CHAPTER IV

A GENERALISED THEORY OF ONE DIMENSIONAL CONSOLIDATION OF CLAYS

The fundamental requirements for a realistic theory of consolidation of clays are (i) consideration of the deformation characteristics of the constituent phases and (ii) application of a consistent mathematical technique. On the basis of the physical and experimental analysis of clays colloid water system due to various workers, it is possible to ascribe the state changes that occur in colloid particle, double layer and clay skeleton configuration consequent upon a consolidating stress. The basic necessity of the mathematical téchnique for analysing the deformation of this kind of material is that it should be valid even for large deformation.

4.1. Physical Background

4.1.1. Nature of Change in Clay Colloid

Grim (1948) argued that the properties of clays are difficult to account for without postulating some changes in the physical state of adsorbed compound in the lattice and surrounding the colloidal particle. On the basis of an analysis of the phase relations of water, Winterkorn (1943) showed that the change in state of water with distance from clay mineral surface is an exponential one. Since the interparticle spacing vary with time under pressure change, a time deformation relationship for the clay colloid could also be represented by an exponential function.

4.1.2. Nature of Change in Fabric Structure

A particular type of contacts forming a structure in clay material is dependent on the net resultant potential at the colloidal surfaces. A change in the contacts occurs as the stress is applied altering the value of the resultant electrical potential. The value of the resultant potential varies exponentially proportional to the applied stress which is gradually acting upon the particles. Hence, it will be reasonable to assume an exponential law for the change in fabric structure with time.

4.1.3. Nature of Flow of Pore Fluid

While considering the flow of pore fluid through an element of soil, at least two factors must be appreciated; one, the continuous contraction of the pore space and two, the drag forces. Gibson et al (1967) expressed the classical

Darcy's law in a more general form conforming to the experimental evidence due to Schiedegger (1957).

4.1.4. Nature of Effective Stress Law

In the Terzaghi (1923) classical principle of effective stress the compressibility of the individual grains is ignored. Bishop (1963) examined the influence of compressibility of water relative to the soil structure and the soil grains on the effective stress law for consolidation. For compressible grains, there is an excess pore water stress over and above the usual excess pore water stress due to distortion and displacement of individual grains. Thus the volume changes due to compressibility of the soil structure are controlled not by the classical effective stress law $\sigma' = \sigma - u$ but by; $\sigma - (1 + a_s)\, u$ where a_s denotes the grain area per unit cross sectional area. Lambe and Whitman (1959) suggested that the contact area of an expansive soil is a function of water content and further argued that areas of influence of hydrostatic pore water may overlap the adsorbed pore water particularly when there is air present in the voids. Skempton (1960) reported an expression of effective stress to be used for volume change as $\sigma' = \sigma - (1 - \frac{c_s}{c})\, u$ where c_s denotes the compressibility of the particle and c is the compressibility of the soil skeleton. For the specialized case of saturated soils of incompressible grains the equation reduces to the classical one.

4.2. Mathematical Formulation

The basic framework adopted for the present mathematical treatment is that of Gibson et al (1967); in fact, it represents the extension of their work and attempts to further generalisation based on physical consideration.

4.2.1. Governing Equations

(i) Coordinate Scheme :

Since in this problem the exact location in a space of the boundary at any time will not be known; the most advantageous choice will be that of the Lagranian scheme over the Eulerian scheme. We shall follow the history of every particle at all instants through coordinates x , y , z of any particle which are functions of the independent variables. a , b , c and t (Lamb, 1932). For the case of one dimensional movement, we may consider an element of the soil skeleton of unit cross sectional area normal to the direction of pore fluid which at time $t = o$ lies between planes embedded at distances a and $(a + \delta a)$ from an embedded datum plane. (Fig. 4.1a). At some subsequent time the same planes will be located at unknown distance $z(a, t)$ and $z(a + \delta a , t)$ from this datum plane. As per Lagrang co-ordinate each plane of particles shall be labelled throughout its subsequent motion by its initial distance 'a' from the datum plane; for example, the upper boundary of the layer is always at $a = a_o$ (Fig. 4.1a,b).

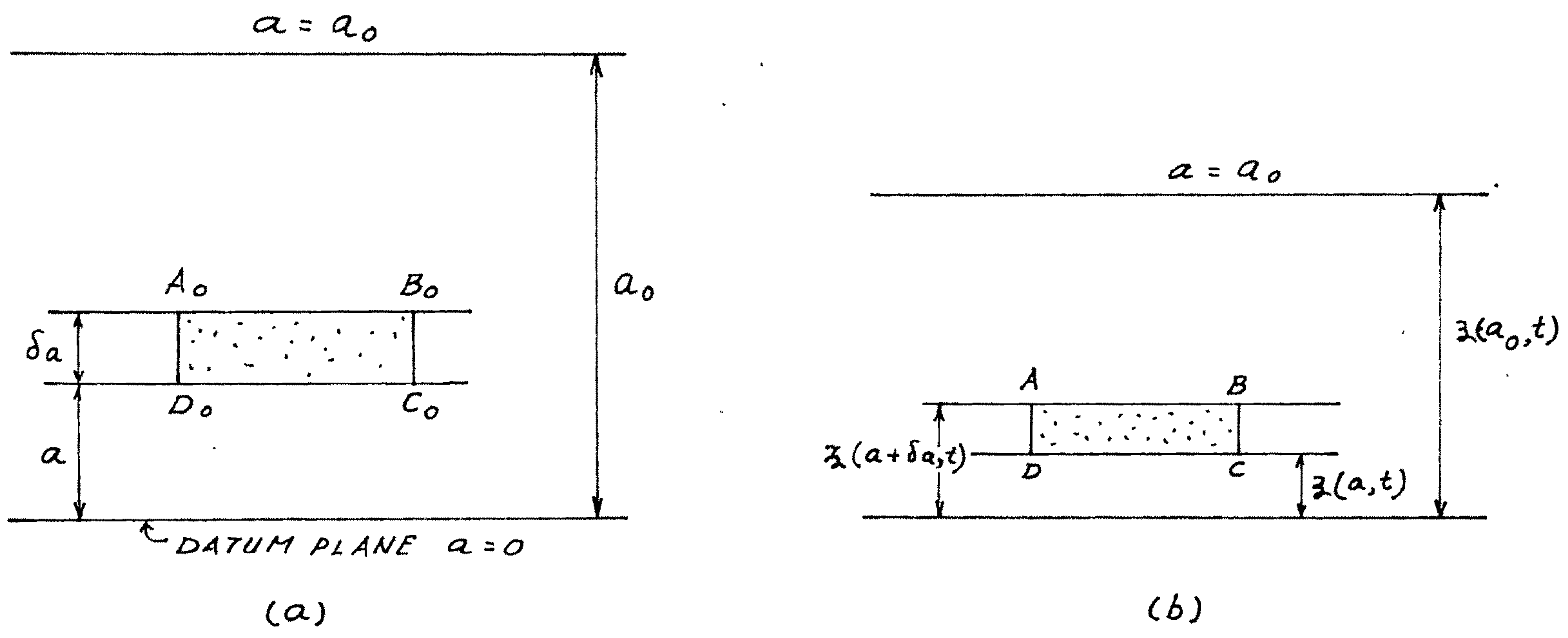

FIG. 4·1 CONFIGURATION OF CLAY ELEMENT

(a) INITIAL CONFIGURATION $t=0$ (b) CURRENT CONFIGURATION AT TIME t

(ii) Equilibrium Equation :

The vertical equilibrium equations of the soil grains and fluid are established by considering an elemental cylinder ABCD (Fig. 4.1b). Neglecting the inertia terms (Mandel, 1953) the equilibrium equation is :

$$\frac{\partial \sigma}{\partial a} \pm \left[n \rho_f + (1-n) \rho_s \right] \frac{\partial z}{\partial a} = 0 \qquad \ldots \quad \ldots \quad (4.1)$$

where σ the vertical total stress

n the initial volume porosity

ρ_f, ρ_s weights per unit volume of fluid and solid phases and are functions of a and t. (a has positive sign if measured against gravity).

(iii) Equation of Continuity :

In consideration of the volume changes occuring in the clay colloid and clay skeleton the equation of continuity for the solid phase will be written as :

$$\rho_s (a,0) \quad 1-n (a,0) = \rho_s (a,t) \quad 1-n (a,t) \cdot \frac{\partial z}{\partial a} \cdot \exp(-\tanh \qquad \ldots \quad \ldots \quad (4.2)$$

where the unit weight of the clay colloid changes with time as :

$$\rho_s (a,t) = \rho_s (a,0) \exp (\alpha \tanh t)$$

To determine the equation of continuity of fluid phase, the concept of relative velocity between solid phase , V_s and that of pore fluid, v_f will be used to eliminate the limitation imposed from continuous contraction of the pore space. Thus, the rate of weight in flow of fluid into the element ABCD will be equal to $n\,(v_f - v_s)\,\rho_f$ if we assume equality of area and volume porosities, while the rate of weight outflow of fluid will be :

$$\frac{\partial}{\partial a}\left[n\,\rho_f\,(v_f - v_s)\right]\delta a \quad .$$

Since then the rate of weight outflow of fluid equals the rate of change of fluid in the element,

$$\frac{\partial}{\partial a}\left[n\,\rho_f\,(v_f - v_s)\right] + \frac{\partial}{\partial t}\left[n\,\rho_f \cdot \frac{\partial \xi}{\partial a}\right] = 0 \qquad \ldots\ \ldots (4\cdot 3)$$

(iv) Flow Equation :

The flow of pore fluid through soil skeleton is in accordance with Darcy's law, but it is expressed in a general form to be consonant with the physics of the problem. Gibson et al (1967) proposed a following expression :

$$n\,(v_f - v_s) = -\frac{k}{\rho_f} \cdot \frac{\partial u}{\partial \xi} \qquad \ldots\ \ldots (4\cdot 4)$$

The influence of compressibility of space and drag forces does not invalidate the above expression.

The excess fluid pressure gradient could be expressed as

$$\frac{\partial u}{\partial z} = \frac{\partial p}{\partial z} \pm \rho_f \qquad \ldots \quad \ldots \quad \ldots \quad (4.5)$$

where p is the fluid pressure (above atmospheric) and the positive sign is taken if z is measured against gravity.

Since z is a dependent variable of 'a' , we will use the relation :

$$\frac{\partial p}{\partial a} = \frac{\partial p}{\partial z} \cdot \frac{\partial z}{\partial a} \qquad \ldots \quad \ldots \quad \ldots \quad (4.6)$$

Thus, uniting equations 4.4 and 4.5 to be consistent with our frame work, we get,

$$n \, (v_f - v_s) \frac{\partial z}{\partial a} = - \frac{k}{\rho_f} \left[\frac{\partial p}{\partial a} \pm \rho_f \frac{\partial z}{\partial a} \right] \qquad \ldots \ldots (4.7)$$

(v) Effective Stress Law :

The classical effective stress law need be rigoursly expressed in consideration of the variations in clay colloid and clay skeleton. The expression due to Gibson et al (1967) is of similar type to that of Skempton (1960) which is adopted in this treatment. The effective stress law used in the present approach is :

$$\sigma' = \sigma - \eta u \qquad \ldots \quad \ldots \quad \ldots (4.8)$$

in which $\eta = 1$ when the solid phase is of constant density and soil is completely saturated. Otherwise, it could be a function of a and t.

(vi) Permeability, Porosity and Fluid Density Relationship:

In regard to the physical considerations discussed in 4.1 the physical quantities, such as, coefficients of permeability, porosity or void ratio and fluid density need to be defined and given mathematical qualification.

k will be a function both of the porosity n or void ratio e and the location of the particular portion of the soil skeleton to account for possible non-homogenity. Therefore,

$$k = k\,(n, a) \qquad \text{.. .. .. (4.9)}$$

n will be a function of factors for nonhomogenity, stress history and time effects. Thus,

$$n = \phi\,(\sigma', a, t) \qquad \text{.. .. .. (4.10)}$$

where ϕ is a functional.

We shall consider valid the isothermal equation of state for the physical quantity ρ_f, thus,

$$\rho_f = \rho_f\,(p) \qquad \text{.. .. .. (4.11)}$$

4.2.2. Transformed Equations

For convenience we shall now resort to transformation to a new independent variable z to replace a.

$$z(a) = \int_0^a [1 - n(a',0)]\, da' \qquad \dots \dots (4.12)$$

and therefore,

$$\frac{\partial z}{\partial a} = 1 - n(a,0) \qquad \dots \dots (4.12\,a)$$

This means that any point of the soil skeleton is now identified by the volume of solids z in a prism of unit cross sectional area lying between the datum plane and the point. Needless to mention that just as with a , this new variable z is independent of t . Further, if we substitute the porosity by void ratio, simplified forms of the governing equations are possible.

Expressed in these new variables the previous equations become :

$$\frac{\partial \sigma}{\partial z} \pm \frac{e\rho_\gamma + \rho_s}{1+e} \cdot \frac{\partial \zeta}{\partial z} = 0 \qquad \dots \dots (4.1)\text{ bis}$$

$$\frac{\partial \zeta}{\partial z} = (1+e)\, \exp(\beta - \alpha) \tanh t \qquad \dots \dots (4.2)\text{ bis}$$

$$\frac{\partial}{\partial z}\left[\frac{e\rho_\gamma}{1+e}(v_\gamma - v_s)\right] + \frac{\partial}{\partial t}\left[\frac{e\rho_\gamma}{1+e} \cdot \frac{\partial \zeta}{\partial z}\right] = 0 \qquad \dots \dots (4.3)\text{ bi}$$

$$\left[\frac{e(v_\gamma - v_s)}{k(1+e)} \pm 1\right]\frac{\partial \zeta}{\partial z} + \frac{1}{\rho_\gamma} \cdot \frac{\partial p}{\partial z} = 0 \qquad \dots \dots (4.7)\text{ b}$$

Further simplification,

Using (4.2) bis in the transformed equations, we get,

$$\frac{\partial \sigma}{\partial z} \pm (e\rho_\gamma + \rho_s) \quad \exp(\beta - \alpha) \tanh t \qquad \cdots \cdots (4.1)*$$

(Note : * against the equation number denotes the simplified form of the previous equation).

$$\frac{\partial}{\partial z}\left[\frac{e\,\rho_\gamma}{1+e}(V_\gamma - V_s)\right] + \frac{\partial}{\partial t}\left[e\,\rho_\gamma \exp(\beta-\alpha)\tanh t\right] = 0 \qquad \cdots \cdots (4.3)*$$

$$\frac{\partial p}{\partial z} + \frac{e\,\rho_\gamma (V_\gamma - V_s)}{k} \cdot \exp(\beta-\alpha)\tanh t \pm \rho_\gamma (1+e)\exp(\beta-\alpha)\tanh t = 0 \qquad \cdots \cdots (4.7)*$$

Using equations 4.8, 4.9, 4.10 and further considering $\exp(\beta-\alpha)\tanh t$ equal to unity, the various relations previously written can now be combined to yield the following equations :

$$\frac{\partial}{\partial z}\left[\frac{k}{\rho_\gamma (1+e)} \cdot \frac{d\sigma'}{de} \cdot \frac{\partial e}{\partial z}\right] \pm \frac{\partial}{\partial z}\left[k \cdot \left\{\frac{e\rho_\gamma + \rho_s}{\rho_\gamma (1+e)}\right\} - \eta\right] + \frac{\partial e}{\partial t} = 0 \qquad \cdots \cdots (4.13)$$

4.2.3. Final Differential Equation

Returning to the original space variable a by using eqn. 4.12a we get,

$$\frac{\partial e}{\partial t} = C_f \cdot \frac{\partial^2 e}{\partial a^2} \mp C_e \frac{\partial e}{\partial a} \quad \ldots \ldots (4\cdot13) *$$

where,

$$C_f = \frac{k}{\rho_f} \cdot \frac{d\sigma'}{de} \cdot \frac{(1+e_o)^2}{1+e}$$

$$C_e = k\,(1+e_o) \left\{ \frac{\rho_s/\rho_f + e}{1+e} - \eta \right\}$$

in which,

$$e = e\,(\sigma', a, t)$$

$$k = k\,(e, a)$$

C_f is a quantity similar to the familiar coefficient of consolidation C_v of Terzaghi theory. From the expression it is evident that it is not a constant but a function of k, e and t Whereas C_e is a quantity which represents mainly void ratio as coefficient of permeability, k and η are dependent on e

By putting $Z = \frac{z}{h}$, $T = \frac{C_f t}{h^2}$ and $P = h \cdot \frac{C_e}{C_f}$

further transformation of the differential equation (4.13)bis is made to a familiar form :

$$\frac{\partial e}{\partial T} = \frac{\partial^2 e}{\partial Z^2} - P \frac{\partial e}{\partial Z} \quad \ldots \ldots (4\cdot3\cdot1)$$

We shall regard P as a constant in which all the factors causing deviation of the experimental observations from the classical Terzaghi theory are taken into account. The assumption of regarding ρ as a constant is purely from the considerations of mathematical simplicity but it may prove as well to be so for most soils. Referring to publications of mathematical physics it may be noted that it has an identical form to that of a differential equation for nonsteady one dimensional flow of heat through moving media. (Bateman, 1964).

4.3. Analytical Solution

Verma (1969) published a Laplace Transform solution of a one dimensional groundwater recharge. This technique is employed in the solution of the above equation for the boundary conditions of standard one dimensional consolidation of clays.

4.3.1. Boundary Conditions

In case of standard one dimensional consolidation test with top and bottom drainage, at the commencement of the test void ratio e_o is uniformly distributed over the depth of the sample and after some time void ratio will be e_1, at the top and bottom of the sample. Mathematically boundary conditions may be expressed as :

$$e(0,T) = e_1 \quad (T>0) \;;\; e(1,T) = e_1 \quad (T>0) \qquad \ldots (4.3.2)$$

$$e(z,0) = e_o \qquad \ldots (4.3.3)$$

4.3.2. Mathematical Treatment

On multiplying each term of equation (4.3.1) by $\exp\{-(sT)dT\}$ and integrating the result from zero to infinity, and further using condition $e(z,0)=0$ we obtain

$$\frac{d^2\bar{e}}{dz^2} - \frac{d\bar{e}}{dz} - s\bar{e} = -e_0 \quad \ldots \ldots (4.3.4)$$

where

$$\bar{e}(z,s) = \int_0^\infty \exp\{-(sT)\,dT\} \,.\, e(z,t)\,dT$$

represents the Laplace transform of $e(z,T)$

The Laplace transformation of the boundary conditions (4.3.2.) yields,

$$\bar{e}(0,s) = \frac{e_1}{s}, \quad \bar{e}(1,s) = \frac{e_1}{s} \quad \ldots \ldots (4.3.5)$$

Since equation (4.3.4) is a linear equation with constant co-efficient, we may write its general solution as :

$$\bar{e}(z,s) = \left[M \cosh\left(z\sqrt{P^2/4+s}\right) + N \sinh\left(z\sqrt{P^2/4+s}\right)\right]\exp\{(P/2)z\} \quad \ldots \ldots (4.3.6)$$

where M and N are constants of integration.

For evaluating M and N, we apply condition 4.3.5, so that, after some simplification, we have

$$M = \frac{e_1 - e_0}{s}, \qquad N = \frac{\frac{1}{s}\exp\left\{-(p/2)\right\} - \frac{e_1 - e_0}{s}\cosh\left(\sqrt{p^2/4 + s}\right)}{\sinh\left(\sqrt{p^2/4 + s}\right)}$$

Substituting these values in equation (4.3.6), we have :

$$\bar{e}(z, s) = \exp\left\{(p/2)z\right\}\frac{e_1 - e_0}{s}\left[\left\{\frac{\sinh(1-z)\left(\sqrt{p^2/4 + s}\right)}{\sinh\sqrt{p^2/4 + s}}\right\}\right] +$$

$$\exp\left\{-(p/2)(1-z)\right\}\frac{e_1 - e_0}{s}\left[\left\{\frac{\sinh z\sqrt{p^2/4 + s}}{\sinh\sqrt{p^2/4 + s}} + \frac{e_0}{p}\right\}\right] \qquad \ldots \ldots (4.3.7)$$

The inverse transform ($\overset{-1}{L}$) of the right hand side terms in equation (4.3.7) may be determined by recalling a standard result (Mickley et al, 1957), viz.,

$$\overset{-1}{L}\left|\frac{J(s)}{\eta(s)}\right| = \sum_{n=0}^{\infty}\frac{J(s_n)}{\eta(s_n)}\exp(s_n T) \qquad \ldots \ldots (4.3.8)$$

where $J(s)$ and $\eta(s)$ represents two entire transcendental functions such that degree of $\eta(s)$ is atleast one greater in

S (when expressed as power series) than that of $\mathcal{J}(s)$

S_n is a simple pole of $\frac{\mathcal{J}(s)}{\eta(s)}$, and $\eta^1(S_n)$ denotes the value of $\frac{d\eta(s)}{ds}$ at $s = S_n$

Putting

$$\frac{\mathcal{J}(s)}{\eta(s)} = \frac{\sinh(z\sqrt{p^2/4+s})}{s\sinh(\sqrt{p^2/4+s})} = \frac{\sinh(iz\sqrt{p^2/4+s})}{\sinh(i\sqrt{p^2/4+s})} \quad \ldots (4\cdot3\cdot9)$$

and noting that the roots of equation

$$\sinh\left(\sqrt{p^2/4+s}\right) = 0$$

are given by

$$S_n = -p^2/4 - n^2\pi^2$$

We may write :

$$\mathcal{J}(S_n) = \sin(n\pi z), \qquad \mathcal{J}(0) = i\sinh(p/2)z$$

$$\eta^1(s_n) = \frac{(-1)^n\left(p^2/4+n^2\pi^2\right)}{2n\pi}, \qquad \eta^1(0) = i\sinh(p/2) \quad \ldots (4\cdot3\cdot10)$$

From equations (4.3.8), (4.3.9) and (4.3.10), we get:

$$\overset{-1}{L} \quad \frac{\sin h\, Z\sqrt{P^2/4+s}}{s\,\sin h\sqrt{P^2/4+s}} =$$

$$\frac{\sin h\, \frac{P}{2} Z}{\sin h\, P/2} + 2\pi \sum_{n=1}^{\infty} \frac{(-1)^n\, n \sin(n\pi Z)}{P^2/4 + n^2\pi^2} \exp - \left\{ \left(P^2/4 + n^2\pi^2 \right) \right\} T \qquad \cdots (4.3.11)$$

Similarly, we have :

$$\overset{-1}{L} \left[\frac{\sin h(1-Z)\sqrt{P^2/4+s}}{s\,\sin h\sqrt{P^2/4+s}} \right] = \frac{\sin h\, \frac{P}{2}(1-Z)}{\sin h\,(P/2)} -$$

$$2\pi \sum_{n=1}^{\infty} \frac{n \sin(n\pi Z)}{P^2/4 + n^2\pi^2} \cdot \exp\left\{ -\left(P^2/4 + n^2\pi^2 \right) \right\} T \qquad \cdots (4.3.12)$$

The inverse transformation of equation 4.3.7 with the help of equation (4.3.11) and (4.3.12) yields :

$$e(Z,T) = (e_1 - e_0)\exp\left\{ (P/2) Z \right\} \frac{\sin h(\frac{P}{2})Z}{\sin h(P/2)} + 2\pi \sum_{n=1}^{\infty} (-1)^n \frac{n \sin n\pi Z}{P^2/4 + n^2\pi^2}$$

$$\exp\left\{ -\left(P^2/4 + n^2\pi^2 \right) T \right\} + (e_1 - e_0)\exp\left\{ -(P/2)(1-Z) \right\}$$

$$\left[\frac{\sin h(P/2)(1-Z)}{\sin h\,(P/2)} - 2\pi \sum_{n=1,3,5}^{\infty} \frac{n \sin n\pi Z}{P^2/4 + n^2\pi^2} \exp\left\{ -\left(\frac{P^2}{4} + n^2\pi^2 \right) \right\} T \right] \cdot \qquad \cdots (4.3.13)$$

as $\overset{-1}{L}\left(\frac{1}{p}\right) = 1$

$$U_Z = \frac{e - e_o}{e_1 - e_o}$$

$$= \exp\left(\frac{p}{2}\right) \frac{\sinh \frac{p}{2} Z}{\sinh \frac{p}{2}} + 2\pi \sum_{n=1,3,5}^{\infty} \frac{(-1)^n \cdot n \sin n\pi Z}{\frac{p^2}{4} + n^2\pi^2} \exp - \left(\frac{p^2}{4} + n^2\pi^2\right) T$$

$$+ \exp\left(-\frac{p}{2}\right) \frac{\sinh \frac{p}{2} (1-Z)}{\sinh \frac{p}{2}} - 2\pi \sum_{n=1,3,5}^{\infty} \frac{n \sin n\pi Z}{p^2/4 + n^2\pi^2} \exp - \left\{\left(\frac{p^2}{4} + n^2\pi^2\right\} T\right.$$

... (4.3.14)

when $p = 0$

$$\lim_{p \to 0} \frac{\sinh \frac{p}{2} Z}{\sinh \frac{p}{2}} \to Z$$

and

$$\lim_{p \to 0} \frac{\sinh \frac{p}{2} (1-Z)}{\sinh p/2} \to 1 - Z$$

$$= 1 - 4\pi \sum_{n=1,3,5}^{\infty} \frac{n \sin n\pi Z}{n^2\pi^2} \exp\left\{-(n^2\pi^2 T)\right\}$$

... (4.3.15)

4.3.3. Theoretical Relationships

To obtain theoretical relationships for the process of consolidation a Fortran programme for the expression (4.3.14) of the previous section was run on IBM-1402 Computer. Isochrones for various values of parameter ρ has been portrayed in Volume II from Figures G.1 to G.33. Figures 4.2 to 4.6 represents the degree of consolidation at various depths against the time factor from which the pore pressure dissipation at various depths is possible to deduce by the expression : $U_z = 1 - \frac{u_z}{u_i}$ where u_z denotes the pore pressure at any depth z and u_i is the initial value of pore pressure while in experimental studies will be the value of increment of pressure applied. Fig. 4.7 shows the theoretical relationship between the average degree of consolidation and Time factor generally used for comparing the experimental results. The value of average degree of consolidation is computed using Simpson's rule as explained in Taylor (1948) and Lambe - Whitman (1969).

Experimental data obtained from the laboratory studies to investigate the influence of various factors affecting the consolidation characteristics of clays are analysed using these theoretical relationships.

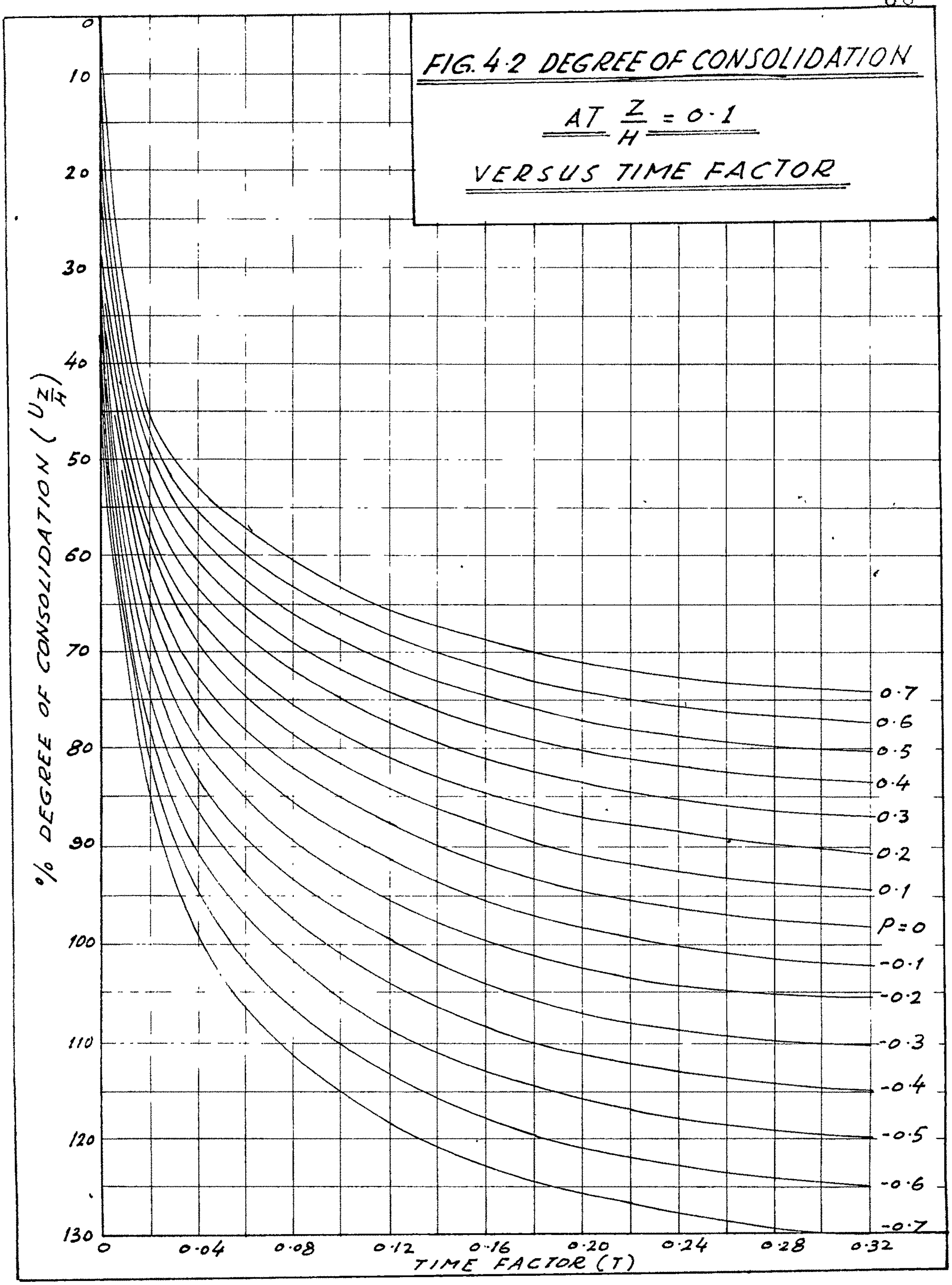
FIG. 4·2 DEGREE OF CONSOLIDATION
AT Z/H = 0·1
VERSUS TIME FACTOR
% DEGREE OF CONSOLIDATION (U Z/H)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
0
0·04
0·08
0·12
0·16
0·20
0·24
0·28
0·32
TIME FACTOR (T)
0·7
0·6
0·5
0·4
0·3
0·2
0·1
P=0
-0·1
-0·2
-0·3
-0·4
-0·5
-0·6
-0·7

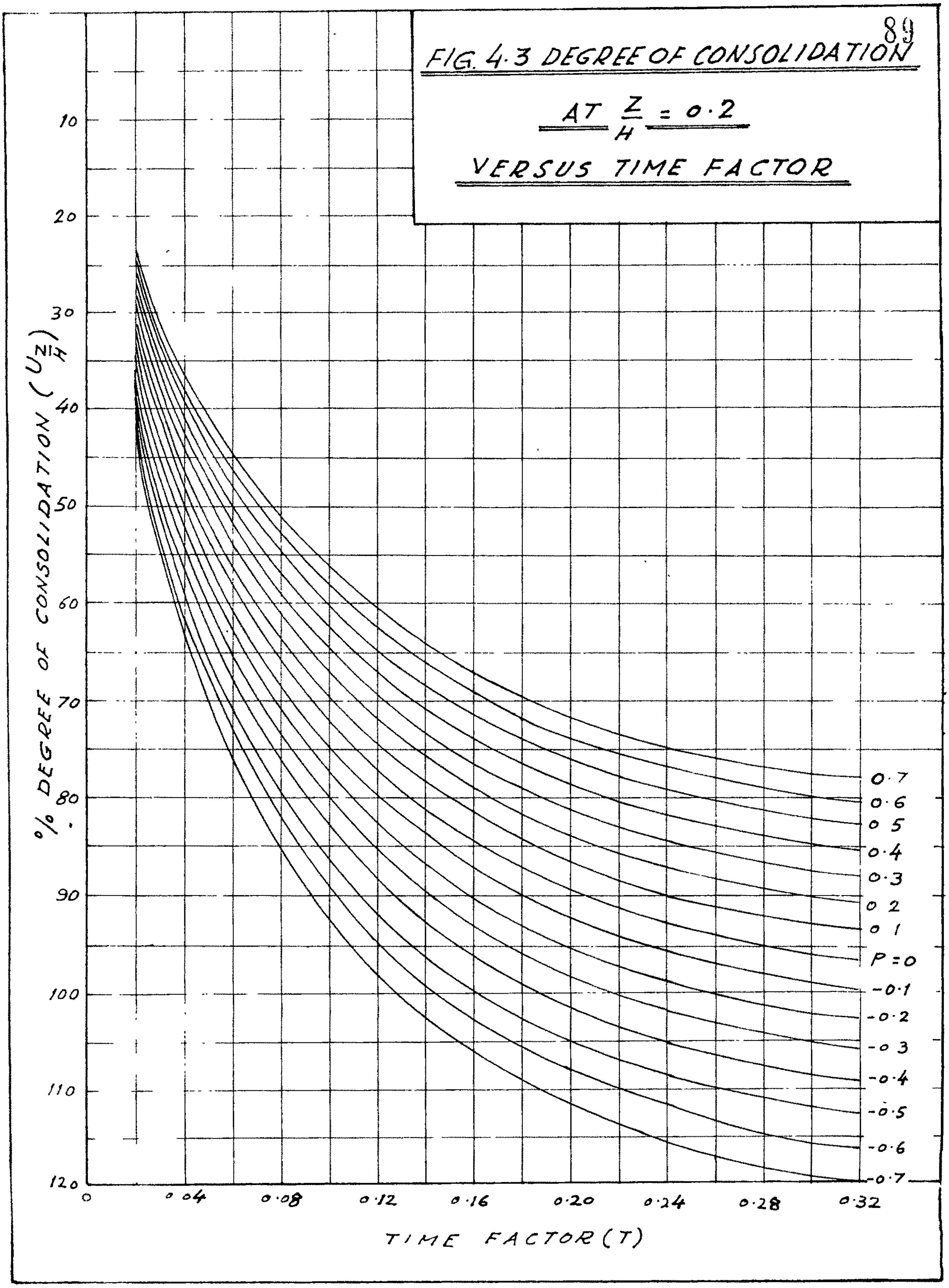
FIG. 4.3 DEGREE OF CONSOLIDATION
AT Z/H = 0·2
VERSUS TIME FACTOR
% DEGREE OF CONSOLIDATION (U Z/H)
10
20
30
40
50
60
70
80
90
100
110
120
0
0·04
0·08
0·12
0·16
0·20
0·24
0·28
0·32
TIME FACTOR (T)
0·7
0·6
0·5
0·4
0·3
0·2
0·1
P = 0
-0·1
-0·2
-0·3
-0·4
-0·5
-0·6
-0·7

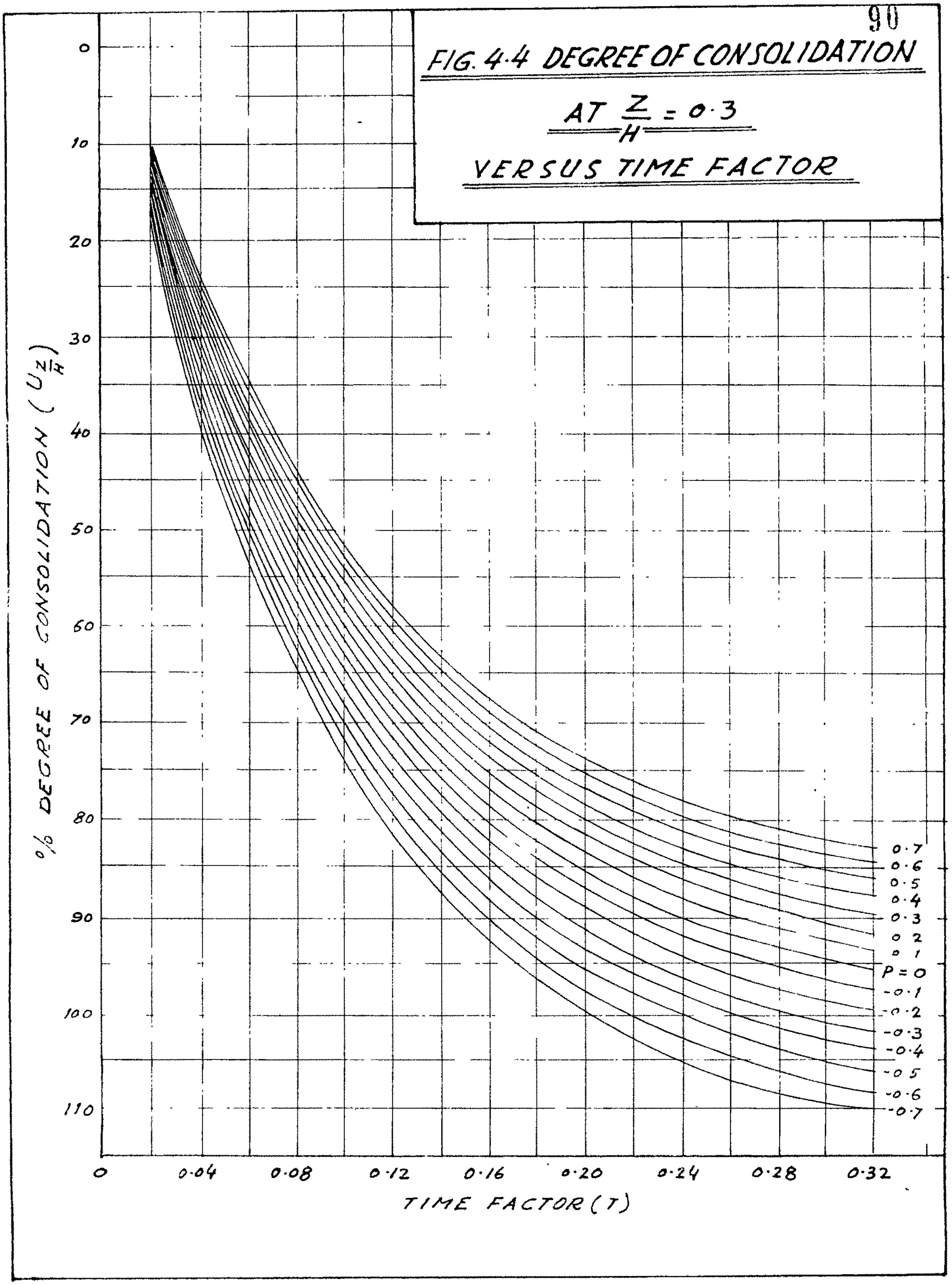

FIG. 4·4 DEGREE OF CONSOLIDATION AT $\frac{Z}{H} = 0{\cdot}3$ VERSUS TIME FACTOR

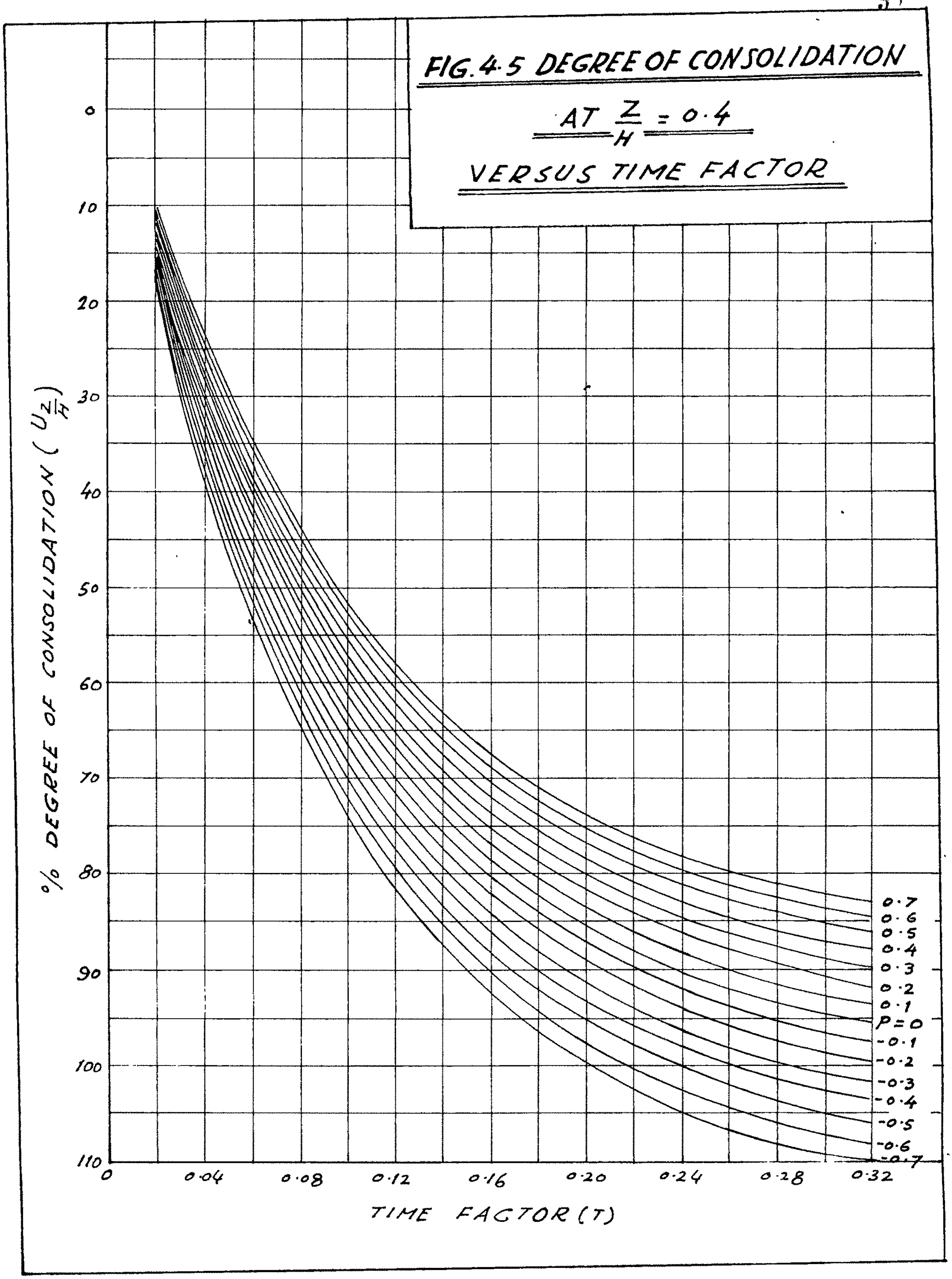
FIG. 4.5 DEGREE OF CONSOLIDATION
AT Z/H = 0.4
VERSUS TIME FACTOR
% DEGREE OF CONSOLIDATION (U Z/H)
0
10
20
30
40
50
60
70
80
90
100
110
0
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
TIME FACTOR (T)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
P = 0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7

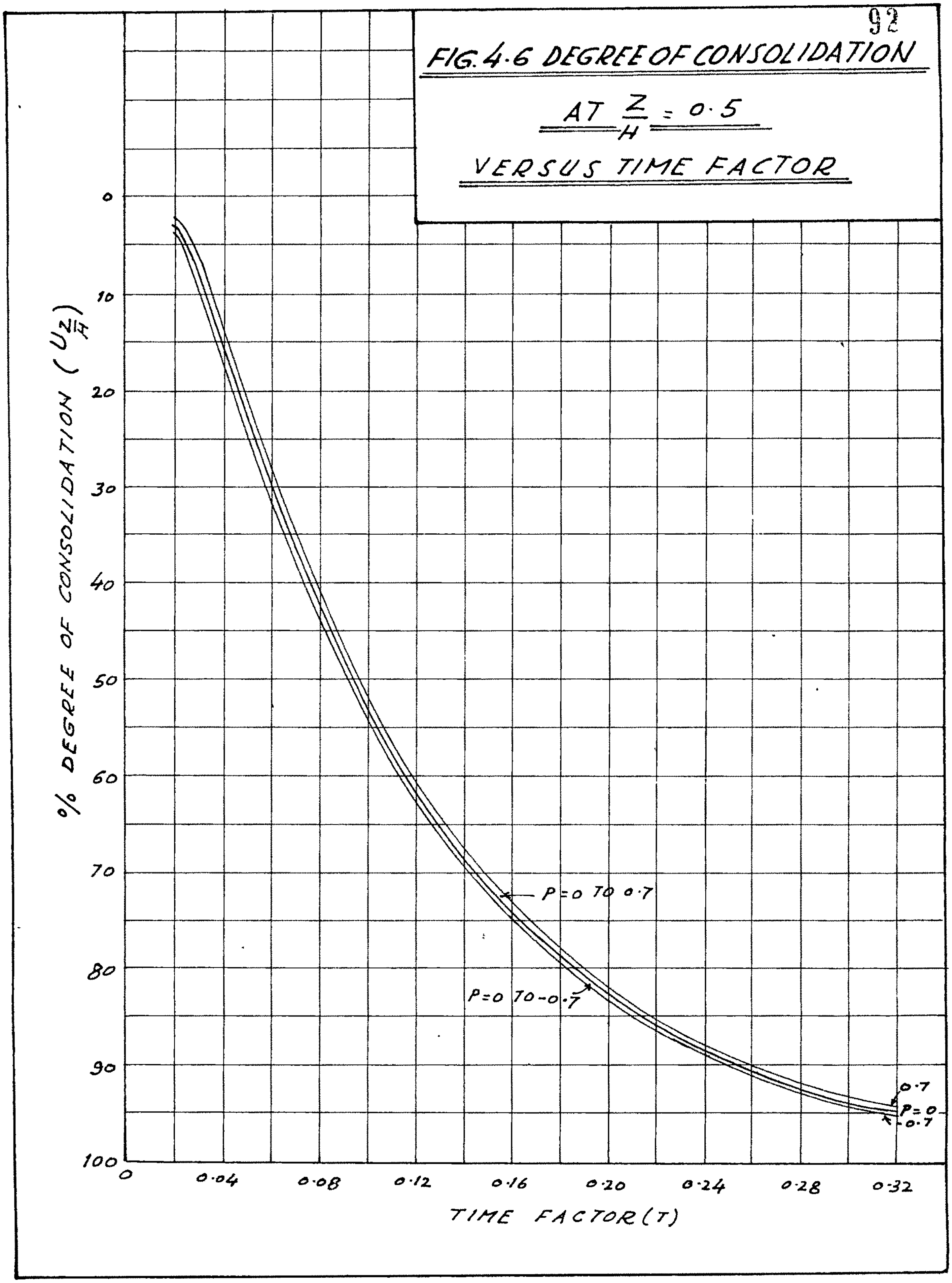
FIG. 4·6 DEGREE OF CONSOLIDATION
AT Z/H = 0·5
VERSUS TIME FACTOR
% DEGREE OF CONSOLIDATION ($U_{Z/H}$)
0
10
20
30
40
50
60
70
80
90
100
0
0·04
0·08
0·12
0·16
0·20
0·24
0·28
0·32
TIME FACTOR (T)
P = 0 TO 0·7
P = 0 TO −0·7
0·7
P = 0
−0·7

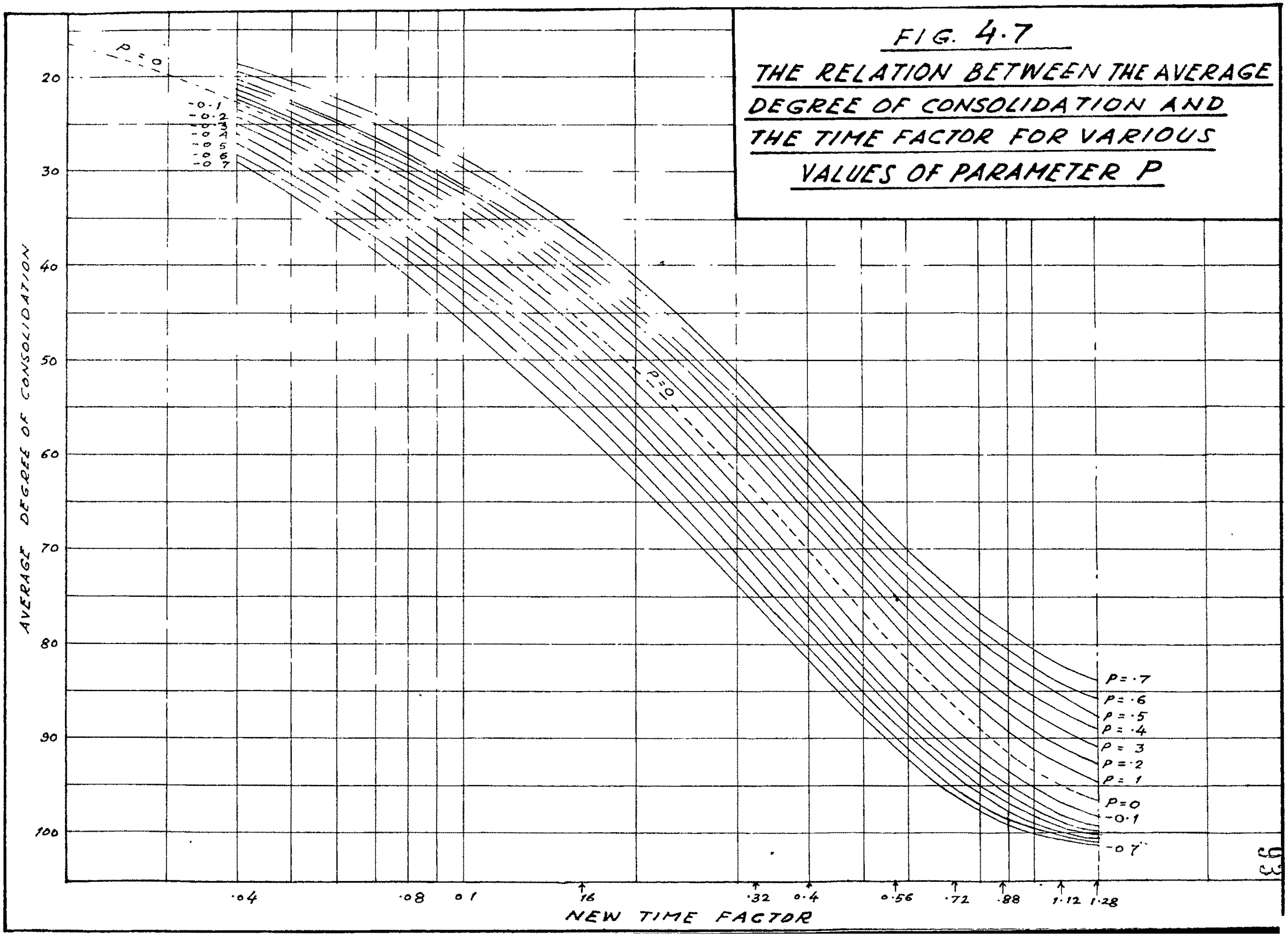
FIG. 4·7
THE RELATION BETWEEN THE AVERAGE DEGREE OF CONSOLIDATION AND THE TIME FACTOR FOR VARIOUS VALUES OF PARAMETER P
AVERAGE DEGREE OF CONSOLIDATION
20
30
40
50
60
70
80
90
100
NEW TIME FACTOR
·04
·08
0·1
·16
·32
0·4
0·56
·72
·88
1·12
1·28
P = 0
P = ·7
P = ·6
P = ·5
P = ·4
P = 3
P = ·2
P = 1
P=0
-0·1
-0·7

NOTATIONS

Symbol	Meaning	Dimensions
σ	Total Stress	FL^{-2}
σ'	Effective stress	FL^{-2}
u	Pore water pressure	FL^{-2}
a_s	Contact area per unit cross-section	L^2
a	Independent variable	L
a_0	Initial height of the soil element	L
z	Unknown distance from datum plane at any time	L
t	Time	T
c_s	Compressibility of particle	$F^{-1}L^2$
c	Compressibility of soil skeleton	$F^{-1}L^2$
n	Initial volume porosity	L^3
ρ_f	Weight per unit volume of fluid	FL^{-3}
ρ_s	Weight per unit volume of solid	FL^{-3}
α	Constant	-
v_s	Velocity of soil solid	LT^{-2}
v_f	Velocity of pore fluid	LT^{-2}
k	Coefficient of permeability	LT^{-1}
p	Fluid pressure (above atmospheric)	FL^{-2}
η	Pore water coefficient	-

Symbol	Meaning	Dimensions
z	Volume of solids	L^3
β	Constant	-
e_o	Initial void ratio	-
e	Void ratio at any time	-
Z	Depth dimension	Non-dimensional
c_f	Coefficient of consolidation as per generalised theory	LT^{-2}
c_v	Coefficient of consolidation as per Terzaghi theory	LT^{-2}
c_e	Lumped coefficient of compressibility	
$P = h\frac{c_e}{c_f}$	Lumped parameter	Non-dimensional
T	Time factor	Non-dimensional
h	Thickness	L
M	Constant of Integration	-
N	Constant of Integration	-
$\zeta(s)$	Entire transcendental function	-
$\eta(s)$	Entire transcendental function	-
s_n	Simple pole of	
U	Average degree of consolidation	
U_z	Percentage consolidation at depth dimension	
u_i	Initial pore water pressure	FL^{-2}
u_z	Pore water pressure at depth at any time	FL^{-2}