Chapter II.

HEAT TRANSFER BY FREE CONVECTION

Introduction

Whenever heat is transferred by natural or free convection from a solid body to a fluid it is generally, believed that a functional relationship of the from $Nu = f(Gr \times Pr)$ exists between the Nusselt number $\left(Nu = \frac{H\ell}{k\Delta\theta}\right)$ and the product of the Grashof number $\left(Gr = \frac{\beta \ell^3 \rho^2 \Delta\theta g}{\mu^2}\right)$ and the Prandtl number $\left(Pr = \frac{C_p \mu}{k}\right)$ Recently however, some doubts have been expressed regarding the generality of this relationship (Epelboin, Pham, and Vapaille,1956) and it was therefore thought desirable to make some additional and accurate measurements of heat transfer for different ranges of (GrxPr.). Again very little work has been done so far, in the range of (GrxPr.) $< 10^{-3}$ and therefore the following experiments were undertaken in the range of (GrxPr.) below 10^{-3} to test the generality of the above relation.

Theory

We know that in any problem of thermal convection, the heat transfer H can be denoted by a relation between the dependent variable Nu(the dimensionless group $Nu = \frac{H\ell}{k\Delta\theta}$) and the independent variable —

— dimensionless groups Re, Pr., and Gr., where Nu = Nusselt number, Re = Reynolds number, Pr = Prandtl number and Gr = Grashof number. (Fishenden & Saunders 1950). We can thus write

$$\frac{H l}{k \Delta\theta} \propto \left(\frac{\rho l}{\mu} v\right)^{n_1} \left(\frac{c_p \mu}{k}\right)^{n_2} \left(\frac{\beta g \Delta\theta\, l^3 \rho^2}{\mu^2}\right)^{n_3}$$

Here the quantities within the three brackets on the right hand side represent Re, Pr, and Gr, respectively. $\frac{H}{\Delta\theta} = h$ is known as the heat transfer coefficient. In the case of free convection, the motion being entirely due to heat, the forced velocity v may be dropped out. Hence putting $n_1 = 0$ in the above expression, the Reynolds number Re. is eliminated so that Nu becomes dependent on Grashof number Gr. and Prandtl number Pr. only. Of course v = 0 does not mean that the fluid has no velocity. It simply means that the forced velocity is zero and that the velocity of the fluid is not controlled from outside the system. Thus we have

$$Nu = C\,(Gr)^{n_3}(Pr)^{n_2}$$

Where C is a constant.

Theory indicates that unless Pr. is very small, only the product (Gr×Pr.) = $\frac{\beta g \Delta\theta\, l^3 \rho^2 c_p}{k\mu}$ need be considered provided the fluid currents are slow enough for inertia stresses to be negligible compared with viscous stresses. It is in fact found that for stream line flow the results are well expressed in terms of (Gr×Pr.).

We can therefore write

$$Nu = C\,(Gr \times Pr)^{n}$$

<u>Experimental arrangement</u>

The apparatus (fig.1) consists of a thin metal wire (Platinum, Copper etc.) stretched horizontally between two vices fixed at the lower ends of two copper rods. The streched wire was maintained at a convenient depth in a glass vessel full of the liquid under examination.

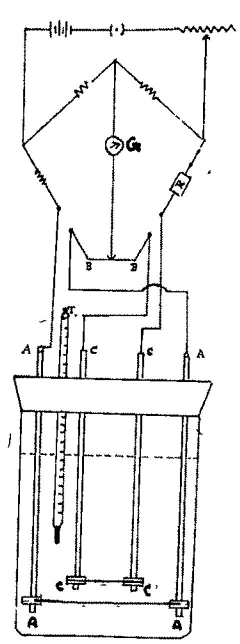

Fig 1

Now to find the value of the Nusselt number we have to determine experimentally the amount of heat developed in the wire by the passage of an electric current through it and also the difference of temperature $\Delta\theta$ between the wire and the fluid surrounding it. The heat developed in the wire by a current I amperes is given by $\frac{I^2R}{J}$ where R is the wire resistance in ohms and J is the mechanical equivalent of heat. To measure the resistance accurately, the wire was included in the third arm of a Wheatstone bridge, equal resistance coils of 100 ohms each being inserted in arms 1 and 2 (fig.I). In the beginning a very small current of the order of 0.01 amp. was passed through the experimental wire A.A producing some deflection in the galvanometer G. The null point was then obtained by adjusting the resistance R in the fourth arm keeping the slider in the middle of the bridge wire B B. A very small additional resistance ΔR was then introduced in the fourth arm. This naturally disturbed the balance and caused some deflection in the galvanometer. The current passing through the wire was then slightly increased thus bringing about a slight increase in the resistance of the wire until the balance was again restored and the null point was obtained on the sensitive galvanometer G. The whole arrangement was then allowed to remain in the same condition so that the current passing through the wire heated the latter to a higher temperature and the heat transfer between the wire and the surrounding fluid went on freely. After some time the heat balance was established and this was indicated by a steady deflection of the galvanometer G. The correction due to the cooling effect at the ends of the experimental wire was eliminated by inserting a compensating wire C C in the fourth arm. The effective length of this wire for the maximum temperature was estimated and it was found to be always less than 1.5 cm. For finer experimental wires the effect was still less and a thick copper wire could be used for the compensating wire C C. The difference of temperature $\Delta\theta$ between the wire and the surrounding fluid was then calculated from the relation $\frac{\Delta R}{R_0 \alpha} = \Delta\theta$ where R_0 is the resistance of the wire at $0^\circ C$ and α is the temperature coefficient of resistance of the wire. The current passing through the circuit was measured by the fall of potential across a standard resistance of one ohm. The value of α was determined by measuring R at room temperature by means of Carey Foster's low resistance bridge. The increment of resistance ΔR was obtained by using a uniform thick Eureka wire of known length in series with the fixed resistance R in the fourth arm. The resistance per unit length of this wire was first determined by a separate experiment using 100 cm. of this wire and finding its resistance on a Carey Foster's low resistance bridge. A correction to $\Delta\theta$ was applied by inserting a sensitive thermometer in the body of the liquid and noting the small rise of temperature. This rise was deducted from the value of $\Delta\theta$ obtained from the relation $\Delta\theta = \frac{\Delta R}{R_0 \alpha}$

The Nusselt number could then be obtained by the formula

$$Nu = \frac{HL}{K\Delta\theta} = \frac{I^2 R}{4.18 \times \pi \times K \times l \times \Delta\theta}$$

Where L is the characteristic length (here, the diameter) of the experimental wire, l is the length of the experimental wire, R its resistance at the temperature at which heat transfer occurs and K is the coefficient of thermal conductivity of the liquid and H is the amount of heat in Calories developed per unit area per second. The values of the Grashof number $\left(Gr = \frac{\beta g \Delta\theta l^3 \rho^2}{\mu^2}\right)$ and the Prandtl number $\left(Pr = C_p\mu/k\right)$ could be found by knowing $\Delta\theta$ and thermal constants of the liquid like the coefficient of cubical expansion β, the coefficient of thermal conductivity K etc.

Results and Discussion.

In figure 2, the values of log Nu against ~~log~~ log(Gr×Pr.) are exhibited using the following liquids: Olive oil, Paraffin oil, Glycerin, Turpentine, Toluene, Benzene, CCl_4 etc., for the following temperature

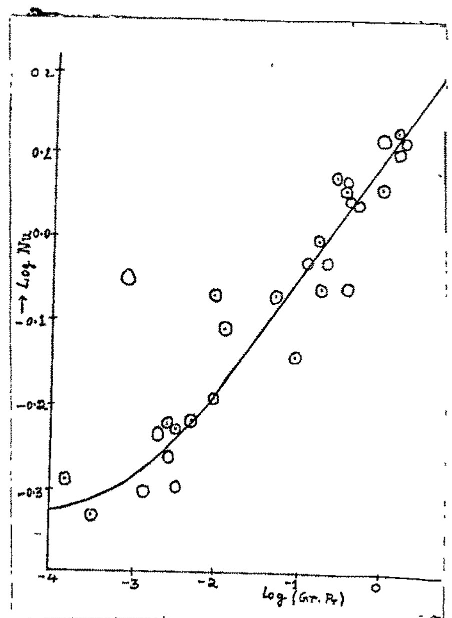

Fig 2

differences $\Delta\theta = 6^{\circ}, 11^{\circ}, 22^{\circ}, 33^{\circ}C$. The value of the slope was found to be 0.0919. In figure 3, a plot of Nu against log (Gr×Pr.) is given.

Though radiation is an important factor in the loss of heat from a surface, where the fluid ~~moments~~ movements are due to natural convection, it can be safely neglected in the case of thin wires; for, the value of Nu is so large that substantially the entire heat loss is due to free convection. Even with the large temperature differences, the radiation loss is 100 times smaller than that due to convection.

Our curve compares favourably with those of MacAdams and Rice (1942) in the range of Gr×Pr. $> 10^{-2}$,

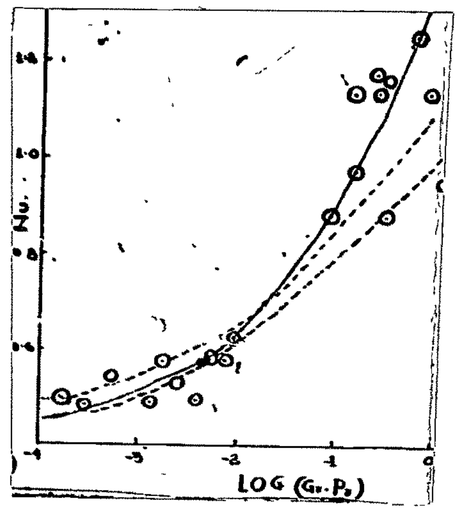

Fig 3

but below Gr×Pr.= 10^{-2}, it fits nicely with that of Senftleben (1951). Max Jacob (1953) has shown that for values of (Gr×Pr $\sim 10^{-5}$), the Nusselt number seems to approach the constant value of 0.4. In our curve also Nu tends towards the constant value of 0.4. From this we can conclude that below Gr×Pr.=10^{-5}, the mode of heat transfer is conduction rather than convection. ($h \simeq 0.4\frac{K}{L}$).

In the case of forced convection, when we plot log Nu, against log Re (where Re, Reynolds number, is a dimensionless group), it is found that most of the points fall almost on a straight line in a limited region. However, if the region taken is sufficiently large, there is some deviation from linearity, The bend in the resulting curve is attributed to the change in flow patterns of the fluid. Now Gr.(Grashof Number) plays a role in free convection similar to that of Re. (Reynolds number) in forced convection. Pr. (Prandtl number) which is a mere property of the substance, has only an additional influence. It appears therefore that the deviation from linearity in the curve obtained by plotting log Nu against log (GrxPr.) is due to the change in the flow pattern of the fluid for values of (GrxPr.) $< 10^{-3}$.

If we represent the relation between Nu and (GrxPr.), by the equation $Nu = C(Gr \times Pr)^n$, we get the values of C and n as shown in table I. Sigurds Arajs and Sam Legvold (1958) have performed experiments in the low range of (GrxPr.) and have arrived at the validity of the same fundamental relationship $Nu = f(Gr \times Pr)$

Table I.

Range of Gr.Pr.	Author	n	C
10^{-4} to 10^{0}	H.S.Desai	0.092	1.057
"	MacAdam	0.086	1.18
"	Nusselt	0.075	0.94
"	Senftleben	0.094	0.96
"	Rice	0.091	1.10
"	Hermann	0.092	1.06
		0.089	0.89
"	V:d:Hegge Zijnen	0.081	0.96
"	Kyte Madden Pirel	0.108	0.91

Table 2.

Difference of temperature = 6.0°C

Liquid	Nusselt Number	Gr.Pr.	log.Nu.	log.Gr.Pr.
Olive oil	0.583	1.66×10^{-3}	-0.204	-2.779
Paraffin Oil	0.469	1.35×10^{-3}	-0.304	-2.869
Turpentine	0.893	9.56×10^{-2}	-0.149	-1.019
Tolune	1.88	0.23	0.074	-0.630
Benzene	1.15	0.170	0.061	-0.769
Glycerine	0.50	1.47×10^{-4}	-0.293	-3.832
Carbontetrachloride	0.853	0.21	-0.068	-0.476
M.Alcohol	0.724	2.35×10^{-2}		-1.628

Table 3.

Difference of temperature = 11°C

Liquid	Nusselt Number Nu.	Gr.Pr.	log.Nu.	log.Gr.Pr.
Olive oil	0.499	3.27×10^{-3}	-0.302	-2.48
Paraffin	0.540	2.65×10^{-3}	-0.268	-2.57
Turpentine	0.918	0.171	-0.037	-0.767
Tolune	1.15	0.291	0.961	-0.536
Benzene	1.19	0.304	0.078	-0.516
Glycerine	0.466	2.88×10^{-4}	-0.33	-3.539
Carbon-tetra-chloride	1.120	4.43×10^{-1}	0.049	-0.353
M.Alcohol	0.845	5.07×10^{-2}	-0.073	-1.294

Table 4.

Difference of temperature = 22°C

Liquid	Nusselt Number Nu	Gr.Pr.	log.Nu.	log.Gr.Pr.
Olive Oil	0.588	5.60×10^{-3}	-0.230	-2.3517
Paraffin Oil	0.586	4.94×10^{-3}	-0.224	-2.30
Turpentine	0.298	0.35	-0.453	-0.45
Tolune	1.152	0.83	0.061	-0.071
Benzene	1.269	0.639	0.103	-0.19
Glycerine	0.545	5.08×10^{-4}	-0.263	-3.29
Carbon-tetra-chloride	1.320	9.08×10^{-1}	0.120	-0.04
M.Alcohol	0.925	1.01×10^{-1}	-0.033	-0.99

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