CHAPTER - 6

HARDNESS ANISOTROPY OF ALKALI HALIDES

(NaCl, KCl AND KBr)

6.1 INTRODUCTION :

In the earlier chapter the hardness of NaCl, KCl and KBr was experimentally studied by considering their variation with applied load and quenching temperature for different but constant orientations of the indenter with the crystal lattice. The present work aims at studying the microhardness anisotropy of NaCl, KCl and KBr by employing Knoop indenter of low symmetry. An important feature of the Knoop hardness test is that the hardness value is dependent on the orientation of the major axis of the indenter in a given plane as well as on the orientation of the plane itself with reference to the principal axis of anisotropy /1/. Further the depth of penetration of the indenter is shallow. Hence brittle materials like glass or mineral could be indented without causing premature fracture. Besides, the indenter shape is non-symmetric, the variations in hardness along different directions on a given surface can be determined. For such a study, single crystals can serve as ideal materials to establish the orientation dependence of hardness values. It is from this point of view that hardness anisotropic study of synthetic single crystals of NaCl, KCl and KBr is carried out and reported here.

It is clear from the hardness studies presented here that macroscopically there are five parameters affecting hardness viz. (i) Applied load, (ii) Orientation of the indenter diagonal (major) with reference to the crystal lattice, (iii) Crystal plane under indentation, (iv) Impurity concentration, (v) Quenching temperature. The emperical formula derived in the present work should be valid for majority of crystals of different materials /2/.

6.2 OBSERVATIONS:

For studying the anisotropic behaviour of cubic crystals of NaCl, KCl and KBr, the observations recorded in chapter V are used for considering in a quantative manner the effect of five major factors, namely, (1) Applied load (P), (2) Orientation of the indenter diagonal (major) with reference to the crystal lattice (A), (3) Crystal plane / face for indentation (f), (4) Quenching temperature (T_q), (5) Impurity concentration (I). For the purpose of quantitative studying of the relations amongst P, A, F, T_q and I ; variations between any two factors are considered by keeping remaining parameters constant. The applied load 'P' should be considered as constant. However, it was shown (vide chapter V) that it represents a range of applied loads in HLR where hardness (H) is constant and independent of load. The range of applied load was from 20 to 160 gm. In this range of applied loads, there is a slight change in values of hardness. In the discussion, mean value of hardness was considered. The hardness anisotropy for different orientations is studied at different constant temperatures, viz., room temperature T and different quenching temperatures T_q's . Thus the approach to hardness study is basically phenomenological. This approach is likely to be useful for development of model theory of hardness of crystalline materials.

6.3 RESULTS AND DISCUSSION:

6.3.1 Variation of hardness with orientation of major diagonal with direction [100] on cleavage faces of NaCl, KCl and KBr at constant temperature :

It is observed from the plots of H Vs. P (Fig. 5.1, 5.2, 5.3) for NaCl, KCl and KBr that for higher loads greater than 20 gm in HLR, Knoop hardness is almost constant. Considering the mean hardness value, the plots of $\bar{H}$ Vs. A for room temperature and different quenching temperatures are shown in Fig. 6.1a, 6.1b & 6.1c where 'A' is the orientation of the longer diagonal of the Knoop indenter from [100] direction on cleavage faces of (100) of NaCl, KCl and KBr. The plot of $\bar{H}$ Vs. A depict the hardness anisotropy as the orientation changes. The plots indicate that the variation of hardness on either side of direction [110] is symmetrical. For all the three crystals, observations were taken for all values of 'A', i.e, from 0° to 90°. The plots show very clearly that hardness changes with A and quenching temperature. In this section only the variation of $\bar{H}$ with A at constant temperature will be considered.

There are certain basic characteristics of these plots (Fig. 6.1a, 6.1b and 6.1c). They are as follows:

(1) For NaCl the hardness range is from 16.66 to 19.62 Kg/mm^2, for KCl it is from 7.71 to 9.86 and for KBr it is from 7.25 to 8.002 $Kg./mm^2$.

(2) The minimum value of hardness occurs at 45°, i.e., along the direction [110] whereas the maximum values on either side of the above direction are at 0° and 90°, i.e., along the direction [100] and [010].

Since all these crystals have perfect cleavages $\{100\}$, hardness along direction $\langle 100 \rangle$ should have a maximum value and a minimum value along $\langle 110 \rangle$.

(3) The plots do not exhibit sharp minima.

(4) For NaCl and KCl hardness values show increasing tendency with increasing T_q whereas KBr shows decrease in values upto a certain value of temperature and then increases with increasing T_q.

(5) The curve is an imperfect parabola.

Since hardness is inversely proportional to the square of the major diagonal of the Knoop indentation mark, it is also possible to draw a plot of d Vs. A. Because of the inverse proportion, these plots should exhibit minima corresponding to maximum values of $\bar{H}$ in the various plots of $\bar{H}$ Vs. A for various, but constant, room temperature and quenching temperatures. This is indeed found to be the case and the above three conclusions are valid. Since due to broadening of the curve, it is desirable to have a change in the mathematical approach so as to obtain straight line plots .

A careful study of the curve of $\bar{H}$ Vs. A indicates that to a greater extent it corresponds to a parabola. The curvature near 0° to 90° shows that the latter part of the parabolic curve turns into a bell shape. It is thus a mixture of these curves. Hence by following mathematical combinations in a judicious manner, it is possible to obtain a straight line plot. The graph of $\sqrt{\bar{H}\ A}$ Vs. A is linear with intercepts on the axis. For NaCl, KCl and KBr the plots are straight lines (vide table 6.1A, 6.1B, 6.1C) (Fig. 6.2a, 6.2b, 6.2c). The general equation for such a straight line plot is,

$$\sqrt{\bar{H}\ A} = mA + C \quad \text{.........} \quad (1)$$

where $\sqrt{\bar{H}\ A}$ and A are along Y and X-axis respectively and m and C are the slope and intercept respectively. Squaring both sides of (1) yields,

$$\bar{H}\ A = m^2 A^2 + 2\ mAC + C^2 \quad \text{...........}(2)$$

Division of both sides by A gives,

$$\bar{H} = m^2 A + 2\ mC + C^2/A \quad \text{..........} \quad (3)$$

Differentiation of (2) and (3) with respect to A yields,

$$\frac{d\bar{H}}{dA}.A + 1.\bar{H} = 2m^2 + 2mC \quad \text{..........} \quad (4)$$

and

$$\frac{d\bar{H}}{dA} = m^2 - C^2/A^2 \quad \text{.............}(5)$$

For an entremum (maximum or minimum) of the curve at a point, say, $\bar{H} = h_o$, $A = a_o$,

$$\frac{d\bar{H}}{dA} = 0 \text{ at}$$

$$\bar{H} = h_o, \quad A = a_o$$

Substitution of this value in (4) and (5) yields,

$$0.a_o + h_o.1 = 2m^2 a_o + 2mC$$

$$h_o = 2m^2 a_o + 2mC \quad \ldots\ldots\ldots\ldots (7)$$

and

$$m^2 - C^2/a_o = 0$$

$$a_o = \pm C/m \quad \ldots\ldots\ldots\ldots (8)$$

Elimination of a_o from (7) by using (8) gives

$$h_o = 2m^2 (\pm C/m) + 2mC$$

$$= \pm 2mC + 2mC$$

$$= 4mC \text{ or } 0$$

Obviously zero value of ho is inadmissible,
Hence,

$$h_o = 4mC \quad \ldots\ldots\ldots\ldots (9)$$

Multiplying (8) and (9), one gets -

$$a_o h_o = \pm C/m.4mC$$

$$= 4 C^2$$

$$\therefore C = \pm \sqrt{a_o h_o/4} \quad \ldots\ldots\ldots (10)$$

Division of (8) by (9) gives

$$a_o/ho = 1/4m^2$$

$$\therefore\ m = \pm \sqrt{h_o/4a_o} \quad \ldots\ldots\ldots \quad (11)$$

Thus the following values are of importance for testing straight line and parabolic plots (Fig. 6.1 a, b, c and 6.2 a, b, c):

$$a_o = C/m \quad \ldots\ldots\ldots\ldots\ldots(12)$$

$$h_o = 4mC \quad \ldots\ldots\ldots\ldots\ldots(13)$$

$$C = + \tfrac{1}{2} \sqrt{a_o\ h_o} \quad \ldots\ldots\ldots\ldots\ldots(14)$$

$$m = + \tfrac{1}{2} \sqrt{h_o/a_o} \quad \ldots\ldots\ldots\ldots\ldots(15)$$

The values of a_o and h_o can be obtained from the parabolic plot where $d\bar{H}/dA = 0$ at the point 'P' having coordinates (a_o, h_o). These values can be utilized to obtain the slope 'm' and intercept 'C' of the linear plots.

For a straight line plot following a well - established relation between the variables in the plot, the conventional method is to select any two points on the straight line, say, $(A_1, \bar{H}_1)$ and $(A_2, \bar{H}_2)$ and to follow the normal procedure of calculating the slop and intercept from the general equation -

$$\sqrt{\bar{H}\ A} = mA + C$$ of the straight line.

Thus,

$$m = \frac{\sqrt{H_1A_1} - \sqrt{H_2A_2}}{A_1 - A_2} \quad \ldots \quad \ldots \quad \ldots (16)$$

$$C = \frac{\sqrt{H_1A_1} - \sqrt{H_2A_2}}{2} - m(A_1 + A_2) \quad \ldots (17)$$

The values of m and C obtained from (16) & (17) should agree with the values obtained from (12), (13), (14) and (15) and also from the statistical method of the best fit of straight line. In the present case more emphasis is given on the statistical method rather than the conventional method in view of the fact that the relations between H and A are in the developmental stage. For NaCl, KCl and KBr, the values of m and C determined by using statistical method are compared with the values obtained from a few distinguishing characteristics of the parabolic plots. These values obtained at room temperature and different quenching temperatures are given in table 6.2 A, 6.2 B, and 6.2 C. A glance at these values indicates a fairly good agreement between values calculated by statistical method and determined from parabolic plots. It should be noted that due to combination of variables in the general equation (1), it is clear that the entire analysis becomes invalid for $\bar{H} = 0$ and $A = 0$.

The above formulae which are based on experimental observations for cleavage faces of NaCl, KCl and KBr are derived without any direct reference to crystal structure or micro-structures developed on a crystal surface due to indentation. The basic requirement is the availability of a highly clean crystal face of low indices free from different growth features. Hence these formulae should be applicable to similar type of hardness studies of different crystals reported in the literature /2/.

It should be noted that in case of NaCl, KCl and KBr, the symmetry direction in the parabolic curve of $\bar{H}$ Vs. A was [110]. Such a direction exists for curved plots of $\bar{H}$ Vs. A for crystals studied by different workers.

6.3.2 Variation of hardness with orientation(A) and quenching temperature(T_q).

The earlier studies were carried out by considering the variation between two parameters, $\bar{H}$ and A, out of $\bar{H}$, P, T_q, A, F and I, by keeping remaining parameters constant. In the present case the variation of hardness with other parameters is reported. In this case the values of hardness corresponding to those values in the HLR where hardness is constant and independent of load, but depends on T_q and A are taken and that their mean hardness value ($\bar{H}$) is used here.

Several combinations of $\bar{H}$, T_q and A were tried to obtain the straight line plot. A plot of log $T_q \sqrt{\bar{H} A}$ Vs. log T_q (fig. 6.3 a, 6.3 b, 6.3 c; (vide table 6.3 A, 6.3 B, 6.3C) is a straight line. The regression coefficient based on statistical consideration for obtaining the best fit of a straight line has a value much nearer to unity. Further the graph of 6.3 a, b, c consists of a series of straight lines parellel to each other corresponding to different orientations A with respect to direction [100].

The general equation for such a straight line plot is

$$\log T_q \sqrt{\bar{H}\ A} = m_1 \log T_q + \log C_1 \quad ....(18)$$

where, as usual, m_1 and C_1 represent slope and intercept. Simplification of the the above yields,

$$\bar{H}\ T_q^{2(1-m_1)} \Rightarrow C_1^2/A$$

$$\bar{H}\ T_q^P = C_1^2/A \quad \text{where } P = 2(1-m_1)$$

or

$$\bar{H} = \frac{C_1^2}{A}\left(\frac{1}{T_q^P}\right) \quad ....(i)$$

From the earlier studies on the variation of H with T_q by keeping other parameters constant, the general equation was derived.

It is

$$\bar{H}\ T_q^k = C$$

or

$$\bar{H} = \frac{C}{T_q^k} \quad ....(ii)$$

Comparison of the above two equations suggests that

$$\frac{C_1^2}{A}\left(\frac{1}{T_q^P}\right) = \frac{C}{T_q^k}$$

$$(C_1^2/A)/C = \frac{T_q^P}{T_q^k} \quad \text{or } T_q^{P-k}$$

Since 'P' and 'k' which depends on A are obtainable from the graphs (Fig. 6.3 abc and 5.4, 5.5, 5.6), T_q^{P-k} for different values of T_q can be calculated. Similarly using (C_1^2/A) and C values obtained from the above graphs $(C_1^2/A)/C$ can also be calculated. These two sides should be equal. This is actually found to be the case.

It is thus obvious that when the orientation A and quenching temperature T_q are simultaneously changed, the hardness number follows the equation-

$$\bar{H} A \, T_q^P = \text{constant},$$

where the exponent P is given by

$$P = 2(1 - m_1)$$

where m_1 is the slope of the straight line plot between $\log T_q\sqrt{\bar{H} A}$ Vs. $\log T_q$.

6.3.3 Variation of standard hardness with orientation(A)at constant temperature and at different quenching temperatures(T_q).

It was shown in chapter IV that b_2 and w_2 are anisotropic constants characterising the alkali halide crystals. Further in the literature b_2 is referred to as standard hardness and the exponent value of the diagonal in modified Kick's law is 2. Looking to the law and and variation of b_2 with A and T_Q, it is conjectured that if $\bar{H}$ is replaced by b_2 in equation (1) and (18), the plots of (i) $\sqrt{b_2 A}$ Vs. A and (ii) log $T_q \sqrt{b_2 A}$ Vs. log T_q should be linear, similar to those obtained for the plots of $\sqrt{\bar{H} A}$ Vs. A and log $T_q \sqrt{\bar{H} A}$ Vs. log T_q. Following the procedure of equation (1) and (18), the general equation for plot of $\sqrt{b_2 A}$ Vs. A (Fig 6.4 and Table 6.4) is

$$\sqrt{b_2 A} = m_2' A + C_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

The values of slope m_2' and intercept C_2 are obtained by two different independent plots. From parabolic plots

$$m_2' = \tfrac{1}{2} \sqrt{b_{2o}/4a_o} \qquad \ldots\ldots\ldots\ldots(20)$$

$$C_2 = \tfrac{1}{2} \sqrt{b_{2o} a_o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

where b_{2o} and a_o are obtained from the parabolic plot of b_2 Vs. A (Fig. 4.8).

The values of m_2' and C_2 are also obtained from the plot of b_2 A Vs. A (Fig. 6.4). They are given in table 6.5 for room temperature and for different quenching temperatures. There is fairly good agreement between the above values calculated from two independent plots with only a little variation. However this variation appears to be due to experimental errors in the set up and measurements.

For studying the variation with quenching temperature, equation similar to (18) is used.

$$\log T_q \sqrt{b_2 A} = m'_3 \log T_q + \log C_3$$

where m'_3 and C_3 are the slope and intercepts respectively (fig. 6.5).

Simplification of the above yields,

$$b_2 T_q^{2(1-m'_3)} = C_3^2 /A$$

or

$$b_2 T_q^{P'} = C_3^2 /A \quad \text{where} \quad 2(1-m'_3) = P'$$

$$b_2 = C_3^2 /A \left[\frac{1}{T_q^{P'}}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots(23)$$

From the general equation of b_2 with T_q (Chapter IV equation.9),

$$g\ T_q^{(1-m)} = C_r$$

$$b_2 \quad T_q^{(1-m_3)} = C_{r3}$$

or $$b_2\ T_q^{k'} = C_{r3}$$

or $$b_2 = C_{r3}\left(\frac{1}{T_q^{k'}}\right) \qquad \ldots\ldots\ldots\ldots\ldots(24)$$

Comparing (23) and (24)

$$\frac{C_3^2 /A}{C_{r3}} = \frac{T_q^{P'}}{T_q^{k'}} = T_q^{P'-k'} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(25)$$

Calculation of $T_q^{P'-k'}$ and $(C_3^2/A)/C_{r3}$ of equation (25) by independent plots are made and presented in table 6.6

The table indicates fairly good agreement between the two sides of equation (25). When the orientation A and quenching temperature T_q are simultaneously changed the standard hardness b_2 obeys the equation :

$$b_2 \; A \; T_q^{\;P'} = \text{constant.}$$

It should be remarked here that b_2 in modified Kick's law is proportional to hardness and hence the above analysis has come out correctly. Since b_2 is anisotropic, hardness is also anisotropic. In the modified Kick's law another anisotropic constant is w_2. In the whole analysis of hardness anisotropy, there is not a single quantity which corresponds to w_2. This simply points to the limitation of the present analysis in which equations (1) and (19) are not valid for A = 0.

In chapter IV, while analysing the observations on the basis of Kick's law, it was shown that (n_1, a_1) and (n_2, a_2) are pair of anisotropic constants in LLR and HLR. On comparing this analysis with analysis based on hardness data, it is found that a_2 or a_1 can not replace H, indicating that they are not proportional to H. This is likely to be due to the following reasons :

(1) The variation of a_2 or a_1 with A is not parabolic

(2) The exponents n_1 and n_2 are having values different from 2.

(3) The variation of a_2 or a_1 with quenching temperature appears to be complicated.

TABLE - 6.1 A

FOR NaCl CRYSTALS

Orientation A	Temperature in °K <— $\sqrt{\overline{H}}$ A —>					
	303	473	573	673	773	873
0						
10	13.9678	14.0854	14.2197	14.3561	14.6833	15.5338
20	19.2500	19.5064	20.0640	20.2682	20.3420	21.7301
30	23.1343	23.1991	24.4397	24.4581	24.8233	26.2220
40	25.8843	26.3438	27.4881	28.2276	28.5096	29.9599
50	28.8617	29.2403	30.7083	31.6306	31.8747	33.3991
60	32.3570	32.9085	34.4586	34.9084	34.9771	37.0351
70	35.8008	36.6128	37.4773	37.8813	38.0473	40.5067
80	39.4664	40.0998	40.1995	40.6054	41.5403	43.9454
90	42.0214	42.7012	42.7012	43.4741	44.7694	47.2059

TABLE - 6.1 B

FOR KCl CRYSTALS

Orientation A in degree	Temperature in °K <—— $\sqrt{H}$ A ——>					
	303°K	473°K	573°K	673°K	773°K	873°K
0						
10	9.5594	9.6120	10.1163	10.6260	11.0905	11.5407
20	13.2664	13.2181	14.0356	14.7292	15.4330	16.0896
30	15.5320	15.6953	16.7749	17.6550	18.5337	19.3432
40	17.5720	17.6111	18.7723	19.8645	20.8518	21.8124
50	19.6341	19.7610	20.9845	22.2087	23.2379	24.3554
60	22.0997	22.2609	23.7259	24.9799	26.2106	27.3433
70	24.7432	24.7562	26.2689	27.5526	28.8738	30.0918
80	27.0998	27.2235	28.7054	30.0644	31.4579	30.6606
90	29.8043	29.6318	31.1191	32.6073	34.0073	35.3351

TABLE - 6.1 C

FOR KBr CRYSTALS

Orientation A in degree	Temperature T in °K ← $\sqrt{\bar{H}}$ A →					
	303°K	473°K	573°K	673°K	773°K	873°K
0						
10	8.9100	9.170	8.9670	8.8374	10.0399	10.3004
20	12.4200	12.5700	12.4730	12.2637	14.2407	14.5602
30	14.8990	15.1780	15.1958	14.7682	17.3030	17.4110
40	17.0500	17.4900	17.4126	16.8160	19.1728	20.2970
50	19.0390	19.5500	19.4170	18.7700	21.4476	22.6825
60	21.0700	21.6050	21.4632	20.8420	24.2239	25.0990
70	23.3660	23.5300	23.5158	22.9280	26.3912	27.2525
80	25.2190	25.9200	25.3298	24.7870	28.4956	29.3530
90	26.8160	25.5600	27.0330	26.6139	30.2985	32.0730

TABLE - 6.2 A

FOR NaCl CRYSTALS

VALUES OF SLOPE m (CALCULATED BY FORMULA, STATISTICAL, OBSERVED) INTERCEPT c (CALCULATED BY FORMULA, STATISTICAL, OBSERVED) OF THE PLOT OF $\sqrt{\bar{H}}$ A Vs A; HARDNESS h_o IN Kg/mm^2 AT ao - 45° FROM THE PLOT OF $\bar{H}$ Vs A.

Temp. °K	Slope = m			Intercept = C			
	$\frac{1}{2}\sqrt{\frac{ho}{ao}}$ mc	Statistically obtained ms	Observed mo	$\frac{1}{2}\sqrt{a_o h_o}$ Cc	Statisitically obtained Cs	Observed Co	h_o
303	0.3032	0.3411	0.2777	13.6450	11.9165	14.6111	16.70
473	0.3068	0.3493	0.3272	13.2400	11.9458	13.8089	17.00
573	0.3214	0.3129	0.3384	14.4654	14.9853	13.4769	18.60
673	0.3312	0.3523	0.3285	14.9059	13.0305	14.1714	19.60
773	0.3456	0.3614	0.3250	15.5520	12.9917	14.9500	20.10
873	0.3519	0.3816	0.3571	15.8390	13.7575	15.2428	22-20

TABLE - 6.2 B

FOR KCl CRYSTALS

VALUES OF SLOPE m (CALCULATED FROM FORMULA, STATISTICAL, OBSERVED) INTERCEPT c (CALCULATED, STATISTICAL, OBSERVED) OF THE PLOT $\sqrt{\bar{H}}$ A Vs A; HARDNESS ho IN $Kg\text{-}mm^{2}$ AT ao=45° FROM THE PLOT OF $\bar{H}$ Vs A.

Temp. °K	Slope = m			Intercept = C			
	$\frac{1}{a}\sqrt{\frac{h_o}{a_o}}$ mc	Statistically obtained ms	Observed mo	$\frac{1}{2} a_o h_o$ Cc	Statistically obtained Cs	Observed Co	h_o
303	0.2041	0.2423	0.2250	9.1855	7.8380	8.3500	7.50
473	0.2054	0.2413	0.2400	9.2466	7.9047	7.6000	7.70
573	0.2198	0.2532	0.2333	9.8931	8.5034	9.1330	8.70
673	0.2309	0.2647	0.2500	10.3923	9.0173	9.7000	9.60
773	0.2438	0.2763	0.2800	10.9715	9.4845	9.2000	10.70
873	0.2544	0.2865	0.2444	11.4482	9.9625	11.9770	11.65

TABLE - 6.2 C

FOR KBr CRYSTALS

VALUES SLOPE m (CALCULATED BY FORMULA, STATISTICAL, OBSERVED) INTERCEPT c: (CALCULATED, STATISTICAL AND FOR OBSERVED) OF THE PLOT OF $\sqrt{\bar{H}}$ A Vs A; HARDNESS IN Kg/mm^2 AT ao = 45° FROM THE PLOT OF $\bar{H}$ Vs A .

Temp. °K	Slope = m			Intercept = C			
	$\frac{1}{2}\sqrt{\frac{h_o}{a_o}}$ mc	Statistically obtained ms	Observed mo	$\frac{1}{2}\sqrt{h_o a_o}$ Cc	Statistically observed Cs	Observed Co	ho
303	0.2000	0.2207	0.2181	9.0000	7.6872	7.4900	7.20
473	0.2054	0.2240	0.2250	9.2460	7.9740	8.8500	7.60
573	0.2034	0.2185	0.2142	9.1549	8.0452	8.2857	7.45
673	0.1972	0.2150	0.2000	8.8741	7.7640	8.6000	7.00
773	0.2248	0.2448	0.2285	10.1180	9.0458	9.7714	9.10
873	0.2386	0.2599	0.2444	10.7383	9.1192	9.9777	10.25

TABLE - 6.3 A

FOR NaCl CRYSTALS

$\text{Log } T_q$	<—— $\text{Log } T_q \sqrt{H}\ A$ ——>								
	10	20	30	40	50	60	70	80	90
2.4814	3.6265	3.7658	3.8456	3.8944	3.9417	3.9914	4.0353	4.0776	4.1049
2,6748	3.8236	3.9650	4.0403	4.0955	4.1408	4.1921	4.2384	4.2780	4.3053
2.7581	3.9110	4.0605	4.1462	4.1972	4.2454	4.2954	4.3319	4.3623	4.3885
2.8280	3.9850	4.1348	4.2164	4.2786	4.3281	4.3709	4.4064	4.4365	4.4662
2.8880	4.0550	4.1965	4.2830	4.3431	4.3916	4.4319	4.4685	4.5066	4.5391
2.9410	4.1322	4.2780	4.3596	4.4175	4.4647	4.5096	4.5485	4.5839	4.6150

m_1	1.1724	1.1428	1.1110	1.1110	1.1110	1.1110	1.1428	1.1000	1.1000
C_1	4.8667	8.3176	12.2712	13.7680	15.4480	17.3330	15.1411	22.3872	23.9800
$P=2(1-m_1)$	-0.3448	-0.2856	-0.2220	-0.2220	-0.2220	-0.2220	-0.2856	-0.2000	-0.2000

Contd...Table - 6.3 A

K and C are the taken from the graph of Log H T_q Vs log T_q

K=1-m	-0.1666	-0.2250	-0.3000	-0.3330	-0.3330	-0.3330	-0.1650	-0.1647	-0.1660
C	7.1647	4.5708	2.8840	2.2353	2.3850	2.3313	6.4595	7.1647	7.4448

K, C Values are taken from the plots of log $\bar{H}$ T_q Vs Log T_q

Temp. °K T_q	T_q^{P-K}	$\frac{C_1^2}{A}$ /C
303	0.3612	0.3305
473	0.7567	0.6884
573	1.6411	1.7404
673	2.0602	2.1200
773	2.0933	2.0010
873	2.1248	2.1478

TABLE - 6.3 B

FOR KCl CRYSTALS

$\log T_q$	$\log T_q \sqrt{\bar{H}}\,A$ — Angle 10	20	30	40	50	60	70	80	90
2.4814	3.4618	3.6041	3.6726	3.7262	3.7744	3.8258	3.8748	3.9144	3.9557
2.6748	3.6576	3.7960	3.8706	3.9206	3.9706	4.0224	4.0685	4.1098	4.1466
2.7581	3.7631	3.9053	3.9828	4.0316	4.0800	4.1333	4.1775	4.2161	4.2511
2.8280	3.8540	3.9961	4.0748	4.1260	4.1745	4.2256	4.2681	4.3060	4.3413
2.8880	3.9331	4.0766	4.1561	4.2073	4.2543	4.3066	4.3486	4.3959	4.4197
2.9410	4.0032	4.1475	4.2275	4.2797	4.3281	4.3778	4.4194	4.4550	4.4892

m_1	1.1148	1.1148	1.1110	1.1110	1.1110	1.1110	1.1110	1.1200	1.1200
C_1	5.1116	6.8954	4.4957	9.7543	11.0150	12.0780	13.5518	13.9315	14.9279
$P=2(1-m_1)$	-0.2296	-0.2296	-0.2220	-0.2220	-0.2220	-0.2220	-0.2220	-0.2400	-0.2400

Contd...Table - 6.3 B

K=1-m	-0.1333	-0.1200	-0.3000	-0.3330	-0.3330	-0.3000	-0.2220	-0.1330	-0.1300
C	4.6425	2.8840	1.4454	1.1030	1.1030	1.4621	2.5644	4.6960	4.8013

K, C Values are taken from the plot of Log $\overline{H}$ T_q Vs log T_q

Temp.°K T_q	T_q^{P-K}	$\frac{C_1^2/A}{C}$
303	0.5768	0.5628
473	0.5526	0.8243
573	1.6411	1.6645
673	2.0602	2.1563
773	2.0921	2.2000
873	1.6958	1.6628

TABLE - 6.3 C

FOR KBr CRYSTALS

$\log T_q$	← $\log T_q \sqrt{\bar{H}}$ A →								
	10	20	30	40	50	60	70	80	90
2.4814	3.4310	3.5750	3.6540	3.7130	3.7610	3.8050	3.8500	3.8830	3.9090
2.6748	3.6370	3.7740	3.8560	3.9170	3.9660	4.0090	4.0460	4.0880	4.1150
2.7581	3.7100	3.8540	3.9370	3.9990	4.0460	4.0890	4.1290	4.1610	4.1900
2.8280	3.7740	3.9160	3.9970	4.0530	4.1010	4.1460	4.1880	4.2220	4.2530
2.8880	3.8890	4.0410	4.1260	4.1700	4.2190	4.2720	4.3090	4.3420	4.3890
2.9410	3.9530	4.1040	4.1810	4.2480	4.2960	4.3400	4.3760	4.4080	4.4471

m_1	1.1250	0.9540	0.9540	1.0470	1.0470	1.0769	1.0769	1.0588	1.1666
C_1	4.3650	17.4769	21.3206	14.0734	15.7900	14.2760	15.6559	18.8492	10.0770
$P=2(1-m_1)$	-0.2500	+0.0920	0.0920	-0.0940	-0.0940	-0.1538	-0.1538	-0.1178	-0.3330

Contd...Table - 6.3 C

K=1-m	-0.1386	+0.0586	+0.0586	-0.1386	-0.1386	+0.0586	+0.0586	-0.1386	-0.1386
C	3.3641	11.0842	11.0842	3.9303	2.9303	11.0842	11.0842	3.3641	3.3641

K, C Values are taken from the plot of log $\bar{H}$ T_q Vs log T_q.

Temp. °K T_q	T_q^{P-K}	$\frac{C_1^2/A}{C}$
303	0.5291	0.5063
473	1.2284	1.3348
573	1.2362	1.3778
673	1.3369	1.6897
773	1.3452	1.7016
873	1.2373	0.3065

TABLE - 6.4

NaCl CRYSTALS

Angle	$\sqrt{b_2 A}$					
A	303°K	473°K	573°K	673°K	773°K	873°K
10	0.1192	0.1200	0.1214	0.1192	0.1268	0.1210
20	0.1616	0.1655	0.1726	0.1718	0.1696	0.1720
30	0.1924	0.1942	0.2022	0.2053	0.2086	0.2099
40	0.2113	0.2113	0.2280	0.2283	0.2325	0.2360
50	0.2362	0.2422	0.2471	0.25.75	0.2575	0.2625
60	0.2721	0.2754	0.2739	0.2903	0.2865	0.2966
70	0.3033	0.3020	0.3097	0.3321	0.3096	0.3249
80	0.3368	0.3336	0.3508	0.3586	0.3541	0.3583
90	0.3574	0.3600	0.3790	0.3823	0.3717	0.3823

TABLE - 6.5

NaCl CRYSTALS

VALUES OF SLOPE m'_2 (OBSERVED AND CALCULATED FROM FORMULA) AND INTERCEPT C_2 (OBSERVED AND CALCULATED FROM FORMULA) OF THE PLOT OF $\sqrt{b_2 A}$ Vs. A; b_{2o} TAKEN FROM THE PLOT OF b_2 Vs. A.

Temp. °K	Slope = m'_2		Intercept = C_2		b_{2o}
	Observed m'_2	Calculated from the equation $m'_2 = \frac{1}{2}\sqrt{b_{2o}/a_o}$	Observed C_2	Calculated from the fromula $C_2 = \frac{1}{2}\sqrt{b_{2o}a_o}$	
303	0.0028	0.0024	0.1060	0.1107	0.0010
473	0.0028	0.0024	0.0980	0.1112	0.0011
573	0.0025	0.0026	0.1425	0.1171	0.0012
673	0.0031	0.0026	0.1057	0.1209	0.0013
773	0.0030	0.0026	0.1050	0.1209	0.0013
873	0.0031	0.0027	0.1057	0.1236	0.0013

TABLE - 6.6

FOR NaCl CRYSTALS

Log T_q	Log T_q $\sqrt{b_2 A}$								
	A=10	A=20	A=30	A=40	A=50	A=60	A=70	A=80	A=90
2.4814	1.557	1.689	1.765	1.806	1.854	1.916	1.963	2.008	2.034
2.6748	1.754	1.893	1.963	1.999	2.059	2.114	2.154	2.199	2.231
2.7581	1.842	1.995	2.064	2.116	2.151	2.195	2.249	2.303	2.337
2.8280	1.904	2.063	2.140	2.186	2.238	2.291	2.349	2.382	2.410
2.8888	1.991	2.117	2.207	2.254	2.299	2.345	2.378	2.437	2.458
2.9410	2.026	2.177	2.262	2.314	2.360	2.413	2.452	2.495	2.523

m_3'	1.125	1.125	1.100	1.222	1.222	1.111	1.111	1.111	1.166
C_3	0.054	0.077	0.104	0.054	0.062	0.140	0.140	0.173	0.127
$P' = 2(1-m_3')$	-0.250	-0.250	-0.200	-0.444	-0.444	-0.222	-0.222	-0.222	-0.332

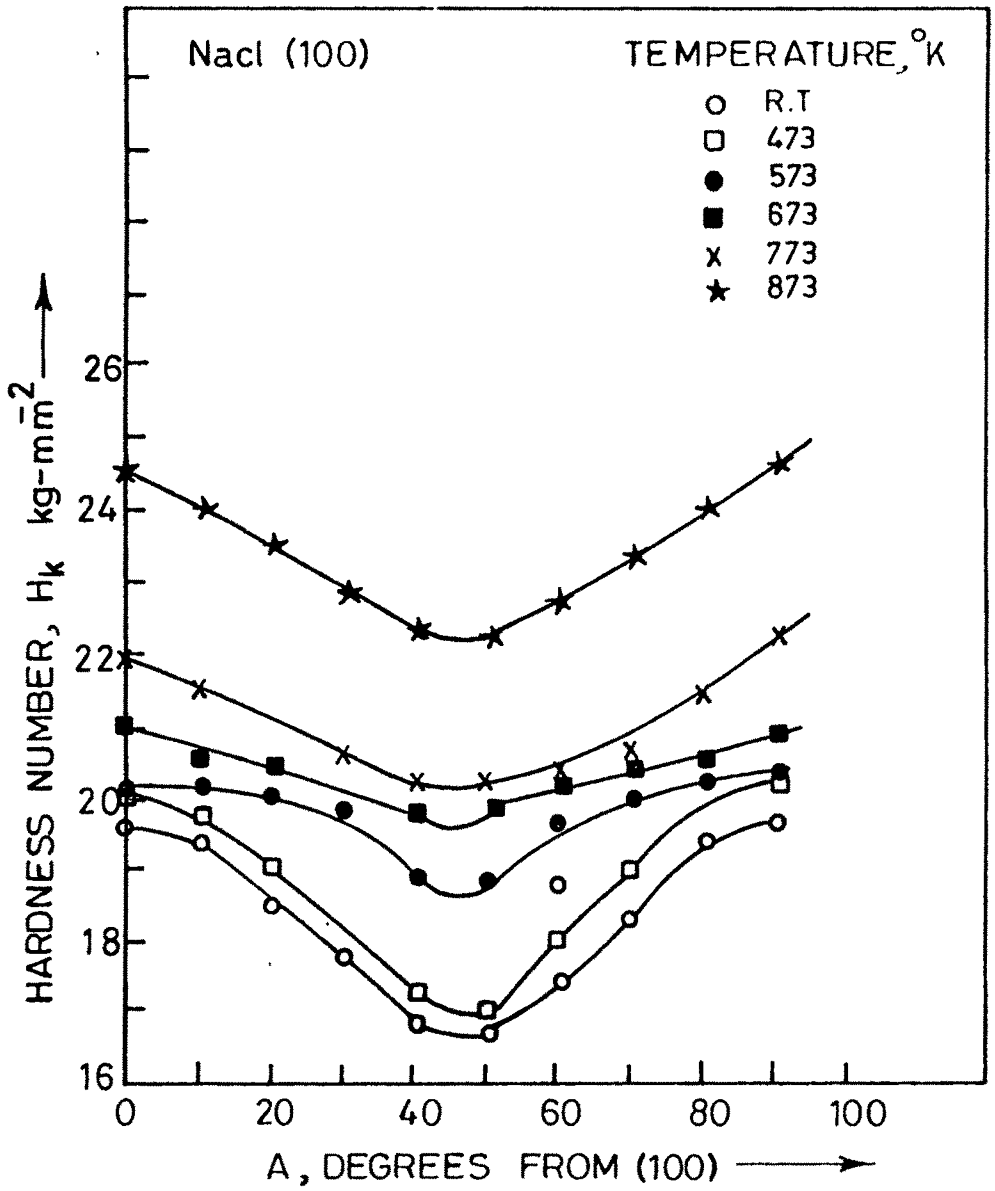
Contd...

Contd...Table-6.6

k' and C'_{r3} are taken from the table : 4.7 A .

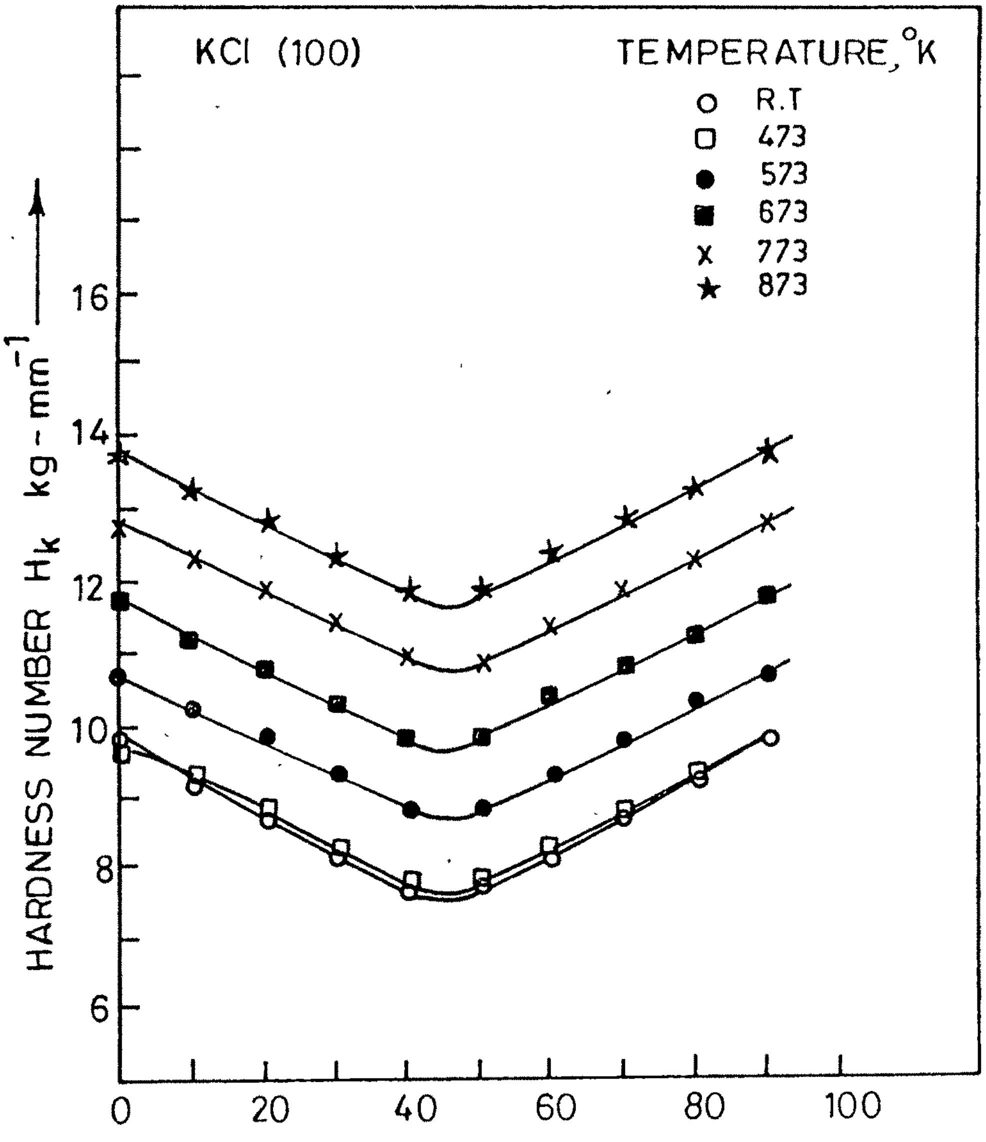
$k' = 1 - m_3$	-0.111	-0.111	0	-0.111	-0.111	0	-0.111	-0.111	-0.111
C_{r3}	0.0007	0.0007	0.0013	0.0005	0.0005	0.0013	0.0007	0.0007	0.0007

Temp °K T_q	$T_q^{P'-k'}$	$\frac{C_3^2/A}{C_{r3}}$
303	0.4519	0.3903
473	0.4248	0.4029
573	0.2807	0.2789
673	0.1143	0.1267
773	0.1092	0.1315
873	0.2223	0.2511



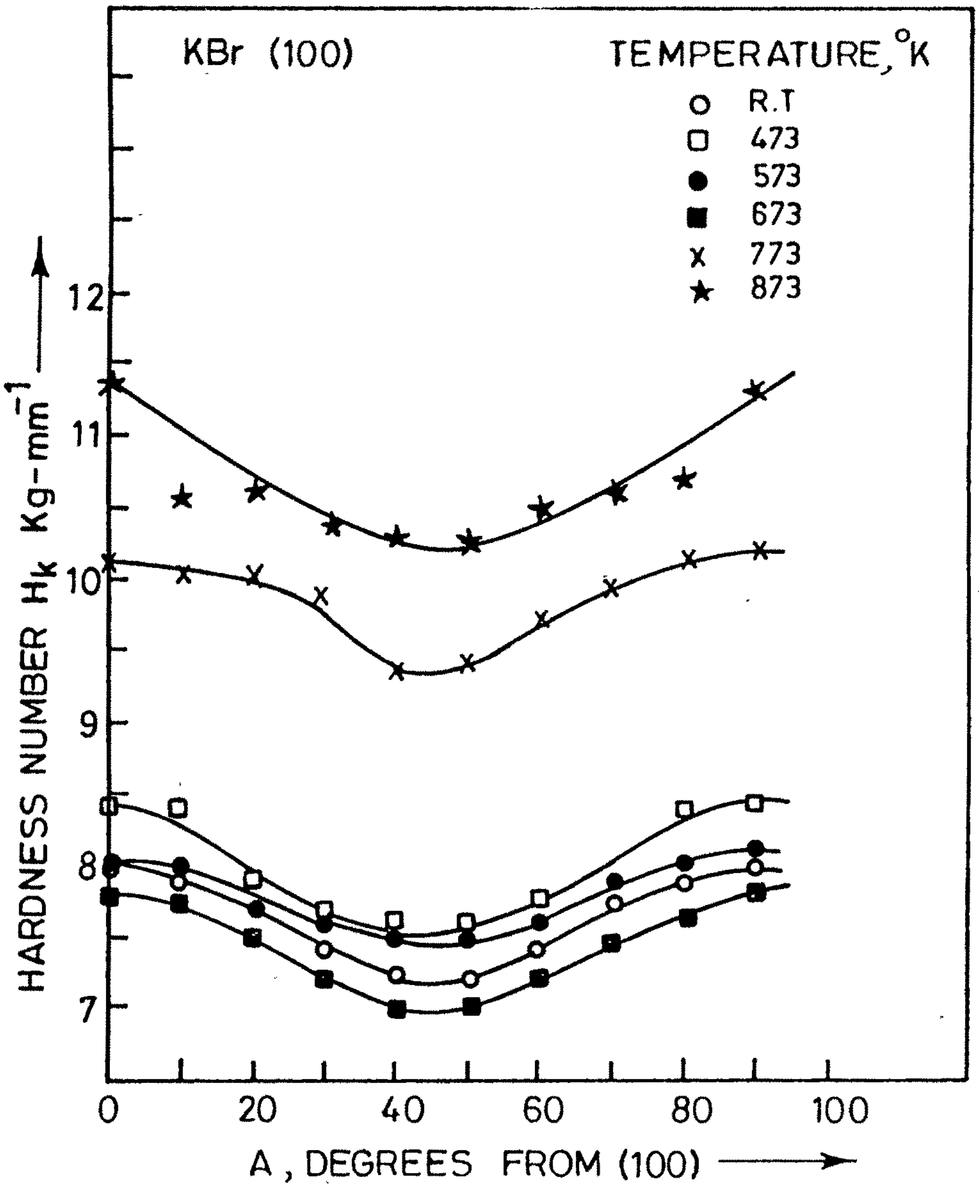
PLOT OF H_k Vs. A.

Fig. 6.1a.



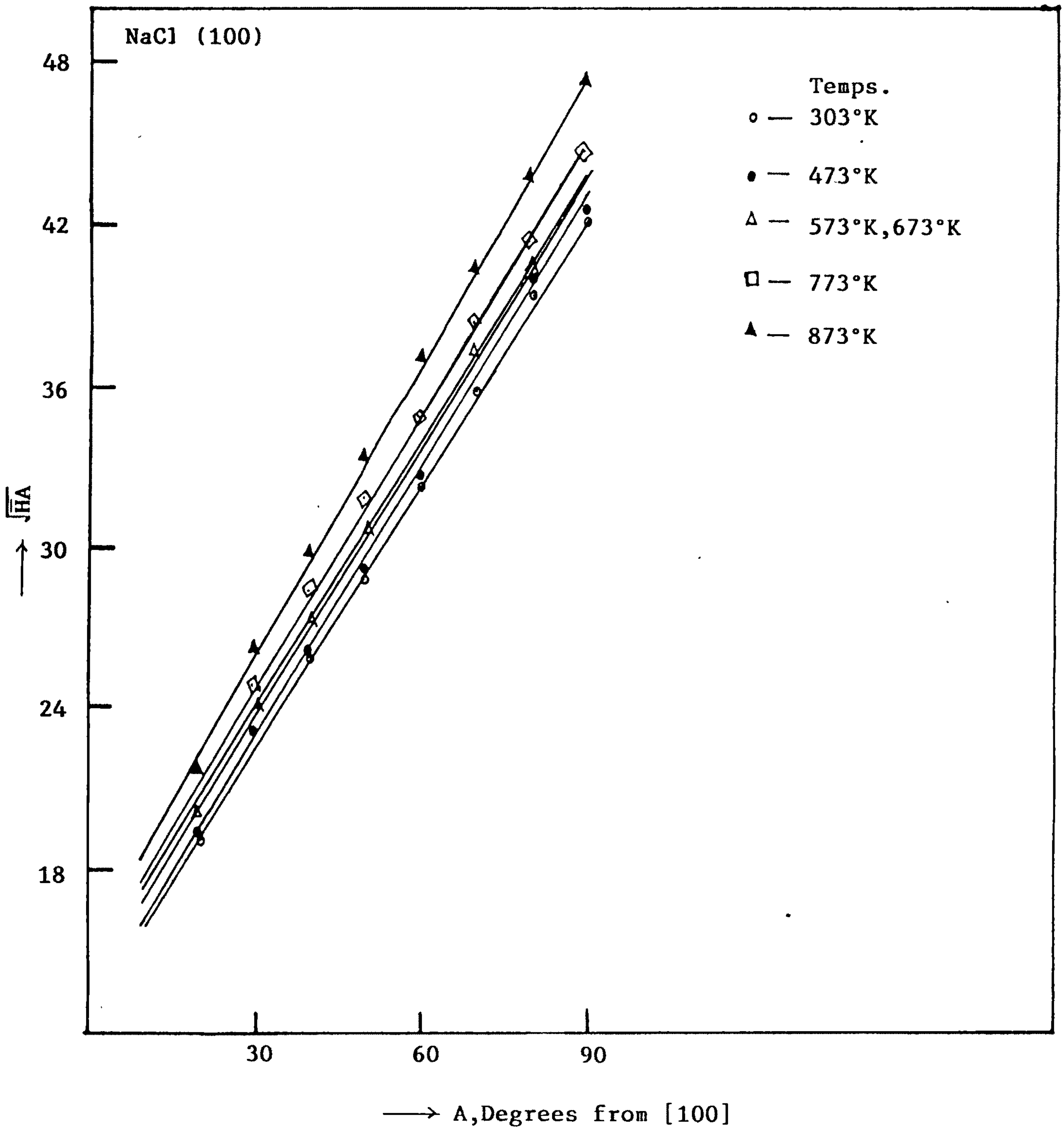
PLOT OF H_k Vs. A.

Fig. 6.1b.

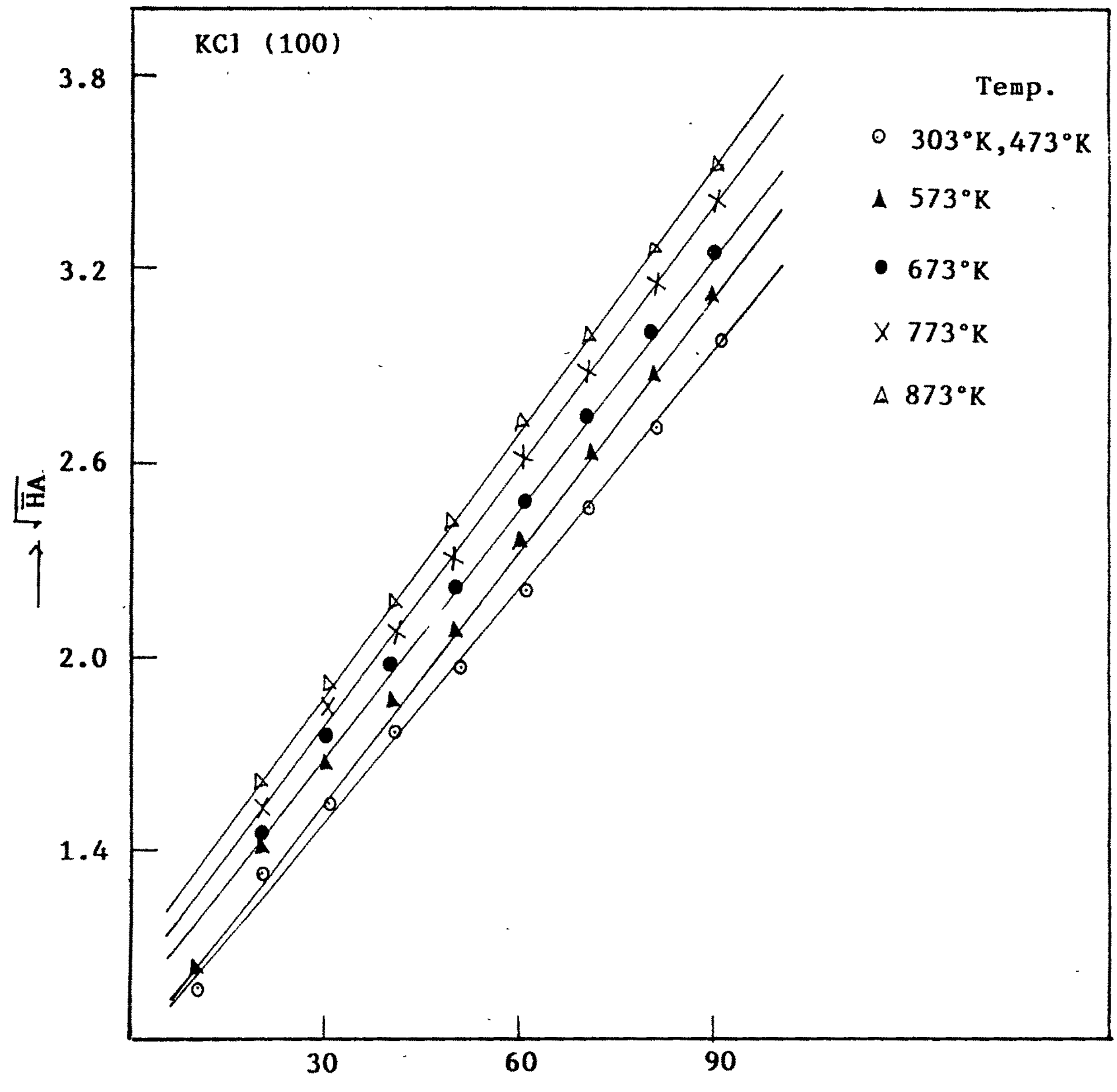


PLOT OF H_k Vs. A.

Fig. 6.1c.



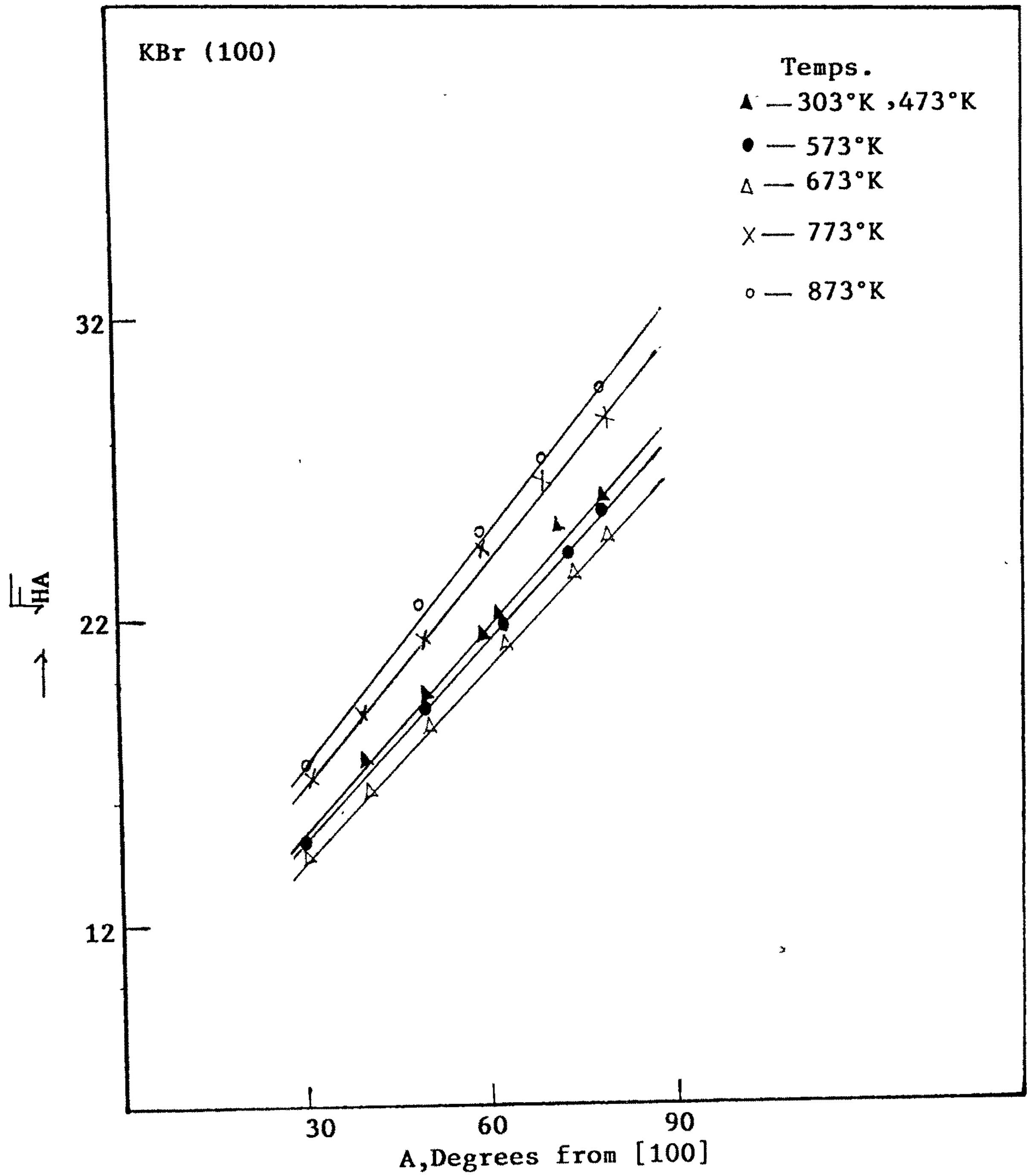
Plot of $\sqrt{HA}$ Vs A
Fig. No.: 6.2 a



A, Degrees from [100]

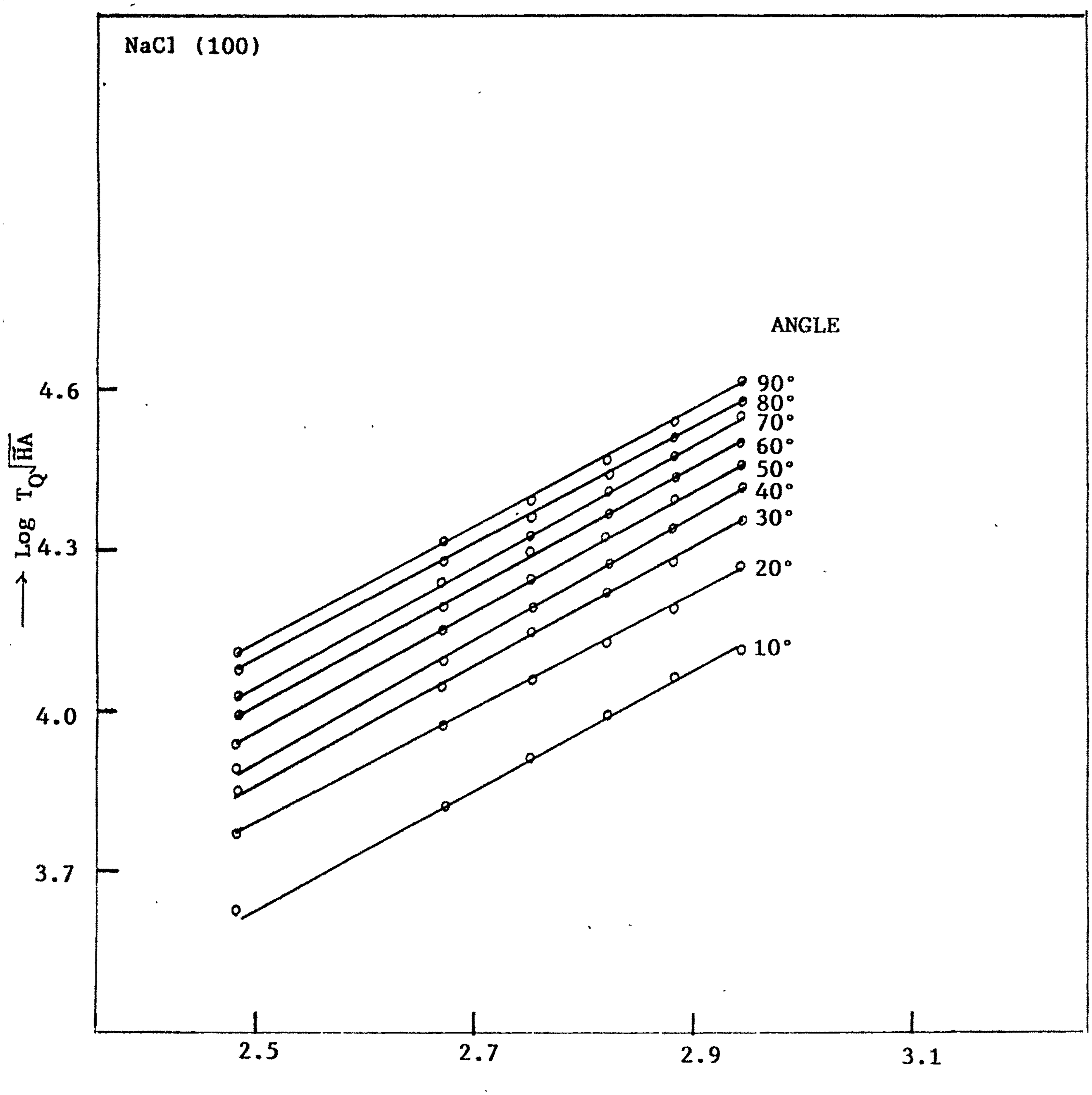
Plot of $\sqrt{HA}$ Vs A

Fig No.:6.2 b



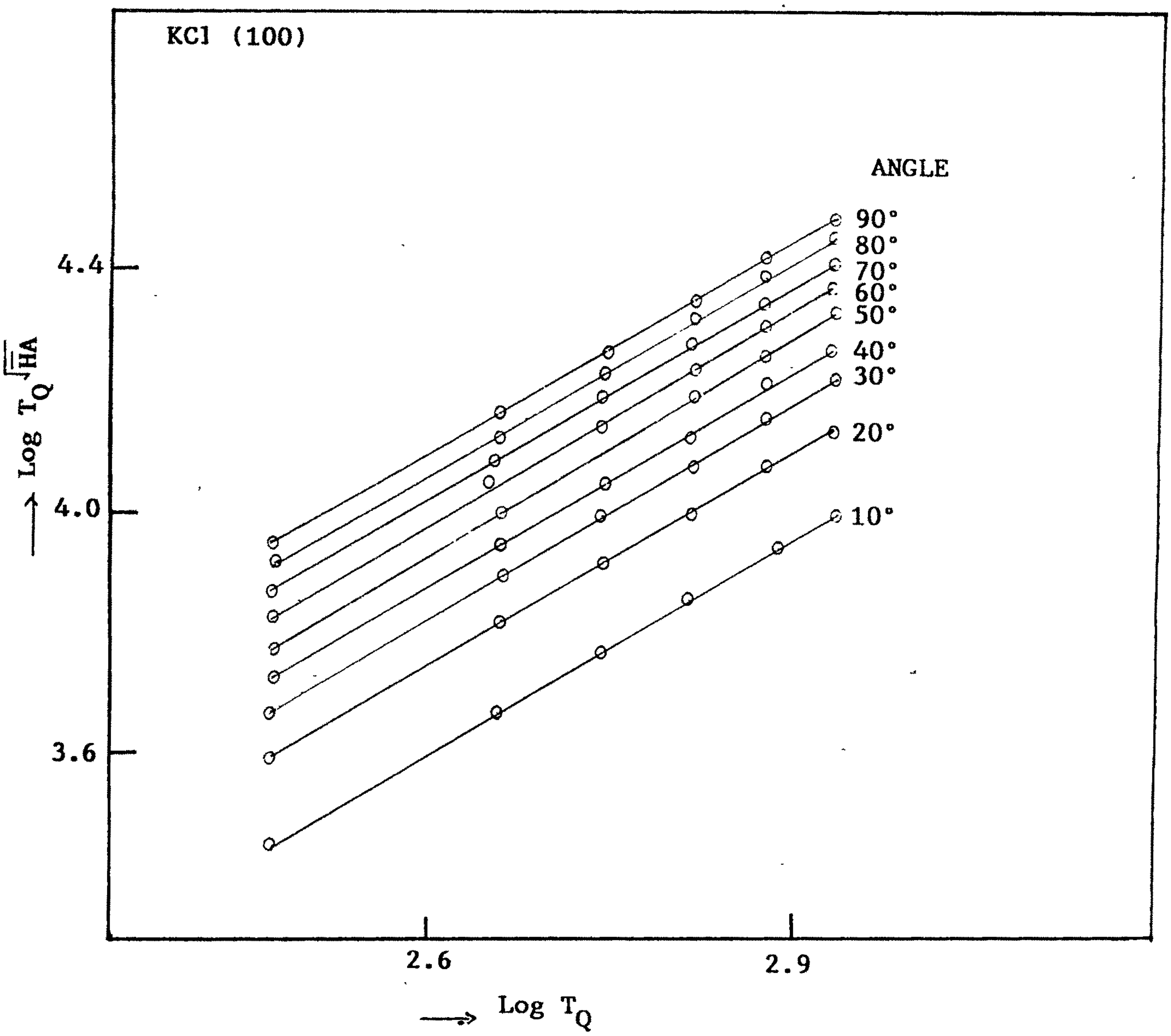
Plot of $\sqrt{HA}$ Vs A

Fig.No.: 6.2 c



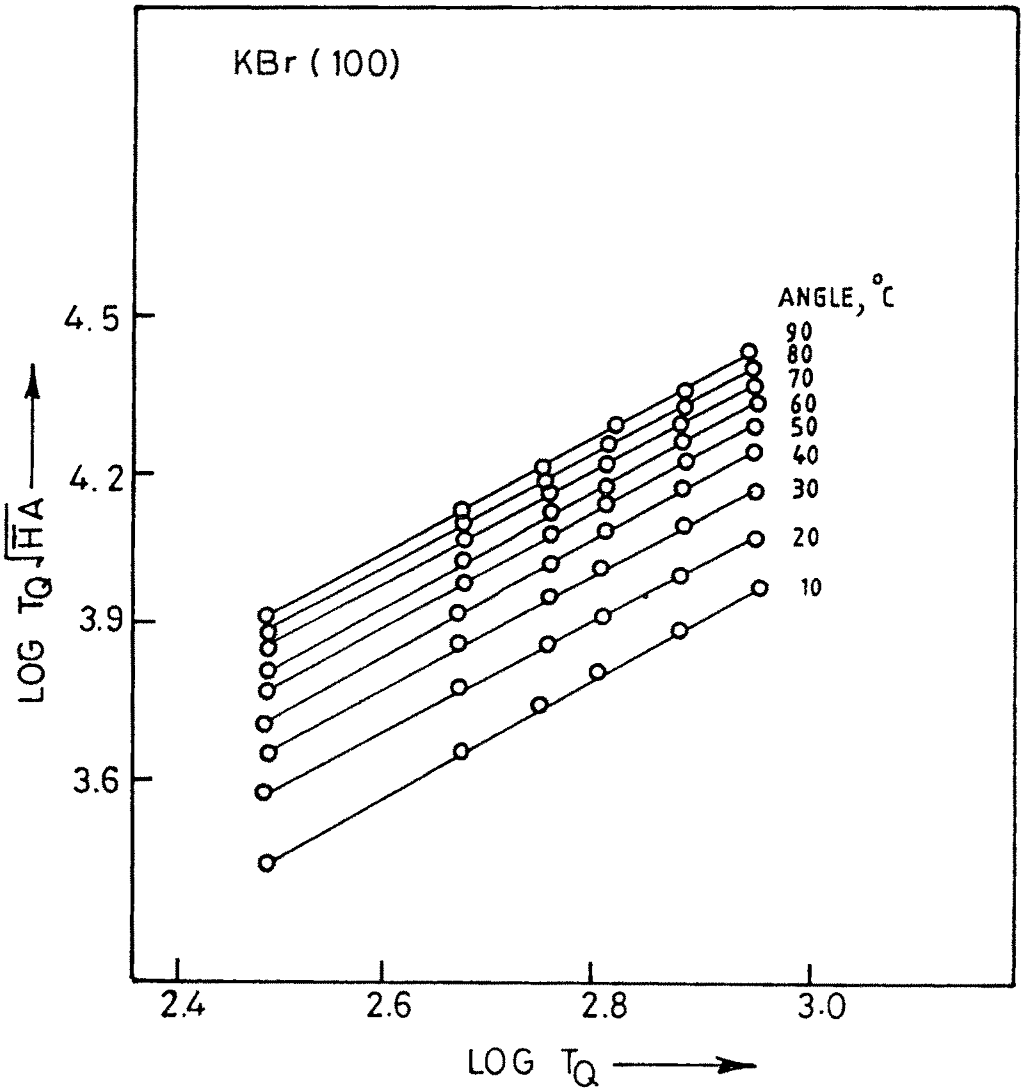
Plot of Log $T_Q\sqrt{\overline{HA}}$ Vs Log T_Q

Fig. No.: 6.3 a



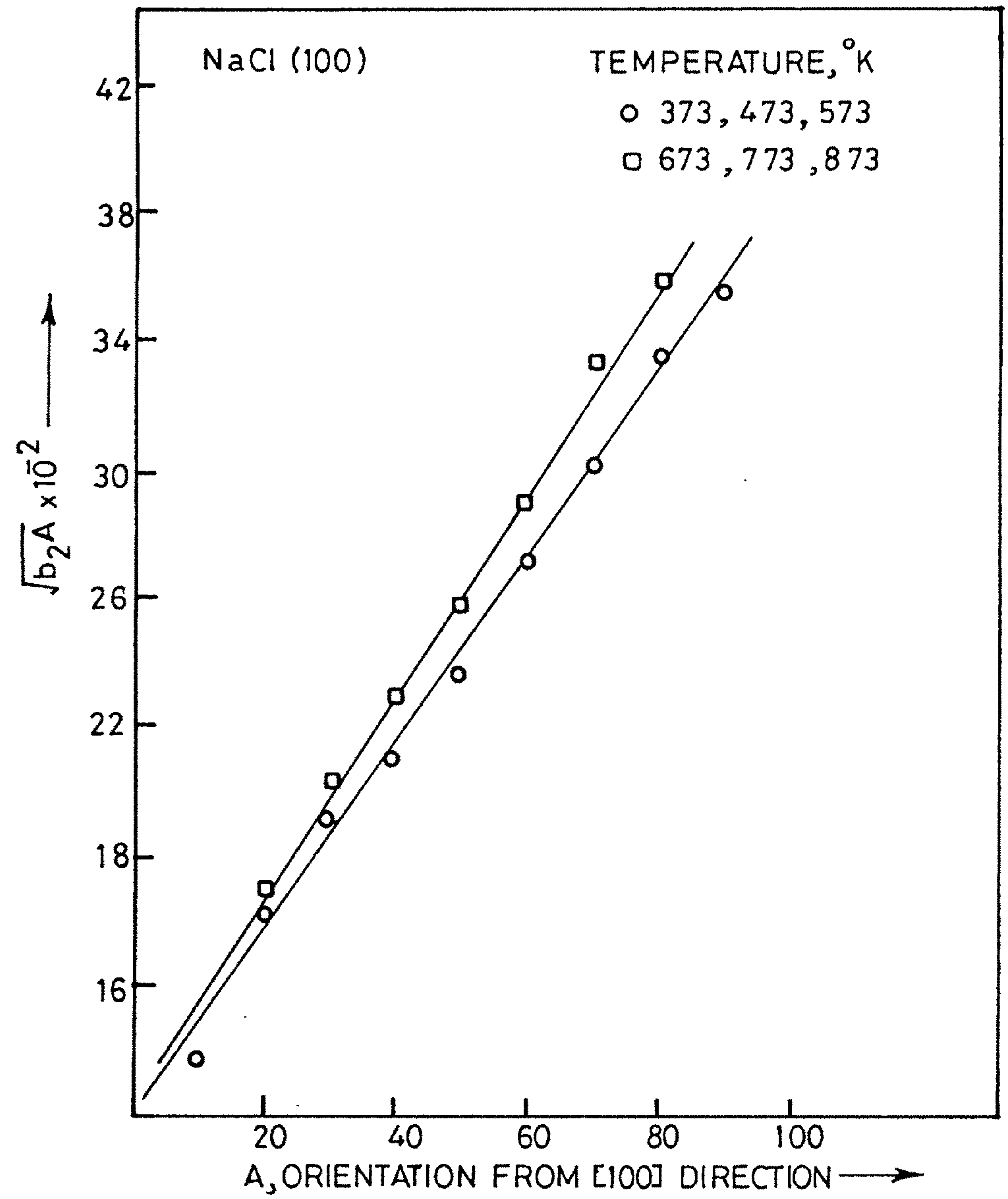
Plot of Log $T_Q\sqrt{HA}$ Vs Log T_Q

Fig.No.: 6·3 b



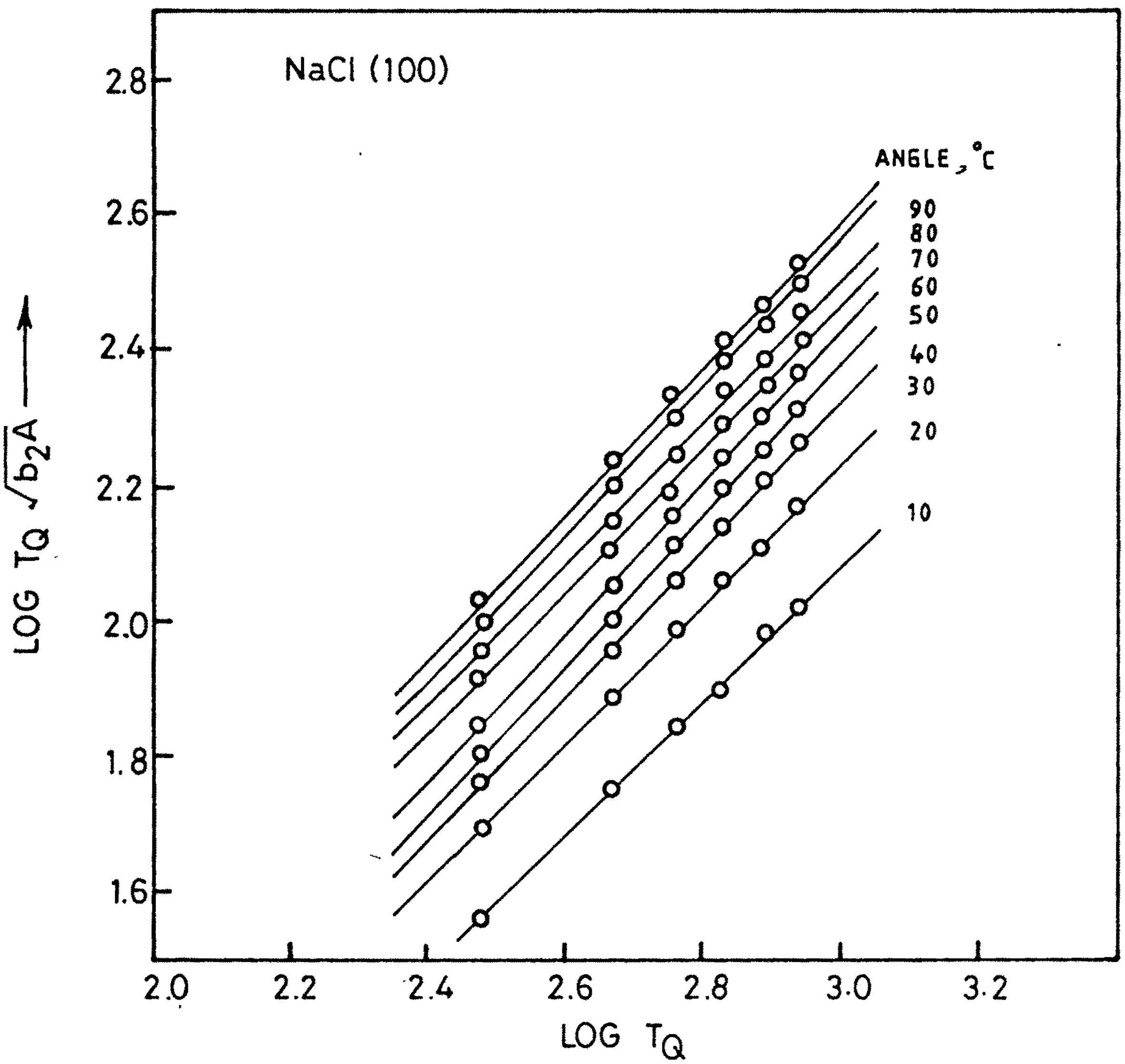
PLOT OF LOG T_Q $\sqrt{\overline{H}A}$ Vs. LOG T_Q

Fig. 6.3c.



PLOT OF $\sqrt{b_2 A}$ Vs. A

Fig. 6.4



PLOT OF LOG $T_Q \sqrt{b_2 A}$ Vs. LOG T_Q

Fig. 6.5

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