CHAPTER III

APPRAISAL OF CONSOLIDATION THEORIES

Not that Terzaghi was unaware of the physical and physico-chemical factors that should influence the consolidation characteristics of soil; in fact, the clarity contained in the very statements which presents the assumptions behind the theory transcends his consciousness to the possible limitations. Essentially, on one hand the classical theory fulfilled the immediate need of a tool for engineering analysis and on the other hand furnished vital clues for future scientific refinements. Interpretation of the classical consolidation process by a spring model led many successive workers to a rheological approach. The obvious fact behind the assumption of highly idealized stress strain law which overcomes the mathematical complexities but at the sacrifice of the material identity induced several mathematicians to resort to rigrous mathematical treatments amply reflecting the material characteristics of the soil. The thermodynamic analogue to the process of consolidation prompted some scientists resort to other physical analogues. Hence, the post Terzaghi developments in consolidation theories could be classified into three distinct approaches namely, Rheological, Analytical and Physical.

3.1. Rheological Approach

The fundamental argument behind this approach is that it is possible to study the complex deformation-time characteristics through mechanical models whose various components are ascribed with certain physical properties. These models, which are generally referred to as Rheological models, consist of combination of elementary units of springs and dashpots in series or in parallel. The general rheological scheme employed for describing the consolidation process of soil material is composed of elastic spring (Hookean body) and spring in parallel to dash pot (Kelvin body).

3.1.1. Taylor Model

Taylor (1942) proposed a modification to the classical spring model by combining a dashpot in parallel to incorporate the phenomenon of plastic resistance. The rheological version of this model is Kelvin body as represented in Figure 3.1. The strain characteristics of a Kelvin body under stress is elastic deformation delayed by time effects.

3.1.2. Taylor-Merchant Model

Taylor-Merchant (1939) evolved a rheological model consisting of Hookean spring connected in series with a Kelvin element (Fig. 3.2.) to represent the complete process of consolidation of saturated soil-water system under a load. The rheological behaviour of proposed scheme is that instantaneously on the application of stress, Hookean body compresses but the Kelvin element exhibits retardation in settlement because of the dashpot. In case of soil material the transference of stress from pore water to the soil skeleton is delayed due to its low permeability and hence the effective stress gradually increases from zero to the full value of the applied stress. Consequently, the compression of spring 'a' is also gradual and is fully accomplished only when the applied stress has become fully effective. Under the gradually increase of the effective stress the Kelvin body commences to compress. At first the full load is borne by the dashpot and as compression proceeds it is progressively transferred to the spring 'b'. This phenomenon of transference corresponds to the process of secondary compression occuring under sustained effective stress. Finally, after the elapse of a long time the full effective pressure is carried through springs 'a' and 'b', the dashpot sustaining no load. Based on the above interpretation Gibson and Lo (1961) produced an exact mathematical solution to the Taylor-Merchant differential equation. Almost similar rheological models have been analysed by various other workers (Goldstein, 1952; Ishii,1951; Tan,1957;

Christie, 1963; Leonard and Altschaeffl, 1964).

3.1.3. Lo Model

Lo (1961) sandwitched an element, designated as S, between the Taylor-Merchant model and another Kelvin element as shown in Figure 3.3. The mechanical behaviour of this new element is to stay rigid upto a certain critical value of either stress or strain and sustain the effective stress from the first Kelvin element without transmitting to the second Kelvin element. At a point, when this critical value is exceeded the interlinking element loses its rigidity and operates to transfer the load to the second Kelvin element. The specific object behind this model is to incorporate the observed accelerated secondary compression in clays of predominantly loose structure. The conception is that at a certain value of stress there occurs a breakdown of bonds which unleashes the compression.

3.1.4. Tan Model

Tan (1957) proposed a rheological scheme as shown in Fig. 3.4 and as the model is meant for three dimensional consolidation it could be interpreted in such a manner that the Hookean spring K is related to the spherical part and the Maxwell body $G - \eta^{d}$ to the distortional part of the stress-strain relationship. Solving for one dimensional case it is concluded that the pore pressures resulting from his theory are greater than the pore pressures obtained by Terzaghi theory.

3.1.5. Wahls Model

Wahls (1962) conceived a versatile model from the Terzaghi concept for primary consolidation and the Lo concept for secondary compression. A dash pot with variable coefficient is suggested to account for secondary compression (Fig. 3.5). An infinite series of such dash pots is shown to develop a linear relation between compression and logarithm of time in the secondary compression range. It is worth noting that the dashpots meant for secondary compression are also subjective to total applied load right from the beginning.

3.1.6. Barden Model

Barden (1965) elaborated Terzaghi-Taylor concept of consolidation process by appropriately weaving in the idea of non-linear structural viscosity. Rheologically, it is represented by a model comprised of a linear spring in parallel with a nonlinear dash pot (Fig. 3.6). The obtained mathematical solution is useful to study the influence of sample thickness, variation in permeability and magnitude of stress increment.

The success of the rheological models demands that the material is homogeneous, isotropic and stress path independent. For material exhibiting large strains the principle of superposition is deprived of its validity. Hence under the

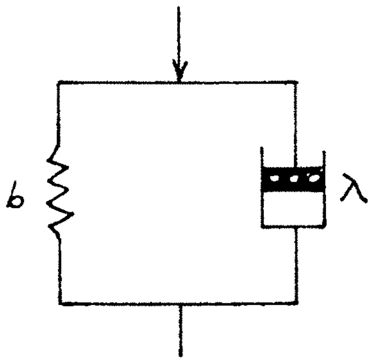

FIG. 3·1 TAYLOR MODEL (1942)

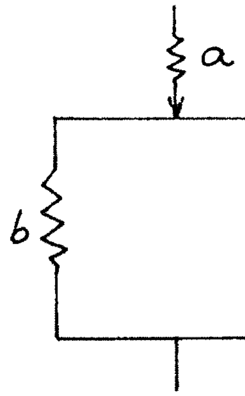

FIG. 3·2 TAYLOR-MERCHANT MODEL (1939)

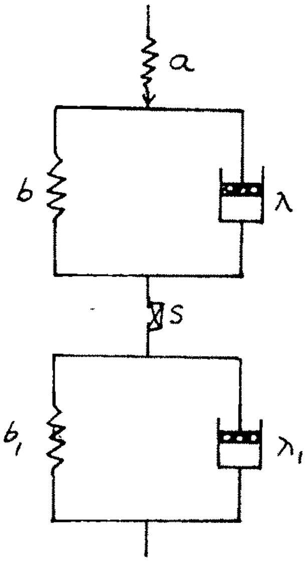

FIG. 3·3 LO MODEL (1961)

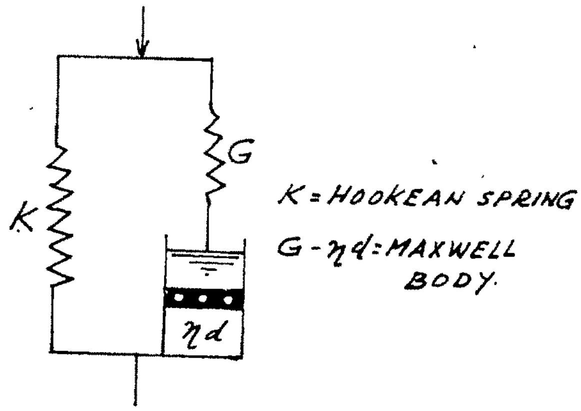

FIG. 3·4 TAN MODEL (1957)

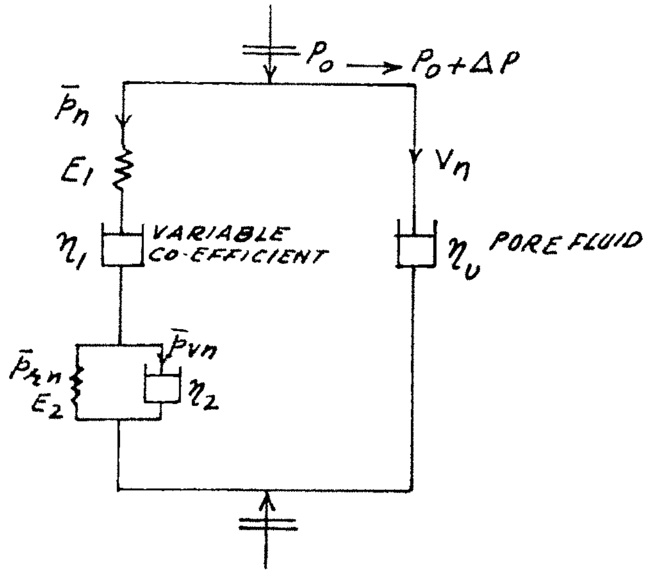

FIG. 3·5 WHALS MODEL (1962)

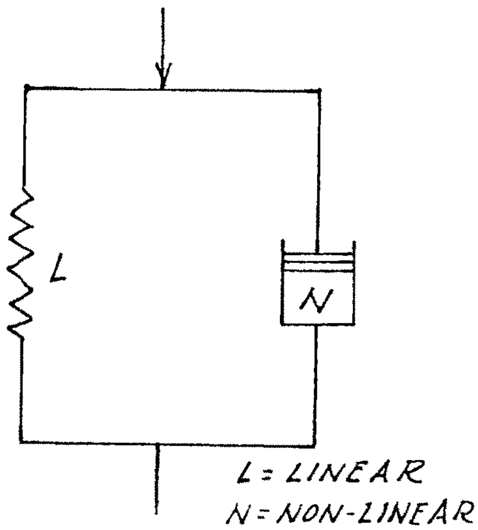

FIG. 3·6 BARDEN MODEL (1965)

circumstances of the observed nature of the soil material the working of this versatile rheological approach has its own limitations.

3.2. Analytical Approach

When a mathematical approach is based on gross idealization avoiding proper qualifications at each stage the value of the treatment eventually turns out to be less significant and often leads to misinterpretation of both physical and mathematical conceptions. In general, for establishing a theoretical relationship from first principles of mechanics, equilibrium equations, compatibility relationship, stress strain law and continuity equations are formulated with a clear definition of the nature of the element. Most soil mechanics workers have sought to extend the classical theory taking into account the variation of permeability and compressibility during consolidation without rigorously examining the mathematical validity. However, some few mathematicians have enlightened upon the various merits and demerits of possible analytical approaches for such an involved physical phenomenon of consolidation in soils.

3.2.1. Biot Analysis

Biot (1956) formulated a theory on the conception that the soil is a porous skeleton filled with water which is elastic but incompressible and flow of water through it follows Darcy's law. Classical elasticity relations are adopted for the development of the equations. Cryer (1962) deduced on mathematical analysis that this theory reduces to that of Terzaghi, though for the case of hydrostatically loaded sphere of soil, the two theories predict quantitatively similar volume changes but the water pressures at the centre of the sphere differ sharply. This theory, like that of Terzaghi, does not explain the secondary compression.

3.2.2. Schiffmann Analysis

Schiffman (1958) generalized the one dimensional theory of consolidation to include the variation in the rate of loading and in the coefficient of permeability during the process of consolidation. The resulting differential equation is nonlinear but an exponential approximation is developed to linearize it thus permitting solution of consolidation boundry value problems. Specific solutions are presented for consolidation of soils for the conditions of constant permeability, and varying permeability subjected to linear and arbitrary loading. Also the approach is further extended for two and three dimensional cases with specific solution.

3.2.3. McNabb Analysis

McNabb (1960) from more general considerations describes the hydrological state of clay mass at any instant by two functions; one states the degree of compaction of the aggregate through a parameter e and another gives the degree of orientatio of particles with respect to degree of consolidation through a distribution function . His analysis leads to a nonlinear type of differential equation. For the boundry conditions of the standard consolidation test it offers a general explanation for the consolidation process. By linearizing the equation in a general manner an expression is obtained for the degree of consolidation which includes secondary compression terms. A similar treatment is also seen in the work of Mikasa (1965).

3.2.4. Richart Analysis

The method due to Richart (1957) is essentially based on the Terzaghi method except that change in void ratio is considered to be appreciable such that the quantity ($1+e$) is not a constant. However, this consideration shows little reflection in void ratio-time factor relationship (Fig. 3.7) exhibiting a slight shift on the left side of the Terzaghi curve. It also does not contribute to the explanation of secondary compression.

3.2.5. Brinch-Hansen Analysis

Brinch Hansen (1961) proposed a model law for simultaneous primary and secondary consolidation for the case of one dimensior consolidation. By combining the logarithmic creep function with the Terzaghi theory a general expression is deduced for the total process of consolidation. This expression shows that the viscous effects only affect the final value of primary consolidation. The final settlement is close to the extended line of secondary compression becoming assymptotic to the primary consolidation curve corresponding to any layer thickness. However, the applicibility is limited to the loading displacement and drainage conditions of Oedometer tests.

3.2.6. Janbu Method

Janbu (1965) derived a differential equation for the one dimensional primary consolidation on the basis of strain instead of additional stress. It is evident from Fig. 3.8 that rate of consolidation by this method is generally more rapid particularly for the first phase of the consolidation than that obtained from the conventional Terzaghi method. One of the strong points of this method is that it is capable of accounting for the important influence of the stress history hitherto neglected in conventional approach.

3.2.7. Raymond Davis-Analysis

One of the useful extension of the Terzaghi classical theory is contained in the publication of Raymond-Davis (1965). It considered non-linear void ratio effective pressure law for soil, retaining the assumption of constant coefficient of consolidation and validity of Darcy's law. The solutions show that for Oedometer boundry conditions the Terzaghi theory predicts satisfactorily settlement but not the value of dissipation of pore pressure (Fig. 3.9). It shows good agreement with

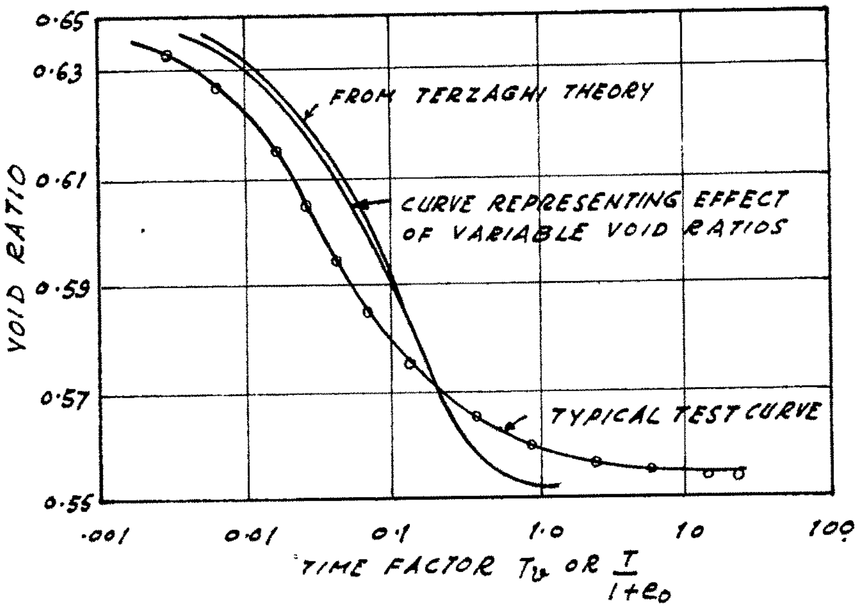

FIG.3·7 VOID RATIO-TIME FACTOR RELATION (RICHART 1957)

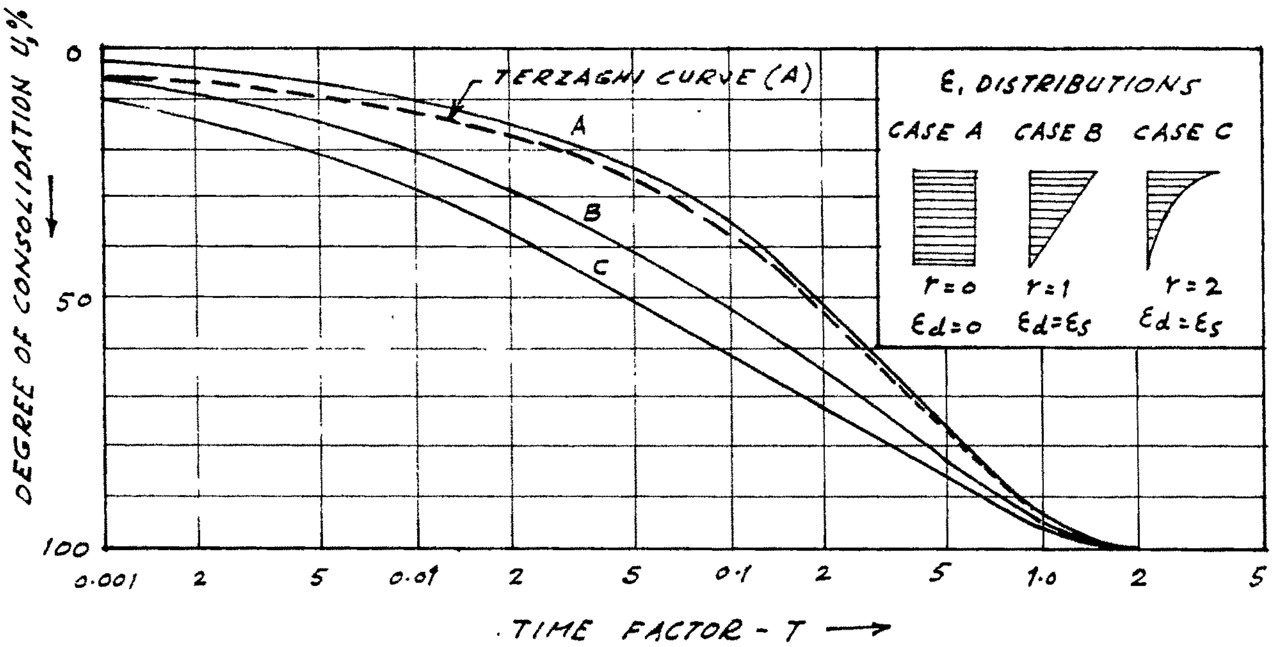

FIG.3·8 COMPARISON BETWEEN CONVENTIONAL TERZAGHI STRESS METHOD AND NEW JANBU (1965) STRAIN METHOD

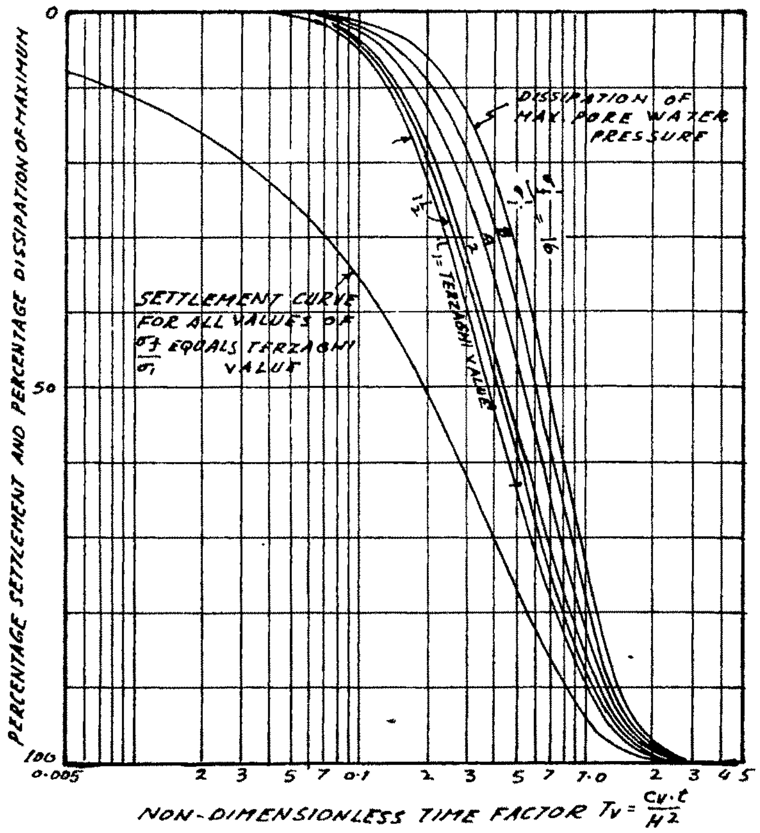

FIG. 3·9 SOLUTION FOR PERCENTAGE SETTLEMENT AND PERCENTAGE DISSIPATION OF MAXIMUM PORE-PRESSURE.

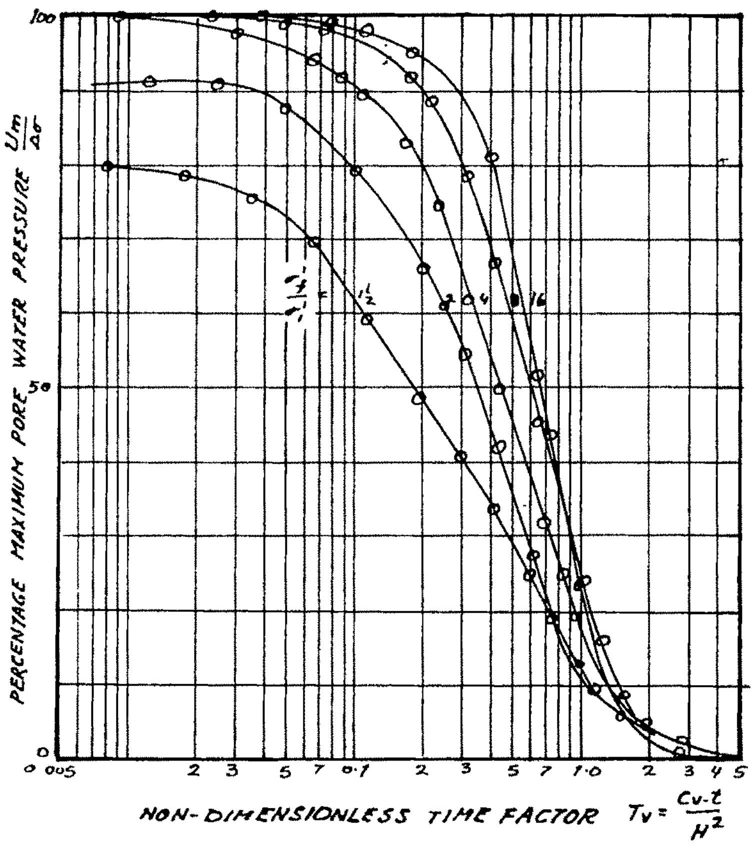

FIG. 3·10 REMOULDED PORT KEMBLA CLAY. PERCENTAGE MAXIMUM PORE WATER PRESSURE VERSUS LOG TIME FACTOR

(AFTER RAYMOND & DEVIS, 1965)

experimental results for normally consolidated clay (Fig. 3.10).

3.2.8. Barden - Berry Analysis

Barden-Berry analysis (1965) from the classical theory incorporating a general function for variation in the permeability produced a non-linear differential equation. Utilizing the method of finite difference a solution of the above nonlinear equation in a suitable form has been obtained for verification of experimental results. Fig. 3.11 portrays theoretical curves in terms of dimensionless parameters, one describing the variation in permeability and another defining and magnitude of pressure increment ratio. A comparisons of experimental results of derwent clays, kaolinite and Frodshaw clays with the Terzaghi and the proposed theory are respectively shown in Figures 3.12, 3.13 and 3.14.

3.2.9. Schiftmann Gibson Analysis

Schiffmann-Gibson (1964) formulated a method within the frame work of Terzaghi's one dimensional consolidation theory without secondary effects to examine the influence of continuous variation of the coefficients of permeability and compressibility on the progress of consolidation in a stratum of clay. The procedure consists in ascertaining a simple fundamental relationship for the actual variation of the consolidation parameters with depth to gain the benefits of numerical and computational methods.The results of the

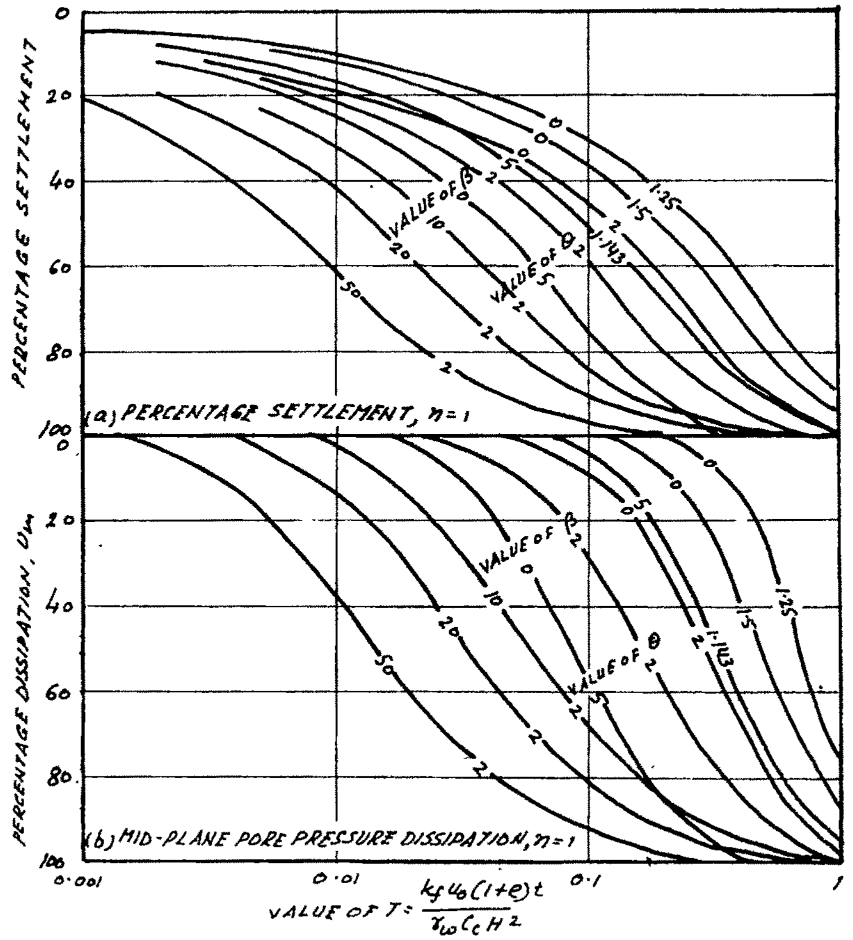

FIG. 3·11 THEORETICAL CURVES

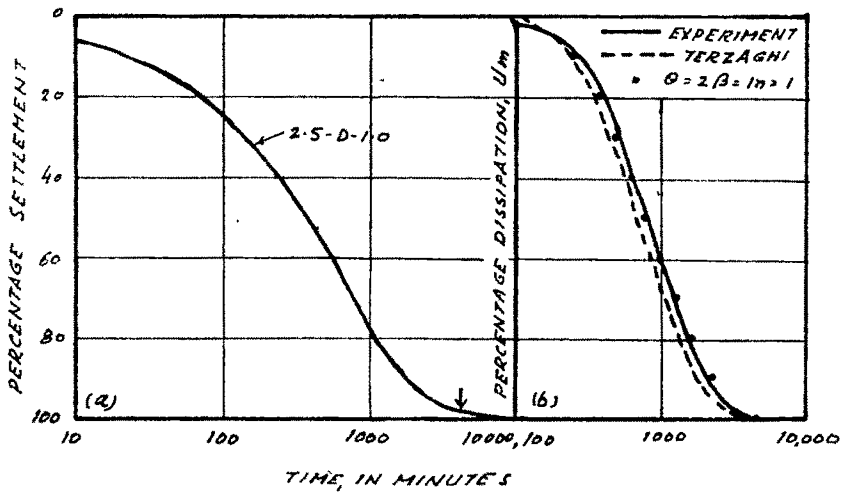

FIG. 3·12 EXPERIMENTAL RESULTS FOR DERWENT CLAY FITTED TO TERZAGHI & NONLINEAR THEORY.

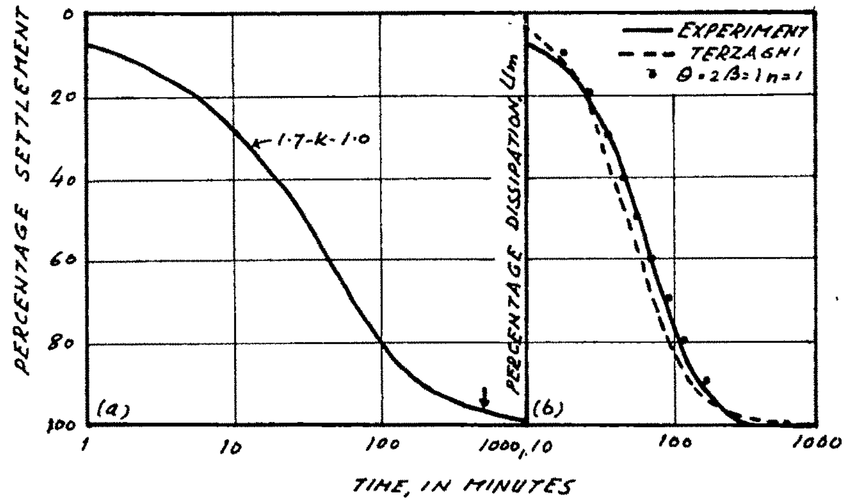

FIG. 3·13 EXPERIMENTAL RESULTS FOR KAOLINITE FITTED TO TERZAGHI AND NONLINEAR THEORY

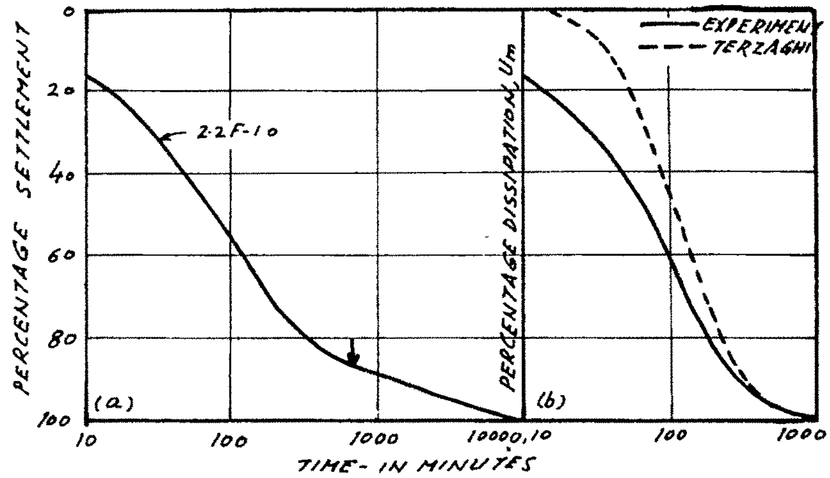

FIG. 3·14 EXPERIMENTAL RESULTS FOR FRODSHAM FITTED TO TERZAGHI THEORY (AFTER BARDEN-BERRY-1965) analysis are represented in Figures 3.15 to 3.28. The high influence of variation in permeability on the excess pore pressure relationships is evident. The compressibility variation affects predominantly the settlement relationship, coefficient of compressibility and coefficients of consolidation.

3.2.10. Gibson Analysis

The work of Gibson et al (1967) is of genuine value as it is based on a complete knowledge of both real characteristics of the soil material and exact requirements of mathematical analysis. Essentially, the argument of McNabb (1960) and Mikasa (1965) has been developed in extenso in a manner so as to reveal clearly the physical significance behind the prospositions. The equations governing the process of one dimensional consolidation in a saturated thin homogeneous clay layer in a form sufficiently general for most applications without imposing the limitations of small strain, are derived. The analysis concludes that void ratio is a privileged variable while other quantities are controlled by much more complicated equations. It could be mentioned that the method possess a potential for detailed analysis of the factors affecting the process of consolidation. Fig. 29 - 32.

3.2.11. Poskit Analysis

Poskit (1969) followed the perturbation method to analyse

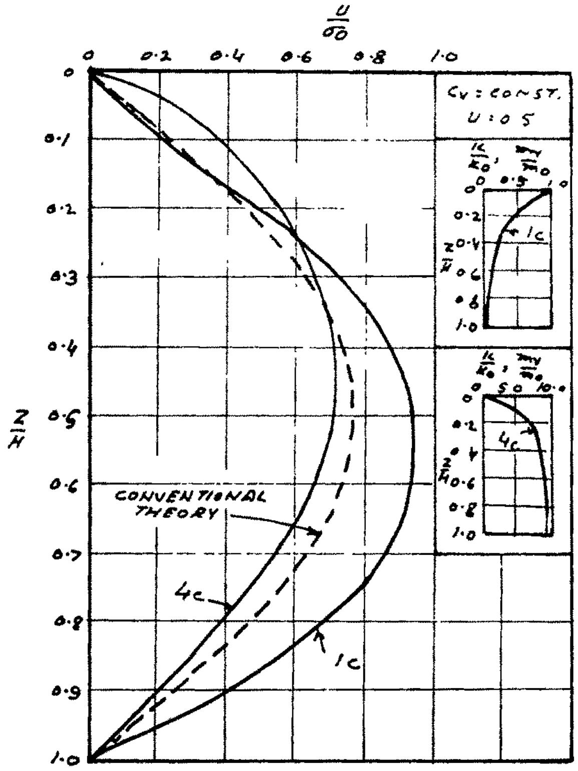

FIG. 3·15 PORE PRESSURE ISOCHRONES: CONSTANT COEFFICIENT OF CONSOLIDATION, POLYNOMIAL VARIATION.

AFTER SCHIFMANN & GIBSON (1964)

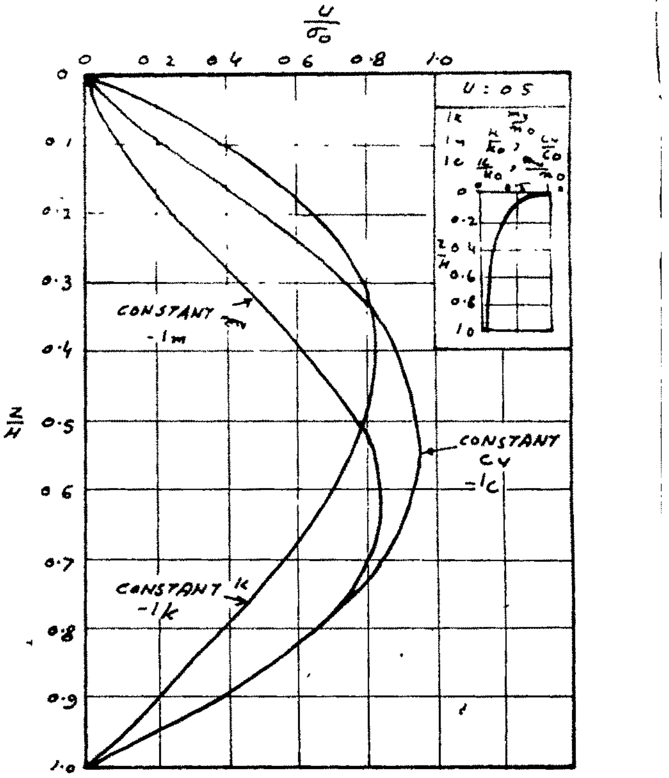

FIG. 3·16 PORE PRESSURE ISOCHRONES: POLYNOMIAL VARIATION.

AFTER SCHIFMANN & GIBSON (1964)

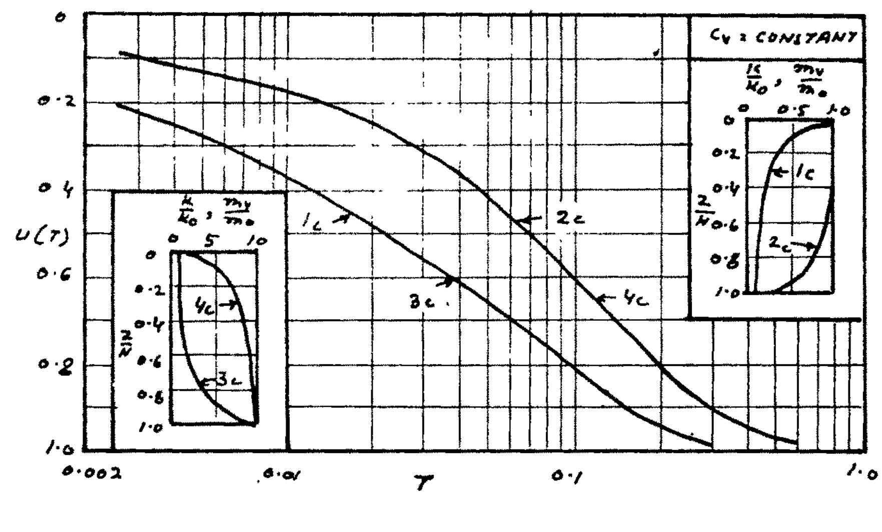

FIG. 3·17 CONSOLIDATION TIME-SETTLEMENT RELATIONS: CONSTANT COEFFICIENT OF CONSOLIDATION, POLYNOMIAL VARIATION. AFTER SCHIFMANN AND GIBSON (1964)

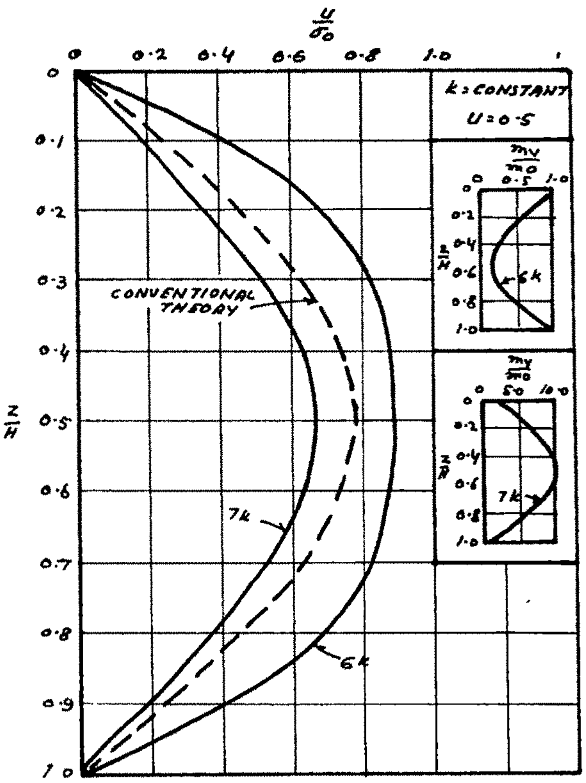

FIG. 3·18 PORE PRESSURE ISOCHRONES: CONSTANT PERMEABILITY, SINUSOIDAL VARIATION.

AFTER SCHIFMANN AND GIBSON (1964)

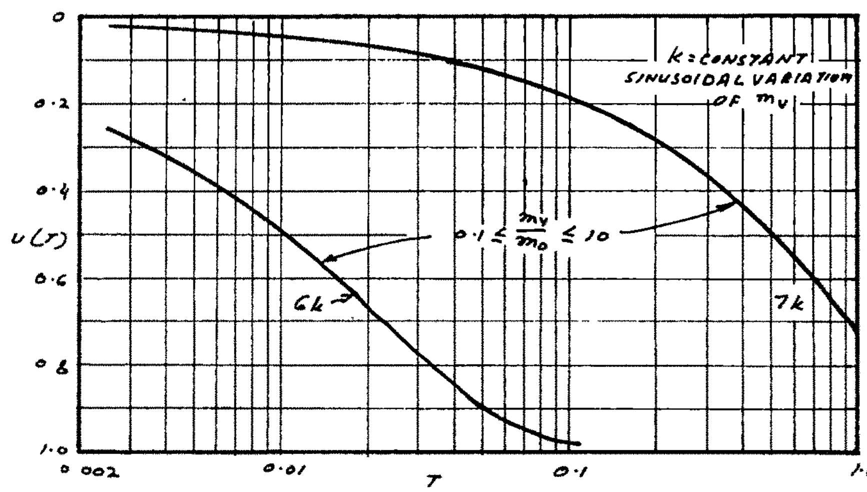

FIG. 3·19 CONSOLIDATION TIME-SETTLEMENT RELATIONS: CONSTANT PERMEABILITY, SINUSOIDAL VARIATION.

AFTER SCHIFMANN AND GIBSON (1964)

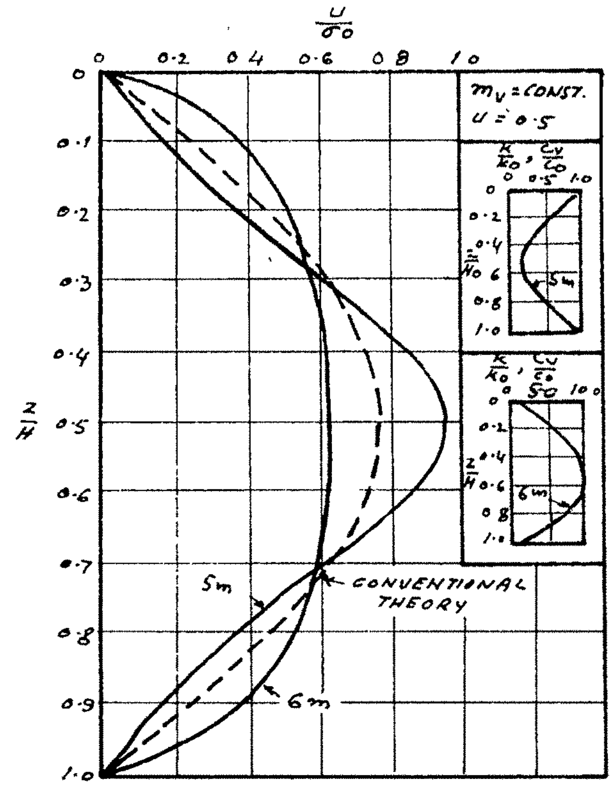

FIG. 3·20 PORE PRESSURE ISOCHRONES: CONSTANT COMPRESSIBILITY, SINUSOIDAL VARIATION

AFTER SCHIFMANN AND GIBSON (1964)

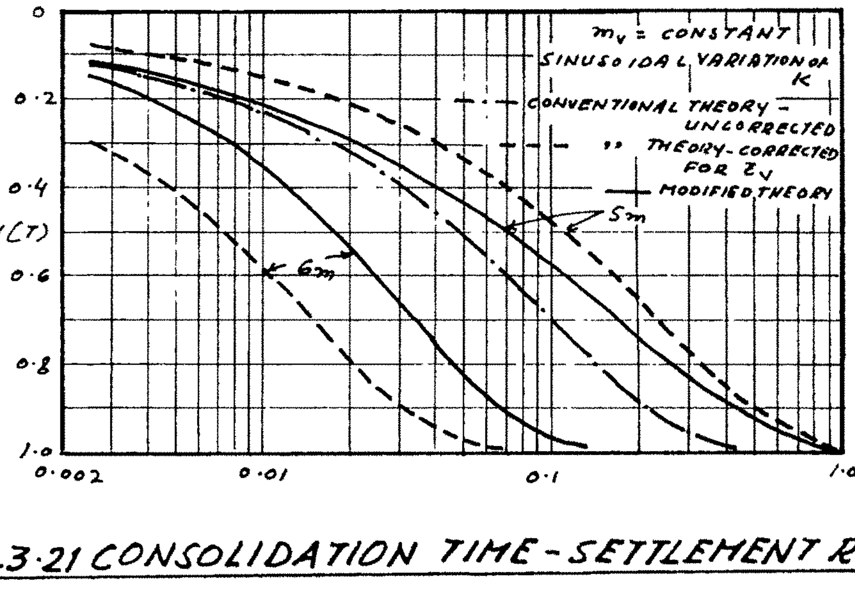

FIG. 3·21 CONSOLIDATION TIME-SETTLEMENT RELATIONS: CONSTANT COMPRESSIBILITY, SINUSOIDAL VARIATION.

AFTER SCHIFMANN AND GIBSON (1964)

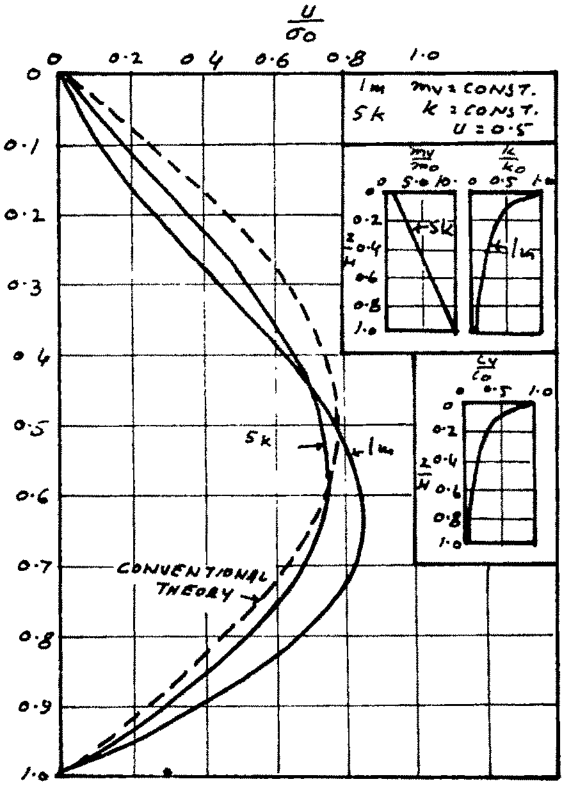

FIG. 3·22 PORE PRESSURE ISOCHRONES: EQUAL COEFFICIENT OF CONSOLIDATION.

AFTER SCHIFMANN AND GIBSON (1964)

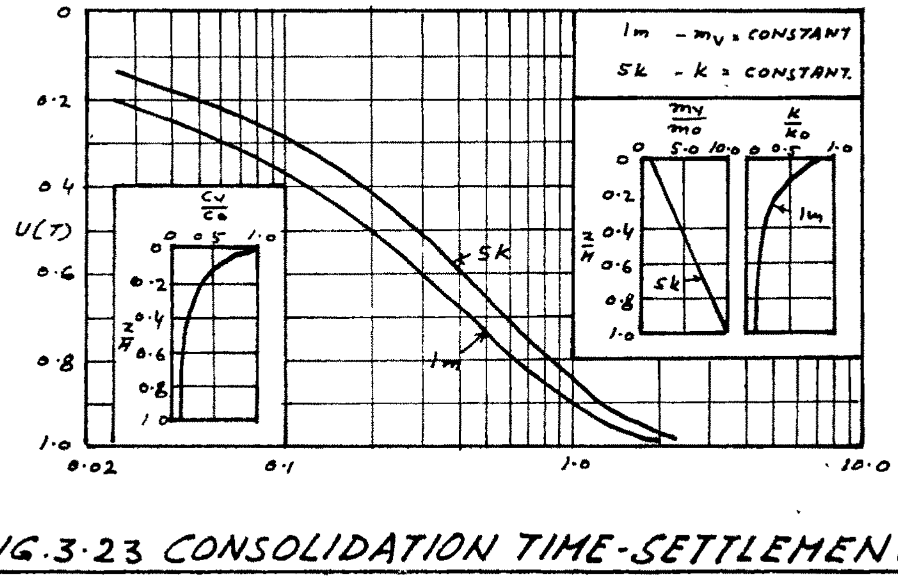

FIG. 3·23 CONSOLIDATION TIME-SETTLEMENT RELATIONS: EQUAL COEFFICIENT OF CONSOLIDATION.

AFTER SCHIFMANN AND GIBSON (1964)

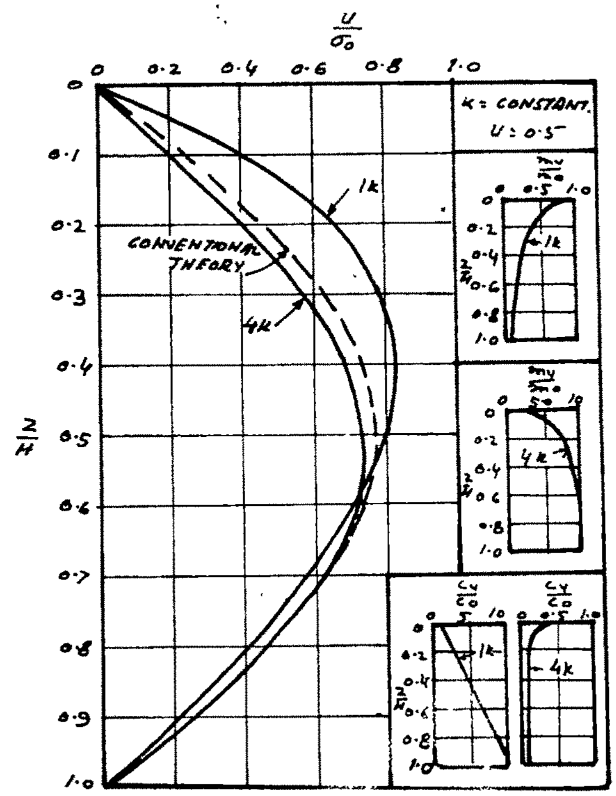

FIG.3.24 PORE PRESSURE: CONSTANT PERMEABILITY, POLYNOMIAL VARIATION

AFTER SCHIFMANN AND GIBSON (1964)

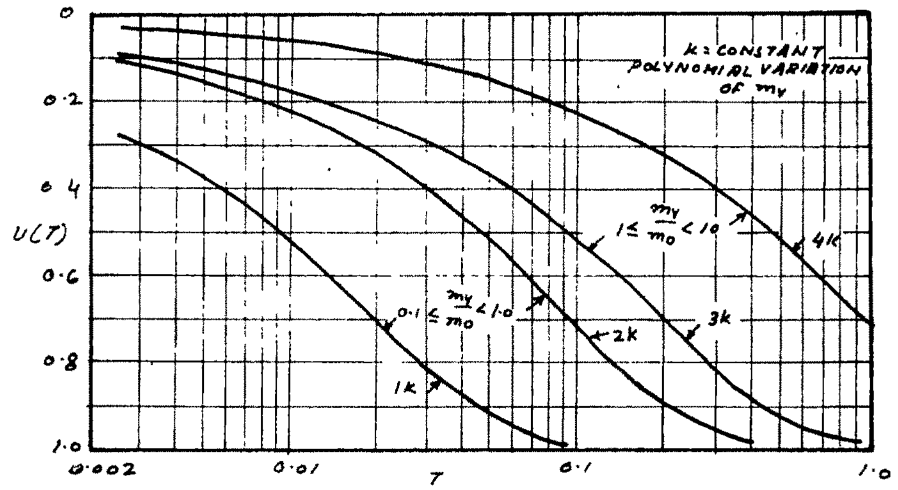

FIG.3.25 CONSOLIDATION TIME-SETTLEMENT RELATIONS: CONSTANT PERMEABILITY, POLYNOMIAL VARIATION.

AFTER SCHIFMANN AND GIBSON (1964)

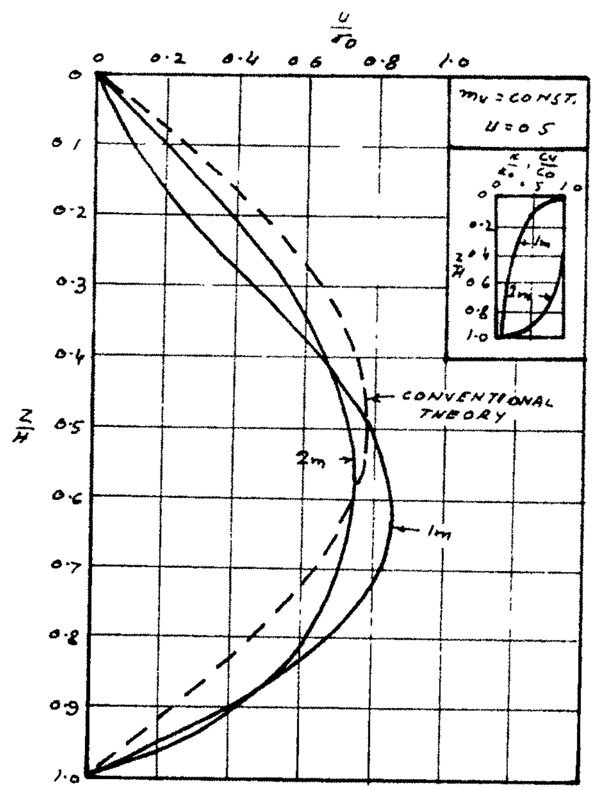

FIG.3·26 PORE PRESSURE ISOCHRONES: CONSTANT COMPRESSIBILITY, POLYNOMIAL VARIATION.
AFTER SCHIFMANN AND GIBSON (1964)

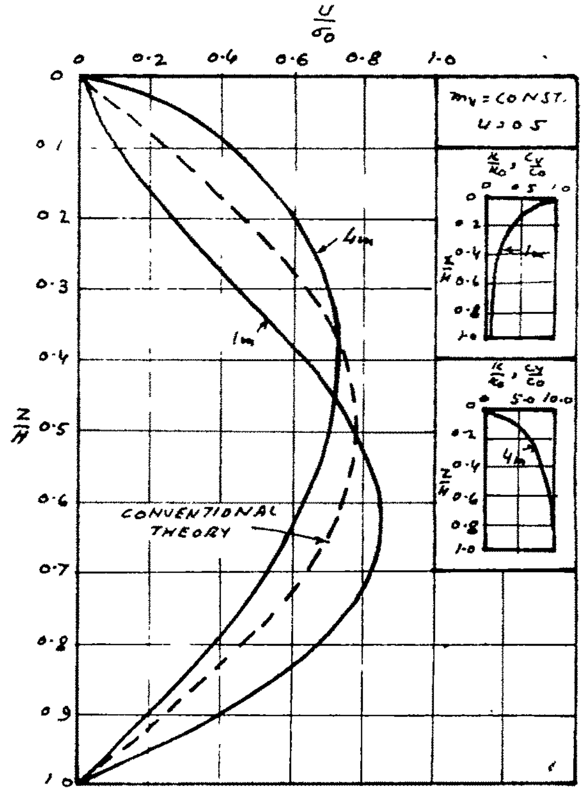

FIG.3·27 PORE PRESSURE ISOCHRONES: CONSTANT COMPRESSIBILITY, POLYNOMIAL VARIATION.
AFTER SCHIFMANN AND GIBSON (1964)

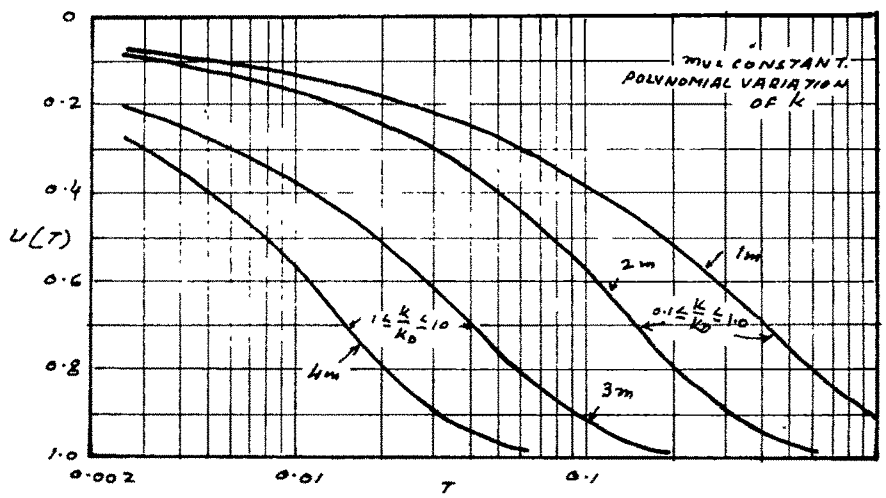

FIG.3·28 CONSOLIDATION TIME-SETTLEMENT RELATIONS: CONSTANT COMPRESSIBILITY, POLYNOMIAL VARIATION.
AFTER SCHIFMANN AND GIBSON (1964)

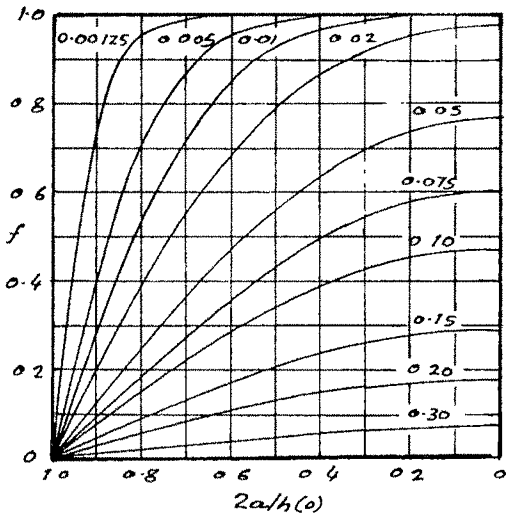

FIG. 3.29 VARIATION OF THE LOCAL DEGREE OF CONSOLIDATION f THROUGH THE THICKNESS OF THE CLAY LAYER WITH TIME FACTOR T_0 (NUMERICAL VALUES ON CURVES), FOR

$\lambda = 0;\ 0 < e_1/e_0 < 1$

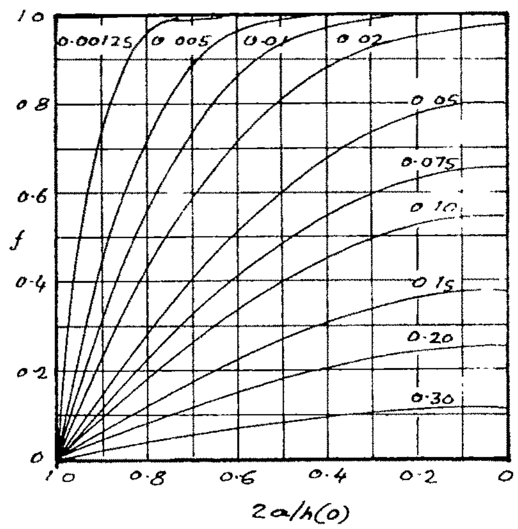

FIG. 3.30 VARIATION OF THE LOCAL DEGREE OF CONSOLIDATION f THROUGH THE THICKNESS OF THE CLAY LAYER WITH TIME FACTOR T_0 (NUMERICAL VALUES ON CURVES), FOR $\lambda = 0.4$,

$e_1/e_0 = 0.2$

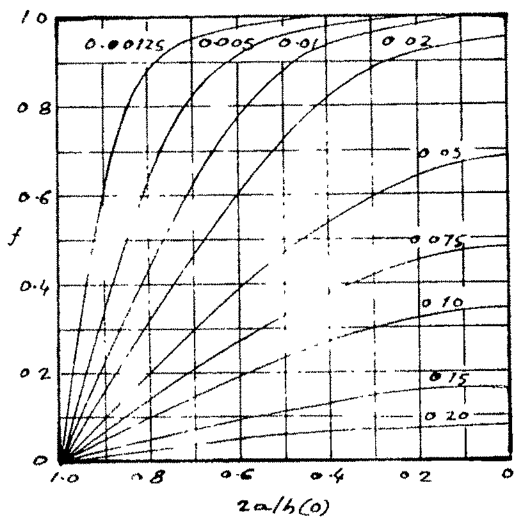

FIG. 3.31 VARIATION OF LOCAL DEGREE OF CONSOLIDATION f THROUGH THE THICKNESS OF THE CLAY LAYER WITH TIME FACTOR T_0 (NUMERICAL VALUES ON CURVES), FOR $\lambda = -0.4$;

$e_1/e_0 = 0.2$

(AFTER GIBSON et al. 1967)

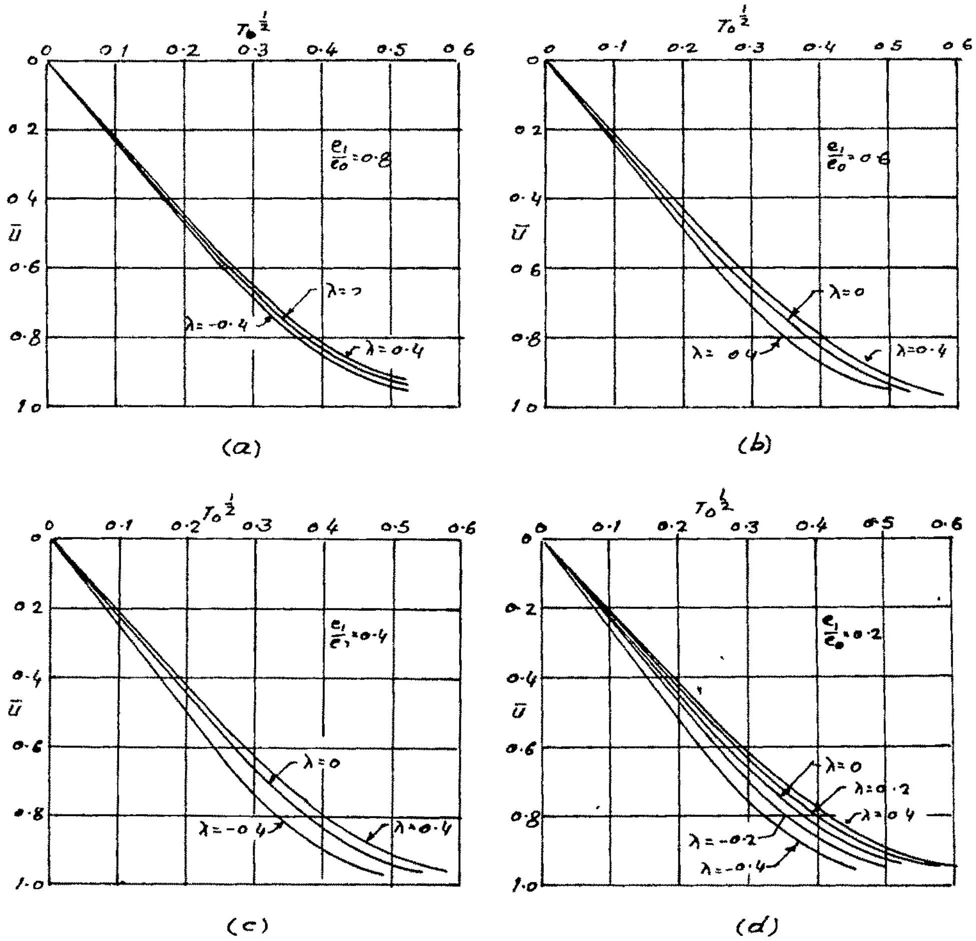

FIG. 3.32 THE RELATION BETWEEN THE AVERAGE DEGREE OF CONSOLIDATION $\bar{U}$ AND THE TIME FACTOR T_0 FOR A VOID RATIO CHANGE FROM e_0 TO e_1 AND FOR $-0.4 \leq \lambda \leq 0.4$ (AFTER GIBSON et al. 1967)

the influence of variable permeability and compressibility on the consolidation of saturated clay. The proposed theory is e-log effective pressure and e-log permeability laws. The perturbation technique has the advantage over numerical methods and it has a capacity to present an immediate and a clear picture of the influence of each variable on the total process. The analysis has attempted to explain the existing differences between various theories. The analysis of Gibson et al (1967) method shows good agreement in all the essentials. In figures 3.33, 3.34, 3.35 the comparison of experimental results is presented.

The consolidation phenomenon in soils cannot be tackled by mathematical treatments based on gross idealizations. Biot (1941) himself remarked that from the mathematical standpoint of view the consolidation problem is different from a elasticity problem and is not sufficiently solved by simply satisfying a heat conduction type equation. One of the distinct feature of the consolidation phenomenon is the occurance of large volume changes during the process. For evolving a better picture of the phenomenon a proper linkage between the physico chemical characteristics of clays and analytical procedure is desirable.

3.3. Physical Approaches

One of the important approach generally sought for the analysis of any unknown physical process is to identify in

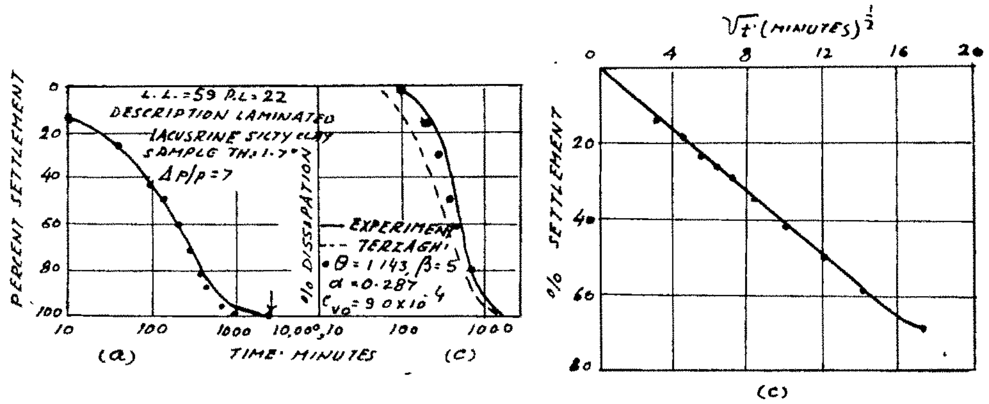

FIG. 3·33 EXPERIMENTAL & THEORETICAL RESULTS FOR DERWENT CLAY

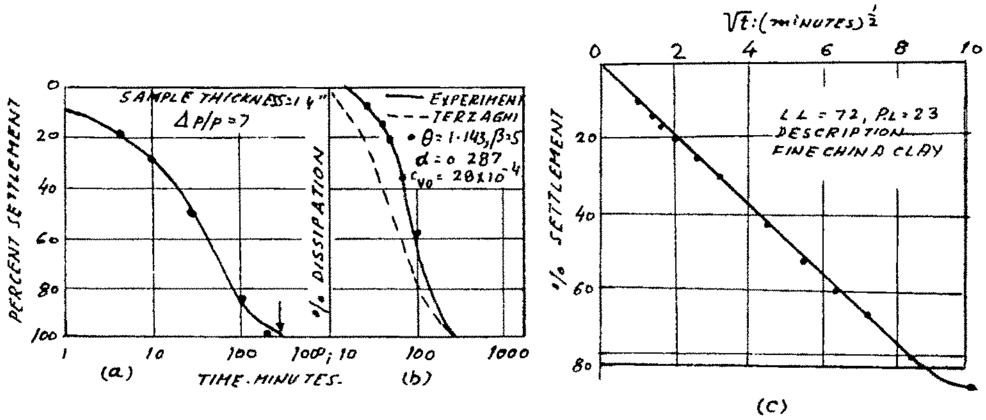

FIG. 3·34 EXPERIMENTAL & THEORETICAL RESULTS FOR KAOLINITE

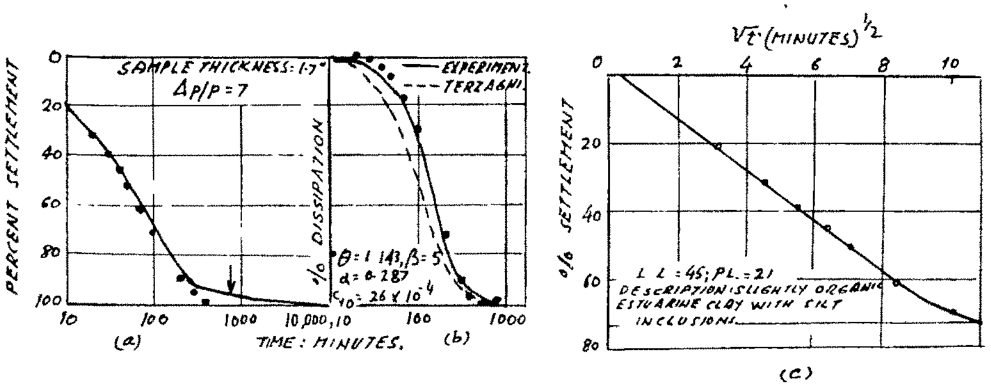

FIG. 3·35 EXPERIMENTAL & THEORETICAL RESULTS FOR FRODSHAM CLAY

terms of a recognized analogue. Once the anology is established, its entire frame work could be exploited to unfold the characteristics of parallel physical phenomenon. After Terzaghi Thermodynamic analogue attempts have been made to apply Thermodynamic laws, Osmotic principle and physical models.

3.3.1. Geuze Approach

Geuze (1948) applied the first law of Thermodynamics to evaluate the influence of loading speed on the amount of work done to obtain a definite amount of consolidation. The energetic point of view behind the theory is that the work done on a sample of soil saturated with water is mainly used to overcome (i) the side friction between the sample and the ring (ii) the internal frictional resistance during consolidation and (iii) the potential energy in the sample. Energy spent in the compression of solids and water molecule and also the kinetic effects are ignored. Based on the above analysis relation ship between load settlements and time settlements are obtained. Experimental results (Figures 3.36 and 3.37) support the argument that the slowest loading speed yields the largest settlement for a definite amount of work done. It shows that the phenomenon of retardation by hydrodynamic effects does not influence the results markedly.

3.3.2. Bolt Approach explained

Bolt (1956) exploited the analogy of osmosis in a water system according to which hydrostatic pressure gradient in double

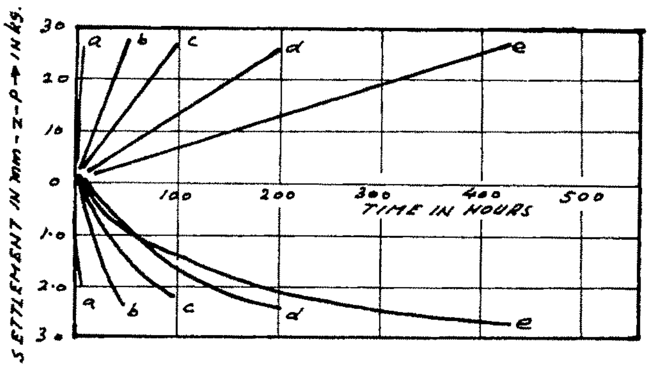

FIG. 3·36 SETTLEMENT TIME RELATIONSHIP (AFTER GEUZE AND BRUYN-1948)

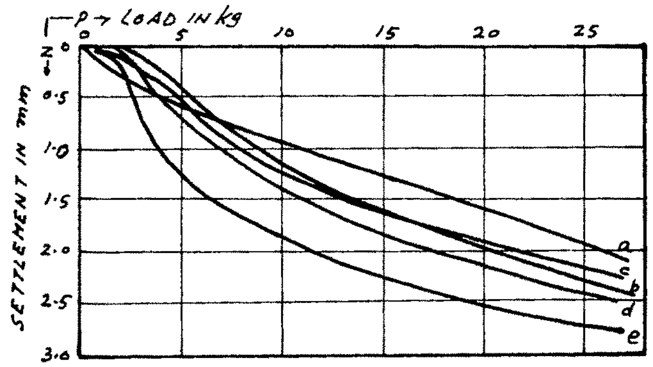

FIG. 3·37 SETTLEMENT LOAD RELATIONSHIP (AFTER GEUZE AND BRUYN-1948)

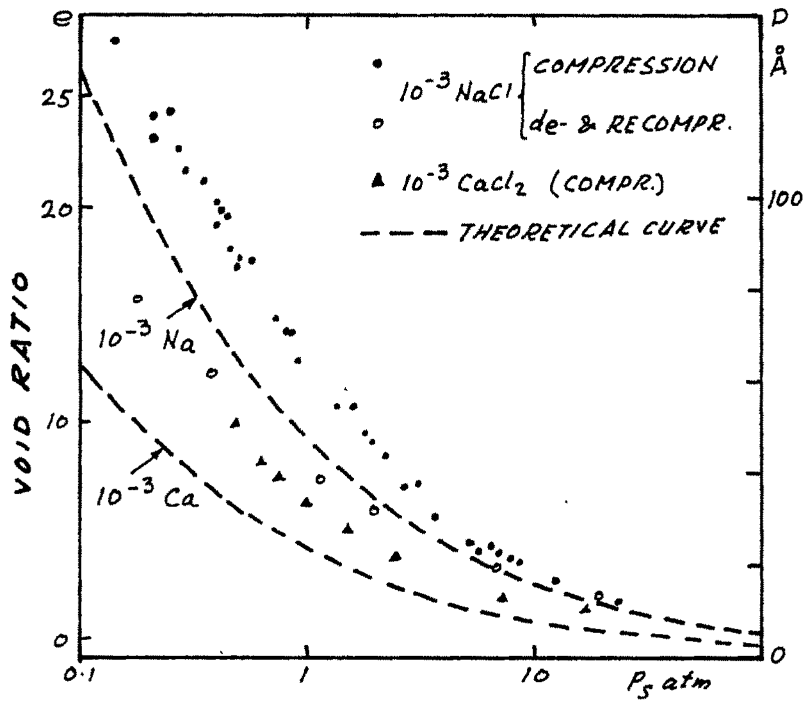

FIG. 3·38 COMPRESSION CURVES OF Na-MONTMORILLONITE & Ca-MONTMORILLONITE, FRACTION < 0·2 μ, IN EQUILIBRIUM WITH 10^{-3} MOLAR NaCl AND $CaCl_2$, RESPECTIVELY layer is balanced by the electric forces due to concentration of ions on particles. On this basis an approximate expression for the distance between the clay particles and the void ratio of the saturated clay water system is formulated. The above expression assumes the validity of Gouy theory, existence of parallel alignment of clay particles and applicability of Van't Hoff relationship. Experimental data compares well with the theoretical curve (Fig.3.38) except in the initial portion where a deviation is evident.

3.3.3. Josselin-de-Jong Analysis

Josselin-de-Jong (1969) employed the stochastic process to analyse the consolidation of the assembly of spring dash pot elements (Taylor and Merchant, 1939) and assembly of Cavity channel elements (Buisman, 1940). A continuous frequency function to characterise the mechanical behaviour of the system is determined through Laplace transform technique with power product law in complex plane. As the two systems behave very similarly the characteristics of the frequency function should remain mathematically identical. However, physically the two systems differ in the interpretation of excess pore pressure and permeability. In the former the excess pore pressure is well defined throughout the body at any moment while in the later case only the average value of the excess pore pressure is possible. The effect of the difference in the permeability is apparent only at the beginning of the settlement process.

Essentially the basic form of the differential equation remains of the same structure as in classical one.

A physical concept, however, attractive it may be, loses its value if it cannot be mathematically exploited to evaluate the parameters for the engineering analysis. In particular the thermodynamic approach becomes a futile exercise as the phenomenon cannot be ever said to satisfy the basic requirements of physics. A proper gearing between the osmotic analogy and mathematical method is still lacking to be of use even for laboratory verification.