#### Chapter I

#### A Study of Heat-Transfer by free and forced convection

#### Introduction

It is well known that Heat transfer takes place by the processes of conduction, convection and radiation. We are concerned here exclusively with convection. In this process, the fluid and the heat contained in it move in a systematic manner so as to transport heat from one region to another. Heat transfer by convection means the exchange of heat energy between moving parts of the fluid or between these parts and other surfaces at a different temperature. Now convection can be of two types (i) natural or free convection and (ii) Artificial or forced convection. In natural or free convection the motion of the fluid is caused solely by gravity forces due to difference of density between the hotter and cooler parts. The lighter parts which generally are those of higher temperature, move upwards while the heavier ones fall down under the action of gravity. In forced convection the fluid motion is caused by forces independent of the temperature of the fluid, such as externally applied pressure differences as in the flow through a tube. Streams of air rising about warm surfaces like that of a hot metal cylinder are examples of free(natural) convection whereas the heating of a vessel by a current of hot air and the cooling of a surface with an electric fan are examples of forced convection.

A knowledge of the convective processes of heat transfer and the laws governing them is very important and useful in various branches of engineering like electrical, chemical, mechanical etc. It is finding an ever increasing application in nuclear engineering because in almost all nuclear reactors it is the process of forced convection by which heat is removed from the fuel. . 6

In the following sections we shall discuss the fundamental and basic equations of thermal convection and obtain from them the various dimensionless groups that are frequently used in our investigations of heat transfer by free and forced convection. The dimensionless groups will be first derived by means of differential equations and later by using the dimensional equations. The last section of this chapter will deal with the importance of these dimensionless groups.

#### Thermal convection:-

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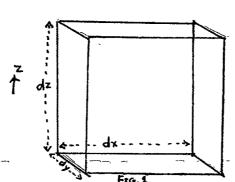
Thermal convection may be regarded as a combination of fluid flow and heat conduction, or it can be looked upon as a hydrodynamical flow accompanied by a thermal flow. In obtaining basic equations of thermal convection therefore, we must consider the equations of mechanical (hydrodynamical) flow and the equation of thermal conductivity.

### Equations of Fluid Flow:-

The mechanical flow of fluids is governed by Newton's Second law of motion Viz:-

Force=Mass X acceleration . . . . . . . . . . (1)

-2-



-3-

We can obtain an expression for the acceleration by considering the change of velocity  $v_z$  of a fluid in a path element dz and in time interval dt. Let us imagine a parallelopiped of volume dx dy dz at a distance x, y, z, from the origin. If we consider the fluid flow along the direction of z axis with velocity  $v_z$  at distance z and at time t, we can express the change in the velocity  $v_z$  as

$$dV_z = \frac{\partial V_z}{\partial t} dt + \frac{\partial V_z}{\partial z} dz - - - (2)$$

The first term in the above expression for  $dv_z$ represents the change of velocity at a fixed distance z in the time interval dt while the second term denotes the difference of velocity at time t between two points in the fluid separated by a distance dz

Dividing by dt throughout we get

$$\frac{dV_z}{dt} = \frac{\partial V_z}{\partial t} + V_z \frac{\partial V_z}{\partial z}$$
(3)

In the above equation the first term on the right,  $\left(\frac{\partial V_z}{\partial t}\right)$  may be called local acceleration, since it arises from a change of state of flow at the point z while the second term  $\left(V_z \frac{\partial V_z}{\partial z}\right)$  may be called convective acceleration because it arises from the fact that the particle under consideration comes from a neighbouring place where it had a different velocity. For simplicity we restrict ourselves to unidirectional flow and neglect the velocity components  $V_{\mu} \stackrel{k}{\to} V_{y}$  in the directions x and y.

Now the force in Newton's second law of motion is composed of three different types of forces (a) Inertia force  $F_g$  due to gravity (b) frictional force  $F_\mu$  due to viscosity and (c) dynamical force  $F_p$  due to pressure drop. Thus Newton's equation (1) applied to the motion of unit volume of the fluid in the direction of z-axis becomes,

$$f \frac{dV_z}{dt} = p \frac{\partial V_z}{\partial t} + p V_z \frac{\partial V_z}{\partial z} = F_q + F_{\mu} + F_{\mu} - - - -(4)$$

The inertia force  $F_g$  generally occurs as a buoyancy and according to Archimedes principle, the unit volume of fluid of density f in a medium of density  $f_o$  is subjected to a buoyancy force given by  $(f_o - f)g$ . Thus we get  $F_{c_s} = (f_o - f)g$  ---- (5) The difference  $(f_b - f)$  in a homogeneous fluid is

caused by a corresponding temperature difference( $T - T_0$ ) and we have

 $f_{o} = f \left\{ I + \beta (T - T_{o}) \right\} \text{ ov } (f_{o} - f) = f \beta (T - T_{o}) = f \beta \theta$ where  $\theta$  stands for  $(T - T_{o})$ . Thus we get

 $F_{G} = (f_{b}-P)g = \beta P \beta g - - - - - - (6)$ The frictional force  $F_{\mu}$  due to viscosity  $\mu$ . may be obtained as follows:-

The force due to viscous drag over an element of surface dy dz at distance x is ( $\mu \frac{\partial V_z}{\partial x}$  dy  $\cdot$  dz ) and that

at a distance (x + du) is  $\mu_{\partial x}^2 \left( V_z + \frac{\partial V_z}{\partial x} \right) dy dz$ Thus the net force over the element of surface  $(dy \cdot dz)$ will be  $\left( \mu \frac{\partial V_z}{\partial x} + \mu \frac{\partial^2 V_z}{\partial x^2} dx - \mu \frac{\partial V_z}{\partial x} \right) dy dz = \mu \frac{\partial^2 V_z}{\partial x^2} dx dy dz$ Hence for unit volume, the viscous force is given by

 $F_{\mu} = \mu \frac{\partial V_2}{\partial x^2}$  -----(7) The dynamical force  $F_p$  due to pressure drop of a fluid which flows in the direction z apparently is  $-\left(\frac{\partial P}{\partial z}\right)dz$  related to the length dz and the unit of the cross-sectional area perpendicular to z. For unit volume it becomes

$$F_{p} = -\left(\frac{\partial F}{\partial z}\right) - - - - - - (8)$$
ng for  $F_{g}$   $F_{\mu}$  and  $F_{p}$  from (6),(7),(8) in

Substituting for  $F_g F_{\mu}$  and  $F_p$  from (6),(7),(8) equation (4) we get

$$\rho \frac{\partial V_2}{\partial t} + \rho V_2 \frac{\partial V_2}{\partial z} = \beta \rho \theta g + \mu \frac{\partial V_2}{\partial x} = \frac{\partial \beta}{\partial z} - - - - (9)$$

# Equations of Heat Convection

If T be the temperature at a distance z and (T - dT)at a distance (z + dz), the quantity of heat absorbed by the volume element  $dx \cdot dy \cdot dz$  per second will be

$$\int dx \, dy \, dz \, q \, \frac{dT}{dt} = \int Q \, V_z \, \frac{\partial I}{\partial z} \cdot dx \, dy \, dz.$$
where  $V_z = \left(\frac{\partial Z}{\partial t}\right)$ . Now the amount of heat entering per second  
at z will be K dx  $dy \frac{\partial T}{\partial z}$  and the heat leaving per second  
at  $(z + dz)$  will be k dx  $dy \frac{\partial}{\partial z} \left(T - \frac{\partial T}{\partial z} dz\right)$  Hence the net  
amount of heat absorbed will be  $k \frac{\partial T}{\partial z^2} dx \, dy \, dz$ . Writing

-5-

the heat balance equation, we get,

Again the heat transfer by conduction and convection is given by the general expression

$$dQ = -k \frac{dT}{dz} dA = h \Delta T \cdot dA - - - - (")$$

Where z is the direction of heat normal to surface element and K is the thermal conductivity of fluid close to the surface. Here  $\frac{k}{2}$  is replaced by h, which is known as heat transfer coefficient. Equations (9),(10) and (11) may be regarded as the fundamental or basic equations of thermal convection from which we can derive the various dimensionless numbers (groups) which have been used in the present work.

## Derivation of dimensionless groups

According to the principles of similarity, the following equations should be valid for two similar systems, denoted by suffixes (1) and (2)

 $V_{z_{2}} = \omega V_{z_{1}}, x_{2} = \lambda x_{1}, z_{2} = \lambda z_{1}, \theta_{2} = \varphi \theta_{1}, T_{2} = \eta T_{1}$   $f_{2} = \sigma f_{1}, \mu_{2} = \eta \mu_{1}, g_{2} = \eta g_{1}, \beta_{2} = \theta f_{1}^{3}, C_{p_{2}} = \lambda e_{p_{1}}^{3}, \dots (2)$   $k_{2} = i k_{1}, h_{2} = \gamma h_{1}, t_{2} = \delta t, \text{ and } p_{2} = \alpha p_{1}^{3}$ 

Here the Greek letters  $\alpha, \delta, \omega, \lambda, \varphi, \eta, \sigma, \psi$  and the

letters n, r, b, s, and i without subscript designate constants. We rewrite the basic equations (9), (10) and (11) for systems (2) and (1) as follows.  $f_2 \frac{\partial V_{z_2}}{\partial t_2} + f_2 V_{z_2} \frac{\partial V_{z_2}}{\partial z_2} = f_2 \beta_2 \theta_2 g_2 + \mu_2 \frac{\partial V_{z_2}}{\partial z_2} - \frac{\partial \beta_2}{\partial t_2} - (i) \\f_2 \frac{\partial V_{z_2}}{\partial t_2} = k_2 \frac{\partial T_2}{\partial z_2} - (ii) \quad k_2 \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (iii) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_2) \int_{t_2}^{t_2} (t_2 - t_2) \frac{\partial T_2}{\partial t_2} = h_2 \Delta T_2 - (t_$ 

and

$$\begin{cases} \frac{\partial V_{z_{i}}}{\partial t_{i}} + f_{i}V_{z_{i}}\frac{\partial V_{z_{i}}}{\partial z_{i}} = f_{i}\beta_{i}\theta_{i}\theta_{i} + \mu_{i}\frac{\partial V_{z_{i}}}{\partial x_{i}^{2}} - \frac{\partial b_{i}}{\partial z_{i}} - \frac{(i)}{\partial z_{i}} \end{cases}$$

$$\begin{cases} f_{i}\frac{\partial V_{z_{i}}}{\partial t_{i}} + f_{i}V_{z_{i}}\frac{\partial V_{z_{i}}}{\partial z_{i}} = f_{i}\beta_{i}\theta_{i}\theta_{i} + \mu_{i}\frac{\partial V_{z_{i}}}{\partial x_{i}^{2}} - \frac{\partial b_{i}}{\partial z_{i}} - \frac{(i)}{\partial z_{i}} \end{cases}$$

$$\begin{cases} f_{i}\frac{\partial V_{z_{i}}}{\partial z_{i}} - \frac{\partial V_{z_{i}}}{\partial V_{z_{i}}} - \frac{\partial V_{z_{i}}}{\partial V_{z_{i}}} - \frac{\partial V_{z_{i}}}{\partial V_$$

Substituting from (12) in (13) we get  

$$\frac{\varpi\sigma}{\delta} \frac{\partial V_{z_{i}}}{\partial t_{i}} + \frac{\sigma\varpi}{\lambda} \frac{\partial V_{z_{i}}}{\partial t_{i}} = \sigma \frac{\partial \varphi r}{\partial t_{i}} \frac{\partial \rho}{\partial t_{i}} + \frac{\eta\omega}{\lambda^{2}} \frac{\partial V_{z_{i}}}{\partial t_{i}} - \frac{\chi}{\lambda} \frac{\partial h}{\partial z_{i}} - (i)$$

$$\frac{\sigma}{\lambda} \frac{\Delta \omega}{\lambda} \frac{\rho}{f_{i}} \frac{Q_{i}}{V_{z_{i}}} \frac{\partial T_{i}}{\partial z_{i}} = \frac{i}{\lambda^{2}} \frac{k_{i}}{\lambda^{2}} \frac{\frac{\partial^{2} T_{i}}}{\partial z_{i}^{2}} - \cdots - (ii)$$

$$\frac{\eta i}{\lambda} \frac{k_{i}}{\partial z_{i}} \frac{\partial T_{i}}{\partial z_{i}} = \eta \frac{\eta h}{\lambda} \Delta T_{i} - \cdots - (iii)$$

Now equations (15) must be indentical with equation (14) and this will occur only if the following relations are satisfied:-

$$\frac{\omega\sigma}{\delta} = \frac{\omega\sigma}{\lambda} = (\sigma \phi \varphi r) = \frac{\eta\omega}{\lambda^2} = \frac{\alpha}{\lambda} - \dots - (16)$$

$$\frac{\sigma - \lambda \omega}{\lambda} = \frac{i}{\lambda^2} - \dots - (17) \qquad \frac{i}{\lambda} = + \dots - \dots - (18)$$
Now from (16) 
$$\omega^2 = h \varphi r \lambda - \dots - \dots - \dots - (19)$$

and 
$$\frac{r\omega^2}{\lambda^4} = \sigma^2 \omega^2 \varphi^2 \omega^2$$
 where  $n^2 = \lambda^2 \varphi \sigma^2 b \left(\frac{\lambda \varphi r b}{\omega^2}\right)^{-(20)}$ 

From (19) and (20)  $n^2 = \lambda \phi \sigma^2 b$  \_\_\_\_ (21)

, ,

Substitutions from (12) gives  $\frac{\mu_{2}^{2}}{\mu_{1}^{2}} = \frac{l_{2}^{3}}{L_{3}^{3}} \cdot \frac{\theta_{2}}{\theta_{1}} \cdot \frac{f_{2}}{f_{1}^{2}} \cdot \frac{g_{2}}{g_{1}} \cdot \frac{\beta_{2}}{f_{1}^{3}}$ 

where  $\mathbf{k} = \mathbf{z}$  or  $\mathbf{x}$  stands for any considered length (e.g. diameter of cylinder or sphere)

$$\frac{L_2^3 \theta_2 f_2^2 g_2 \beta_2}{\mu_2^2} = \frac{L_1^3 \theta_1 f_1^2 g_1 \beta_1}{\mu_1^2} = \frac{L_0^3 \theta_1 \rho_2^2 g_1 \beta_1}{\mu_2^2} = \frac{L_0^3 \theta_1^2 \rho_2^2 g_1 \beta_2}{\mu_2^2} = \frac{L_0^3 \theta_1^2 \rho_2^2 g_1 \beta_1}{\mu_2^2} = \frac{L_0^3 \theta_1^$$

or 
$$\frac{\mu_2 C_{12}}{\mu_1 C_{p_1}} = \frac{k_2}{\kappa_1}$$
 or  $\frac{C_{p_2} \mu_2}{\kappa_2} = \frac{C_{p_1} \mu_1}{\kappa_1} = \frac{C_{p_1} \mu_1}{\kappa} = \frac{C_{p_2} \mu_2}{\kappa_1}$  Number)

Similarly from (12) and (18) we have  $\frac{k_2}{k_1} = \frac{h_2}{h_1} \text{ or } \frac{h_2 l_2}{k_2} = \frac{h_1 l_1}{k_1} = \frac{h_1^2}{k_1} = \frac{h_1^$ 

and substituting (12) in (16) we obtain

$$\frac{P_1 L_1}{\mu_1} V_{z_1} = \frac{P_2 l_2}{\mu_2} V_{z_2} = \frac{P l}{\mu} V = Re. (Rey molds Number)$$

Finally from (17) and (18) we get  $\psi = \dot{V}_{\lambda}$ and  $i = \lambda \sigma \Delta \omega$  Hence  $\psi = \sigma \Delta \omega$ Thus

$$\frac{h_2}{h_1} = \frac{P_2}{P_1} \frac{Q_{P_2}}{Q_{P_1}} \frac{V_{Z_2}}{V_{Z_1}} \text{ or } \frac{h_1}{P_1 Q_2 V_{Z_1}} = \frac{h_2}{P_2 Q_2 V_{Z_2}} = \frac{h}{P Q_2 V_{Z_2}}$$

$$= St. (Stanton Number)$$

### Derivation of dimensionless groups by the dimensional method

Let us suppose that for free convection the heat transfer coefficient h' is given by the relation:-

$$[h] = [HT]L \theta] - - - - (23)$$

The letters M, L, T, H and  $\theta$  are used here for the basic units of mass, length, time heat and temperature, respectively.

Substituting the dimensions of  $l, \theta, f, \mu, \beta, \zeta_{\mu}, g$ and K in equation (22) and using (23) we get,  $[HTL\theta^{-1-2-i}] = C \times L^{\alpha-3}f^{-1+\eta-\gamma} \times M^{f+i-m} \times T$ 

Hence,

$$-2 = a - \delta f - i + n - r$$

$$-1 = b - j - m - r$$

$$0 = b + i - m$$

$$-1 = -i - 2n - r$$

$$1 = m + r$$

From these equations we can express the five exponents a,,,, i, r in terms of the remaining three viz:j, m, n thus:

a = 3n - 1, b = j, f = 2n, i = m - 2n, r = 1 - m

Substituting these in equation (22) we get,

$$h = C l^{3n-1} \times O^{\dagger} \times \rho^{2n} \mu^{m-2n} \times p^{\dagger} \times q^{n} \times q^{n-2n}$$

$$or = \frac{hl}{k} = C \left(\frac{l^{3}_{pg}}{\mu^{2}}\right)^{n} \left(O^{3}_{p}\right)^{\dagger} \left(\frac{\mu q}{k}\right)^{m}.$$

Since  $q, \beta$  and  $\theta$  appear only in the form of a product in equations (6), & (9), they must have the same exponent. Therefore j = n and the above equation

becomes 
$$\frac{h\ell}{k} = C \left(\frac{L^3 r^2 g \beta^3 \theta}{\mu^2}\right)^n \left(\frac{\mu c_p}{\kappa}\right)^m$$

It can be easily seen that the terms in brackets have zero dimensions and they are known as Nusselt number  $\left(Nu = \frac{h\ell}{K}\right)$  Grashof number  $\left(f_r = \frac{\ell_r^3 \rho_r^2}{\mu_r^2}\right)$ , and Prandtl number  $\left(f_r = \frac{\ell_r \mu}{k}\right)$  as already indicated in the derivation by the method of differntial equations. Again let us assume that in the case of forced convection, the heat transfer coefficient 'h' is given by:-

$$h = c v^{a} l^{b} \mu^{b} k^{b} p^{c} c_{p}^{n} - - - - (24)$$

where C is a constant,  $\ell$  is the characteristic length,

e.g. diameter of a cylinder or sphere,  $\bigvee$  is the velocity of the fluid and the other symbols have their usual meaning. Thus we write, as in the case of free convection, for the dimensions of `k

$$[h] = [HT]^{2} \theta^{-1} - - - - (25)$$

Comparing indices

$$j+n = 1$$
  
 $-j-n = -1$   
 $-a-f-j = -1$   
 $a+b-f-j - 3m = -2$   
 $f+m-n = 0$ 

from these equations we can express the four exponents,  $\alpha$ , b, b, j in terms of the remaining two viz: m and n. Thus we have

$$a = m$$

$$b = m - 1$$

$$b = n - m$$

$$j = 1 - n$$

Substituting these in equation (24) we get

$$h = C \frac{k}{l} \left(\frac{pl}{\mu}v\right)^{m} \left(\frac{q_{\mu}u}{k}\right)^{n}$$
  
or 
$$\frac{hl}{K} = C \left(\frac{pl}{\mu}v\right)^{m} \left(\frac{q_{\mu}u}{k}\right)^{n}$$

The terms in brackets which have zero dimensions are known as Nusselt Number  $\left(Nv = \frac{k\ell}{\kappa}\right)$  Reynolds number  $(\text{Re} = \frac{fl}{\mu} v)$  and Prandtl Number  $(f_v = \frac{q_{\mu}\mu}{k})$  as already indicated.

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Finally we may assume that in the case of forced convection the force upon the body, placed in a fluid stream depends upon,  $\mu$ , f, l, and V where letters have their usual meanings. Thus we may write

$$F = c \mu^{a} \rho^{b} \ell^{c} v^{d}$$

where ( is a constant

Substituting the dimensions of the various quantities we get

$$MLT = C M \times L \times T$$

Equating indicies

$$a+b=1$$
  
$$a-3b+c+d=1$$
  
$$-a-d=-2$$

Expressing b, c, d in terms of a we have

Thus

or

 $F = C \left(\frac{\mu}{\ell v l}\right)^{a} p v^{2} l^{2}$  $\frac{F}{A\left(\frac{\ell v^{2}}{2}\right)} = 2C \left(\frac{\mu}{\ell v l}\right)^{a}$ where A stands for area having dimensions  $\mathcal{L}_{and}\left(\frac{\rho_{V\ell}}{\mu_{a}}\right)$ 

is the well known Reynolds number He. Now  $\frac{F}{A + tv^2/2}$  is known as the friction drag. Coefficient  $O_{f}$  in aerodynamics. We have further, Stanton number  $\text{St=O}_{f}/2$ or

sr. = 
$$\frac{F}{A f v^2} = C \left(\frac{\mu}{f l v}\right)^2 = C \left(\frac{Re}{r}\right)^2$$

Now  $C(Re)^{-\alpha}$  can be easily shown\* to be equal to  $\frac{N\nu}{Re}$  for which is noting but  $\frac{h}{\rho}c_{\rho}v$  as shown by the diffrential method given before.

### Advantages of dimensionless groups

In the discussion of heat transfer by either free or forced convection, the dimensionless groups like Nusselt Number, Reynolds Number, Grashof Number etc., will occur over and over again. One great advantage of these numbers lies in the fact that any involved magnitudes can be changed in such a way that the numerical value of the dimensionless groups remains the same. Hence the effect of changing each one of these magnitudes in a group can be easily calculated

\* We know  $\frac{N\nu}{Re \cdot P_r} = c'(Re)^{m-1}(P_r)^{n-1}$ Where c' is a constant. Now m is always less than one as found experimentally hence  $m-1 \perp 0$  say equal to -a and for a given fluid  $(P_r)^{n-1}$  is constant = c". Thus  $St = \frac{N\nu}{Re \cdot P_r} = c(Re)^{-\alpha}$ Where  $C = (c' \times c'') = Constant$ . by determining experimentally the influence of the variation of only one magnitude in that group. Equations of dimensionless groups are therefore very useful and they lead to a saving of considerable amount of work and time. For instance let us consider the well known dimensional equation of free convection:

$$Nv = C \left(\frac{4r}{k}\right)^{m} \left(\frac{r}{r}\right)^{n}$$

$$av \quad \frac{k\ell}{K} = C \left(\frac{\ell^{3}r^{3}g\theta}{\mu^{2}}\right)^{m} \left(\frac{4\mu}{K}\right)^{n}$$

If we wish to find the change in the coefficient of heat transfer 'h' in the case of a sphere of diameter 'l' for different variations of the involved magnitudes, we can proceed by changing first 'l' taking say, four different sphere diameters, then conducting experiments with each of these and with fluids of four different thermal conductivities 'k' further varying four times and repeating the process with each of the seven independent variables (, k, f, f, O, pand 4 This would require  $4^7 = 16384$  tests. On the other hand, if we use the dimensional equation given above, it would be sufficient to conduct experiments with four different values of the Grashof number (Gr.), each with four values of the Prandtl number Pr. i.e.  $4^2 = 16$ experiments in order to get about the same result as with more that 16000 tests by using the direct method. In actual practice however more than the minimum of

16 tests may be necessary. Again all the variation of the seven independent variables referred to above would not be found necessary or even possible. Even then, the use of these dimensionless groups leads to an enormous saving of work and time.

Quality	: Symbol	Dimensions in terms of Mass Length Time Heat & Temperature (M)(L) (T) (H) (2)
Uharacteristic length	: L	נאז <i>נוז נדן נאן נפז</i>
Temperature difference between surface and	, <del>O</del>	
fluid Density of fluid	f	
Viscosity of fluid	м	
Coefficient of thermal expansion of fluid	ß -	
Specific heat of fluid at constant pressure	Cp	
Acceleration due to gravity	q	
Thermal conductivity of fluid	K	ראש באשר באשר באשר באשר באשר באשר באשר ב
Forced velocity of fluid.	V	[M] LLJ LTJ LTJ L
Heat transfer coefficient	h	

Dimensions of quantities used in the chapter.

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