3. Technical Background

3.1 Introduction

The chapter briefly covers wavelets, continuous wavelet transform, discrete wavelet transform and stationary wavelet transform. It also includes image retrieval performance measures - Precision and Recall along with Precision-Recall curves presented as performance evaluation measures of various algorithms. The watershed region related issues are covered lastly in the chapter. The prominent boundaries detection and prominent boundaries detection based algorithms incorporates stationary Haar wavelet decompositions at different levels. The watershed transforms and prominent boundary has been utilized in foreground revealing method.

3.2 Signal Analysis

Fourier transform is a time domain to frequency domain transformation that represents a signal under consideration into harmonics of sinusoidal having different frequencies, amplitudes and phases. As the signal is being transformed into frequency domain, the powerful transformation has a serious drawback of not retaining time information. Hence, time localization of an instance of an event cannot be detected and analyzed with Fourier transform. To overcome this shortcoming, Gabor proposed Short-time Fourier analysis using *windowing* a signal under consideration. The windowed signal is analyzed using Fourier analysis giving frequency domain representation of a windowed signal by retaining time information. The accuracy of detection of the instance of an event depends on the size of the selected window and the frequency content of the signal. For all practical non-stationary signals, a fixed sized window is not suitable and adaptively adjusting the window size is difficult. Hence, short-time Fourier analysis is not suitable for many applications.

The solution to the question of detecting time-instance of an event is Wavelet analysis. As the name suggests, Wavelets are small time limited waves having zero average value. Different types of available wavelets are shown in Table 1 and mother

wavelets and corresponding scaling functions in Figure 6. These wavelets are the basis function for wavelet analysis. The wavelet analysis represents signal with scaled and shifted versions of mother wavelets.

3.2.1 Types of Mother Wavelets

Different wavelet families and corresponding mother wavelets are tabulated below. ** following wavelets in the last column of the table indicate a wavelet being a part of an infinite family of wavelets.

Table 1. Types of Wavelets.

Sr. No	Mother wavelet family names	Abbreviations	Wavelets
1	Haar	haar	
2	Daubechies	db	db1 db2 db3 db4 db5 db6 db7 db8 db9 db10 db**
3	Symlets	sym	sym2 sym3 sym4 sym5 sym6 sym7 sym8 sym**
4	Coiflets	coif	coif1 coif2 coif3 coif4. coif5
5	BiorSplines	bior	bior1.1 bior1.3 bior1.5 bior2.2 bior2.4 bior2.6 bior2.8 bior3.1 bior3.3 bior3.5 bior3.7 bior3.9 bior4.4 bior5.5 bior6.8
6	ReverseBior	rbio	rbio1.1 rbio1.3 rbio1.5 rbio2.2 rbio2.4 rbio2.6 rbio2.8 rbio3.1 rbio3.3 rbio3.5 rbio3.7 rbio3.9 rbio4.4 rbio5.5 rbio6.8
7	Meyer	meyr	
8	DMeyer	dmey	
9	Gaussian	gaus	gaus1 gaus2 gaus3 gaus4 gaus5gaus6 gaus7 gaus8 gaus**
10	Mexican_hat	mexh	
11	Morlet	morl	
12	Complex Gaussian	gaus	cgau1 cgau2cgau3cgau4 cgau5 cgau**
13	Shannon	shan	shan1-1.5 shan1-1 shan1-0.5 shan1-0.1 shan2-3 shan**
14	Frequency B-Spline	fbsp	fbsp1-1-1.5 fbsp1-1-1 fbsp1-1-0.5 fbsp2-1-1 fbsp2-1-0.5 fbsp2-1-0.1 fbsp**
15	Complex Morlet	cmor	cmor1-1.5 cmor1-1 cmor1-0.5 cmor1-0.1 cmor**

3.2.2 Example - Mother Wavelets & Scaling Functions

The mother wavelets and corresponding scaling functions are plotted with the help of Matlab functions and shown below in Figure 6.

(a)	db1 Haar	Approximations of the waveletdb1	Scaling functiondb1
(b)	db2	Approximations of the waveletdb2	Scaling functiondb2
(c)	db4	Approximations of the waveletdb4	Scaling functiondb4
(d)	db6	Approximations of the waveletdb6	Scaling functiondb6
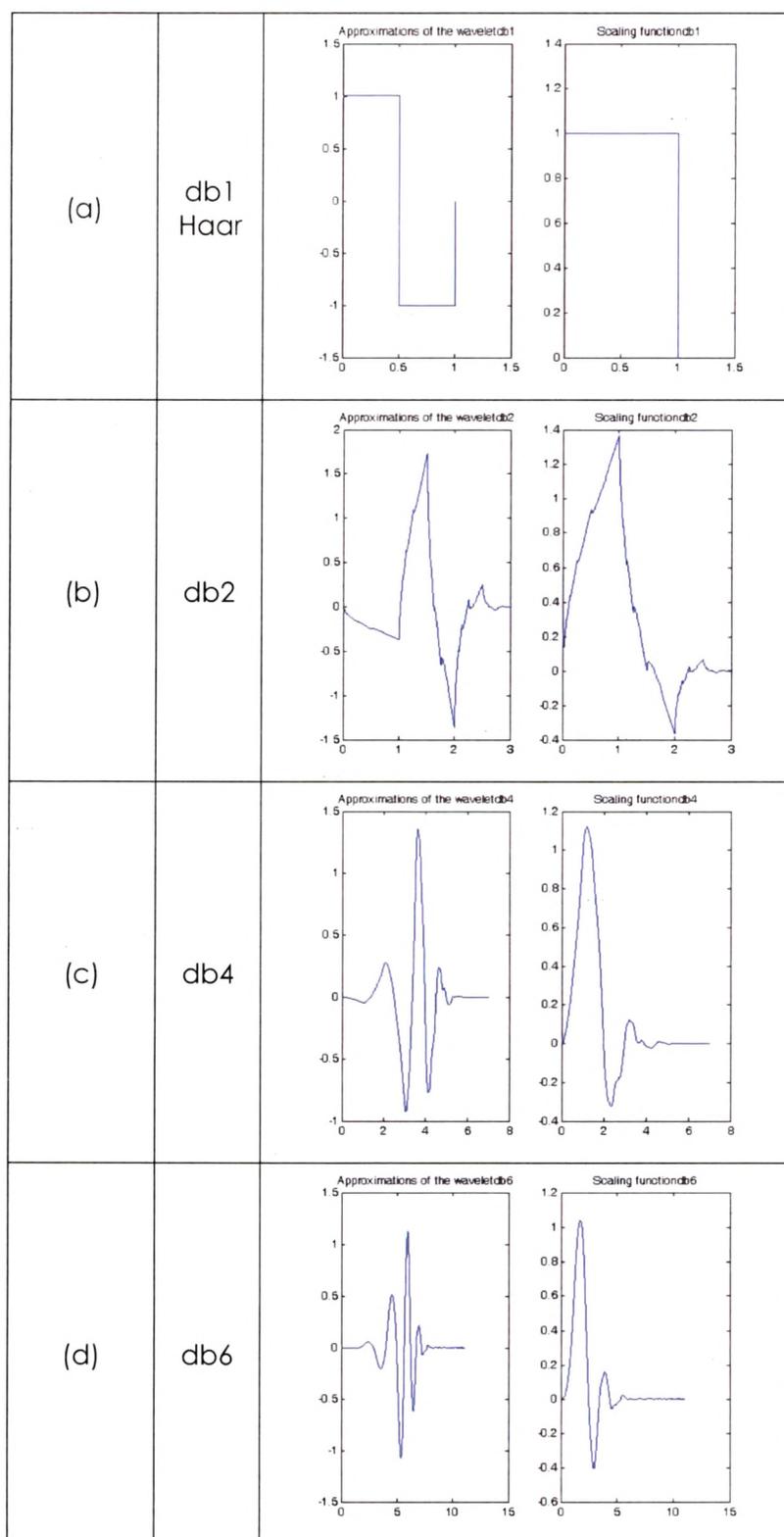
Figure 6. Mother Wavelets and Corresponding Scaling Functions.

(e)	db10	Approximations of the wavelet db10	Scaling function db10
(f)	Sym2	Approximations of the wavelet sym2	Scaling function sym2
(g)	Coif1	Approximations of the wavelet coif1	Scaling function coif1
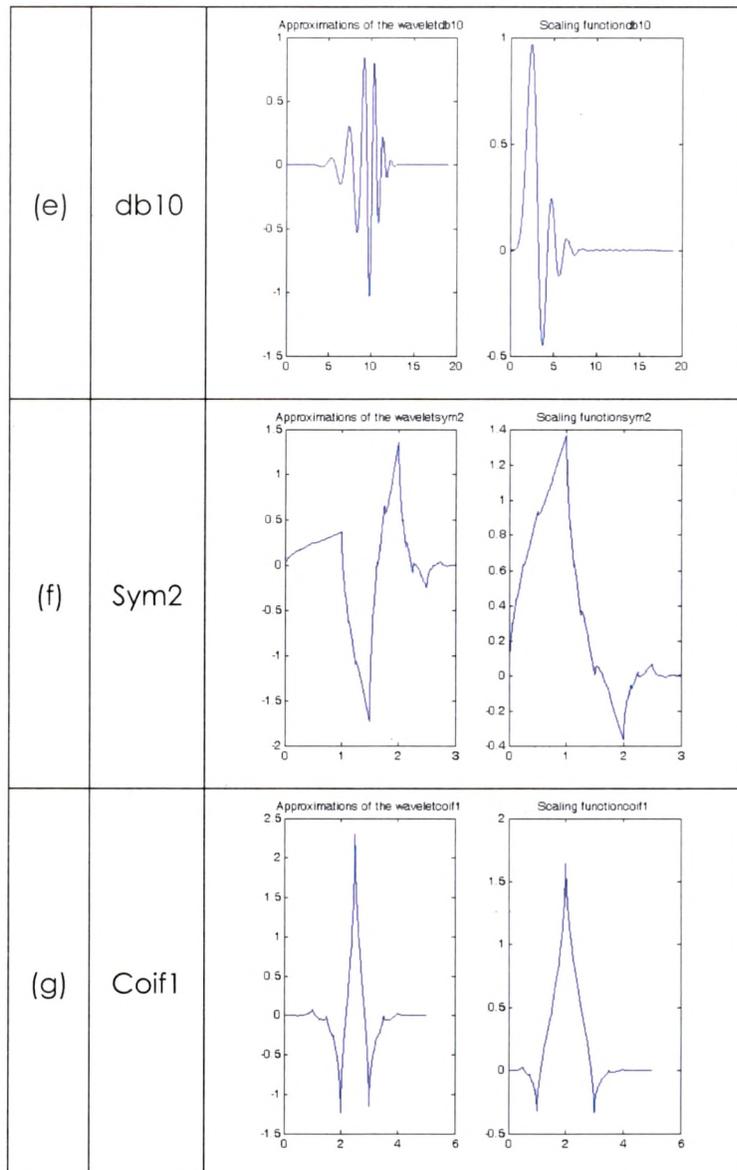
Figure 6 (Contd). Mother Wavelets and Corresponding Scaling Functions.

The Haar mother wavelet function ψ(x), also known as db1 mother wavelet, can be described as

$$\psi(x) = 1, \quad \text{if } x \in [0, 1/2[\,,$$
$$\psi(x) = -1, \quad \text{if } x \in [1/2, 1[\,, \tag{3.1}$$
$$\psi(x) = 0, \quad \text{otherwise.}$$

and its scaling function φ(x) can be given as

$$\varphi(x) = 1, \quad \text{if } x \in [0,1[\,, \tag{3.2}$$
$$\varphi(x) = 0, \quad \text{otherwise.}$$

Haar mother wavelet is the simplest and the first wavelet. The step like wavelet and simple scaling functions are shown in Figure 6 (a).

3.2.3 Continuous Wavelet Transform

Continuous Wavelet transform (CWT) is defined as

$$C (a, b) = \int_{-\infty}^{\infty} \emptyset(t) \, \Psi(a , b, t) \, dt \tag{3.3}$$

Where,

C is the wavelet coefficients characterized by scale a, and shift b.

$\emptyset$ (t) is a function, whose wavelet transform is sought.

Ψ (a, b, t) is a mother wavelet that is scaled and shifted for coefficient computation.

Thus wavelet coefficient at given scale and shift is computed by multiplying $\emptyset$ (t) with scaled and shifted version of mother wavelets and summing up the results in other words, $\emptyset$ (t) is convolved with scaled and shifted version of Ψ(a, b, t) for a given scale a and shift b. Figure 7 [Ha, on line] shows scale and shift operations on mother wavelet.

The scale and shifts are continuous in the CWT resulting into production of a large number of wavelet coefficients. These computations are not only computationally-expensive, but also more than enough for majority of practical applications.
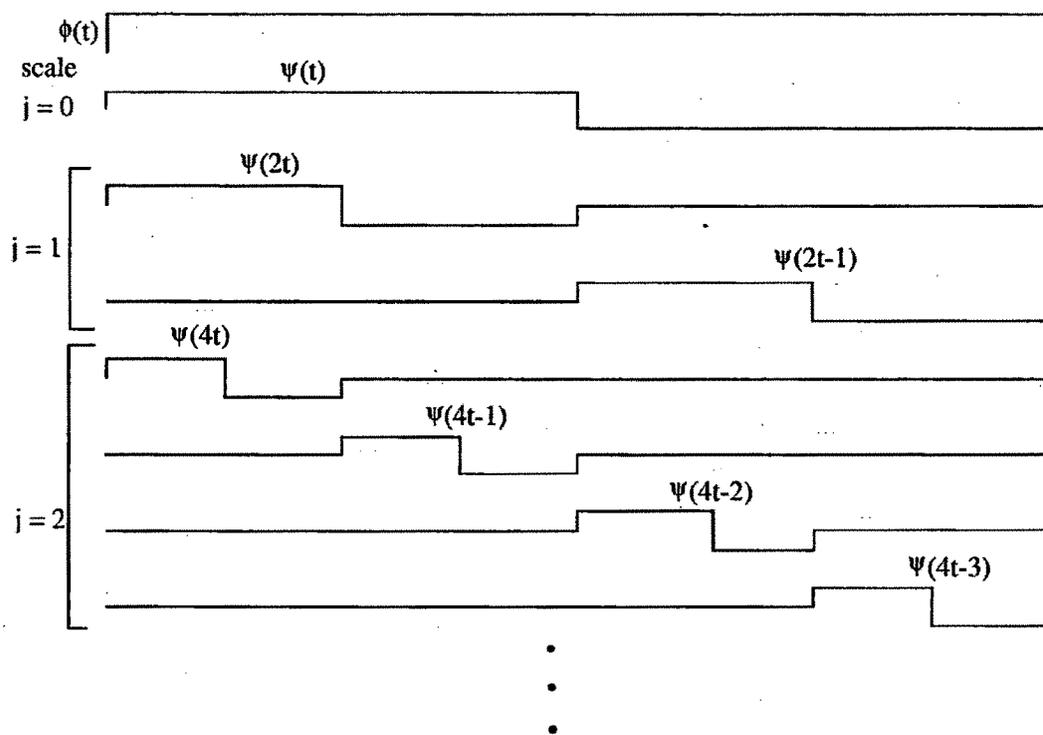

Figure 7. Shift and Scale Operation on Mother Wavelet [Ha, on line].

3.2.4 Discrete Wavelet Transform (DWT) & Stationary Wavelet Transform (SWT)

The Discrete Wavelet transform selects scale and shifts based on power of 2 to have faster and accurate enough transform with reduced number of wavelet coefficients. Figure 8 illustrates DWT operation. The classical Discrete Wavelet Transform (DWT) at each level convolves the signal with low and high pass fitters and then performs decimation operation to generally discard odd coefficients and preserve even ones. The DWT decomposes sampled signal into approximate and detailed components. The filter pair is designed corresponding to the mother wavelet under consideration. These coefficients are down sampled by a factor of 2 by generally discarding every odd coefficient. This process is known as decimation operation. The approximate and detail coefficients have half the length totaling same number of coefficients as original number of samples. The approximate coefficients are low frequency components whereas detail coefficients are high pass counter parts. The mathematical model for computation and interpretation of multi-resolution signal decomposition as wavelet representation and the extension of orthogonal wavelet representation for images was proposed in [Mallat, 1989].
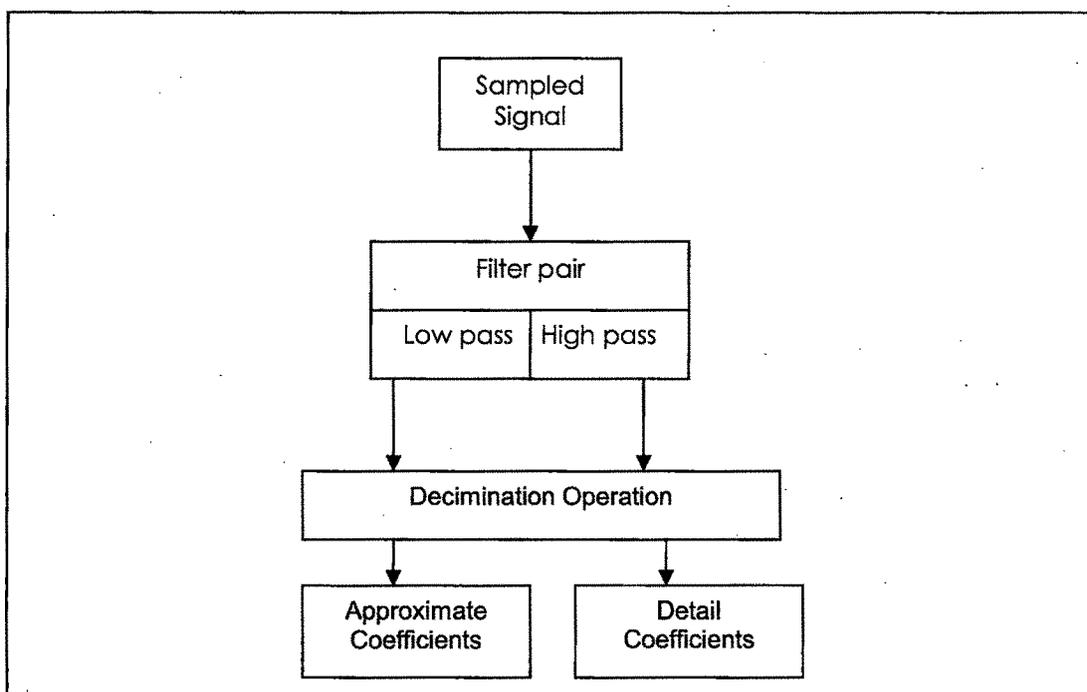

Figure 8. Block diagram for DWT.

The approximate coefficients can be further decomposed into approximate and detailed coefficients of next level, producing multi-level DWT as shown in Figure 9. The resultant tree known as wavelet decomposition tree that is useful for hierarchical analysis. The wavelet synthesis reconstructs the signal using wavelet coefficients, up-sampling and complementary filters.

```
                    ┌──────────┐
                    │ Sampled  │
                    │ Signal   │
                    └────┬─────┘
                         │
                    ┌────▼──────────┐
                    │ Decomposition │
                    │ at level 1    │
                    └──┬─────────┬──┘
                       │         │
              ┌────────▼──┐  ┌───▼──────┐
              │ Approx.   │  │ Detail   │
              │ Coeff.    │  │ Coeff.   │
              └──┬────────┘  └──────────┘
                 │
          ┌──────▼────────┐
          │ Decomposition │
          │ at level 2    │
          └──┬────────┬───┘
             │        │
      ┌──────▼──┐  ┌──▼──────┐
      │ Approx. │  │ Detail  │
      │ Coeff   │  │ Coeff   │
      └──┬──────┘  └─────────┘
         │
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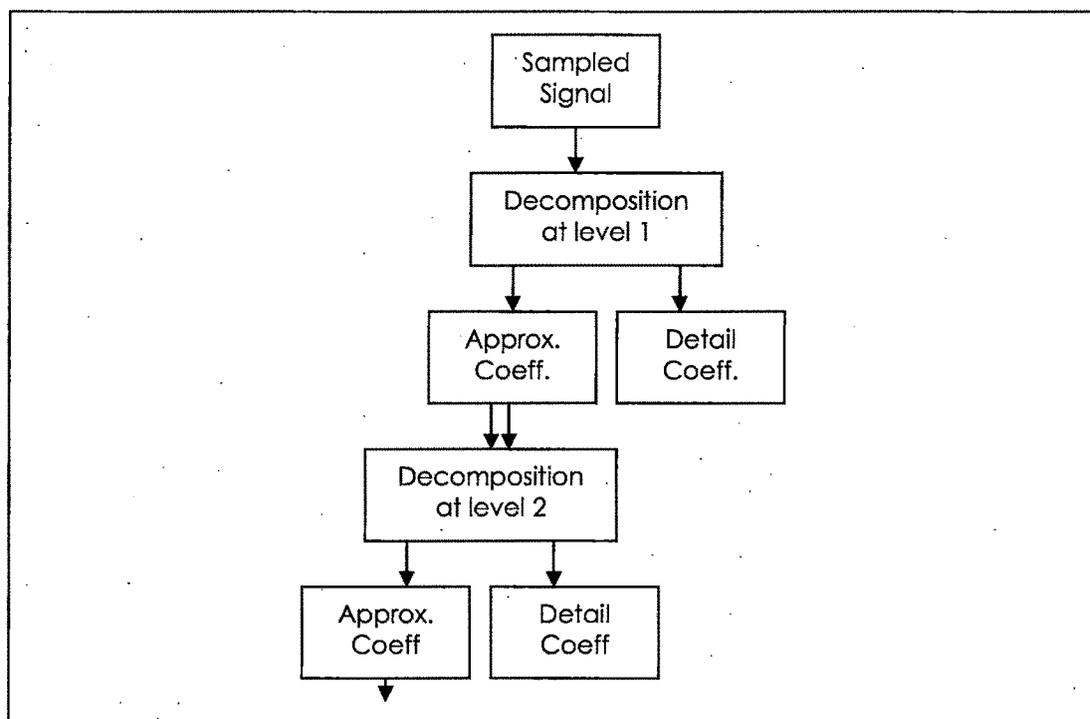
Figure 9. Wavelet - Multi-level Decomposition.

Maximum number of levels in DWT and SWT for given sequence of length N is $J = \log_2 (N)$ as given in [Nason, 1995].

The coefficients for DWT, as denoted in [Nason, 1995],

$$c^j = D_0 H\ c^{j+1} \text{ and } d^j = D_0 G\ c^{j+1} \text{ for } j = J-1, J-2,....,0 \qquad (3.4)$$

Where,

c^J is initialized with original sequence data.

D_0 is a binary decimation operator, keeping even indexed coefficients in the sequence.

H is a low pass filter producing approximate (smooth) coefficients c^j.

G is a high pass filter producing detailed coefficients d^j.

Decimation operation of DWT given in (3.4) causes length reduction of vectors c and d by a factor of 2 at every level which makes DWT unsuitable for the proposed method. The length reduction in the vectors introduces difficulties to exactly map the boundaries on to the image. So, the method incorporates SWT where there is no decimation

operation involved for coefficient computations resulting into same length vectors of coefficients at all levels.

The coefficients for SWT, as denoted in [Nason, 1995],

$$a^{j-1} = H^{[J-j]} a^j \text{ and } b^{j-1} = G^{[J-j]} a^j \text{ for } j = J, J-1, \ldots, 1. \tag{3.5}$$

Where,

a^J is initialized with original sequence data.

$H^{[J-j]}$ is a low pass filter used at level j for producing approximate (smooth) coefficients a^j.

$G^{[J-j]}$ is a high pass filter used at level j for producing detailed coefficients b^j.

The H and G are required to be modified at every level so as to have length N for approximate and detailed coefficients, same as the length of original data.
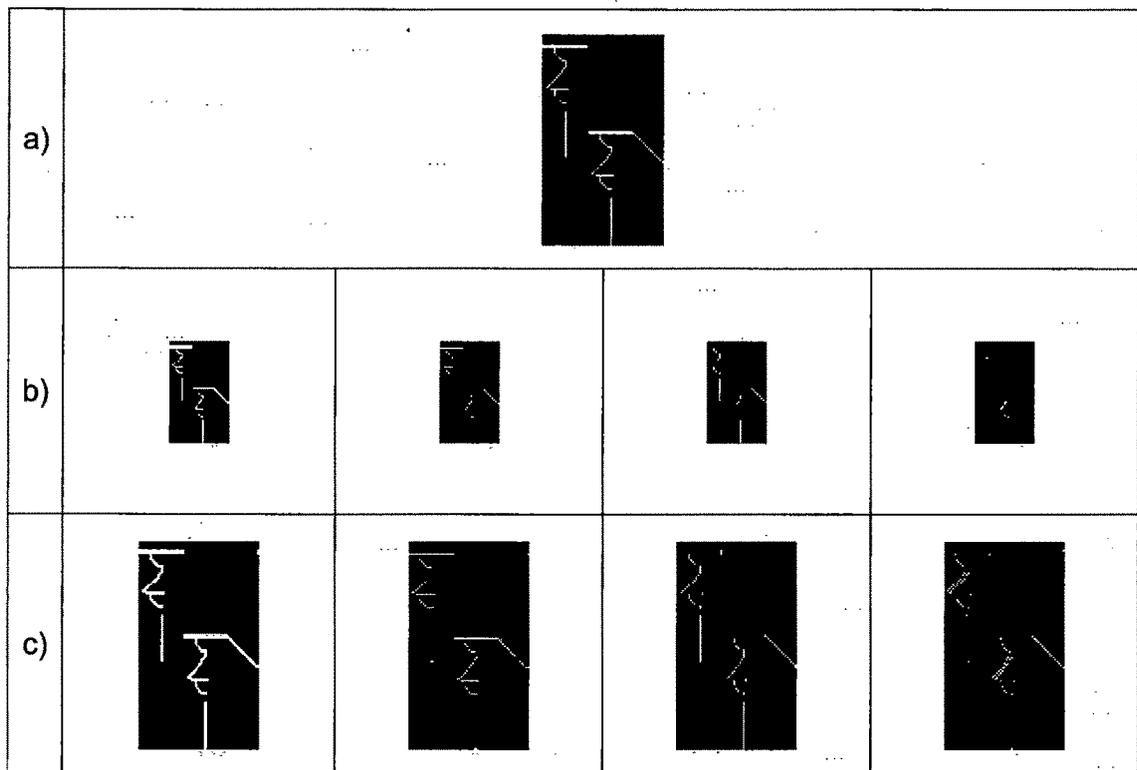
Figure 10. Decomposition with Discrete Haar Wavelet and Discrete Stationary Haar Wavelet. (a) Original Image. (b) DWT – Haar decomposition at level 1, from left to right - Approximate, Horizontal, Vertical and Diagonal coefficients (c) SWT –Haar decomposition at level 1, from left to right Approximate, Horizontal, Vertical and Diagonal coefficients

The qualitative comparison of results of SWT and DWT decompositions with Haar mother wavelet at level 1 are shown in Figure 10. The decimation operation of DWT halves the size of the coefficients discarding half of the information. The effect of

discarding the coefficients can be compared with corresponding coefficients achieved with SWT. The prominent boundary detection technique makes use of SWT with Haar. The performance comparison of other type of mother wavelet in prominent boundaries detection can be a future enhancement of the work.

3.3 CBIR Performance Measures

The precision, recall and F-measure are used to measure performance of CBIR algorithms / systems.

3.3.1 Precision

It is a measure of exactness (accuracy) in the retrieved results indicating how many retrieved images are relevant, given by a ratio of no. of retrieved relevant images over total no. of retrieved images. The maximum valve 1 indicates all retrieved images are relevant.

$$p = \frac{rr}{total} \qquad (3.6)$$

Where,

rr – No. of retrieved relevant images

total – Total no. of retrieved images

3.3.2 Recall

It is a measure of comprehensiveness of correctly retrieved images indicating how many relevant images are retrieved, given by a ratio of retrieved relevant images over total relevant images in the data base. The maximum valve 1 indicates all relevant images in the database have been retrieved.

$$r = \frac{rr}{Total} \qquad (3.7)$$

Where,

rr – No. of retrieved relevant images

Total – Total no. of relevant images in the database

Ideal Precision - Recall Curve
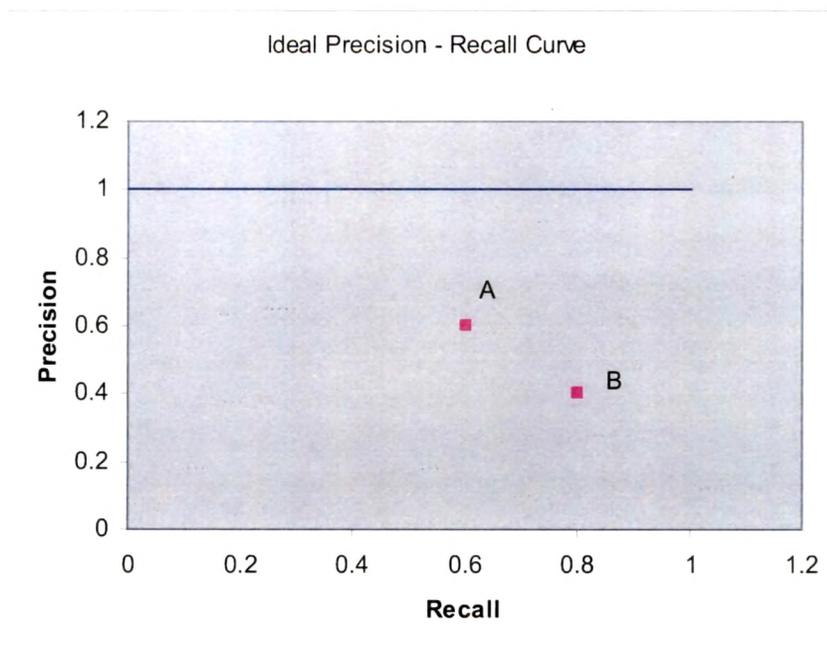
Figure 11. Ideal Precision – Recall Curve

3.3.3 P - R Curves

The precision – recall curve for a given query image is plotted for different parameters / thresholds of the retrieval algorithm. The ideal precision-recall curve is shown in Figure 11. It indicates that all retrieved images must be relevant giving precision as 1 for all values of recall. Recall value less than one indicate that some relevant images of database are missed, but all retrieved images are relevant as per ideal P – R curve. The best case scenario is precision 1 for recall 1 indicating all and only relevant images of the database have been retrieved. But, it is impossible to achieve ideal P-R curve for large image database. The precision falls non-linearly for higher values of recall in practical P – R curves. Increase in recall means an attempt to include missed relevant images. This attempt, in general, does not include only missed relevant but also includes new irrelevant images in the retrieval, decreasing the precision. Thus, better precision for lower recall and lower precision for higher recall is the characteristic of a practical P - R curves. So, precision and recall both cannot be high – ideally approaching to 1. Improvement in one measure compromises the other. The point B (0.8, 0.4) indicates that 80 % of relevant images form the database is retrieved with 40% of precision, i.e. out of total retrieved images, 60% images are irrelevant. The point A (0.6, 0.6) corresponds to better precision at the cost of recall compared to point B, i.e.

lesser irrelevant images are retrieved and missing more relevant images of the database.

3.3.4 F - measure

It is a harmonic mean, combining precision and recall to describe a single numerical value, given as [Petrakis, on line]

$$F = \frac{2}{(1/r + 1/p)}$$ (3.8)

Where,

r - recall

p - precision

The ideal value of precision and recall gives F measure as 1. Though F measure value does not indicate contribution of r and p, it is a single numeric value indicating performance measure of CBIR - higher the F-measure, better the performance.

The other important factor for P – R curve is the size of image database. For smaller databases, the P – R curves are likely to be better than that of larger image databases.

3.4 Watershed Regions and Issues

Watershed algorithm can be applied to segment intensity or binary images. The algorithm finds local minima forming catchment basins to determine segments of the image. The watershed function of Matlab R14 applied to segment intensity / binary images shown in Figure 12. The segmentation results are shown in the adjacent column. The segmentation results are appropriate for image of Figure 12 (a), where region boundaries are horizontal / vertical and of one pixel width. The watershed function does not segment intensity image of Figure 12 (b) appropriately. The regions surrounded by one pixel wide non-vertical and non-horizontal boundaries have not been resulted into segments despite considering 8- pixel connectivity (A Matlab-bug?). Figure 12 (c) is yet another typical image having two intensity regions surrounded by black boundaries. The watershed algorithm does not perform as expected for a region having same intensity value for all pixels belonging to the region. (?) So is the case for binary equivalent image of Figure 12 (c), shown in Figure 12 (d). The same binary image, if labeled with bwlabel function, gives correct segmentation as shown in Figure 13. The

watershed algorithm implementation of Matlab R14 does produce expected segmentation for region boundaries of width greater than 1 pixel. But it is prone to erode a patch of region having same intensity values. Thus, watershed algorithm implementation of Matlab R14 adds to the challenges for achieving proper segmentation.
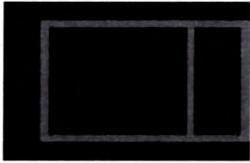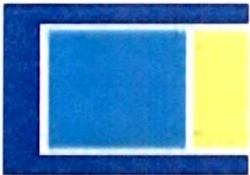
(a) Intensity Image & Corresponding Watershed Regions		
(b) Intensity Image & Corresponding Watershed Regions		
(c) Intensity Image & Corresponding Watershed Regions		
(d) Binary Image & Corresponding Watershed Regions		
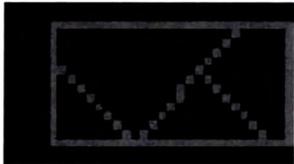
Figure 12. Watershed Regions – Issues

Binary Image & Corresponding labeled regions without using watershed function		

Figure 13. Correctly Labeled Regions without using watershed transform

3.5   Concluding Remark

Major related technical issues have been briefly presented …