C H A P T E R - II

CONCEPTS OF FAILURE

2.0 General:

The problem of failure of solid materials is the central problem of mechanics. The first concepts on failure mechanics are associated with the names of Galileo (1400), Hooke (1678), Coulomb (1776), Saint Venant (1855), and Mohr (1900). These concepts initiated extensive studies on deformation properties of solids and led to the development of various failure critera termed as strength theories. These theories state that failure starts at a moment when at a certain point in a body, a particular combination of parameters such as stress, strain reaches a definite critical value. In recent years the new concepts in strength analysis evolved as a result of the works of Griffith (1921), Taylor (1940), Orawan (1951), Irwin (1957) and others. The new concepts suggest to include the influence of defects existing in a body and their propagation during loading.

2.1 Classical concepts of failure:

The basic aim of all classical theories is to postulate critical conditions at failure in terms of stress, strain or energy parameters. These theories do not take in to consideration the material properties and also disregard the actual boundary conditions. The principal expressions from classical concepts are presented below.

2.1.1 Maximum stress theory:

The maximum stress theory is the oldest theory of failure some times known as Rankine's theory. It postulates that maximum principal stress in the material determines failure regardless of the magnitudes and nature of the other two principal stresses. According to the theory, yielding in a stressed body commences when the absolute value of the maximum stress reaches the yield point stress of the material in simple tension or compression which when plotted in principal stress space represents yield surface as a cube.

2.1.2 Maximum elastic strain theory:

The maximum elastic strain theory is attributed to St.Venant which assumes that a material begins to yield when either the maximum strain equals to the yield point strain in simple tension or the minimum strain equals to the yield point strain in simple compression. Mathematically it can be expressed as:

In tension,

$$\left| \frac{\sigma_1}{E} - \frac{\mu}{E} (\sigma_2 + \sigma_3) \right| = \frac{\sigma_{yp}}{E} \text{ (Ten)} \quad . \quad . \qquad 2.1 \text{ A}$$

In compression,

$$\left| \frac{\sigma_3}{E} - \frac{\mu}{E} (\sigma_1 + \sigma_2) \right| = \frac{\sigma_{yp}}{E} \text{ (Comp)} \quad . \quad . \qquad 2.1 \text{ B}$$

In principal stress space the yield surface corresponding to this theory consists of two straight three sided pyramids in inverted positions relative to each other having equilateral triangles as sections normal to the axis coinciding with one of the space diagonals. The theory is in contradiction to material behaviour under hydrostatic tensile or compressive stress.

2.1.3 Constant elastic strain energy theory:

The quantity of strain energy per unit volume of the material is used as the basis for determining failure in this theory. Equating the strain energy for given state of stress at failure to the energy stored at yield in simple tension the criterion can be expressed as:

$$\left[\frac{(\sigma_{yp})^2}{2E} \right] = \frac{1}{2E} \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right) - \frac{u}{E} \left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \right) \quad . \quad 2.2$$

2.1.4 Maximum tensile stress theory:

The material is assumed to fail by brittle fracture in tension if least principal stress equals to minus uniaxial tensile strength. It is expressed as:

$$\sigma_3 = -\,To \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 2.3$$

2.1.5 Maximum shear stress theory:

The material is assumed to fail when the maximum shear stress reaches a value So. It is expressed as:

$$\sigma_1 - \sigma_3 = 2\,So \quad . \quad . \quad . \quad . \quad . \quad . \qquad 2.4$$

This is well known as Tresca's failure criterion.

2.1.6 Maximum octahedral shear stress theory:

The material is assumed to fail when the octahedral shear stress reaches a value k which represents characteristic of the material. It is expressed as:

$$(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 = 9\ k^2 \quad . \quad . \quad . \qquad 2.5\ A$$

In terms of invariant of deviator stress it can be expressed as:

$$J_2 = 3\ k^2/2 \quad . \quad . \quad . \quad . \quad . \quad . \qquad 2.5\ B$$

A linear relationship between octahedral normal and shear stresses was used by Freudenthal (1951), Bresler and Pister (1957). It is expressed as:

$$\tau_{oct} = \tau_1\ \sigma_{oct} + C_2 \quad . \quad . \quad . \quad . \quad . \qquad 2.6$$

Where C_1 and C_2 are constants. The failure surface corresponds to a circular cone. Bresler and Pister further proposed a third invariant and is expressed as:

$$\tau_{oct}/\ C_o = f_1\,(I_1/Co) + f_2\ (I_3/Co^3) \quad . \quad . \qquad 2.7$$

2.2 Neo-classical concepts of failure:

Neo-classical concepts are the extension of classical concepts based on realistic behaviour of materials. These theories have potential to include the various aspects of materials and the capable to consider the various boundary conditions. Following are the most notable expressions, generally employed for analysis of failure in materials.

2.2.1 Coulomb's theory:

Coulomb (1776) postulated that the shear stress tending to cause failure is resisted by the cohesion of the material plus a constant times the normal stress across the plane of failure. It is expressed as:

$$\tau = So + \mu \sigma \quad . \quad . \quad . \quad . \quad . \qquad 2.8$$

2.2.2 Mohr's theory:

Mohr (1900) conceptulized that failure occurs by yielding and fracture. He further assumes slippage as mode of failure and provides a functional relationship between normal and shear stresses on the failure plane. It can be expressed as:

$$\tau = f(\sigma) \quad . \quad . \quad . \quad . \quad . \qquad 2.9$$

Graphically it is termed as Mohr rupture envelope, which represents the locus of all points and defines the limiting values of both components of stress in the slip plane subject to different states of stress.

2.2.3 Griffith's theory:

Griffith (1921) hypothesized that fracture is caused due to stress concentrations at the tips of minute cracks pervading the material. The fracture is initiated when the maximum stress near the tip of the most favourably oriented crack reaches a critical value. It is expressed as:

$$(\sigma_1 - \sigma_3)^2 = -8\, To\, (\sigma_1 + \sigma_3) \quad . \quad . \qquad 2.10$$

2.2.4 Weibull's theory:

Weibulls (1939) weakest link theory is perhaps the earliest statistical theory for failure of general solids. It works out the distribution of strength of general solids within a body and further works out the probability that the solid would fail under a given stress. The vital point of the theory is in its capability to predict the size effect.

2.3 Current concepts of failure:

As a consequence of research work on failure of materials chiefly based on the pioneering work of Griffith a number of mathematical and statistical approaches have been developed. The prominent approaches particularly for rock material are outlined below.

2.3.1 McClintock and Walsh Extension of Griffith theory:

McClintock and Walsh (1962) and Brace (1969 b) modified Griffith's theory by assuming that in compression Griffith's cracks close and a frictional force develops across the cracks surface. Failure occurs when

$$\mu(\sigma_1+\sigma_3-2\sigma_c) + (\sigma_1-\sigma_3)\ (1+\mu^2)^{\frac{1}{2}} = 4To \left\{\frac{(1-\sigma_c)}{To}\right\}^{\frac{1}{2}} \qquad 2.11$$

Brace pointed out that σ_c is small and can be neglected. Hence equation becomes:

$$\mu(\sigma_1+\sigma_3) + (\sigma_1-\sigma_3)\ (1+\mu^2)^{\frac{1}{2}} = 4To \qquad 2.11\ A$$

2.3.2 Murrell extension of Griffith theory:

Murrell (1963) extended the Griffith theory in to three dimensions. It provides a simple criterion for studying the effects of polyaxial stresses.

For the triaxial test $\sigma_2 = \sigma_3$ it becomes:

$$(\sigma_1-\sigma_3)^2 = 12\ To\ (2\ \sigma_1 + \sigma_3) \quad . \quad . \qquad 2.12$$

Which is a parabola which intersects $\sigma_3 = 0$ when $\sigma_1 = 24$ To and at this point $d\sigma_1/d\sigma_3 = 2.5$

For biaxial stress it gives:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 12\ To\ (\sigma_1+\sigma_2) \quad . \qquad 2.13$$

Which is an ellipse whose slope when $\sigma_2=0$ is $d\sigma_1/d\sigma_2=2.0$ it intersect the line $\sigma_1=\sigma_2$ when $\sigma_2 = 24$ To

the equation becomes:

$$(\sigma_1-\sigma_3)^2 = 24\ To\ (\sigma_1+\sigma_3) \text{ if } \sigma_2=\tfrac{1}{2}\ (\sigma_1+\sigma_3) \qquad 2.14$$

2.3.3 <u>Wiebols and Cook theory:</u>

The strain energy theory given by Wiebols and Cook (1968) suggest that the process of failure may by controlled by total amount of strain energy stored by the deformed crack and fracture occurs when this strain energy reaches to a maximum. The calculation of the strain energy of the crack system in the theory takes full account of the effect of the stress system on a closed crack. The energy stored within the material prior to failure is computed from elastic deformation and more specifically from the shear distortion of closed microcracks. The total distortion energy W for a large number of cracks will be sum of all the shear energy. It is expressed as:

$$W = K \int w\ (\eta \lambda)\ N\ (\eta \lambda)\ d\eta\ d\lambda \quad . \quad . \qquad 2.15$$

where K is a constant whose value depends upon the elastic constants of surrounding material. η and λ are orientation parameters of azimuth and latitude of cracks respectively.

2.3.4 John's theory:

John (1969) observed that failure in biaxial stress field occurs due to slippage along the joint sets having lower angle of friction which may be considered as critical sliding plane. Accordingly the critical stress conditions for the various joint groups can be superimposed in the form of a polar diagram. The results obtained supports the Muller and Pacher (1965) observations and further by Ladanyi and Archambault (1972) and Lama (1978).

2.3.5 Brady's theory:

Brady's (1970) theory incorporates both theoretical and empirial aspects of prefailure stress-strain behaviour of rock and in this sense it is an integral crack extension concept. It is developed from a prediction of the permanent strains generated by cracks undergoing failure and the failure criterion makes use of the predicted strains. The total volumetric strain is given by

$$|d\epsilon_1| = \int A_p \frac{d\epsilon_1}{(\sigma \{-\sigma\}^m)} \qquad . \qquad . \qquad . \qquad 2.16$$

Brady states that the above work criterion is equivalent to density criterion: $\rho = C$ . . . 2.16 A where C is a material constant.

2.3.6 Gramburg ellipse with notch theory:

The ellipse with notch theory due to Gramburg (1970) is based on the phenomenon of axial cleavage fracturing. He suggested that Griffith theory can be extended with the beak shaped cusp notch on the ellipse as a model for the edge of the fracture considering the axial direction of primary fracturing verified during experimentation.

2.3.7 Lundborg theory:

Lundborg (1971) applied the idea of statistics as used by Weibull to explain the volume effect and polyaxial stress effect on the tensile strength of materials. Lundborg introduced an effective stress τ_e that appears over a certain angle and stated that the probability of rupture is a function of τ_e and the size of the angle. He considered that μ value is dependent on the normal stress expressed as:

$$\mu = \frac{\mu_o}{\{1+(\mu_o\sigma_n/\tau_x)\}} \quad . \quad . \quad . \quad . \qquad 2.17$$

2.3.8 Lajtai's stress gradient theory:

Lajtai (1972) proposed a simple theory which incorporates the maximum stress concentration and the stress gradient. He postulated that fracture will initiate when maximum tensile stress at a certain distance beyond the periphery but along the fracture path reaches the uniaxial tensile strength. The general form of the fracture criterion is expressed as:

$$\sigma_d = \sigma_m + d\ (\partial\sigma/\partial r)_{r = R} = To \quad . \quad . \quad 2.18$$

The fracture criterion for an ellipse having its minor axis oriented parallel to the applied major principal stress becomes:

$$\sigma_1 = \frac{To}{d/a(2+3b/a)} + \sigma_2 \frac{d/a[4(b/a)^2+3b/a]-(1+2b/a)}{d/a(2+b/a)-1} \quad 2.19$$

where '2d' is the length of the crack and 'a' and 'b' are major and minor axis size of the ellipse, when a = b = R the ellipse becomes a circle and the fracture criterion becomes:

$$\sigma_1 = \frac{To}{5d/R-1} + \sigma_2 \frac{7d/R - 3}{5d/R - 1} \quad . \quad . \quad . \quad 2.20$$

2.3.9 Bieniawski's extension to Murrel theory:

Murrell(1965) proposed an empirical failure criterion based on his work. It is expressed as:

$$\sigma_1 = k_1\ (\sigma_3)^k + C_o \quad . \quad . \quad . \quad . \quad 2.21$$

where k_1 and k are constants. Bieniawski (1974) proposed some changes in the above equation. The new equation is written as:

$$\sigma_1/Co = A\ (\sigma_3/Co)^k + 1.0 \quad . \quad . \quad . \quad 2.22$$

where A and K are constants. It was found by him that the values of A and K were 3 and 0.75 respectively.

2.3.10 Simple extension strain criterion:

A number of research workers notably Brown and Trollope (1967), Kriston and Klokow (1979), Waldect (1979), Maso and Lernan (1980) and Stacey (1981) extended the Griffith criterion specifically for rock well known as simple extension strain criterion. The criterion is that the fracture in brittle rock initiates when total extension strain in the rock exceeds a critical value which is the characteristic of a particular rock type. The fracture is formed in planes normal to the direction of the extension strain corresponding to the direction of minimum principal stress.

2.4 Concluding remarks:

The review of various failure theories clearly establish that is is not possible to develop any generalize approach for the failure of rock material since the structure of the rock is too complex. The most rational approach is therefore to conduct experimental investigations and analyse from various theoretical considerations.