CHAPTER VII

GRAPHICAL PROCEDURE FOR THE EASY EXECUTION OF SAMPLING PLANS BY ATTRIBUTES

7.1 In this chapter we give a graphical procedure for the easy execution of two class and three class attributes sampling plans - uncurtailed as well as curtailed. The procedure is illustrated with numerical examples.

7.2 For complicated sampling schemes quite often an inspector experiences difficulty in its execution. The level of inconvenience in the execution depends upon the type and form of the sampling plan. For example, multiple sampling plans are more complicated than single sampling plans. Similarly, three class attributes sampling plans are more complicated than two class attributes sampling plans and curtailed sampling plans are more complicated than uncurtailed sampling plans. Hence there is a need for some special devices which can reduce the complexity and make the execution of a sampling scheme simpler. One such device is to use a grid or the so called storage board [13 ,pp.423]. This device can be used for the easy execution of any multiple two class or three class attributes sampling plans-curtailed or uncurtailed.

In case of a two class attributes sampling plan this device is useful not only for the easy execution of a plan but is also useful in determining the ASN, the probability of acceptance etc. Lloyd and Lipow [31] have used this device for finding the ASN of a curtailed single two class attributes sampling plan and its execution. They have also used this device for the execution of a truncated sequential ratio test. Using the device, we have illustrated the execution of a fully-curtailed triple two class attributes sampling plan and obtained the ASN and the probability of acceptance.

We have extended the use of this device for the execution of three class attributes sampling plan - curtailed as well as uncurtailed. However, it is found that this device does not help in evaluating the ASN, the probability of acceptance etc. We have illustrated the use of this device in executing a fully-curtailed single sampling plan and a semi-curtailed double sampling plan.

7.3 Easy Execution of Two Class Attributes Sampling Plan :

7.3.1 Execution Procedure :

The device which reduces the complexity in the execution is the usual graph (grid) in the two dimensional space whose ordinates denote defectives and the abscissa denote the number of units inspected. A point with co-ordinates (Y, D) represents the occurrence of D defectives in the inspection of Y units.

For a given sampling plan firstly one plots the boundary points of the region in accordance with the statement of a given sampling plan. These boundary points are identified into two mutually exclusive groups, boundary points of the acceptance region and boundary points of the rejection region. As the sampling inspection of units of a lot proceeds, starting from the origin one plots a point one unit horizontally to the right when a nondefective occurs and one unit horizontally to the right and one unit vertically up (i.e. one unit diagonally up) if a defective occurs. The process of plotting the points on the graph (and thereby the process of inspection) is continued till the path joining the points plotted reaches one of the boundary points of the rejection and acceptance regions. When a path reaches to the boundary point of the acceptance region, the sampling is terminated with the decision of the acceptance of a lot. When a path hits a boundary point of the rejection region, the sampling is terminated with the decision of the rejection of a lot.

7.3.2 Probability of Acceptance and the ASN :

Having prepared the graph with the boundary points marked on it one can determine the number of paths for reaching to every boundary point from the origin (0,0). Once this is obtained, the expressions for the probability of acceptance and the ASN can be evaluated which are as given below:

$$\text{Probability of Acceptance} = P_a = \sum_{\alpha} P_{\gamma}(\alpha) \qquad \ldots(7.3.1)$$

where (i) $P_{\gamma}(\alpha)$ = Probability of accessing to boundary point α with coordinates (γ, D)

$$= N(\alpha)\, p^{D} q^{\gamma - D} \qquad \ldots(7.3.2)$$

(ii) $N(\alpha)$ = The number of paths from the origin to point α.

(iii) $\sum_{\alpha}$ denotes the summation over the boundary points of acceptance region.

$$\text{ASN} = \sum_{\gamma} \gamma P_{\gamma}(\alpha) \qquad \ldots(7.3.3)$$

where $\sum_{Y}$ denotes the summation over all the boundary points.

7.3.3 A Numerical Illustration :

We illustrate the procedure described in the previous sections in executing the following fully-curtailed triple sampling plan :

$$N = 150$$

$$n_1 = 4,\ n_2 = 5,\ n_3 = 6$$

$$r_1 = 3,\ r_2 = 5,\ r_3 = 6$$

$$g_1 = 3,\ g_2 = 6,\ g_3 = 10$$

The execution of the sampling plan follows in accordance with the statement of the sampling plan which can be had from the statement of a multiple sampling plan, given in Section 2.2.3 of Chapter II, for k=3. Figure 7.1 is self-explanatory. It is the grid or the storage board described in Section 7.3.1. Boundary points of the rejection region are shown by the symbol X and those of the acceptance region are shown by the symbol O. Numeric figure at a boundary point is the number of paths required to reach that boundary point from origin (0,0). For example the point with co-ordinates $\gamma=9, \mathfrak{D}=5$ represents a boundary point of the rejection region which is accessible from (0,0) by 36 different paths.

The probability of accessing this point is, then, $36p^5q^{9-5}$. The ASN of the plan is the summation of the following six terms :

$$3(p^3q^0) + 4(3p^3q),$$
$$7(6p^5q^2) + 8(18p^5q^3) + 9(36p^5q^4),$$
$$11(60p^6q^5)+12(120p^6q^6)+13(180p^6q^7)+14(240p^6q^8)+15(300p^6q^9),$$
$$3(p^0q^3) + 4(3pq^3),$$
$$8(6p^2q^6) + 9(24p^3q^6),$$
$$14(60p^4q^{10}) + 15(300p^5q^{10}). \qquad ...(7.3.4)$$

The probability of rejection of a lot is the sum of the quantities involved in brackets in first three terms of the above display (7.3.4) whereas the probability of acceptance of a lot is the sum of the quantities involved in brackets in the last three terms of the same display.

7.4 Easy Execution of Three Class Attributes Plan :

7.4.1 Execution Procedure :

The graphical procedure developed for executing the two class attributes acceptance sampling plan can be modified in executing the three class attributes sampling plan-curtailed as well as uncurtailed. The graphical procedure in this case consists of plotting points simultaneously on two graphs, Graph-1 and Graph-2.

In case of a single three class attributes sampling plan points (Y,D) are plotted on Graph-1 whereas points (Y,D_1+D_2) are plotted on Graph-2. Thus a point (Y,D_2) on Graph-1 represents the occurrence of D_2 bad units in the inspection of Y units and a point (Y, D_1+D_2) on Graph-2 represents the occurrence of D_1+D_2 nongood units in the inspection of Y units.

In case of a multiple three class attributes sampling plan the plotted points are $(Y, \sum_{j=1}^{i} D_{2j})$ on Graph-1 and $(Y, \sum_{j=1}^{i} D_{2j}+ \sum_{j=1}^{i} D_{1j})$ on Graph-2. The point $(Y, \sum_{j=1}^{i} D_{2j})$ on Graph-1 represents the occurrence of $\sum_{j=1}^{i} D_{2j}$ bad units in the inspection of first i samples when the total number of units inspected is Y. Similarly the point $(Y, \sum_{j=1}^{i} D_{2j}+ \sum_{j=1}^{i} D_{1j})$ represents the occurrence of $\sum_{j=1}^{i} D_{2j}+ \sum_{j=1}^{i} D_{1j}$ nongood units in the inspection of first i samples and the total number of units inspected is Y.

Next, for a given plan one plots the boundary points (X) of the rejection region and the boundary points (O) of the acceptance region on both the graphs in accordance with the statement of a given sampling plan. For instance, the boundary points (0) of the acceptance region on Graph-2 for the fully-curtailed single sampling plan are given below :

$$(n-k_2+1,0),\ (n-k_2+1,1),\ldots,(n-k_2+1,k_1-k_2),$$
$$(n-k_2+2,1),\ (n-k_2+2,2),\ldots,(n-k_2+2,k_1-k_2+1),$$
$$\ldots \qquad \ldots \qquad \ldots$$
$$\ldots \qquad \ldots \qquad \ldots$$
$$(n,k_2-1),\quad (n,k_2),\quad \ldots\ ,\ (n,k_1-1).$$

Then during the execution of the given sampling plan, as the inspection of units of a lot progresses, starting from (0,0) one plots a point, simultaneously on both the graphs, either one unit horizontally to the right or one unit diagonally up (i.e. one unit horizontally to the right and one unit vertically up) depending upon the classification of the unit. On Graph-1, a point is plotted one unit horizontally to the right if the unit inspected is nonbad and one unit diagonally up if the unit inspected is bad. On Graph-2, a point is plotted one unit horizontally to the right if the unit inspected is good and one unit diagonally up if the unit inspected is nongood. It is clear then that at the end of each inspection of a unit a new point is plotted on both the graphs. The process of plotting the points on the graphs (and thereby the process of inspection) is continued till the path joining the points plotted hits

(a) one of the boundary points (X) of the rejection region on either of the Graph-1 and 2, or

(b) the boundary points (O) of the acceptance regions of both the Graphs-1 and 2. In the former case the lot is rejected and in the latter the lot is accepted.

7.4.2 A Numerical Example for Fully-Curtailed Single Three Class Attributes Sampling Plan :

In this section we illustrate the execution of a fully-curtailed single three class attributes sampling plan. The Figures 7.2 and 7.3 give the skeleton of Graphs 1 and 2 respectively with the boundary points marked for the following fully-curtailed three class attributes sampling plan :

$$n=10,\ k_1=7,\ k_2=4,\ n-k_1+1=4,\ n-k_2+1=7,$$

in accordance with the statement of the sampling plan given in Section 5.5.1 of Chapter V. Consider the following sequence of inspections :

g g b g g b m g

where g indicates good unit, b indicates bad unit and m indicates marginal unit. A path for this sequence meets the boundary points of the acceptance region of Graph-2 but does not meet any of the boundary points of Graph-1. Hence, at this stage one cannot stop the inspection. However, for the following sequence of inspections

b g b g g g m g m

one can conclude to stop the inspection with a decision to accept the lot since the paths hit the boundary points (9,2) and (9,4) of the Graph-1 and Graph-2 respectively. Just for information we state that this sequence of inspections implies the occurrence of the event (e_1) of Section 5.5.1 of Chapter V.

7.4.3 A Numerical Example for Semi-Curtailed Double Three Class Attributes Sampling Plan :

To illustrate the execution in case of a multiple three class attributes sampling plan, consider the following semi-curtailed double three class attributes sampling plan.

$n_1=5$, $n_2=5$, $a_1=4$, $a_{21}=0$, $a_{22}=2$.

The execution of the sampling plan follows according to the statement of the sampling plan which can be had from the statement of MTCAP, given in Section 6.3.2 of Chapter VI, for k=2.

Figures 7.4 and 7.5 give the Graphs 1 and 2 respectively with the boundary points marked. Consider the following sequences of inspection for two different lots :

(i) b g g g g m m g m g

(ii) g g g g b b g b

It is clear that the path for the sequence (i) hits the boundary points (10,1) and (10,4) of the acceptance regions on both the graphs. Hence one stops the inspection with a decision to accept the lot.The path for the sequence (ii) hits the boundary point (8,3) of the rejection region on Graph-1. Hence one stops the inspection to reject the lot. The curtailment in the inspection is resulted when the second sample was being inspected. Both the sequences illustrate that the decision of acceptance or rejection can not be made at the completion of the inspection of the first sample. Sequence (i) is shown on the Graphs.

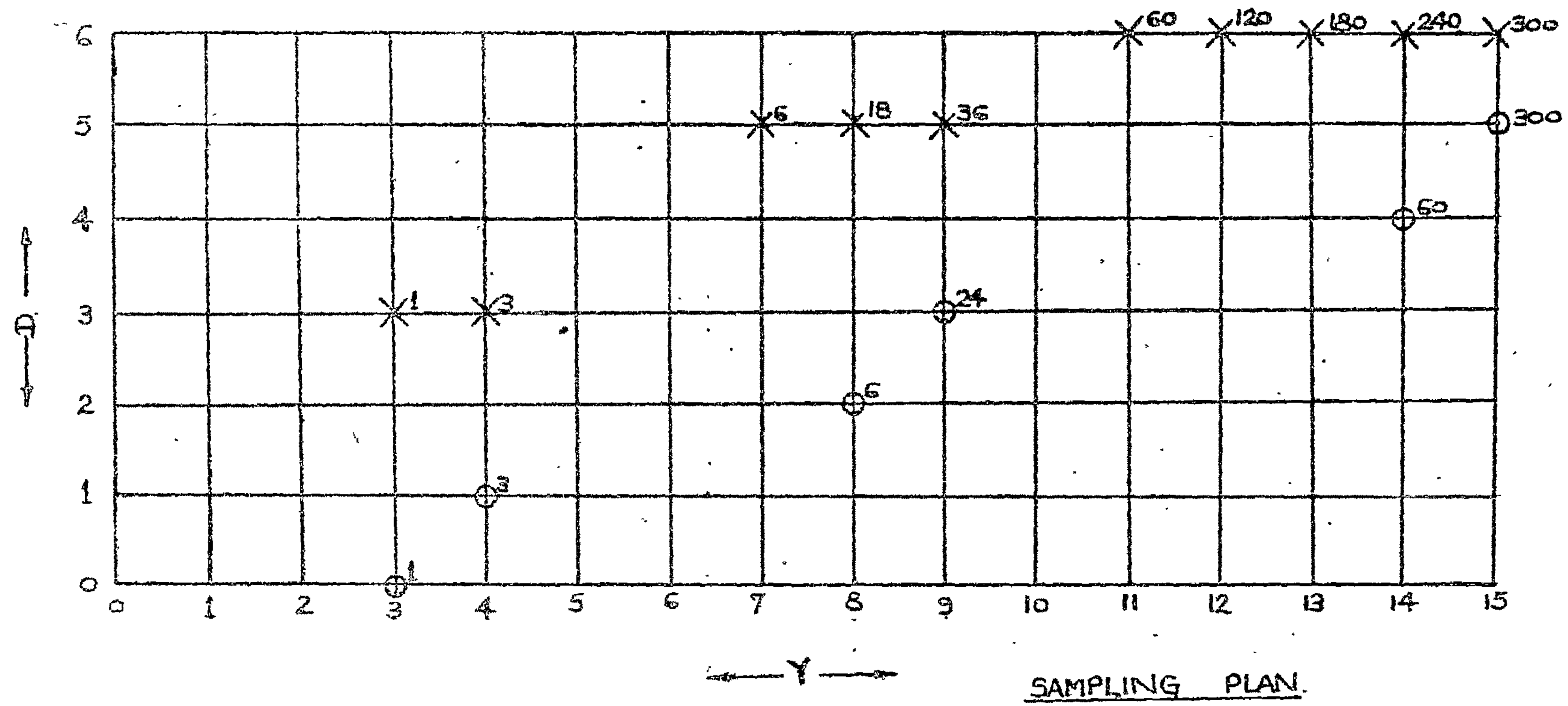

FIGURE - 7-1.

SAMPLING PLAN.

$n_1 = 4$, $n_2 = 5$, $n_3 = 6$.

$z_1 = 3$, $z_2 = 5$, $z_3 = 6$.

$g_1 = 3$, $g_2 = 6$, $g_3 = 10$

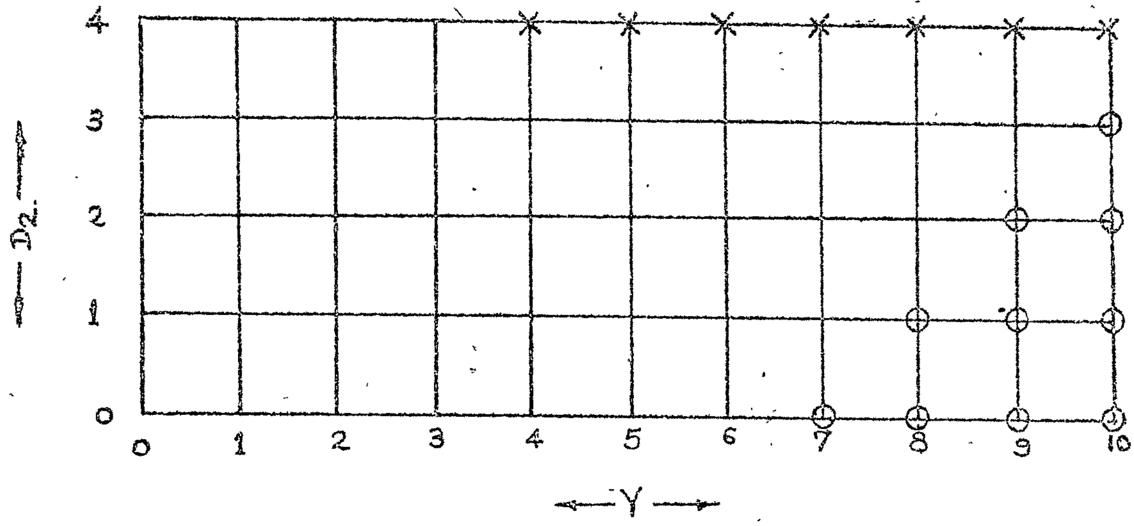

FIGURE - 7·2

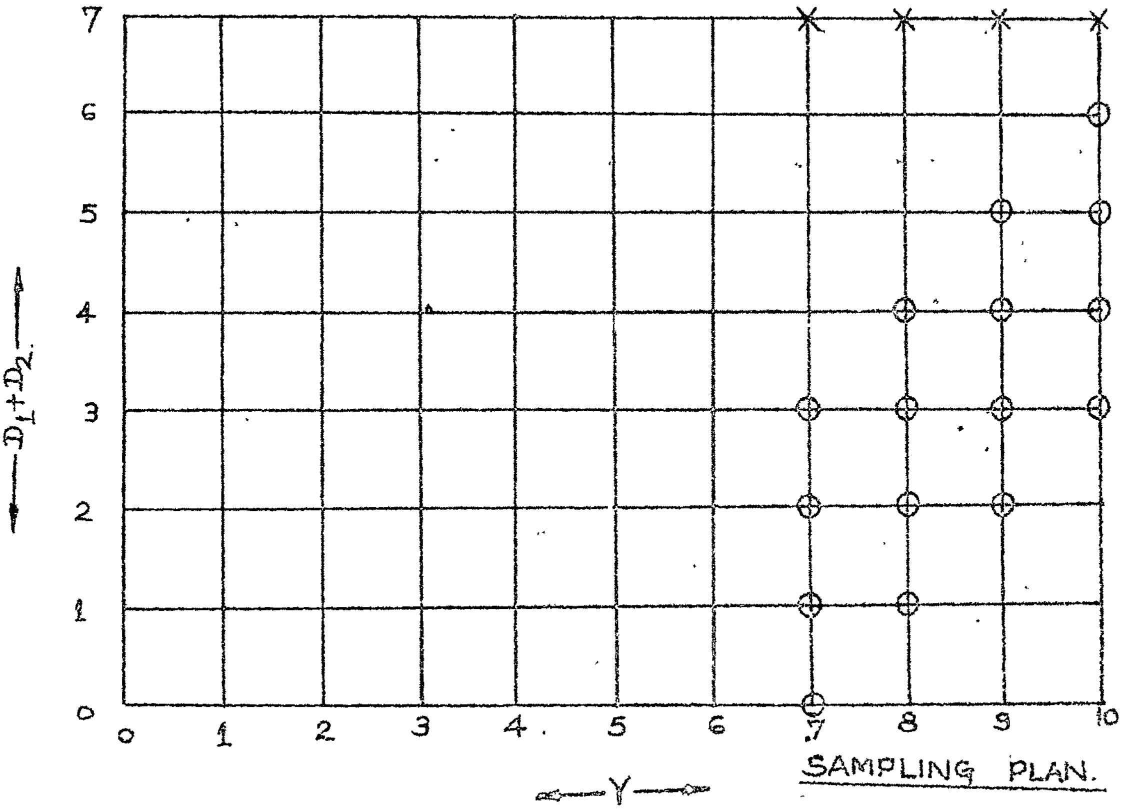

FIGURE - 7·3.

SAMPLING PLAN.

$n_1=10,\ K_1=7,\ K_2=4.$

$n-K_1+1=4,\ n-K_2+1=7.$

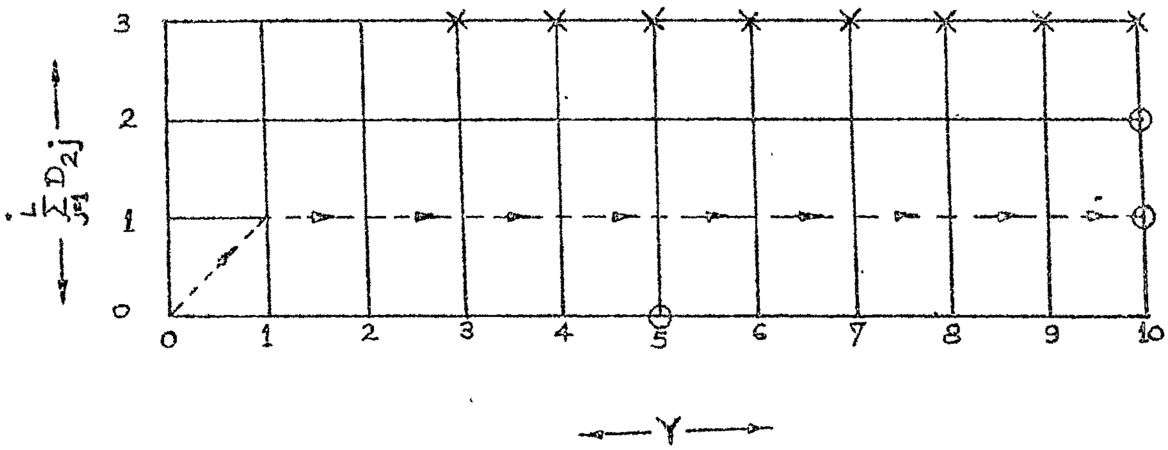

FIGURE - 7·4

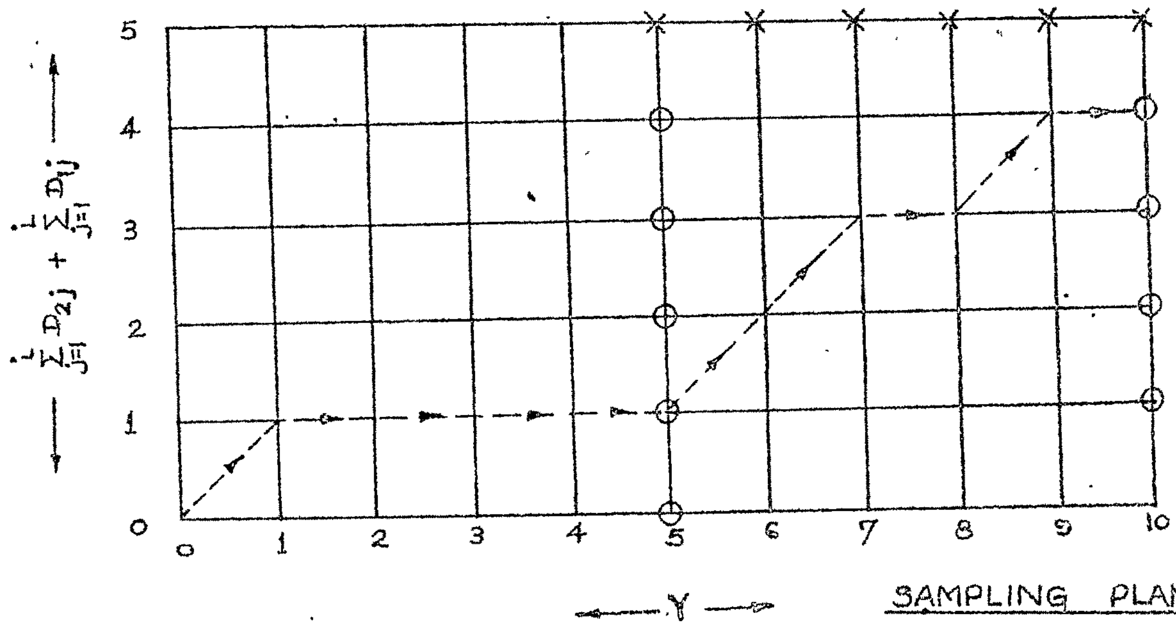

FIGURE - 7·5

SAMPLING PLAN.

$n_1 = 5$, $n_2 = 5$.

$a_1 = 4$, $a_{21} = 0$, $a_{22} = 2$.