CHAPTER - V

COBB-DOUGLAS PRODUCTION FUNCTION

1. About the Function :

As noted at the end of Chapter III, given the elasticity of substitution between say, labour and capital, we can always examine the relationship between wage share on the one hand and variables like capital/labour ratio and relative prices of factors of production on the other hand for a given industry. The constant elasticity of substitution (CES)[1] production function (under certain assumptions) provides a formula to estimate the (constant) value of elasticity of substitution between labour and capital. Before we estimate such production function it would not be out of place first to examine and test the validity of the most celebrated production function, namely, the Cobb-Douglas production function where elasticity of substitution is always equal to unity.[2]

1. Kenneth J.Arrew; Hollis B.Chenery, Bagicha Minhas and Robert M.Solow: "Capital Labor Substitution and Economic Efficiency", Review of Economics and Statistics, Aug., 1961.

2. This is true even if we do not specify constant returns to scale (see mathematical note given in Appendix). R.G.D. Allen's proof starts with the assumption of constant returns to scale, see his Mathematical Analysis for Economists, For the earlier version of the Cobb--Douglas production function, see Cobb and Douglas, "A Theory of Production", American Economic Review, March, 1928.

The Cobb-Douglas type of function has been tried throughout the world in the case of manufacturing sector. The choice in favour of this function appears to have been due to many interesting properties of the function (see mathematical note in the Appendix). While testing $V=bL^{\alpha}C^{1-\alpha}$ (V = product, L=Labour, and C= capital) for the American manufacturing industries during the first quarter of 20th century, the authors of the production function estimated the value of α, the share of labour (also the production elasticity with respect to labour), leaving the share of capital to be determined by $(1-\alpha)$. Interestingly enough, the product estimated for each year did not show a significant divergence from the actual product. The actual share of labour also did not diverge much from the value of the labour exponent.

Following the criticism by David Durand[3], Cobb and Douglas later on fitted the modified version of the function, namely, $V = bL^{\alpha}C^{\beta}$, making the exponent of capital (β) to be determined independently, so as to give the value of $(\alpha+\beta)$ either greater or less than unity, indicating the presence of either increasing or decreasing returns to scale respectively, as against the assumption of constant returns to scale under $\beta = 1-\alpha$.

3. D.Durand: "Some thoughts on Marginal Productivity with special reference to Professor Douglas Analysis", Journal of Political Economy, Dec.1937, pp.740-758.

Apart from the aggregation and measurement problems of the variables, the Douglas type of production function is attacked on various grounds. The first objection against the function is that it suffers from the inter--correlation among different factors of production. Such multi-collinearity is defined as the general problem which arises when some or all of the explanatory variables in a relation are so highly correlated with one another that it becomes very difficult, if not impossible, to disentangle their separate influences and obtain reasonably precise estimate of their relative effects.[4] However, as Klein points out, the multicollinearity will create a problem only when the degree of interrelation between say capital and labour is so high that it exceeds the overall degree of multiple correlation among all variables simultaneously. "Production functions with overall correlations much in excess of 0.95, as often occur in practice, can be well estimated with inter-correlations between labor and capital as high as 0.8 to 0.9. If these functions were not well-estimated, we would tend to find high sampling errors of the estimated coefficient ..... It does not appear that the Douglas type of research is open to the charge that the estimates are plagued by multicollinearity".[5]

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4. J.Johnston: Econometric Methods, (New York: McGraw-Hill, 1960), p.201.

5. L.R.Klein, An Introduction to Econometrics,(New Delhi: Prentice-Hall,1969),p.101.

Another attack on the function is in connection with the identification problem. The Cobb-Douglas production function, they argue, is not capable of identifying when considered in relation to the cost function, namely, $V=x_1P_1 + x_2P_2$ (P_1 and P_2 being the prices of the factors x_1 and x_2 respectively) under equilibrium conditions.[6] To meet the objection, what is required is, for example, the shifts in the function. Without such shifts it would not be possible to tell whether the plotted points represent a production function of the Cobb-Douglas type or the cost function. However, an introduction of a new variable representing say, technical change (the time element in our time-series analysis) helps overcome the difficulty of identification mentioned above.

The Cobb-Douglas production function is also criticised on the ground that the variables which appear in the function are all endogenous variables and hence they are subject to simultaneous determination. The implicit assumption of the function is that output depends

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6. J.Marschak and W.H.Andrews: "Random Simultaneous Equations and the Theory of Production": Econometrica, July-Oct.1944. See also M.Bronfenbrenner: "Neoclassical Macro--Distribution Theory" in The Distribution of National Income, Proceedings of a Conference held by International Economic Association, ed.by Jean Marchal and Bernard Ducros, (London: Macmillan, 1968), p.486, and E.H.Phelps Brown: "The Meaning of the Fitted Cobb-Douglas Production Funbtion", Quarterly Journal of Economics, Nov., 1957.

on inputs and not vice-versa. As Klein points out "The assumption of this unique line of causation, when it is, infact, not the case, will lead to statistical bias in the estimates of parameters..... that Douglas' results are open to the pit falls of single equation biases", but, as he further points out, "we are not able to judge the seriousness of this charge".[7]

2. Various Estimates :

Based on the Douglas type of production function, a number of time-series and cross-section studies for manufacturing have been carried out in the countries like the United States, Canada, Australia, New Zealand, South Africa and others. Most of these studies agree with the results derived by Cobb and Douglas. The following Table V-1 displays some of the estimates mentioned above :

Table V-1

Different Estimates of Cobb-Douglas Production Function.

Country/State	Period covered	Value of labour exponent	Value of capital exponent
1	2	3	4
U.S.A.	1899-1922(time series)	0.63	0.30
U.S.A.	1889-1919(average of six cross-section studies)	0.63	0.34
Australia	1912-1937(average of nine cross-section studies)	0.60	0.37
South Africa	Cross-section study.	0.65	0.37
Victoria	1907-1929(time-series)	0.84	0.23
New South Wales	1901-1927(time-series)	0.78	0.20

Source: L.R.Klein: An Introduction to Econometrics,(New Delhi: Prentice-Hall,1969)p.95.

7. L.R.Klein, Op.cit.,p.102.

In the case of Victoria and New South Wales, the labour exponent has been on the high side while that of capital on the low side. This has happened because time-series data in these two states were not adjusted for the trend. The presence of technological progress would introduce a bias in the output and other variables over time, unless the trend is extracted from the variables. Douglas' estimates of the exponents show that either they add up to one or give a figure slightly less than one; this indicates the presence of either constant returns or decreasing returns to scale.

In India very few attempts have been made to estimate the factor shares and test the validity of the Douglas type of production function.[8] The following Table V-2 summarises such estimates of factor shares in India. Murti and Sastry's study is based on the balance-sheets of the joint-stock companies, and hence it considers firms rather than industrial aggregates. They have used labour input as the value of labour in terms of wages and salaries. Other

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8. See, for example, J. Tewari: "Productivity of Capital Investment in U.P.". Bulletin of the International Statistical Institute, Vol.XXXIII, Part III, 1951; M.M. Dutt, "The Production Function for Indian Manufactures", Sankhya, Vol.15, Part IV, 1955; R.J. Bhatia: "The Production Function for Indian Manufactures, 1948", Journal of Bombay University, January, 1954; V.N. Murti and Sastry V.K., "Production Functions for Indian Industry", Econometrica, Vol. 25, No. 2, April, 1957.

studies have considered employees as labour input. Bhatia has also examined averages per firm in each industry.

Table V-2

Estimates of Cobb-Douglas Production Function in India.

Year	Author	Exponent of labour	Exponent of capital
1	2	3	4
1946	M.M.Dutt	0.77	0.23
	J.N.Tewari	0.66	0.31
1947	M.M.Dutt	0.57	0.50
	J.N.Tewari	0.68	0.47
1948	R.J.Bhatia	0.59	0.44
	M.M.Dutt	0.67	0.26
1951	V.N.Murti & V.K.Sastry	0.59	0.40
1952	V.N.Murti & V.K.Sastry	0.53	0.50

Source: V.N.Murti and V.K.Sastry: "Production Functions for Indian Industry", Econometrica, April,1957,p.212.

The greatest drawback of all these studies, however, lies in the fact that they have used the book values of fixed capital to arrive at the measure of capital input. The book value of fixed assets, as noted in Chapter IV, does not reflect the true value of capital. The present method of depreciating the fixed assets in fact, does not meet the requirements for economic analysis. Murti and Sastry's study, however, could be considered relatively superior, as it is based on firm to firm variations within an industry.

3. Scope of the Present Analysis :

The analysis here attempts to estimate the Douglas type of production function by considering both cross-section and time series data. The cross-section study refers to the large-scale (two-digit) manufacturing industries for the year 1964, the data for which are derived from the Annual Survey of Industries. The time-series study relates to the total of 28 ASI industries which are comparable to CMI industries (see section-3 Chapter II). The period covered is 1946 to 1964.

The product used in the analysis refers to the gross value added.[9] The physical measure of output would be a superior variable. But in the absence of availability of such figures, we have to rely on value added figures. The labour input when defined as the amount of labour which a unit of currency can buy, turns out to be the total wage bill accruing to the workers (or wages and salaries accruing to the employees). Total number of workers (or employees), where one year is the length of time, is another measure which can be used for labour input. Since the data on man-hours worked (where one hour is the length of time) are available from the Annual Survey of Industries, they are used as labour

9. Since we consider capital gross of depreciation as representing the true value of capital as a factor of production (as per discussion in Chapter IV), the relevant concept of output would also be gross of depreciation i.e. gross value added.

input for the cross-section analysis. "In general, when man-hour measures are available, they are preferable from an econometric point of view.".[10] The number of man-hours worked during a particular year is calculated by multiplying the number of workers employed in each shift by the number of hours in the shift and aggregating the products for all shifts on all the working days in the year.

The total of fixed capital as reported in different sources of industrial statistics is actually a combination of many items like building, improvement to land and other construction, plant, machinery, tools, transport equipment and other fixed assets.The components of working capital are materials, stores, fuels, semi-finished goods byproducts etc. Of these two broad categories of capital, as noted in Chapter IV, working capital or inventories do not require any adjustment. But the fixed assets being accumulated over long period and being subject to accumulated depreciation need to be adjusted before their use as fixed capital. It is the gross values (replacement values) of fixed capital (adjusted for prices) which have been used in the present analysis.

One can arrive at the total value of productive capital by adding up the value of inventories to the

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10. L.R.Klein: Op.cit.,p.85.

adjusted value of gross fixed capital. However, in the present analysis, the inventories are treated as a separate factor of production. The production function fitted in its traditional way without the breakdown of detailed components of the total value of capital does not give the idea of relative importance and the roles played by different categories of capital. Such a breakdown of total capital into different categories assumes a still greater importance in a developing country like India where these categories grow at different rates during the program of industrialisation. The extent of the relative roles played by the major categories of capital in a given situation, it is hoped, would provide a suggestive analysis of change and direction in investment pattern.

4. Cross-Section Analysis :

The cross-section analysis, which refers to all two-digit ASI industries for the year 1964, estimates the Douglas type of production function by considering the gross value of fixed capital in two ways: (i) at current prices, and (ii) at purchase prices. The first value refers to the replacement cost of assets, while the second refers to the value of fixed asset at the price of the year in which it was purchased.[11] In the case of first relation,

11. It may be pointed out that the proper concept of capital relevant to the analysis here is replacement cost only. However, for different categories of assets it was not possible to adjust for prices after obtaining the value of assets at purchase prices.

the total value of capital is divided into (a) gross value of fixed capital (at current prices) and (b) inventories; while in the second relation the gross value of fixed capital (at purchase price) has been further divided into (a) buildings, improvement to land etc., (b) plant, machinery tools, etc. (See Table V-3).

The following multiple regressions in their logarithmic forms are fitted:

$$V = \beta_0 \, x_1^{\beta_1} \, x_2^{\beta_2} \, x_3^{\beta_3} \qquad \ldots(i)$$

and $$V = \beta_0 \, x_1^{\beta_1} \, x_{2.1}^{\beta_{2.1}} \, x_{2.2}^{\beta_{2.2}} \, x_3^{\beta_3} \qquad \ldots(ii)$$

where V = value added (gross)

x_1 = Man-hours worked

x_2 = value of gross fixed capital at current prices.

$x_{2.1}$ = Building, improvement to land and other construction (at purchase price)

$x_{2.2}$ = Plant, machinery, tools, transport equipment and other fixed assets (at purchase price)

x_3 = Inventories.

The results of the logarithmic relations are as follows:

(i) $$Y = 0.206 + \underset{(0.108)}{0.270\, X_1} + \underset{(0.076)}{0.253\, X_2} + \underset{(0.150)}{0.453\, X_3}$$

$R^2 = 0.9814.$

(ii) $Y = 0.332 + 0.282\,X_1 + 0.168\,X_{2.1} + 0.129\,X_{2.2} + 0.398\,X_3$
$\qquad\qquad (0.134) \quad (0.153) \quad (0.119) \quad (0.168)$

$R^2 = 0.9805$

where $Y = \log V$, $X_1 = \log x_1$, $X_2 = \log x_2$

$X_{2.1} = \log x_{2.1}$, $X_{2.2} = \log x_{2.2}$ and $X_3 = \log x_3$

Figures in brackets indicate the standard errors of the regression coefficients.[12]

The production function fits well in both the relations.Each of the regression coefficients estimated indicates the production elasticity with respect to the factor of production concerned.[13] The value of the coefficient

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12. If we run the regressions by considering the value added net of depreciation, the two relations give the following results:

(i) $Y = 0.201 + 0.327X_1 + 0.167X_2 + 0.481X_3$; $R^2 = 0.976$
$\qquad (0.121) \quad (0.086) \quad (0.169)$

(ii) $Y = 0.304 + 0.336X_1 + 0.157X_{2.1} + 0.056X_{2.2} + 0.427X_3$
$\qquad (0.150) \quad (0.171) \quad (0.134)$

$R^2 = 0.975$

13. Marginal productivity of x_1, for example, is given as

$$\frac{\partial V}{\partial x_1} = \beta_1\, V/x_1$$

$$\therefore\ \beta_1 = \frac{\partial V}{\partial x_1} \cdot \frac{x_1}{V} = \text{Production elasticity w.r.t. } x_1.$$

Similarly for other factors.The marginal productivity of any factor is a constant proportion of its average productivity; if the factor is apid according to its marginal productivity, the total payment to the factor will be constant proportion of the total output.

shows the average percentage change in gross value added, given the increase in the amount of factor of production by one percent. A change of one per cent in man-hour worked, for example, assuming no change in other factors, would mean a change of about 0.27 per cent in gross value added (relation-i). The highest value of regression coefficient is that of inventories, being 0.453 in the first relation and 0.398 in the second relation.And this is quite expected looking to the fact that output and inventories are much more interdependent than output and other factors.

The insignificant role played by the machinery in 1964 seems to have been caused by the under utilisation of capacity. This reduces the importance of machinery as a factor of production. In such a situation any addition of labour and inventories made to work with a given quantity of idle machinery will certainly make the production to rise. But an increase of extra machine in this situation will only mean an addition to idle capacity and no increase in output.

The goodness of the fit of the production function is tested by calculating the square of the multiple correlation coefficient (R^2). This has turned out to be highly significant at little more than 0.98 in both the relations.The explanatory variables taken together, thus, explain more than 98 per cent of the variations in value added.

Table V-3

Observations on Regression Variables - 1964

(Rs. in '00000)

Industry No.	Industry	Value added (gross)	Man-hours worked	Gross value of fixed capital at current prices	Gross value of buildings, construction etc. at purchase prices	Gross value of plant machinery etc. at purchase prices	Working capital
1	2	3	4	5	6	7	8
20	Food (except beverage)	14332	8113	50104	9985	26896	20408
21	Beverage	736	179	1569	394	845	491
22	Tobacco	2691	1257	1885	587	944	3157
23	Textiles	44278	30673	105123	18188	59973	40875
24	Footwear & wearing apparel	364	280	350	537	240	448
25	Wood & Cork	548	478	1321	299	811	655
26	Furniture & Fixtures	640	380	850	403	333	687
27	Paper & Paper products	3664	1266	18764	2822	11673	3511
28	Printing & publishing	3480	1699	7024	1394	4418	1891
29	Leather & Fur products	277	197	539	144	288	577
30	Rubber	3120	866	4987	1032	3342	2878
31	Chemicals	15322	2874	55432	10681	32814	14099

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Table V-3 (concluded)

Observations on Regression Variables - 1964 (Rs. in '00000)

Industry No.	Industry	Value added (gross)	Man-hours worked	Gross value of fixed capital at current prices	Gross value of buildings construction etc.at purchase prices	Gross value of plant machinery etc. at purchase prices	Working capital
1	2	3	4	5	6	7	8
32	Petroleum & Coal	1600	245	14496	2225	10214	1668
33	Non-metallic mineral products	6786	3907	27529	6165	14446	5805
34	Basic metal	24576	6548	185011	19602	126313	29804
35	Metal products	3765	1586	6913	1307	4541	4068
36	Machinery	8244	3121	17029	4548	10795	9853
37	Electrical machinery	7160	2229	15788	5415	8634	9694
38	Transport equipment	15983	6844	32722	9326	18584	17903
39	Miscellaneous industries	2059	1003	3797	859	2423	1770

Source: Annual Survey of Industries, 1964.
Columns 5,6,7 are calculated on the basis of the methodology discussed in Chapter IV.

The relations fit very well to the Indian manufacturing industries. The sum of the regression coefficients (factor exponents) is little more than 0.976.

The regression coefficient of x_1, as noted earlier, can also be interpreted as the estimated share accruing to labour.[14] The following table V-4 summarises the estimates of the parameters of the two relations:

Table V-4

Results of the Estimated Relations

Relation	No. of observations	Degrees of freedom	Estimated exponent of labour (β_1)	Sum of the estimated exponents (β)	β_1/β	Observed wage-share W/V
1	2	3	4	5	6	7
(i)	20	16	0.270	0.976	0.277	0.330
(ii)	20	15	0.282	0.977	0.289	00.330

Source: Estimated on the basis of data shown in Table V-3.

β in the above table is the combined estimated share going to all factors. Under the conditions of constant returns to scale it should exhaust the total product. β_1/β is the estimated relative share of labour. This assumes that the value added between labour and other factors is divided such that no residual is left over either in the form of profits or losses. When we compare β_1 or

14. Wages = $W = x_1 \frac{\partial V}{\partial x_1} = x_1 \cdot \beta_1 \frac{V}{x_1} = \beta_1 V$

Thus, β_1 = W/V = wage share.

β_1/β with W/V, the observed share of labour, we find that the difference between the two is not ~~found~~ statistically significant. Labour on an average, therefore, is getting what its marginal productivity would warrent for.

5. <u>Time Series Analysis</u> :

While estimating the Douglas type of production function to the time series data, the difficulty arises about the technological change over time. "In a period--to-period variations over a long historical stretch, there will be much technical progress. In the cross-section sample, the non-uniform technology varies in a haphazard way, but in the time series sample it changes gradually in a strictly chronological fashion".[15] Cobb and Douglas tried to overcome this difficulty by extracting trends from the arithmetic values of variables before fitting the production function to the logarithmic values of the variables.An alternative would be to introduce the explicit trend variables in the equation..The function then becomes:

$$V = f\,(x_1, x_2, t)$$

where t is trend variable representing the technological change.[16] Solow, while considering the aspect of technological

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15. L.R.Klein, Op.cit.,p.100. According to E.H.Phelps Brown "The fitting of the Cobb-Douglas function to time series has not yielded, and can not yield, the statistical realization of a production function. It can describe the relations between the historical rates of growth of labour, capital, and product, but the coefficients that do this do not measure marginal productivity". See his article, op.cit.,p.551.

16. The Cobb-Douglas production function when expressed in this way meets the objection regarding its identification from the cost function.The cost identity, $V=x_1P_1+x_2P_2$,

change, has suggested the following form of the function:

$$V = A^t x_1^{\beta_1} x_2^{\beta_2}$$

Although he prefers a modified version of the function in the form of

$$\frac{V}{x_1} = A^t \left(\frac{x_2}{x_1} \right)^{\beta_2}$$

An alternative to this is not to estimate the share of capital (β_2) directly, but to substitute the observed share of capital and then fit the relation to estimate β_2. Incidentally, this also enables to isolate and measure the technological progress.[17]

Solow assumes the neutral technological progress where the shifts in the production function leave the marginal rate of substitution between labour and capital untouched, but simply increase or decrease the output attainable from given inputs. The marginal rate of substitution between labour and capital being independent of t, every change in technology is neutral.

If the factors are paid according to their marginal products, the assumption of neutral technological change would give the form of production function as :

(P_1 and P_2 are the factor prices) does not include the t term.

17. R.M.Solow: "Technical change and the aggregate production function". The Review of Economics and Statistics, August, 1957; reprinted in Growth Economics, ed. by Amartya Sen (Penguin Book Ltd., 1970).

$$V = A(t)\ f(x_1, x_2)$$

Or $$V = A(t)\ x_1^{1-\beta_2}\ x_2^{\beta_2}$$ (under the conditions of constant returns to scale).

A(t) measures the cumulated effect of shifts over time; it changes over time and affects the changes in output for any given factor inputs.

The above function can also be written as :

$$\frac{V}{x_1} = A(t) \left\{\frac{x_2}{x_1}\right\}^{\beta_2}$$

Or $$\log \left(\frac{V}{x_1}\right) = \log A(t) + \beta_2 \log \left(\frac{x_2}{x_1}\right)$$

Taking the difference between two adjacent periods, the equation in its incremental form[18] can be written as :

$$\Delta \log \left(\frac{V}{x_1}\right) = \Delta \log A(t) + \beta_2 \Delta \log \left(\frac{x_2}{x_1}\right)$$

i.e. $$\frac{\Delta (V/x_1)}{V/x_1} = \frac{\Delta A(t)}{A(t)} + B_2 \frac{\Delta (x_2/x_1)}{x_2/x_1}$$ (approximately)

Thus, $$\frac{\Delta A(t)}{A(t)} = \frac{\Delta (V/x_1)}{V/x_1} - \beta_2 \frac{\Delta (x_2/x_1)}{x_2/x_1}$$

where V = Gross value added,

x_1 = Total number of employees

x_2 = Gross value of capital

β_2 = Profit share

The above ratios along with the series of A(t) are presented in Table V-5. The series of A(t) has been arrived

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18. See L.R.Klein, op.cit.,p.106.

at from the series $\frac{\Delta A(t)}{A(t)}$ by assuming the initial value of A(t) as one. A(t) series reveals that production function has shifted by about 52 per cent over the period 1946-1964. The corrected productivity of 1964, that is productivity net of technological progress, is Rs.2403 (i.e., productivity in 1964 divided by value of A(t) in 1964). The rise in productivity due to capital intensity, therefore is Rs.299, (i.e. the corrected productivity minus the productivity in 1946). The total rise in productivity during 1946-1964 is Rs.1539. Thus out of an increase in total productivity of Rs.1539, Rs.1240 is due to technical progress and Rs.299 due to capital intensity, giving the two percentages as 80.6 per cent, and 19.4 per cent respectively. It is striking to note that Solow's own estimate of the contribution of technical progress over a 40 years' period (1909-1949) in United States (Non-farm Economy) is 87.5 per cent. (op.cit).

6. Conclusion :

The greatest draw back of the earlier estimates of the Douglas type of production function in India has been the use of book values for capital input. The written-down book value of fixed capital, as noted, does not reflect the true value of capital. The present study removes the defect by considering the gross value of fixed capital both for cross-section as well as time-series analysis. Further,

Table V-5

Labour Productivity, Capital/Labour Ratio, Profit Share, and Shifts in Production Function :1946-1964.

Year	Gross value added per employee (at 1950 prices) Rs.	Total capital per employee (at 1950 prices) Rs.	Profit share in gross value added	A(t)
1	2	3	4	5
1946	2104	13790	0.5545	1.000
1947	2018	12772	0.4754	0.994
1948	2063	12284	0.5075	1.036
1949	1777	12352	0.3898	0.890
1950	1895	12981	0.4353	0.969
1951	1944	12930	0.4900	0.992
1952	2012	13025	0.4045	1.024
1953	2286	13081	0.4299	1.165
1954	2387	12641	0.4559	1.234
1955	2592	12547	0.4856	1.345
1956	2578	12415	0.4879	1.331
1957	2481	13054	0.4599	1.249
1958	2746	14061	0.4947	1.335
1959	2944	14516	0.5145	1.409
1960	2889	14830	0.4925	1.367
1961	3030	15319	0.5162	1.411
1962	3305	18046	0.5146	1.410
1963	3535	18741	0.5222	1.479
1964	3643	18955	0.5131	1.516

Source: Calculated on the basis of the data derived from the reports of the Census of Manufacturing Industries and the Annual Survey of Industries. The adjustments of the data are discussed in Chapters II and IV.

to analyse the relative importance of different categories of capital, the cross-section study for 1964 also takes into account the detailed breakdown of capital rather than aggregate value of capital as a measure of capital input.

The Douglas type of production function fits well to Indian industries.The estimated share of labour is not significantly different from the observed labour share. The exponent of inventories (the production elasticity with respect to inventories) is found to be as high as 0.48. To take care of the technological progress over-time, the trend variable has been introduced while testing the function over the period of 1946-1964. The contribution of technological progress to the growth in productivity has worked out to be 80.6 per cent.

APPENDIX

Properties of Cobb-Douglas Production Function

In the Cobb-Douglas Production Function of the form $V = A\, a^{\alpha}\, b^{\beta}$ (where V = output, a and b are inputs), it is not necessary to specify the degree of scale in advance. The estimated values of α and β will decide about the degree. If $\alpha + \beta = 1$, for example, then it is homogeneous of degree one.

(i) Marginal productivity of a is

$$f_a = \frac{\partial V}{\partial a} = \frac{\alpha V}{a} \quad \text{and} \quad f_b = \frac{\beta V}{b}$$

when $(\alpha + \beta) = 1$, the marginal productivities are homogeneous of degree zero:

$$\text{i.e. } f(a,b) = A\, a^{\alpha}\, b^{1-\alpha}$$

$$\therefore f_a(a,b) = A\,\alpha\, a^{\alpha-1}\, b^{1-\alpha}$$

$$\therefore f_a(\lambda a, \lambda b) = A\,\alpha\,\lambda^{\alpha-1}\, a^{\alpha-1}\, \lambda^{1-\alpha}\, b^{1-\alpha}$$

$$= A\,\alpha\, a^{\alpha-1}\, b^{1-\alpha}$$

$$= f_a(a,b).$$

(ii) Cross partial derivatives, f_{ab} and f_{ba} are of degree -1

$$f_a = A\,\alpha\, a^{\alpha-1}\, b^{1-\alpha}$$

and $f_{ab} = A\,\alpha(1-\alpha)\, a^{\alpha-1}\, b^{-\alpha}$

$$\therefore f_{ab}(\lambda a, \lambda b) = A\,\alpha(1-\alpha)\,\lambda^{\alpha-1}\,a^{\alpha-1}\,\lambda^{-\alpha}\,b^{-\alpha}$$

$$= \lambda^{-1} f_{ab}$$

(iii) Relative shares :

In equilibrium, $\frac{f_a}{f_b} = \frac{P_a}{P_b}$

where P_a and P_b are the prices of factors a and b respectively.

i.e. $$\frac{\frac{\alpha V}{a}}{\frac{\beta V}{b}} = \frac{P_a}{P_b}$$

$$\therefore \frac{aP_a}{bP_b} = \frac{\alpha}{\beta} = \text{relative shares.}$$

(iv) Production elasticities :

Production elasticity with respect to a is

$$\frac{\partial V}{\partial a} \cdot \frac{a}{V} = \frac{\alpha V}{a} \cdot \frac{a}{V} = \alpha$$

Similarly production elasticity with respect to b is β.

(v) The elasticity of demand for the factor is defined as the proportionate change in the demand for quantity of the factor with respect to the proportionate change in the marginal productivity of the factor.

i.e. $$e_a = \frac{\frac{\partial a}{a}}{\frac{\partial f_a}{f_a}}$$ (ignoring the sign) i.e. $\frac{1}{e_a} = \frac{\partial f_a}{\partial a} \cdot \frac{a}{f_a}$

$$= \frac{(\alpha-1)\,\alpha V}{a^2} \cdot \frac{a}{\frac{\alpha V}{a}}$$

$$= \alpha - 1$$

$\therefore\ e_a = \frac{1}{\alpha-1}$

considering the -Ve sign, $e_a = \frac{1}{1-\alpha}$

(vi) Elasticity of substitution :-

marginal rate of substitution, $R = \frac{f_a}{f_b} = \frac{\alpha V}{a} \cdot \frac{b}{\beta V}$

i.e. $R = \frac{\alpha}{\beta} \cdot u$ (where $u = b/a$)

$\therefore\ \log R = \log\left(\frac{a}{b}\right) + \log u$

$\therefore\ d \log R = d \log u$

$\therefore\ \frac{d \log u}{d \log R} = 1$ = the elasticity of substitution.
