# CHAPTER I

# INTRODUCTION

This thesis on "Further Contributions to Attributes Acceptance Sampling Plans" consists of a study of problems pertaining to attributes acceptance sampling plans.

## 1.1 Various Aspects of Attributes Acceptance Sampling Plans :

1.1.1 Inspection for acceptance purposes is carried out at many stages in manufacturing. There may be inspection of incoming materials and parts, process inspection at various points in the manufacturing operations, final inspection by a manufacturer of his own product, and ultimately inspection of the finished product by one or more purchasers. Inspection for acceptance is carried out either on a screening basis or on a sampling basis. When each submitted item (or unit) of a product is inspected and all defectives found during the inspection are rejected the inspection procedure is called "screening". Screening is the only programme (procedure) that can possibly guarantee the rejection of all the defective items and only defective items. Furthermore, such

100% rejection of defective items can be approached only if suitable automatic machines are available to do the inspection or the inspector is not subjected to a load so excessive as to impair his accuracy since the percentage of erroneous classification of items increases sharply with the volume of inspection. Suitable automatic machines are rarely available. So also screening is ordinarily expensive in personnel and time. It is experienced that when there are a great many items of product to be inspected and if all are to be inspected then the inspection is likely to be influenced by 'inspection fatigue'. So screening in this case will rarely eliminate all defective items. Furthermore, if the quality test for each item is destructive, screening is out of the question. Due to these reasons screening is not so widely usable method of inspection. Evaluation of the quality of product by inspecting some but not all of the product is called 'Sampling Inspection'. Sampling inspection is widely applicable as a method for determining the quality and acceptability of the product. It is almost always possible to use sampling inspection for this purpose, and if the inspection is destructive or costly sampling inspection may be the only feasible method. An important advantage of modern acceptance sampling systems is that they exert more effective pressure for quality improvement than is possible with 100% inspection [44].

1.1.2 Sampling inspection is of two kinds, namely, lot-by-lot sampling inspection and continuous sampling inspection. In lot-by-lot sampling inspection, the product is divided into appropriate inspection lots. (Here onwards the term 'lot' implies 'inspection lot'.) The lot may or may not be exactly the number of items in a container, or the number of items produced in a day, or the number submitted by the supplier at a given time. One sample or several samples are drawn from the lot, and the lot is accepted or rejected according to the quality of the lot reflected in the sample or samples. This is most appropriate for acceptance inspection. The continuous sampling inspection is followed when production is continuous. In this case formulation of lots for lot-by-lot acceptance is somewhat artificial. Moreover, where conveyor lines are used, it may be impracticable or unduly costly to form lots. In continuous sampling inspection, current inspection results are used to determine whether sampling inspection or screening inspection is to be used for the next articles to be inspected.

1.1.3 Sampling inspection plans are further classified depending on the quality characteristics of an item. The most common methods of describing the quality of an item are

(i) by variable, that is, either by measurement of some characteristic of an item along a continuous scale, or by counting along a discrete scale.

(ii) by attributes, that is, by classification of the quality of an item into one of two, three or any number of classes.

When the quality of an item is determined according to (i), the sampling plans are called sampling plans for variables. When the quality of an item is determined according to (ii), the sampling plans are called attributes sampling plans. Generally the term "attributes" in acceptance sampling inspection means the classification of an item into one of the two classes, defective or nondefective. Recently Bray, Lyon, and Burr (1973) [4] introduced new types of attributes sampling plans in which an item is classified into three classes; bad, marginal, and good. They called these new plans as three class attributes sampling plans. With reference to these plans, former are then called two class attributes sampling plans. In this thesis we have considered lot-by-lot attributes acceptance sampling plans - two class as well as three class.

1.1.4 Any lot-by-lot sampling plan has as its primary purpose the acceptance of good lots and the rejection of bad lots. It is important to define what is meant by a good lot. Naturally, the consumer would like all of his accepted lots to be free of defectives. On the other hand, the manufacturer will usually consider this to be an unreasonable request since some defectives are bound to appear in the manufacturing process. So the consumer can think of tolerating some defectives in his lot, provided the number is not too large. Consequently, the manufacturer and the consumer get together and agree on what constitutes good quality of a lot. If lots are submitted at this quality or better, the lot should be accepted; otherwise rejected.

1.1.5 Having fixed the quality of a good lot, the manufacturer tries to maintain his production run at this quality level. In case of a two class attributes sampling plan the quality of the lot is characterized by the fraction defective p, which is nothing but proportion of defectives in a submitted lot for inspection. In case of three class attributes sampling plan the quality of the lot is characterized by the proportion of marginals $p_1$ and the proportion of bads $p_2$ in a submitted lot. We assume that p or $p_1$ and $p_2$ remain constant over the entire production run. But they would change as the time goes on because of some wear and tear in the machinery or some reasons and the results of the inspection obtained during the execution of the sampling plans may be used to assess them and to be sure that the quality remains the same. A part of the thesis is devoted to the estimation of the quality defined on the basis of sampling inspection results.

1.1.6 There are many kinds of lot-by-lot sampling inspection plans where inspection is by attributes. These plans are grouped into three types such as (i) single sampling, (ii) double sampling, (iii) multiple sampling. A single sampling procedure can be characterized by the following: one sample of n units is drawn from a lot of N units; the lot is accepted if the number of defectives d in the sample does not exceed a. Here a is referred to as the acceptance number. A double sampling procedure can be characterized by the following: A sample of $n_1$ items is drawn from a lot; the lot is accepted if there are no more than $a_1$ defectives and is rejected if there are more than $a_2$ defectives. If there are between $a_1+1$ and $a_2$ defectives, a second sample of size $n_2$ is drawn; the lot is accepted if there are no more than $a_2$ defectives in the combined sample of $n_1+n_2$; the lot is rejected if there are more than $a_2$ defectives in the combined sample of $n_1+n_2$. The multiple sampling plan is a straight extension of double sampling plan in which three or more samples of a stated size are permitted.

1.1.7 Having decided to administer a particular sampling plan, one can have two alternative forms of sampling inspection, namely, (i) complete inspection of each sample, and (ii) curtailed inspection of each sample. If inspection had no other purpose than to determine which lot to accept and which to reject, it would be feasible to stop inspection as the rejection number is reached or as soon as it is known that the acceptance number will not be exceeded, since further inspection would not affect the acceptance or rejection of the lot. Stopping of inspection under the situation described above is known as curtailment of the inspection. Reference to curtailment of the inspection appears in Sampling Inspection Tables Single and Double

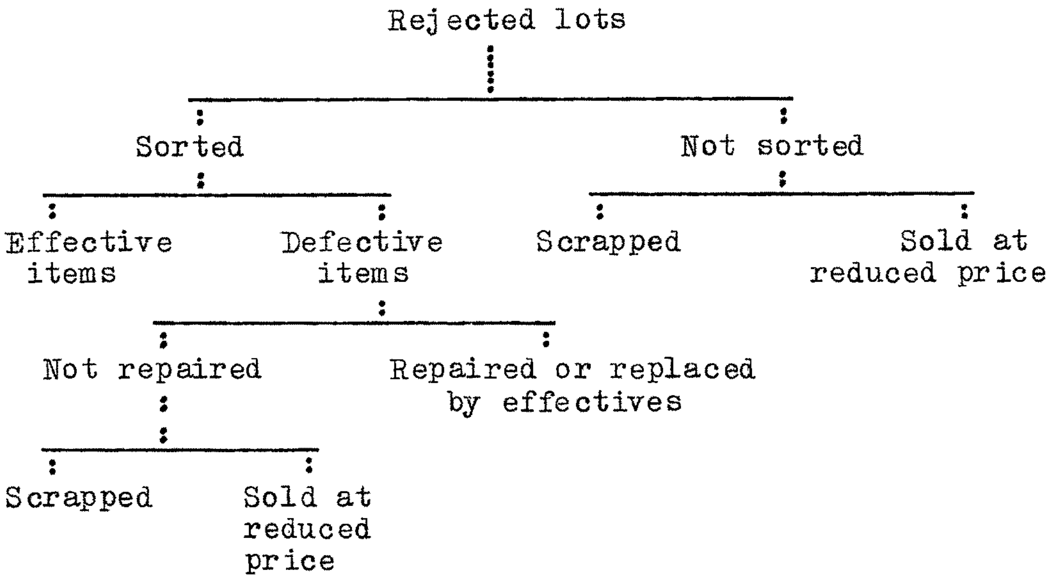
Sampling by Dodge and Romig as early as in 1944 [9]. It may be noted that curtailed inspection is not always advisable. The main reason for uncurtailed inspection is that it may be desirable to obtain information about the quality of product, in addition to deciding whether to accept or reject each lot. Knowledge of the number of defective items in entire sample facilitates estimation of the quality of the inspection lot. There are formulas by which the quality of the lot can be estimated under curtailed inspection, but these formulas require not only that the units be selected at random from the lot but that they be inspected in a random order. Random selection is necessary for valid acceptance or rejection; but random order of inspection within samples is necessary only if the process average is to be estimated and sampling is curtailed. Ordinarily, random order of inspection within samples will not be burdensome, since it is only necessary to inspect the items in the order drawn. Thus in many situations retention of a random order within samples is not a difficult task and hence curtailment in the inspection whenever desired is not a problem. Sometimes, however, retention of this order (or randomizing if the order is lost) may cost more than is justified by the saving achieved from curtailing inspection. In such situations curtailment is not advisable [44] . A major part of the thesis is dealt with curtailed sampling inspection.

1.1.8 It appears that the estimation of the fraction defective under curtailed sampling plan was introduced by Girshick, Mosteller, and Savage (1946) [12] . Their main problem was to find the unique unbiased estimator of the fraction defective under binomial sampling. As an application to the theory developed by them, they have considered curtailed single and double sampling plans which take into consideration the curtailment of the inspection of a lot both at the rejection and the acceptance stages. They have considered the problem of estimation when one lot is inspected and they have considered one trivial case where estimation is based on two lots. One of the problems raised by them, but not solved, is related with the estimation of p when one is faced with the results of several lots. Phatak and Bhatt (1967) [40] have solved the problem of estimation of the fraction defective where there is curtailment of the inspection during the execution of single sampling plan by attributes and the results of the inspection of several inspected lots are on hand. They have given the maximum likelihood estimator of the fraction defective when several lots are inspected and the inspection is curtailed. It may be noted that the estimator given by Phatak and Bhatt [40] is biased even for one lot.

1.1.9 One of the characteristics of the curtailed sampling plans is to have a reduction in the average sample number (ASN). This problem was dealt with, as early as 1948, in Chapter 17 of Sampling Inspection by the Statistical Research Group, Columbia University [44] . Later Burr (1957) [5] , Patil (1963) [38] , Phatak and Bhatt (1967) [40] , and Craig (1968) [8] , worked out the ASN for some sampling plans. Later Cohen (1970) [7] , Guenther (1969),(1971),[14],[16], and many others have discussed the problem of the ASN in curtailed sampling plans. However, it is worth while to note from Craig's remark [8] that curtailed sampling plan has not yet been given that importance in usual textbooks as it ought to have [redacted] received.

1.1.10 The chief problem in acceptance sampling inspection is the determination of the sampling plan that is best suited to the producer and consumer. Most of the methods for the determination of sampling plan, depend upon the points on operating characteristic curve, the lot quality and the amount of inspection. Alternatively, the full significance of a sampling plan can be developed on the basis of the prior distribution and the economic consequences associated with the decision of acceptance or rejection of a lot. Sampling plans based upon a prior distribution are known as Bayesian sampling plans. Many papers have appeared on Bayesian sampling plans. Some of the pioneer papers on Bayesian sampling plans are due to Barnard (1954) [2], Wetheril (1960) [48], and Hald (1960, 1965, 1967, 1968) [19],[21],[22],[23],[24]. A chapter of the thesis is devoted to Bayesian sampling plans.

1.1.11 We have noted that (Section 1.1.1) one of the advantages of sampling inspection is the quality improvement. However, a sampling alone cannot guarantee that the quality of the product finally accepted will be high. The quality of the product finally accepted depends upon the quality of the product submitted, the sampling plan itself, and the method used to dispose of lots that have been rejected on the basis of a sampling plan. Furthermore, the decision to have curtailed inspection or uncurtailed inspection for each sample and the construction of cost models for Bayesian sampling plans are also dependent upon the method used to dispose of rejected lots. Hence it would be worth while to consider the different alternative methods used to dispose of rejected lots. These alternative methods are as given below :




The Dodge and Romig case, for example, corresponds to sorting of rejected lots and replacement of defective items by effective items. For the construction of a linear cost model used for the determination of Bayesian sampling plan we have assumed this case. If sampling inspection is destructive or inspection procedure is a very costly affair, sorting is naturally out of question. This is the case under which curtailment of inspection has a meaning. The major part of the thesis is dealt with curtailed sampling under the assumption that rejected lots are not sorted.

The points discussed so far give an idea of the areas discussed in the thesis. In the next section the specific problems dealt with in the thesis are stated.

1.2 <u>Problems Studied in the Thesis</u> :

(i) Maximum likelihood estimation of the fraction defective under curtailed multiple two class attributes sampling plan.

(ii) Maximum likelihood estimation of the proportion of marginals and proportion of bads under single and multiple three class attributes sampling plans - curtailed as well as uncurtailed.

(iii) Misclassification of a defective as a non-defective under fully-curtailed double two class attributes sampling plan.

(iv) Average Sample Number for two class and three class attributes sampling plans.

(v) Relation between saving in inspection and loss in efficiency in estimation.

(vi) Easy execution of attributes sampling plans - two class as well as three class.

(vii) Bounds on the variance of the unique unbiased estimator in case of a fully-curtailed single two class attributes sampling plan.

(viii) MVUE of the proportion of marginal units and proportion of bad units under single three class attributes sampling plans - curtailed as well as uncurtailed.

(ix) Determination of a single three class attributes sampling plan based upon a linear cost model and a prior distribution.

1.3 Scheme of the Work Presented in the Thesis :

The present work runs in terms of the following nine chapters :

Chapter I presents the introduction to some of the areas of attributes acceptance sampling plan, the problems dealt with in the thesis, the scheme of the work, main results and conclusions in the thesis.

Chapter II discusses the problem of the maximum likelihood estimation (MLE) of the fraction defective p under curtailed multiple two class attributes sampling plan. The statement of a fully-curtailed multiple sampling plan is given. A particular case of curtailed multiple sampling plan, namely, curtailed double sampling plan is studied extensively under two different situations, Situation-A and Situation-B. Situation-A takes into consideration the reporting of complete information of the sampling inspection records whereas Situation-B occurs when censored information of Type-I on inspection records is reported. In Situation-B, incomplete information can be reported in two different manners when the sampling inspection is according to fully-curtailed double sampling plan. The maximum likelihood estimator of the fraction defective and the asymptotic variance of the MLE are obtained under both the situations. In Situation-B the evaluation of the MLE needs iterative method, hence a SUBROUTINE which can evaluate both the MLE and the asymptotic variance, is given in the appendix. Using this SUBROUTINE two examples are worked out, one for each case.

Chapter III deals with the problem of misclassification under fully-curtailed double two-class attributes sampling plan. There are four possible situations under which an inspector may misclassify a defective as a non-defective which leads to the acceptance of a lot. Assuming the fact that misclassification should not lead to either curtailment in the inspection or avoid the inspection of second sample, we discard the three situations. In the situation, which we have considered, the inspector observes $(r_2-1)$ defectives in $(n_1+n_2-1)$ inspections and a defective at the $(n_1+m_2)$th inspection and he misclassifies this last defective as a nondefective with probability $\theta$. The MLEs of the fraction defective p and the probability of misclassification and the asymptotic variances and covariances of the MLEs are obtained in this chapter. A numerical example is provided to illustrate the results of this chapter.

Chapter IV discusses the problem of the ASN under curtailed two class attributes sampling plan. Two probability laws are considered: Hypergeometric and Binomial. Under hypergeometric probability law the expressions of the ASN for a semi-curtailed single two class attributes sampling plan and a fully-curtailed double two class attributes sampling plan are obtained. Furthermore, in case of a fully--curtailed double sampling plan we have given the expressions of the ASN when both samples have (i) a common rejection number and (ii) different rejection numbers. We have also given the expression of the ASN for a semi-curtailed double sampling plan when two samples have different rejection numbers. Under binomial probability law we have given, following Craig's procedure, a simplified form of the ASN under fully-curtailed single sampling plan. Expressions of the ASN under semi-curtailed and fully-curtailed double sampling plan are also derived. The percent saving in inspection is illustrated by numerical examples under both the probability laws.

Chapter V deals with a single three class attributes sampling plan where the unit of the lot is classified as either bad, marginal or good. Semi-curtailed and fully-curtailed forms of single three class attributes sampling plan are introduced. Statement of the sampling plan and probability function are given under both the forms. The expressions of the average sample number are also given. Furthermore, we have obtained the maximum likelihood estimators of the proportion of marginal units and the proportion of bad units and the asymptotic variances-covariances of these estimators under both the forms when several lots have undergone the sampling inspection. The relation between the percent saving in inspection and the loss in the efficiency of the estimator is established. Furthermore, the percent saving in inspection as one passes from uncurtailed sampling plan to a semi-curtailed or a fully-curtailed sampling plan is illustrated with a numerical example.

Chapter VI discusses the multiple three class attributes sampling plan which is an extension of the single three class attributes sampling plan considered in Chapter V. We have introduced three forms of the multiple three class attributes sampling plan: Uncurtailed, Semi-curtailed, and Fully-curtailed. Statements of the sampling plans under these three forms are given. The description of the double three class attributes sampling plan which is a particular case of the multiple three class attributes sampling plan is given. We have studied the double three class attributes sampling plan under the assumptions that both samples have common rejection numbers (rejection number for bad units plus marginal units and rejection number for bad units) and common acceptance number for bad units plus marginal units. Probability functions and the expressions of the average sample number are given for the double three class attributes sampling plan under the three forms of the sampling inspection. The maximum likelihood estimators of the proportion of marginal units and the proportion of bad units and the asymptotic variances and covariances of these estimators are obtained under the semi-curtailed double three class attributes sampling plan. These results are extended to the other sampling plans such as uncurtailed and fully-curtailed double three class attributes sampling plan and any form of the multiple sampling plan.

Chapter VII presents the easy execution of sampling plans, two class as well as three class and uncurtailed as well as curtailed. The difficulty experienced by an inspector during the execution of a complicated sampling scheme could be overcome by using a graphical procedure. In this chapter we have discussed the graphical procedure to simplify the execution of two-class and three class attributes sampling plans which may be in any form, curtailed or uncurtailed. In case of the two class attributes sampling plan the usual graph whose ordinates denote defectives and the abscissa denote the number of units inspected is used. The execution of the given fully-curtailed triple sampling plan is illustrated by means of this graphical procedure. It may be noted that in case of the two class attributes sampling plan this graphical procedure is also useful in determining the average sample number and the probability of acceptance. In case of the three class attributes sampling plan plotting is done on two graphs simultaneously. On Graph-1 the number of units inspected are presented along the abscissa and the number of bad units along the ordinate. On Graph-2 the difference is that along the ordinate bad units plus marginal units are presented. We have illustrated the graphical execution for a given fully-curtailed single three class attributes sampling plan and semi-curtailed double three class attributes sampling plan.

Two problems are studied in Chapter VIII. The first problem is about the investigation of the bounds on the variance of the MVUE of the fraction defective under fully--curtailed single two class attributes sampling plan. The technique used here is similar to that used by Sathe [42] wherein he has obtained the sharper bounds for the variance of the MVUE of the parameter of the usual negative binomial distribution. The other problem studied is regarding the determination of the MVUE of the proportion of good, marginal and bad units under single three class attributes sampling plan, curtailed as well as uncurtailed. The method used for this purpose is a natural generalization of that of Girshick, Mosteller, and Savage [12]. In both the problems the results are based on the assumption that one has the information on the inspection of a single lot.

Chapter IX deals with the determination of a single three class attributes sampling plan based upon a linear cost model and a prior distribution. Three bivariate distributions for a lot quality given in terms of $p_1$ and $p_2$ are considered as prior distributions. They are the bivariate degenerate, the bivariate two point, and the bivariate beta distributions. The linear cost model formulized by Hald [20], [19], [22] is modified for the three class attributes sampling plan on the lines similar to those used by Guenther [15] as was done by him in his case of two class attributes sampling plan. The expressions for the expected value of the cost function based on this linear cost model and the above prior distributions are obtained. The sampling plan can be determined either by minimizing the expected value of the cost function subject to no side condition or by minimizing the expected value of the cost function subject to any one of the three side conditions in terms of producer's and consumer's risks. The side conditions are : (i) satisfy producer's risk, (ii) satisfy consumer's risk, (iii) satisfy both risks simultaneously. The determination of a single three class attributes sampling plan when prior distribution of lot quality is bivariate degenerate is illustrated by means of a numerical example under two side conditions : (i) satisfying the producer's risk and (ii) satisfying the consumer's risk.

## 1.4 Main Results and Conclusions in the Thesis :

### 1.4.1 Curtailed Two Class Attributes Sampling Plan :

(i) It is possible to obtain an explicit form of the MLE of the fraction defective (p) for a fully-curtailed DSP under the Situation-A. This explicit form of the MLE of p is

$$\hat{p} = \frac{\text{Total number of defectives observed}}{\text{Total number of units inspected}}$$

(ii) The asymptotic variance of the MLE of p given in (i) is inversely proportional to the ASN.

(iii) The features of the MLE and the asymptotic variance of the MLE remain same for a fully-curtailed MSP under the Situation-A. In fact these features remain same for any two class attributes sampling plan, curtailed as well as uncurtailed, under the Situation-A.

(iv) It is not possible to obtain an explicit form of the MLE of the fraction defective p for fully-curtailed DSP under both the cases of the Situation-B. However, the evaluation of the MLE is not difficult. An iterative procedure is to be used for the MLE of the fraction defective under both the cases of the Situation-B.

(v) When a fully-curtailed DSP data are subjected to the misclassification under the Situation-A, the MLEs of the fraction defective and the probability of the misclassification are given explicitly.

(vi) Appreciable saving in inspection is possible when one passes from an uncurtailed sampling plan to a semi--curtailed or to a fully-curtailed sampling plan. However, a mentionable saving in inspection is possible in going from a semi-curtailed sampling plan to a fully-curtailed sampling plan for smaller values of M in case of hypergeometric probability law and for smaller values of p in case of binomial probability law. The saving in inspection is counter balanced by the loss in efficiency in estimation.

(vii) A simple graphical procedure can be used to simplify the execution of any two class attributes sampling plan. The probability of acceptance and the ASN can be determined from the graph.

(viii) It is possible to give bounds for the variance of the unique unbiased estimator of the fraction defective p under a fully-curtailed single sampling plan. These bounds can be evaluated easily with the usual Binomial Probability Tables. It is observed that the lower bound is not better than the C-R lower bound, but the exact variance has a tendency to be nearer to the upper bound.

1.4.2 Three Class Attributes Sampling Plan : Curtailed as well as Uncurtailed :

(i) The maximum likelihood estimators of $p_1$ and $p_2$ for single three class attributes sampling plans, curtailed as well as uncurtailed are

$$\hat{p}_1 = \frac{\text{Total number of marginal units observed}}{\text{Total number of units inspected}}$$

$$\hat{p}_2 = \frac{\text{Total number of bad units observed}}{\text{Total number of units inspected}}$$

(ii) The asymptotic variances and covariances of the MLEs given in (i) are inversely proportional to the ASN.

(iii) The features of the MLEs and the asymptotic variances and covariances of the MLEs remain same for multiple three class attributes sampling plans, curtailed as well as uncurtailed.

(iv) The relation between the efficiency of the MLE $\hat{p}_i$ (semi), for i=1,2, and the ASN (semi) is established in case of a semi-curtailed three class attributes sampling plan. Similarly the same is established for fully-curtailed sampling plan. From these relations it is concluded that the percent saving in inspection is equal to the percent loss in efficiency in estimation. Furthermore, it is also concluded that the price in reduction in sampling inspection is paid by the proportional increase in the variance of the estimator. The conclusions drawn are also true for any multiple three class attributes sampling plan, curtailed as well as uncurtailed.

(v) Execution of three class attributes sampling plans can be simplified by using the graphical procedure. However, in this case the graphical procedure will not be useful to find the probability of acceptance and the ASN.

(vi) The unique unbiased estimators under uncurtailed single three class attributes sampling plan are the same as the MLEs when one lot is submitted for sampling inspection.

(vii) Determination of single three class attributes sampling plan is possible under the assumption of a linear cost model and a prior distribution of the lot quality. Plan can also be obtained when some side conditions in terms of OC or AOC curve are given.

1.5 Some Work of the Thesis has already been published in the following papers :

1. Shah, D.K., and Phatak, A.G. (1972). A Simplified Form of the ASN for a curtailed Sampling Plan, Technometrics, 14, 4, 925-929.

2. Shah, D.K., and Phatak, A.G. (1974). The Maximum Likelihood Estimate of the Fraction Defective under Curtailed Multiple Sampling Plans, Technometrics, 16, 2, 311-315.

3. Shah, D.K.,and Phatak, A.G. (1971). The ASN of the Curtailed Plans under the Hypergeometric Probability law, Estadistica, XXIX, III, 150-157.

4. Shah, D.K., and Phatak, A.G. (1975). Misclassification under Fully-Curtailed Double Sampling Plans by Attributes, Journal of the M.S. University of Baroda, XXIV, 3, 19-25.