CHAPTER-II

GENERAL FORMALISM OF LOW DIMENSIONAL SYSTEMS

In this chapter, we present a general formalism of conductivity in real space by employing Maxwell's equations for different kinds of low dimensional systems, in a unified manner. Calculated conductivity takes into account the screening effects due to electron-electron interactions. Our general formalism is applied to calculate dynamical conductivities of three-dimensional, two-dimensional and one-dimensional and layered free electron gas for all values of wave vector and frequencies. It is found that screening effects become less significant on reduction in dimensionality. Our calculations reproduces well known long wavelength results on three-and-two dimensional free electron gas. Formalism presented in this chapter will be used to calculate collective excitations, optical properties and d.c conductivity of various types of low dimensional systems, in forthcoming chapters

2.1 Introduction

LDS have extensively been investigated both theoretically as well as experimentally over a period of more than three decades because of their fundamental and technological interests [1-5]. These systems have attracted a great deal of recent interest in context of high temperature superconductivity in cuprates which are highly anisotropic and exhibit Q2D behaviour [6]. Artificially prepared semiconductor structures such as SLs, QWS and the QDS form an important class of LDS on which much of the research work has been done including several extensive reviews and books [4,5]. Theoretical research on LDS has largely been motivated by fabrication of materials in which charge carriers can be confined in 2D planes, 1D wires and 0D dots under real experimental conditions [7-10].

The 2DEG can be realised at inversion layers in semiconductor devices, whereas Q2DEG can be realised in modulation doped semiconductor superlattices and quantum well structures Important aspects of 2DEG, Q2DEG and Q1DEG which have been studied in past include collective excitations [11], light scattering [12,13], many body effects mainly focussing on inelastic electron-electron scattering [14-16], screening of hydrogen like impurities [17] and the propagation of electromagnetic waves [18-20]. Large literature exists on collective excitations in 2DEG and Q2DEG dealing elaborately with all kinds of effects which can arise in a real superlattice or quantum well structure [11]. Response of a system to an electromagnetic field comprises of several dynamical processes in a system. It involves single particle and collective excitations, energy loss, optical processes and conduction of charge carriers. Dynamical conductivity, which describes the response of a system to an electromagnetic field, has been a fundamental problem of condensed matter physics from view point of study of collective excitation, optical properties, charge transport and relaxation processes The aim of this chapter is to present a calculation of dynamical conductivity for different types of LDS.

There have been several efforts to calculate and to understand dynamical conductivity of LDS, which significantly differs from the dynamical conductivity of isotropic 3D systems [21-30]. High frequency conductivity, memory function and relaxation time for electron impurity and electron phonon scattering have been studied for semiconductor superlattices [25-31]. Recently, longitudinal as well as transverse conductivities have been calculated for a periodic structure of metallic sheets to study the longitudinal and transverse plasmons in high temperature superconductors [18] We here report a generalised formalism (properly incorporating the screening effects arising from electron-electron interaction) of longitudinal and transverse dynamical conductivities. The general formalism of dynamical conductivity is applicable to solids of full translational reduced symmetry and also of the reduced dimensionality General formalism of conductivity is used to calculate dynamical conductivity of 3DFEG. 2DFEG and 1DFEG, LEG and QWS. The main aim of this chapter is to demonstrate that macroscopic and microscopic dynamical conductivities can be related in a closed form and they can be expressed in terms of density response function and dielectric response function in real space where reduction in symmetry and dimensionality can be handled more accurately and conveniently It is demonstrated that the present formalism is correct and it reproduces the well known results on 3DFEG and 2DFEG The conductivity of LEG differs from that of isotropic 3D or 2D system Development of general formalism of conductivity is given in sec 2.2 In sec.2.3, we report an unified calculation of conductivities of 3DFEG, 2DFEG and 1DFEG. Section 2 4 deals with calculations of conductivity in LEG Summary of this chapter is given in sec 2.5

2.2 General Formalism

Maxwell's equations can be combined with the equation of continuity to derive a self-consistent integral equation relating macroscopic conductivity, $\sigma(\mathbf{r},\mathbf{r}',\omega)$ and microscopic quasi-conductivity, $\tilde{\sigma}(\mathbf{r},\mathbf{r}',\omega)$ for an electromagnetic field. Continuity equation can be written as [21]

$$\nabla.\mathbf{J}_{ind}(\mathbf{r},\omega) = i\omega\rho_{ind}(\mathbf{r},\omega), \tag{2.1}$$

where $\mathbf{J}_{ind}(\mathbf{r},\omega)$ and $\rho_{ind}(\mathbf{r},\omega)$ are induced macroscopic current density and the induced charge density at position vector $\mathbf{r}$. ω is the frequency of the applied field. The $\mathbf{J}(\mathbf{r},\omega)$ is related to macroscopic field, $\mathbf{E}(\mathbf{r},\omega)$ and external field, $\mathbf{E}_{ext}(\mathbf{r},\omega)$ through $\sigma(\mathbf{r},\mathbf{r}',\omega)$ and $\tilde{\sigma}(\mathbf{r},\mathbf{r}',\omega)$, respectively in the following manner [22].

$$\mathbf{J}_{ind}(\mathbf{r},\omega) = \int \sigma(\mathbf{r},\mathbf{r}',\omega)\, \mathbf{E}(\mathbf{r}',\omega)\, d^3\mathbf{r}'. \tag{2.2a}$$

$$= \int \tilde{\sigma}(\mathbf{r},\mathbf{r}',\omega)\, \mathbf{E}_{ext}(\mathbf{r}',\omega)\, d^3\mathbf{r}'. \tag{2.2b}$$

Similarly, $\rho_{ind}(\mathbf{r},\omega)$ can also be related to total potential, $\phi(\mathbf{r},\omega)$ and external potential, $\phi_{ext}(\mathbf{r},\omega)$ through polarization function $\alpha(\mathbf{r},\mathbf{r}',\omega)$ (response function in absence of Coulomb electron-electron interaction) and $\beta(\mathbf{r},\mathbf{r}',\omega)$ (density response function in presence of Coulomb electron-electron interaction).

$$\rho_{ind}(\mathbf{r},\omega) = \int \beta(\mathbf{r},\mathbf{r}',\omega)\, \phi_{ext}(\mathbf{r}',\omega)\, d^3\mathbf{r}'. \tag{2.3a}$$

$$= \int \alpha(\mathbf{r},\mathbf{r}',\omega)\, \phi(\mathbf{r}',\omega)\, d^3\mathbf{r}'. \tag{2.3b}$$

On combining Eqs.(2.2a), (2.3a) and (2.1), one gets

$$\nabla.\nabla'\, \sigma(\mathbf{r},\mathbf{r}',\omega) = i\omega e^2\, \alpha(\mathbf{r},\mathbf{r}',\omega). \tag{2.4a}$$

Similarly, Eqs.(2.2b), (2.3b) and (2.1) gives

$$\nabla.\nabla'\, \tilde{\sigma}(\mathbf{r},\mathbf{r}',\omega) = i\omega e^2\, \beta(\mathbf{r},\mathbf{r}',\omega). \tag{2.4b}$$

The $\sigma(\mathbf{r},\mathbf{r}',\omega)$ describes conductivity in absence of screening effects, whereas $\tilde{\sigma}(\mathbf{r},\mathbf{r}',\omega)$ is the conductivity including screening effects arising from Coulomb electron-electron interactions. Equation (2.4a) and (2.4b) are applicable to 3D, 2D, and 1D systems. For 3D system, where $\mathbf{r}$ and $\mathbf{r}'$ are treated as 3D vectors, Eq.(2.4a) and (2.4b) have solutions [32]

$$\sigma(\mathbf{r},\mathbf{r}',\omega) = -(i\omega e^2/4\pi) \int \alpha(\mathbf{r},\mathbf{r}'',\omega) / |\mathbf{r}''-\mathbf{r}'| \, d\mathbf{r}'' \qquad (2.5a)$$

and

$$\tilde{\sigma}(\mathbf{r},\mathbf{r}',\omega) = -(i\omega e^2/4\pi) \int \beta(\mathbf{r},\mathbf{r}'',\omega) / |\mathbf{r}''-\mathbf{r}'| \, d\mathbf{r}''. \qquad (2.5b)$$

For a 2D system, solutions of Eqs.(2.4a) and (2.4b) are given by [32]

$$\sigma(\rho,\rho',\omega) = -(i\omega e^2/2\pi) \int \ln|\rho-\rho''| \, \alpha(\rho'',\rho',\omega) \, d^2\rho'' \qquad (2.6a)$$

and

$$\tilde{\sigma}(\rho,\rho',\omega) = -(i\omega e^2/2\pi) \int \ln|\rho-\rho''| \, \beta(\rho'',\rho',\omega) \, d^2\rho'', \qquad (2.6b)$$

where ρ, ρ' and ρ'' are 2D vectors Equations (2.1) to (2.6) are valid for both longitudinal and as well as transverse fields [21]. Any solution of Eqs (2.4a) and (2.4b) does not exist in 1D. However, a solution can be obtained for a quantum wire of finite transverse width.

2.21 Longitudinal Conductivity

On application of a longitudinal electromagnetic field. $\rho_{ind}(\mathbf{r},\omega)$ can be related to the induced potential. $\phi_{ind}(\mathbf{r},\omega)$ in following manner;

$$\phi_{ind}(\mathbf{r},\omega) = \int V(\mathbf{r},\mathbf{r}') \, \rho_{ind}(\mathbf{r}',\omega) \, d^3\mathbf{r}', \qquad (2.7a)$$

where

$$V(\mathbf{r},\mathbf{r}') = e^2 / |\mathbf{r}-\mathbf{r}'| \qquad (2.7b)$$

is the Coulomb electron-electron interaction Equations (2.3) and (2 7), with the use of

$$\phi(\mathbf{r},\omega) = \phi_{ext}(\mathbf{r},\omega) + \phi_{ind}(\mathbf{r},\omega), \qquad (2.8)$$

result in the following integral equation [21]

$$\alpha^L(\mathbf{r},\mathbf{r}',\omega) = \beta^L(\mathbf{r},\mathbf{r}',\omega) - \iint \alpha^L(\mathbf{r},\mathbf{r}_1,\omega)\, V(\mathbf{r}_1,\mathbf{r}_2)\, \beta^L(\mathbf{r}_2,\mathbf{r}',\omega)\, d^3\mathbf{r}_1\, d^3\mathbf{r}_2, \tag{2.9}$$

where L stands for longitudinal component of field. The current density for bound electrons, $\mathbf{J}_b(\mathbf{r},\omega)$ can be given by [22]

$$\mathbf{J}_b(\mathbf{r},\omega) = \partial P(\mathbf{r},\omega)/\partial t, \tag{2.10}$$

where $P(\mathbf{r},\omega)$ is the electric polarization. Combining Eqs.(2.1) and (2.10), one gets

$$\nabla.\mathbf{P}(\mathbf{r},\omega) = -\rho_b(\mathbf{r},\omega). \tag{2.11}$$

$\rho_b(\mathbf{r},\omega)$ is the charge density of bound electrons. We combine Eqs.(2.11) with Maxwell's equation

$$\nabla.\mathbf{D}(\mathbf{r},\omega) = 4\pi\rho^f(\mathbf{r},\omega) \tag{2.12}$$

to get

$$\mathbf{E}(\mathbf{r},\omega) = -(4\pi i/\omega)\, \mathbf{J}(\mathbf{r},\omega), \tag{2.13}$$

where $\mathbf{D}(\mathbf{r},\omega)$ is the electric displacement vector and $\rho^f(\mathbf{r},\omega)$ is the charge density of free electrons. Equation (2.13) can be written as

$$\mathbf{E}(\mathbf{r},\omega) - \mathbf{E}_{ext}(\mathbf{r},\omega) = -(4\pi i/\omega)\, \mathbf{J}_{ind}(\mathbf{r},\omega). \tag{2.14}$$

Equation (2.14) and (2.2) can be transformed to

$$\mathbf{E}_{ext}(\mathbf{r},\omega) = \int \varepsilon(\mathbf{r},\mathbf{r}'',\omega)\, \mathbf{E}(\mathbf{r}'',\omega)\, d^3\mathbf{r}'' \tag{2.15a}$$

and

$$\mathbf{E}(\mathbf{r},\omega) = \int \varepsilon^{-1}(\mathbf{r},\mathbf{r}'',\omega)\mathbf{E}_{ext}(\mathbf{r}'',\omega)\, d^3\mathbf{r}'', \tag{2.15b}$$

where we define

$$\varepsilon(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}') + (4\pi i/\omega)\, \sigma(\mathbf{r},\mathbf{r}',\omega) \tag{2.16a}$$

and

$$\varepsilon^{-1}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}') - (4\pi i/\omega)\,\tilde{\sigma}(\mathbf{r},\mathbf{r}',\omega). \tag{2.16b}$$

Here $\varepsilon(\mathbf{r},\mathbf{r}',\omega)$ and $\varepsilon^{-1}(\mathbf{r},\mathbf{r}',\omega)$ are longitudinal dielectric response function and inverse of dielectric function within linear response theory, respectively. This demands

$$\int \varepsilon^{-1}(\mathbf{r},\mathbf{r}'',\omega)\,\varepsilon(\mathbf{r}'',\mathbf{r}',\omega)\, d^3\mathbf{r}'' = \delta(\mathbf{r}-\mathbf{r}') \tag{2.17}$$

which results in

$$\tilde{\sigma}^L(\mathbf{r},\mathbf{r}',\omega) = \sigma^L(\mathbf{r},\mathbf{r}',\omega) - \iint \tilde{\sigma}^L(\mathbf{r},\mathbf{r}_1,\omega)\,V(\mathbf{r}_1,\mathbf{r}_2)\,\alpha^L(\mathbf{r}_2,\mathbf{r}',\omega)\, d^3\mathbf{r}_1\, d^3\mathbf{r}_2. \tag{2.18}$$

Equation (2.18) can be transformed to

$$\tilde{\sigma}^L(\mathbf{r},\mathbf{r}',\omega) = \int [\varepsilon^{-1}(\mathbf{r},\mathbf{r}'',\omega)\,\sigma^L(\mathbf{r}'',\mathbf{r}',\omega)\, d^3\mathbf{r}''. \tag{2.19}$$

Equation (2.5) and (2.16a) can also be written as

$$\varepsilon^L(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}') + \int V(\mathbf{r},\mathbf{r}'')\,\alpha^L(\mathbf{r}'',\mathbf{r}',\omega)\, d^3\mathbf{r}''. \tag{2.20}$$

The macroscopic and microscopic conductivities are contributed by the free and bound charges. Hence σ is sum of σ_e (electric part) as well as σ_i (ionic part). The $\sigma^L(\mathbf{r},\mathbf{r}',\omega)$ is obtained from Eq.(2 5a) on replacing $\alpha(\mathbf{r},\mathbf{r}',\omega)$ by $\alpha^L(\mathbf{r},\mathbf{r}',\omega)$. Evaluation of $\sigma^L(\mathbf{r},\mathbf{r}',\omega)$ and $\tilde{\sigma}^L(\mathbf{r},\mathbf{r}',\omega)$ basically depends on calculation of $\alpha_e^L(\mathbf{r},\mathbf{r}',\omega)$, electronic polarizability, which involves relaxation time, τ for scattering of a charge carrier from impurity potentials, lattice vibration etc. To find accurate expressions for $\alpha_e^L(\mathbf{r},\mathbf{r}',\omega)$ and τ for different scattering mechanisms in LDS is a real theoretical task which would be attempted in our future work. There have been several approaches to evaluate $\alpha_e^L(\mathbf{r},\mathbf{r}',\omega)$ depending upon the approximations made to incorporate various kinds of interactions and scattering mechanisms in a real solid. A simple approach which works well in case of 3D isotropic system is self-consistent Hartree approximation. It yields [23]

$$\alpha_e^L(\mathbf{r},\mathbf{r}',\omega) = \sum_{kk'} \{[f(E_{k'})-f(E_k)]/E_{k'}-E_k-h(\omega+i/\tau)\}\,\phi_k(\mathbf{r})\phi_k^*(\mathbf{r}')\phi_{k'}(\mathbf{r}')\phi_{k'}^*(\mathbf{r}), \tag{2.21}$$

where E_k and $\phi_k(\mathbf{r})$ are the single particle energy and wave function, respectively in the state k. $f(E_k)$ is Fermi distribution function. Here, τ has been introduced, as a parameter, in a phenomenological manner.

2.22 Transverse Conductivity

For a transverse electromagnetic field, we combine Maxwell's equations

$$\nabla\times\mathbf{E} + (1/c)\,\partial\mathbf{B}/\partial t = 0 \tag{2.22a}$$

and

$$\nabla\times\mathbf{B} = 4\pi\mathbf{J}/c + (1/c)\,\partial\mathbf{E}/\partial t \tag{2.22b}$$

to obtain [22]

$$\nabla^2\mathbf{E}^T(\mathbf{r},\omega) = (4\pi i\omega/c^2)\,\mathbf{J}^T(\mathbf{r},\omega) + (\omega/c)^2\,\mathbf{E}^T(\mathbf{r},\omega). \tag{2.23}$$

J in Eq.(2.22b) is defined as

$$\mathbf{J} = \mathbf{J}_f + \partial\mathbf{P}/\partial t, \tag{2.24a}$$

where $\mathbf{J}_f$ and $\partial\mathbf{P}/\partial t$ are the contributions to the current density by free charges and bound charges. respectively [22]. Equation (2.24a) can also be written as

$$\mathbf{J} = (\sigma_e + \sigma_i)\,\mathbf{E}, \tag{2.24b}$$

where σ_e is the conductivity due to free charges (electronic) and σ_i is ionic conductivity due to bound charges. Equation (2.23) is well known Helmeholtz equation whose solution is

$$\mathbf{E}^T(\mathbf{r},\mathbf{r}',\omega) = (i\omega/c^2)\int G(\mathbf{r},\mathbf{r}'',\omega)\,\mathbf{J}(\mathbf{r}'',\mathbf{r}',\omega)\,d^3\mathbf{r}'', \tag{2.25}$$

with

$$G(\mathbf{r},\mathbf{r}',\omega)=\exp(ik|\mathbf{r}-\mathbf{r}'|)/|\mathbf{r}-\mathbf{r}'|, \tag{2.26}$$

where $G(\mathbf{r},\mathbf{r}',\omega)$ is proper Green function and $\mathbf{k}=\omega/c$. Making use of relation $\mathbf{E}(r,\omega)= \mathbf{E}_{ext}(r,\omega) + \mathbf{E}_{ind}(r,\omega)$ and Eq.(2.2) in Eq.(2.23), one gets

$$\mathbf{E}_{ext}(\mathbf{r},\omega) = \int F(\mathbf{r},\mathbf{r}'',\omega)\, \mathbf{E}(\mathbf{r}'',\omega)\, d^3\mathbf{r}'' \qquad (2.27a)$$

and

$$\mathbf{E}(\mathbf{r},\omega) = \int F^{-1}(\mathbf{r},\mathbf{r}'',\omega)\, \mathbf{E}_{ext}(\mathbf{r}'',\omega)\, d^3\mathbf{r}'', \qquad (2.27b)$$

where we define

$$F(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}') - (i\omega/c^2) \int G(\mathbf{r},\mathbf{r}'',\omega)\, \sigma(\mathbf{r}'',\mathbf{r}',\omega)\, d^3\mathbf{r}'' \qquad (2.28a)$$

and

$$F^{-1}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}') + (i\omega/c^2) \int G(\mathbf{r},\mathbf{r}'',\omega)\tilde{\sigma}(\mathbf{r}'',\mathbf{r}',\omega)\, d^3\mathbf{r}''. \quad (2.28b)$$

We recognise $F(\mathbf{r},\mathbf{r}',\omega)$ and $F^{-1}(r,\mathbf{r}',\omega)$ as transverse response function and inverse of response function within linear response theory, respectively. This leads to

$$\tilde{\sigma}^T(\mathbf{r},\mathbf{r}',\omega)=\sigma^T(\mathbf{r},\mathbf{r}',\omega)+(i\omega/c^2)\int\int\tilde{\sigma}^T(\mathbf{r},\mathbf{r}_1,\omega)G(\mathbf{r}_1,\mathbf{r}_2,\omega)\sigma^T(\mathbf{r}_2,\mathbf{r}',\omega)\, d^3\mathbf{r}_1\, d^3\mathbf{r}_2, \qquad (2.29)$$

which can be transformed to

$$\tilde{\sigma}^T(\mathbf{r},\mathbf{r}',\omega) = \int F^{-1}(\mathbf{r},\mathbf{r}'',\omega)\, \sigma^T(\mathbf{r}'',\mathbf{r}',\omega)\, d^3\mathbf{r}''. \qquad (2.30)$$

In Eqs.(2.23) to (2.30) suffix T stands for transverse components of field. $F(\mathbf{r},\mathbf{r}',\omega)$ can be related to $\varepsilon^T(\mathbf{r},\mathbf{r}',\omega)$, transverse dielectric response function which is defined in manner analogous to $\varepsilon^L(\mathbf{r},\mathbf{r}',\omega)$ given by

$$\varepsilon^T(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}') + \int V(\mathbf{r},\mathbf{r}'')\, \alpha_e^T(\mathbf{r}'',\mathbf{r}',\omega)\, d^3\mathbf{r}''. \qquad (2.31)$$

The $\alpha_e^T(\mathbf{r},\mathbf{r}',\omega)$ is density-response function for a transverse field. $\alpha_e^T(\mathbf{r},\mathbf{r}',\omega)$ can be correlated to current-current correlation function. $f^T(\mathbf{r},\mathbf{r}',\omega)$ [21] :

$$\alpha_e^T(\mathbf{r},\mathbf{r}',\omega) = (\nabla.\nabla'/\omega^2)\, f^T(\mathbf{r},\mathbf{r}',\omega) - (ne^2/m\omega^2)\, \nabla^2\delta(\mathbf{r}-\mathbf{r}'), \qquad (2.32)$$

where, n is number of electrons per unit volume. Equations (2.5a) and (2.18) describe generalised longitudinal dynamical conductivity, whereas Eqs.(2.5a) and (2.29) describe generalised transverse dynamical conductivity. These equations being in real space are applicable to all kinds of systems including the systems of lower symmetry and low dimensionality such as thin film, quantum wire and quantum dot. Also, our formalism can be used to compute dynamical conductivity for all values of wave vector and frequency for a LDS The clear advantage of the present formalism over other existing theories of dynamical conductivity is the evaluation of σ and $\tilde{\sigma}$ in terms of polarisation function which can be computed rigorously for a LDS for all frequencies including low frequency regime where several interesting features are observed experimentally.

2.3 Conductivity of 3D, 2D and 1D free electron gas

In this section, we report a unified formalism of dynamical conductivity for 3DFEG, 2DFEG and 1DFEG, in a dielectric medium of dielectric constant, ε_0, which is valid for all values of wave vector and frequency. Fourier transform of Eqs.(2.5a), (2.18) and (2.29) for 3DFEG, 2DFEG and 1DFEG yields

$$\sigma^{L/T}_{SD}(\mathbf{q}_S,\omega) = i\omega e^2\, \alpha^{L/T}_{SD}(\mathbf{q}_S,\omega)/\mathbf{q}_S^2 - (i\omega/4\pi)\,(\varepsilon_0-1), \quad (2.33)$$

$$\tilde{\sigma}^{L}_{SD}(\mathbf{q}_S,\omega) = \sigma^{L}_{SD}(\mathbf{q}_S,\omega) \,/\, \varepsilon^{L}_{SD}(\mathbf{q}_S,\omega), \quad (2.34)$$

$$\tilde{\sigma}^{T}_{SD}(\mathbf{q}_S,\omega) = \sigma^{T}_{SD}(\mathbf{q}_S,\omega) \,/\, F_{SD}(\mathbf{q}_S,\omega), \quad (2.35)$$

where

$$\varepsilon^{L/T}_{SD}(\mathbf{q}_S,\omega) = \varepsilon_0 + V_{SD}(\mathbf{q}_S)\, \alpha^{L/T}_{SD}(\mathbf{q}_S,\omega) \quad (2.36)$$

and

$$F_{SD}(\mathbf{q}_S,\omega) = 1 - (i\omega/c^2) + W_{SD}(\mathbf{q}_S,\omega)\, \sigma^{L/T}_{SD}(\mathbf{q}_S,\omega). \quad (2.37)$$

The first term on right hand side of Eq.(2.33) is contributed by the electrons, whereas second term is contributed by dielectric background. $\alpha^{L/T}_{SD}(q_S,\omega)$ represents electronic polarizability. The suffix S takes values 1, 2 and 3 for 1DFEG, 2DFEG and 3DFEG, respectively $V_{SD}(q_S)$ is Fourier transform of bare Coulomb potential and $W_{SD}(q_S)$ is Fourier transform of proper Green function. For 3DFEG, 2DFEG and 1DFEG is given by

$$V_{3D}(\mathbf{q}_3) = 4\pi e^2/\mathbf{q}_3^2, \quad (2.38a)$$

$$W_{3D}(\mathbf{q}_3,\omega) = 4\pi e^2/\mathbf{p}_3^2, \quad (2.38b)$$

$$V_{2D}(\mathbf{q}_2) = 2\pi e^2/\mathbf{q}_2, \quad (2.39a)$$

$$W_{2D}(\mathbf{q}_2,\omega) = 2\pi e^2/\mathbf{p}_2, \quad (2.39b)$$

$$V_{1D}(\mathbf{q}_1) = 2e^2 \ln(1/\mathbf{q}_1 b) \quad (2.40a)$$

and

$$W_{1D}(\mathbf{q}_1,\omega) = 2e^2 \ln(1/\mathbf{p}_1 b). \quad (2.40b)$$

The $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ are obtained from $\mathbf{p}_S=[\mathbf{q}_S^2-(\omega^2/c^2)]^{1/2}$, where $\mathbf{q}_1$, $\mathbf{q}_2$ and $\mathbf{q}_3$ are 1D, 2D and 3D wave vectors, respectively The b, in Eq.(2.40), is transverse dimension of sample confining 1DFEG. To demonstrate the correctness of our real space formalism of conductivity in a simple manner, we discuss Eqs.(2.23) to (2.40) for long wavelength limit of $\alpha_{SD}^{L/T}(\mathbf{q}_S,\omega)$. In long wavelength limit, $\alpha_{SD}^{L}(\mathbf{q}_S,\omega)\sim\alpha_{SD}^{T}(\mathbf{q}_S,\omega)$ for $\mathbf{q}_S v_{FS} << |\omega+i\gamma|$ and they are given by [33,34]

$$\alpha_{SD}^{L/T}(\mathbf{q}_S,\omega) = -\, n_S\, \mathbf{q}_S^2/m_e^*\omega(\omega+i\gamma), \quad (2.41)$$

where v_{FS} is Fermi velocity The n_3 is number of electrons per unit volume, n_2 is number of electrons per unit area and n is number of electrons per unit length. On substituting Eq.(2 31) into Eq.(2.29) and neglecting ionic interactions, we recover the formula for Drude conductivity (σ^d)

$$\sigma_{SD}^{d}(\omega) = n_S\, e^2/m_e^*(\gamma-i\omega). \quad (2.42)$$

The electric current is confined to 2D and 1D for 2DFEG and 1DFEG, respectively. Substitution of Eqs.(2.38) to (2.40) into Eq.(2.36) suggests that $|\varepsilon_{3D}| > |\varepsilon_{2D}| > |\varepsilon_{1D}|$, at given values of ω and γ in long wavelength limit. Also, ε_{3D} is independent of $\mathbf{q}_3$, whereas ε_{2D} and ε_{1D} reduce to unity, resulting in $\sigma_{2D}^{L/T}=\tilde{\sigma}_{2D}^{L/T}$, $\sigma_{1D}^{L/T}=\tilde{\sigma}_{1D}^{L/T}$ and $\sigma_{3D}^{L/T}>>\tilde{\sigma}_{3D}^{L/T}$, for $\mathbf{q}_S\to 0$. It is interesting to find that screening effects are reduced on the reduction of dimensionality of the system. The effective current in a system involves both $\tilde{\sigma}_{SD}^{L}(\mathbf{q}_S,\omega)$ and $\tilde{\sigma}_{SD}^{T}(\mathbf{q}_S,\omega)$. Our analysis suggests that effective current increase on reducing the dimensionality of a system. On combining Eqs.(2 33) to (2.41), well-known long wavelength results on dynamical conductivity are obtained which are frequently reported [1,11,24].

2.4 Conductivity of layered electron gas

In a modulation doped GaAs/$Al_xGa_{1-x}As$ superlattice, electrons are confined to narrow GaAs layers. Also, dielectric constant of GaAs does not differ much from that of $Al_xGa_{1-x}As$. Therefore, GaAs/$Al_xGa_{1-x}As$ superlattice can be modelled as LEG embedded into a homogeneous dielectric medium of dielectric constant. ε_0. Further simplification can be made by assuming that the thickness of an electron layer is small enough to treat it as a 2D plane. The Eqs.(2.5a), Eq.(2.18) and Eq.(2.29) undergo continuous Fourier transform in x-y plane and a discrete Fourier transform, with respect to layer indices, along z-axis. We obtain

$$\sigma^{L/T}_{LEG}(\mathbf{q},q_z,\omega)=(i\omega e^2/2\mathbf{q})\alpha^{L/T}_{2D}(\mathbf{q},\omega)S(\mathbf{q},q_z)-(i\omega/4\pi)(\varepsilon_0-1), \quad (2.43a)$$

$$\tilde{\sigma}^{L}_{LEG}(\mathbf{q},q_z,\omega) = \sigma^{L}_{LEG}(\mathbf{q},q_z,\omega) / \varepsilon^{L}_{LEG}(\mathbf{q},q_z,\omega), \quad (2.43b)$$

$$\tilde{\sigma}^{T}_{LEG}(\mathbf{q},q_z,\omega) = \sigma^{T}_{LEG}(\mathbf{q},q_z,\omega) / F(\mathbf{q},q_z,\omega) \quad (2.43c)$$

and

$$F(\mathbf{q},q_z,\omega) = [1-(\omega^2 d/2c^2\mathbf{p})S(\mathbf{p},q_z)[\varepsilon^{T}_{LEG}(\mathbf{q},q_z,\omega)-1], \quad (2.44a)$$

where

$$\varepsilon^{L/T}_{LEG}(\mathbf{q},q_z,\omega) = \varepsilon_0 + (2\pi e^2/\mathbf{q})\,\alpha^{L/T}_{2D}(\mathbf{q},\omega)\,S(\mathbf{q},q_z) \quad (2.44b)$$

with

$$S(\mathbf{q},q_z) = \sinh(\mathbf{q}d) / [\cosh(\mathbf{q}d)-\cos(q_z d)], \quad (2.45a)$$

$$S(\mathbf{p},q_z) = \sinh(\mathbf{p}d) / [\cosh(\mathbf{p}d)-\cos(q_z d)] \quad (2.45b)$$

and

$$\mathbf{p} = [\mathbf{q} -(\omega^2/c^2)]^{1/2}. \quad (2.45c)$$

The $\mathbf{q}$ and q_Z are wave vector components along x-y plane and the z-axis, respectively

Equations (2.43a) and (2.43b) are valid for all values of $\mathbf{q}$, q_Z and ω including case of $qv_F \leq |\omega+i\gamma|$, where Drude theory of conductivity is not applicable. Also, Eqs.(2.43) to (2.45) include the possibility of charge transfer along z- axis for non-zero values of q_Z. The points in the plane of $\mathbf{q}$, q_Z and ω, where $\tilde{\sigma}_{LEG}$ shows peaks describing the condition for self-sustaining TEM waves. Equation (2.43c) includes the possibility of propagation along both x-y plane as well as z-axis and ours are more rigorous and accurate results on conductivity of LEG as compared to those reported earlier [18]. In order to show that our results are correct and they reproduce existing calculations in various limiting cases, we consider long wavelength limit of our results.

In case of q_Z=0 and $\mathbf{q}\rightarrow 0$ (in-plane conduction in the long wave length limit) neglecting ionic interactions Eq.(2.43a) goes to

$$\sigma^{L}_{LEG}(\omega) = n_2\, e^2/m^*_e(\gamma\text{-}i\omega)d \qquad (2.46)$$

which reduces to Eq.(2.42) on defining $d\sigma^{d}_{LEG}(\omega) = \sigma^{d}_{2D}(\omega)$. d is the length of unit cell along z-axis. Further, for d→0, Eqs.(2.43a) to (2.43c) reduce to Eqs.(2.33) to (2.35), respectively for the case of S=3, as they should, because for d→0 electron layers merge with each other and LEG behaves like a 3DFEG. However, in general Eqs.(2.43c) differs from $\tilde{\sigma}^{T}_{3D}(\mathbf{q}_3,\omega)$. Also, our Eqs. (2.43b) and (2.43c) differ from the expressions of longitudinal and transverse conductivities of LEG given by Shi and Griffin [18], which in our notation are given as

$$\tilde{\sigma}^{L}_{LEG}(\mathbf{q},q_z,\omega) = \sigma^{L}_{2D}(\mathbf{q},\omega)\,/\,\varepsilon^{L}_{LEG}(\mathbf{q},q_z,\omega) \qquad (2.47a)$$

and

$$\tilde{\sigma}^{T}_{LEG}(\mathbf{q},q_z,\omega) = \sigma^{T}_{2D}(\mathbf{q},\omega)\,/\,\{1+(\,\omega^2/\omega^2\text{-}c^2\mathbf{q}^2)\,[\varepsilon^{T}_{LEG}(\mathbf{q},q_z,\omega)\text{-}1]\}. \qquad (2.47b)$$

Equations (2.47a) and (2.47b) have been obtained using definition of current density as current per unit length. These equations do not correctly describe electromagnetic wave propagation along z-axis.

2.5 Conclusion

General formalism of $\sigma^{L/T}(\mathbf{r},\mathbf{r}',\omega)$ and $\tilde{\sigma}^{L/T}(\mathbf{r},\mathbf{r}',\omega)$ has been developed. Wave vector and frequency dependence of conductivities for 3DFEG, 2DFEG, 1DFEG, LEG and QWS are calculated using our general formalism. Explicit results in long wavelength limit of our calculation are given. Our calculation shows that $|\varepsilon_{3D}|>|\varepsilon_{2D}|>|\varepsilon_{1D}|$, at given values of ω and γ in long wavelength limit. Also, ε_{3D} is independent of q_3, whereas ε_{2D} and ε_{1D} reduce to unity, resulting in $\sigma_{2D}^{L/T}=\tilde{\sigma}_{2D}^{L/T}$, $\sigma_{1D}^{L/T}=\tilde{\sigma}_{1D}^{L/T}$ and $\sigma_{3D}^{L/T}>>\tilde{\sigma}_{3D}^{L/T}$, for $q_s\rightarrow 0$. Our calculated dynamical conductivities of LEG and QWS can significantly differ from Drude theory of conductivity for $qv_F\leq|\omega+i\gamma|$. Calculated $\tilde{\sigma}_{LEG}^{T}$ includes the possibility of propagation of collective excitations along both x-y plane as well as z-axis and it is more rigorous as well as accurate as compared to the transverse dynamical conductivity reported in past.

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