaran and Arapostathis[32]). In Leela [54], the controllability aspect of a linear finite dimensional impulsive systems was investigated. George, Nandakumaran and Arapostathis [32] generalized the controllability result to nonlinear systems with impulse. Recently, Boukhamla and Mazouzi [13] obtained the controllability result for linear systems in infinite dimensional settings. We study the controllability of the impulsive evolution systems of the form:

$$\left.\begin{array}{l}x'(t) = Ax(t) + f(t, x(t), Tx(t), Sx(t)) + Bu(t), \;\; 0 < t < T, \;\; t \neq t_k \\ x(0) = x_0, \\ \Delta x(t_k) = I_k x(t_k), \;\; k = 1, 2, 3, ..., p\end{array}\right\} \tag{1.1.4}$$

in a Banach space X, where $f \in C([0,T]\times X\times X\times X, X)$, A is infinitesimal generator of C_0 semigroup with impulsive condition and B is a bounded linear operator from X to X and the control function $u(\cdot)$ is in $L^2([0,T];X)$.

$$Tx(t) = \int_0^t K(t,s)x(s)ds, \quad K \in C[D, R^+]$$

$$Sx(t) = \int_0^T H(t,s)x(s)ds, \quad H \in C[D_0, R^+]$$

where $D = \{(t,s) \in R^2 : 0 \le s \le t \le T\}$, $D_0 = \{(t,s) \in R^2 : 0 \le t, s \le T\}$ and $0 < t_1 < t_2 < t_3 < ... < t_p < T$

$$\Delta x(t_k) = x(t_k^+) - x(t_k^-)$$

where $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$ respectively and f is a nonlinear function satisfying Lipschitz condition. Anguraj and Arjunan[3] has proved the existence and uniqueness of the solution of the above impulsive evolution equation without control. We obtain controllability results using fixed point theory.

IV. Controllability and Steering Control By Spectral Method.

Consider the following nonlinear n-dimensional first order system:

$$\left.\begin{array}{l} \frac{dx(t)}{dt} = A(t)x(t) + B(t)u(t) + f(t, x(t)) \\ x(t_0) = x_0. \end{array}\right\} \tag{1.1.5}$$

where the state $x(t) \in R^n$, the control $u(t) \in R^m$, $A(t)$ and $B(t)$ are matrices of order $n \times n$ and $m \times n$ respectively. The vector $x_0 \in R^n$ is the initial state and $f(t, x(t))$ is a nonlinear function. The main objective is to obtain controllability result and develop a computational algorithm for the steering control. Here our result depend upon the spectral properties of the controllability Grammian. First we obtain the spectral controllability result for the linear system:

$$\left.\begin{array}{l} \frac{dx(t)}{dt} = A(t)x(t) + B(t)u(t) \\ x(t_0) = x_0. \end{array}\right\} \tag{1.1.6}$$

The solution of (1.1.6) can be written as:

$$x(t) = \Phi(t, t_0)x_0 + \int_{t_0}^{t} \Phi(t, s)B(s)u(s)ds \tag{1.1.7}$$

where $\Phi(t, s)$ is the state transition matrix of the linear system.

Controllability Grammian matrix of the system (1.1.6) is given by

$$W(t_0, T) = \int_{t_0}^{T} \Phi(T, s)B(s)B^*(s)\Phi^*(T, s)ds \tag{1.1.8}$$

The system (1.1.6) is controllable if and only if 0 is not an eigen value of the Controllability Grammian $W(t_0, T)$. Here, the steering control is defined using the eigen values λ_i and eigenvectors v_i of the controllability Grammian matrix as follows:

$$u(t) = B^*(t)\Phi^*(T, t)\sum_i \frac{c_i v_i}{\lambda_i} \tag{1.1.9}$$

where $\{v_n\}$ is the orthonormal basis of R^n generated by eigenvectors corresponding to $\{\lambda_i\}$, the eigenvalues of the matrix $W(t_0, T)$ and $c_i^{\prime}s$ are the coordinates of the vector $\{x_1 - \Phi(T, t_0)x_0\}$ with respect to the orthonormal basis $\{v_n\}$.

In this approach we do not compute the inverse of the controllability Grammian and hence the computation of steering control is comparatively easy. The same approach has been applied to nonlinear system (1.1.5), with Lipschitz continuous nonlinear function f. We provide examples to illustrate this approach. There are many powerful mathematical packages like MATLAB /Maple /Octave /Mathematica to find eigenvectors and eigenvalues of a matrix. The steering control can be easily computed making use of these packages.

V. Controllability of Urysohn Type Integral Inclusion System

In recent years a number of papers appeared in the literature concerning integral inclusions, in particular inclusions of Hammerstein type and Urysohn type (refer Rangimkhannov [66], Gaidarov[29], Angel [1]). This type of inclusions have been used to model many thermostatic devices (refer Glashoff and Spreckels [33], [34]). We consider a controlled nonlinear Urysohn delay integral inclusion of Volterra type given by:

$$x(t) \in (Hx)(t) + \int_0^t g(t,s,x_s)F(s,x_s)ds + \int_0^t K(t,s)u(s)ds. \qquad (1.1.10)$$

where, for each $t \in [0,T]$ the state x(t) is in R^n and the control $u(t) \in R^m$.
For any given real number $0 < r < T$ and for any function $x \in C([-r,T]; R^n)$ and $s \in [0,T]$, we define an element $x_s \in C([-r,0]; R^n)$ by

$$x_s(\theta) = x(s+\theta), \quad -r \le \theta \le 0.$$

The initial conditions are given by

$$x(\theta) = \phi(\theta), -r \le \theta \le 0, \qquad (1.1.11)$$

for a fixed, $\phi \in C[-r,0]$.
Here, $H : L^\infty([-r,T]; R^n) \to C([0,T]; R^n)$ is the Urysohn operator defined by

$$(Hx)(t) = \phi(0) + \int_0^T h(t,s,x_s)ds$$

where, $h : [0,T] \times [0,T] \times L^\infty([-r,0]; R^n) \to R^n$ is a nonlinear function, $g : [0,T] \times [0,T] \times L^\infty([-r,0]; R^n) \to M_{n\times n}$ is also a nonlinear function, where $M_{n\times n}$ is a space of $n \times n$ matrices. For $(t,s) \in [0,T] \times [0,T]$, K(t,s) is $n \times n$ matrix, $F : [0,T] \times L^\infty([-r,0]; R^n) \to 2^R$ is a set-valued mapping. Choung[20] studied a general Urysohn inclusion of Volterra type, without delay and control. The existence result for such systems was established under much stronger hypothesis on the set-valued mappings. The existence of the solution of (1.1.10)-(1.1.11) without control was established by Angel [1].

Here, we convert the controllability problem into a fixed point problem for set-valued mapping. We prove controllability result for the inclusion (1.1.10)-(1.1.11) by using Bohnenblust-Karlin extension of Kakutani's fixed point theorem for set-valued mappings. We impose sufficient conditions on the nonlinear functions g, h and F to guarantee the existence of a fixed point for a set-valued mapping. We provide example to illustrate the theory.

1.2 Layout of the thesis

The thesis is organized as follows:

Chapter 1 deals with general introduction of the thesis.

Chapter 2 focuses on the necessary concepts of control theory and analysis which will be used subsequently in the thesis.

In Chapter 3, we study the controllability of Matrix Second Order Linear and Nonlinear Systems in finite dimensional space. Here we make use of Sine and Cosine matrices to obtain the solutions of the second order systems. An algorithm based on páde approximation to compute Sine and Cosine of a matrix is given. Here we also provide an algorithm for the actual computation of steering control of the MSOL. A sufficient condition of controllability of second order nonlinear systems is proved by invoking the fixed point theorem. For both linear and nonlinear systems, we present numerical experiments which show the applicability of the theory developed in this chapter.

Chapter 4 deals with the study of controllability of a class of second order systems. We prove similar kind of controllability results for both MSOL and MSON in finite dinemsional space by using general matrices Φ and Ψ. Matrices Φ and Ψ, have similar properties as Sine and Cosine matrices.

In Chapter 5, we discuss the exact controllability of nonlinear impulsive systems. Here, we obtain the controllability of a nonlinear impulsive evolution systems. Anguraj and Arjunan[3] has proved the existence and uniqueness of the solution of the same impulsive evolution equations without control. We prove the controllability result by reducing the system into solvability problem and apply the fixed point theorem on this problem by imposing sufficient conditions on the nonlinear function f. The Banach contraction principle is used in our analysis.

In the Chapter 6, we have developed an algorithm for the computation of steering control using spectral analysis. The chapter begins with the controllability result of the linear first order system and the computational algorithm for the steering control of linear system is provided. Then we prove the controllability result for nonlinear system and develop a computational algorithm for steering control. The chapter concludes with the numerical examples for both the linear and nonlinear systems.

Chapter 7 deals with the controllability of a system described by an integral inclusion of Urysohn type with delay. To obtain, the controllability result we employ Bohnenblust-Karlin extension of Kakutani's fixed point theorem for set-valued mappings. We prove the controllability result by reducing the controllability problem into solvability problem and applying the fixed point theorem on this problem. We conclude the chapter by giving numerical example to illustrate the result obtained here.