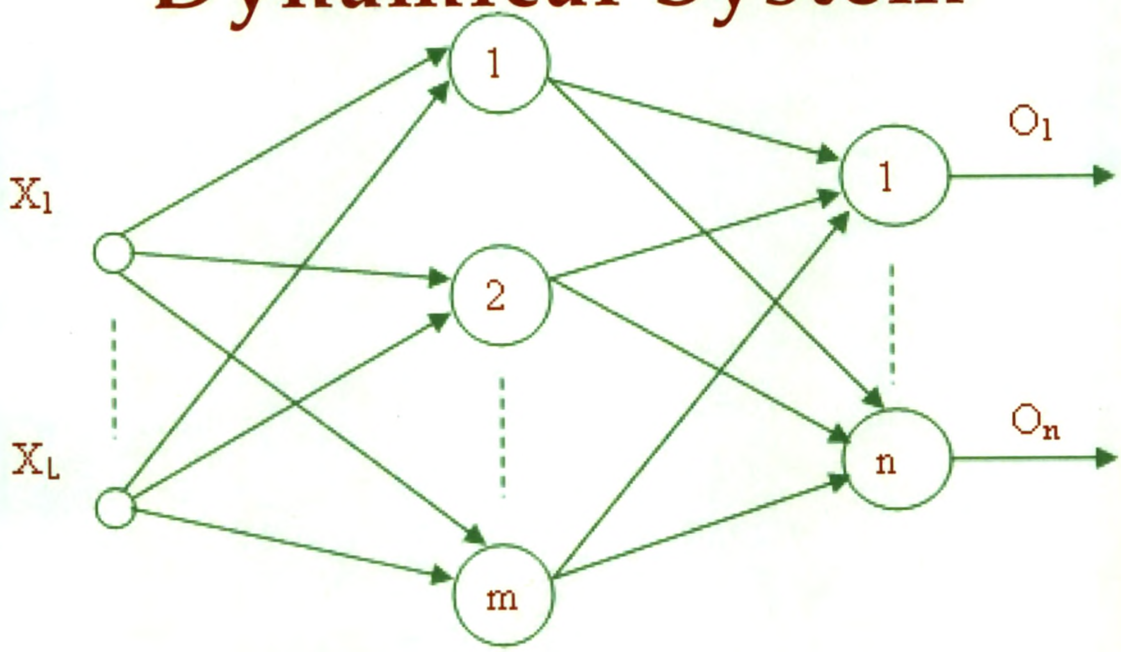
Chapter 6

ANN Implementation of Zonal Controller for Parabolic Semilinear Dynamical System



Chapter 6

ANN IMPLEMENTATION OF ZONAL CONTROLLER FOR PARABOLIC SEMILINEAR DYNAMICAL SYSTEM

6.1 Introduction

In this Chapter, we illustrate the use of ANN as zonal controllers applied to semilinear parabolic systems. In general, the parabolic systems give rise to infinite dimensional state space and hence for the simulation of controller by Neural Network in this setup is difficult. So, we convert the infinite dimensional state space into finite dimension state space by the projection method. In our system we assume that the control is applied only on certain zones (areas), instead of applying control on the whole domain.

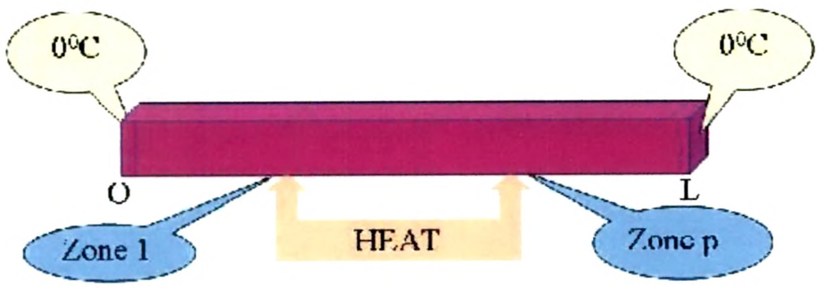


Figure 6.1: Conductive bar coated with more conductive material on the lateral sides

6.2 Mathematical Modeling - Linear Parabolic System

Consider a one dimensional conducting bar of length L with cross sectional area A_c, as shown in Figure 6.1. Let the initial temperature distribution of the bar be uniform. The bar is heated at p-zones. Then the problem is to find the control (heat) to be applied at zone-1,...,zone-p such that the initial temperature distribution is steered to the desired temperature distribution $z_d(x)$ in finite time T.

For the mathematical model, let us denote the temperature distribution of the bar by $y(x,t)$ for any time t and any space point $x \in [0, L]$. Assume, that the bar is placed at the origin in the positive x-direction. Let $e(x,t)$ be the amount of the thermal energy / unit volume (called thermal energy density).

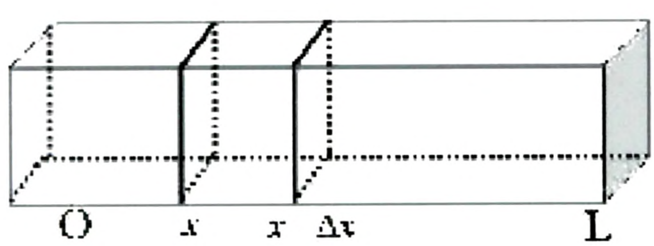


Figure 6.2: The slice of thickness Δx

Then the total thermal energy, in the slice of thickness Δx, as shown in Figure 6.2, is given by

$$\text{Thermal energy density} \times \text{Volume}.$$

That is,

$$e(x,t) \times A_c \times \Delta x \tag{6.2.1}$$

Let $\phi(x,t)$ be the amount of thermal energy per unit time per unit surface area, flowing towards the right direction, known as heat flux. Let $S(x,t)$ be the heat energy per unit volume generated per unit time due to the actuators excited at p-zones. Also, the rate of change of heat energy is given by

$$\frac{\partial}{\partial t}\left[e(x,t)A_c\Delta x\right]$$

By law of conservation of Heat Energy,

Rate of change of heat energy per unit time = Heat energy generated inside / unit time + heat energy flowing across the boundary / unit time

That is,

$$\frac{\partial}{\partial t}\left[e(x,t)A_c\Delta x\right] = \phi(x,t)A_c - \phi(x+\Delta x,t)A_c + S(x,t)A_c\Delta x$$

Dividing by $A_c\Delta x$ and letting $\Delta x \to 0$ we get

$$\begin{aligned}\frac{\partial}{\partial t}e(x,t) &= lim_{\Delta x\to 0}\frac{\phi(x,t)A_c - \phi(x+\Delta x,t)A_c}{\Delta x} + S(x,t)\\ &= -\frac{\partial\phi(x,t)}{\partial x} + S(x,t)\end{aligned} \tag{6.2.2}$$

Using, the relation for thermal energy density in terms of temperature $y(x,t)$ we have

$$e(x,t) = c(x)\rho(x)y(x,t) \tag{6.2.3}$$

where, $c(x)$ is the specific heat (heat energy applied to unit mass to raise its temperature by 1° C)and $\rho(x)$ is mass density.

Also, by the Fourier's law we have the heat flux given as

$$\phi(x,t) = -K\frac{\partial\phi(x,t)}{\partial x} \tag{6.2.4}$$

where, K is the thermal conductivity. Substituting, (6.2.3) and (6.2.4) in (6.2.2) we get,

$$c(x)\rho(x)\frac{\partial y}{\partial t} = -\frac{\partial}{\partial x}\left(K\frac{\partial y}{\partial x}\right) + S(x,t)$$

For special case, when c, ρ and K are constants we get

$$\frac{\partial y}{\partial t} = \alpha\frac{\partial^2 y}{\partial x^2} + Q(x,t) \tag{6.2.5}$$

where, $\alpha = -\frac{K}{\rho c}$ and $Q(x,t) = \frac{S(x,t)}{c\rho}$.

Without loss of generality, let us assume that initial temperature distribution is zero.

That is,

$$y(x,0) = 0,\ x \in [0,L]$$

Let $\Omega_i = [x_i - 0.1, x_i + 0.1] \subset \Omega = (0,1), i = 1,...p$ be disjoint p zones on the surface of the conducting bar, on which we apply the controls. Let $u_i(t)$ be the control functions applied on the i^{th} zone Ω_i.

Let g_i, be the distribution of the control over Ω_i defined by

$$g_i(x) = \begin{cases} 1, & x \in \Omega_i; \\ 0, & \text{otherwise.} \end{cases}$$

Hence, the source term $Q(x,t)$ becomes

$$Q(x,t) = \sum_{i=1}^{p} g_i(x)u_i(t)$$

Thus, we have the controlled model for the conducting bar as:

$$\frac{\partial y(x,t)}{\partial t} = \alpha\frac{\partial^2 y(x,t)}{\partial t^2} + \sum_{i=1}^{p} g_i(x)u_i(t) \tag{6.2.6}$$

with the initial and boundary conditions as

$$y(x,0) = 0,\ x \in [0,L]$$

$$y(0,t) = y(L,t) = 0,\ t > 0$$

The equation (6.2.6) is one-dimensional linear parabolic system excited by p zonal controls (Ω_i, g_i).

These p zonal controls are called actuators which excite the system. Usually, actuators can be found in one of the forms: fixed point, mobile point, zone and filament.

A zonal actuator is the couple (Ω_i, g_i) where, Ω_i is the support of the actuator and g_i defines the spatial distribution of the actuator. The controllability properties of a system are affected by both the positioning of the controls and the distribution functions on their supports.

In this work, our aim is to implement the *zonal actuators* using Artificial Neural Networks, which allows us to steer the initial temperature distribution to the desired final temperature distribution.

Abstract Formulation :

Let

$$u = (u_1, u_2, ..., u_p) \in L^2(0, T; R^p),$$

Define the operator

$$Az = \frac{\partial^2 z}{\partial x^2} \ ; z \in D(A) = H_0^1(\Omega) \cap H^2(\Omega)$$

where,

$H^m(\Omega)$ is a Sobolev space of order m and

$H_0^m(\Omega) = \{f \in H^m(\Omega) : \partial^k f / \partial v^k|_{\partial\Omega} = 0; 0 \leq k \leq m\}$

Here, the operator A is linear and unbounded and let $B : [u_1, u_2, ..., u_p] \to \sum_{i=1}^{p} g_i u_i$.

Then the system (6.2.6) can be represented in the abstract differential equation form:

$$\dot{z} = Az + Bu, \ 0 < t < T, \tag{6.2.7}$$

$$z(0) = z_0$$

where, $z = y(\cdot, t) \in L^2(0, T; \Omega)$. The linear operator A generates a strongly continuous semigroup $(S(t))_{t\geq 0}$ on Hilbert space $L^2(\Omega)$, (refer Pazy [58]). B is a bounded linear operator from $L^2(0, T; R^p) \to L^2(0, T; R^p)$.

The solution of system (6.2.7) can be expressed using the semigroup $S(t)$ as shown in the following result

THEOREM 6.2.1 *Let $z_0 \in D(A)$ and $S(t)$ be the strongly continuous semigroup generated by A then,*
(i) the unique solution of the homogeneous system

$$\dot{z} = Az, \; 0 < t < T, \qquad (6.2.8)$$

$$z(0) = z_0$$

is given by

$$z(t) = S(t)z_0. \qquad (6.2.9)$$

(ii) the unique solution of the non-homogeneous system (6.2.7) is given by using the variation of parameter method as

$$z(t) = S(t)z_0 + \int_0^t S(t-\tau)Bu(\tau)d\tau.$$

Proof: (i) Differentiating (6.2.9) w.r.t. t and using the property of semigroup, we have

$$\begin{aligned} \tfrac{d}{dt}z(t) = \tfrac{d}{dt}S(t)z_0 &= AS(t)z_0 \\ &= Az \end{aligned}$$

and

$$z(0) = S(0)z_0 = Iz_0 = z_0.$$

(ii) Let z be a solution of the non-homogeneous system (6.2.7) then,

$$\begin{aligned} \tfrac{d}{ds}(S(t-s)z(s)) &= -AS(t-s)z(s) + S(t-s)\tfrac{dz}{ds} \\ &= -AS(t-s)z(s) + S(t-s)(Az(s) + Bu(s)) \\ &= -AS(t-s)z(s) + AS(t-s)z(s) + S(t-s)Bu(s) \\ &= S(t-s)Bu(s) \end{aligned}$$

Now, integrating over $(0, t)$ we get

$$z(t) = S(t)z_0 + \int_0^t S(t-\tau)Bu(\tau)d\tau$$

Hence, the proof.

We investigate the approximate controllability for system (6.2.7) on $[0, T]$. That is, for all $z_d \in L^2(\Omega)$ we want to prove the existence of $u \in L^2(0, T; R^p)$ such the corresponding solution z of (6.2.7) satisfies

$$\| z(T) - z_d \| < \epsilon;\ \epsilon > 0.$$

It has been proved in the literature that the exact controllability space for the parabolic system is $H_0^1(\Omega)$ but not $L^2(\Omega)$.

Define an operators $H : L^2(0, T; R^p) \longrightarrow L^2(\Omega)$ given by

$$Hu = \int_0^T S(T - \tau)Bu(\tau)d\tau$$

and

$$M = HH^* = \int_0^T S(T - t)BB^*S^*(T - t)dt$$

where, $H^* : L^2(\Omega) \rightarrow L^2(0, T; R^p)$ is the adjoint operator of H and is given by

$$H^*z(\cdot) = B^*S^*(T - \cdot)z$$

Characterization for Approximate Controllability :

For the distributed parameter system (6.2.7) the concept of approximately controllability can be established using any of the following equivalent statements (refer Pritchard [60]):

- The system (6.2.7) is approximately controllable.
- Range(H) is dense in $L^2(\Omega)$
- $\ker(H^*) = \ker(H^*H) = \{0\}$
- $B^*S^*(s)v = 0$ implies $v = 0$ for $s \in [0, T]$.

We now give result for the construction of the optimal steering control for the system (6.2.7).

THEOREM 6.2.2 *If z_d is in the range of M, then the unique control u which solves the optimal control problem :*

$$J(u) = \min \| u \|^2_{L^2(0,T;R^p)}$$

is given by

$$u^* = H^\dagger z_d$$

where, $H^\dagger$ is the pseudo-inverse of H.

Proof: If z_d is in the range of M then the equation

$$Hu = z_d$$

has a minimum norm solution given by

$$\begin{aligned} u^* &= H^\dagger z_d \\ &= H^*(HH^*)^{-1} z_d \\ &= B^*S^*(T-t)\left(\int_0^T S(T-t)BB^*S^*(T-t)dt\right)^{-1} z_d \end{aligned}$$

Thus, the system (6.2.7) has approximate controllability on a subspace $Z_1 = H_0^1(\Omega) \subset L^2(\Omega)$.

For our system (6.2.6), the semigroup $S(t)$ generated by

$$Az = \frac{\partial^2 z}{\partial x^2} \ ; \ z \in D(A) = H_0^1(\Omega) \cap H^2(\Omega)$$

is given by

$$S(t)z = \sum_n e^{\lambda_n t} \sum_{j=1}^{\gamma_n} < z, \phi_{nj} > \phi_{nj}$$

where,

$$\phi_n(x) = \sqrt{2}\sin(\pi n x) \text{ are the eigen functions}$$

and

$$\lambda_n = -\alpha n^2 \pi^2 \text{ are the eigen values of } A.$$

Because of the infinite dimensional nature of the problem we project the parabolic problem into finite dimensional subspace by using the orthonormal basis generated by the eigen functions to get the form :

$$\dot{z}_n = A_n z_n + B_n u$$

$$z_n(0) = z_{n0}. \tag{6.2.10}$$

where,

$$A_n = diag\{\lambda_1, \lambda_2, ..., \lambda_n\}.$$

And

$$B_n \in BL(R^p, R^n),\ B_n = (B_{ij})\ 1 \leq i \leq n,\ 1 \leq j \leq p$$

where,

$$B_{ij} = < g_j, \phi_i >_{L^2(\Omega_j)} .$$

The semigroup $S(t)$ projected to S_n is defined by

$$S_n \in L(R^n), S_n = (S_{ij})\ 1 \leq i, j \leq n$$

and

$$S_{ij}(x) = \begin{cases} e^{\lambda_i x} & \text{if } 1 \leq i = j \leq n \\ 0 & \text{if } 1 \leq i \neq j \leq n \end{cases} \tag{6.2.11}$$

Because of the controllability to the subspace, the controllability Grammian is defined by the matrix

$$M_n = \int_0^T S_n(T-\tau) B_n B_n^* S_n^*(T-\tau) d\tau \tag{6.2.12}$$

Hence,

$$M_{ij} = \sum_{k=1}^{p} B_{ik} B_{jk} \frac{e^{(\lambda_i + \lambda_j)T} - 1}{\lambda_i + \lambda_j}\ i, j = 1, 2, ..., n. \tag{6.2.13}$$

and thus M_n is invertible.

Let

$$V_n = M_n^{-1} \in R^{n \times n} \tag{6.2.14}$$

The minimum norm controller for the finite dimensional system (6.2.10) can be obtained using the following result

THEOREM 6.2.3 *The control*

$$u_n^* = B_n^* S_n^* (T-t) V_n z_{dn}$$

is the unique control of minimum norm which realizes the minimum of

$$\| z_{dn} - z_n(T) \|_{R^n},$$

where, z_{dn} is the n-dimensional projection of the desired final distribution z by using the orthonormal eigen basis.

The control in the component-wise form is given by

$$U^*_{nj}(t) = \sum_{k=1}^{n} B_{kj} e^{\lambda_k (T-t)} (\sum_{m=1}^{n} V_{km} z_{dm}) \tag{6.2.15}$$

where, z_{dm} is the m^{th} component of the n dimensional projection of the desired temperature distribution z_d.

The controller (6.2.15), steers the approximated initial temperature to the approximated desired temperature profile.

Numerical Computation :

Recall the one dimensional parabolic system

$$\frac{\partial y(x,t)}{\partial t} = \alpha \frac{\partial^2 y(x,t)}{\partial t^2} + \sum_{i=1}^{p} g_i(x) u_i(t) \tag{6.2.16}$$

with the initial and boundary conditions as

$$y(x,0) = 0 \ x \in (0,L)$$

$$y(0,t) = y(L,t) = 0 \ t > 0$$

The operator A defined by

$$Az = \frac{\partial^2 z}{\partial x^2} \ ; \ z \in D(A) = H_0^1(\Omega) \cap H^2(\Omega)$$

has a complete set of eigen functions

$$\phi_n(x) = \sqrt{2} \sin(\pi n x)$$

with eigen values

$$\lambda_n = -\alpha n^2 \pi^2$$

We choose the number of zones $p = 2$ and $T = 1$. For this choice of control we have

$$B_{ij} = \frac{2^{\frac{3}{2}}}{i\pi} \sin\left(\frac{i\pi}{10}\right) \sin(i\pi x_j)$$

Let the desired final temperature distribution be given by

$$z_d = x(1 - x^2)$$

$$\alpha = 0.5$$

and let the system be projected on the 5 dimensional space.

That is,

$$n = 5.$$

Then

$$A_5 = \begin{bmatrix} -4.9348 & 0 & 0 & 0 & 0 \\ 0 & -19.7392 & 0 & 0 & 0 \\ 0 & 0 & -44.4132 & 0 & 0 \\ 0 & 0 & 0 & -78.9568 & 0 \\ 0 & 0 & 0 & 0 & -123.3701 \end{bmatrix}$$

$$B_5 = \begin{bmatrix} 0.2612 & 0.2069 \\ 0.1712 & -0.2631 \\ -0.1196 & 0.1421 \\ -0.2112 & 0.0454 \\ -0.0337 & -0.1564 \end{bmatrix}$$

Controllability Grammian is

$$W_5 = \begin{bmatrix} 0.0112 & -0.0004 & -0.0000 & -0.0005 & -0.0003 \\ -0.0004 & 0.0025 & -0.0009 & -0.0005 & 0.0002 \\ -0.0000 & -0.0009 & 0.0004 & 0.0003 & -0.0001 \\ -0.0005 & -0.0005 & 0.0003 & 0.0003 & 0.0000 \\ -0.0003 & 0.0002 & -0.0001 & 0.0000 & 0.0001 \end{bmatrix}$$

The Figure 6.3 shows the desired temperature distribution and the temperature distribution after projecting it on the 5-dimensional space by using control (6.2.15).

With this setup we proceed to implement the Neural Network steering control

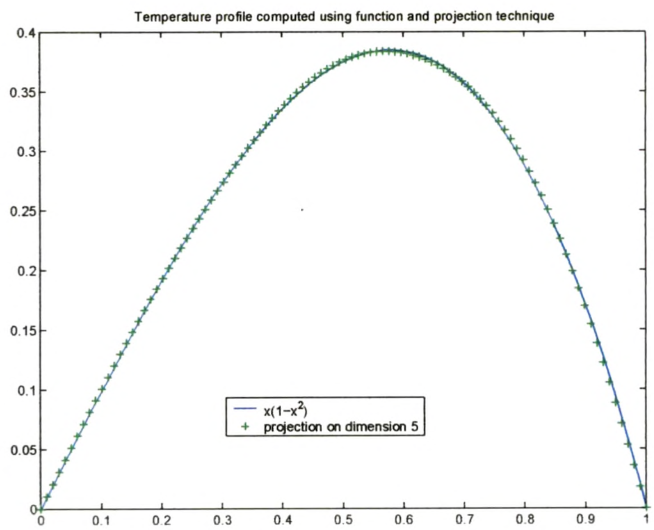


Figure 6.3: The temperature distribution by formula and using the projection technique with $n = 5$.

ANN Zonal Controller - Linear System :

For the implementation of the zonal controller using ANN we obtain the discrete form of the system projected on 5 dimensional space using sampling rate $ST = 0.1$:

$$z_5(k+1) = F_5 z_5(k) + G_5 u_2(k)$$

where,

$$F_5 = \begin{bmatrix} 0.6105 & 0 & 0 & 0 & 0 \\ 0 & 0.1389 & 0 & 0 & 0 \\ 0 & 0 & 0.0118 & 0 & 0 \\ 0 & 0 & 0 & 0.0004 & 0 \\ 0 & 0 & 0 & 0 & 0.0000 \end{bmatrix}$$

and

$$G_5 = \begin{bmatrix} 0.0206 & 0.0163 \\ 0.0075 & -0.0115 \\ -0.0027 & 0.0032 \\ -0.0027 & 0.0006 \\ -0.0003 & -0.0013 \end{bmatrix}$$

In the numerical experiment, we take projection on the 5-dimensional space hence, the complete temperature profile is to be constructed using the 5-dimensional coordinate vector which are actually fourier coefficients in the expansion of the function representing the temperature profile.

In our example, for the final temperature distribution

$$z_d = x(1-x^2)$$

we get these fourier coefficients, that is the coordinate vector as

$$\begin{bmatrix} 0.3870 & -0.0484 & 0.0143 & -0.0060 & 0.0031 \end{bmatrix}$$

For implementing 2-zone controller for the 5-dimensional system which is controlled in 7-time steps, we use multilayered Feed-forward neural network with architecture $N^2_{5,30,14}$.

The I/O patterns used for training the NN are as follows:

Input : Randomly generated 5-dimensional vector normalized in $[-1, 1]$. It represents the coordinate vector for the initial temperature profile.

Output : The 2-zone steering signal generated using definition (6.2.15) which is 7, 2-dimensional vectors. That is, 14-dimensional vector to the neural network.

The ANN training as 2-zone steering control converged in 18 epochs. For details refer program NNZONAL.m in Appendix-A.

This trained ANN acts as zonal controller and steers the initial temperature profile to the desired z_d.

For example for the input $\begin{bmatrix} .1 & .3 & .5 & .7 & .9 \end{bmatrix}$.

The ANN signal is

$$1.0e+003 * \begin{bmatrix} 0.1338 & 0.2233 & 0.3961 & 0.8647 & 2.7809 & -1.0957 & 0.0020 \\ 0.1042 & 0.1643 & 0.2225 & 0.0319 & -2.1166 & -0.0967 & -0.0029 \end{bmatrix}$$

This control signal steers the temperature to the desired temperature in 7 time steps each of .1 seconds.

The final temperature and the desired temperature distribution are shown in the Figure 6.4.

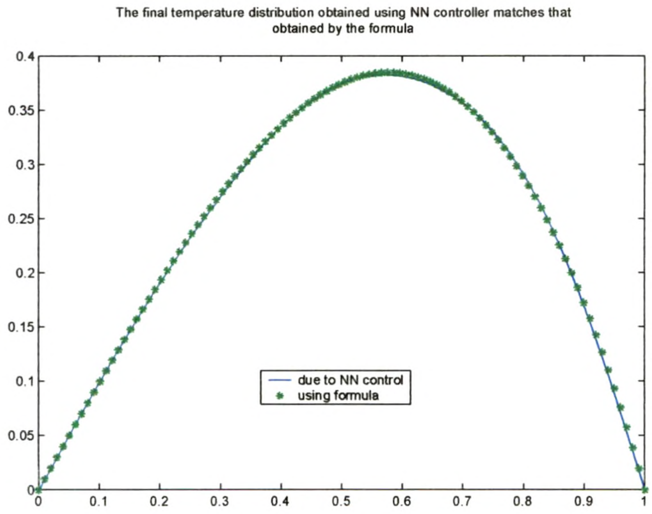


Figure 6.4: The final temperature distribution steered due to the control signal produced by ANN Zonal Controller and the desired temperature distribution.

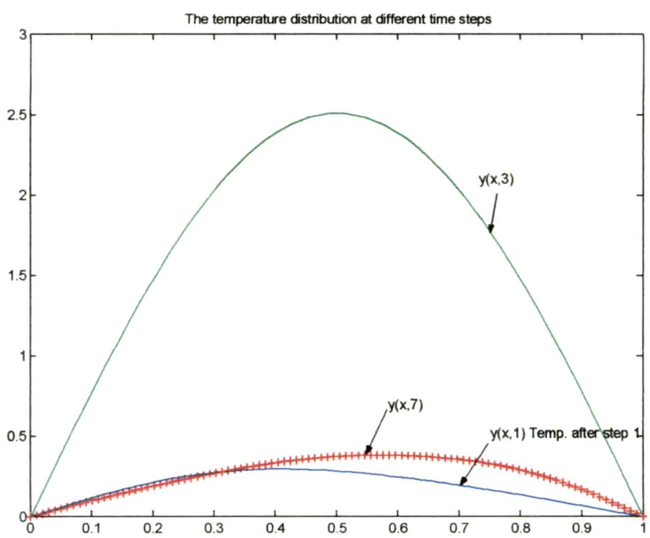


Figure 6.5: The evolution of the temperature distribution at different time-step due to the ANN zonal controller.

The evolution of the temperature distribution due to the steering of the ANN control is as shown in the Figure 6.5 for $n = 5$ and $p = 2$.

6.3 Zonal Controller - Semilinear Parabolic System

Suppose that the model (6.2.6) is nonlinear, which can happen via various situations like a nonuniform conductive material coated on the surface of the bar. In that case, we assume that the effect is added in the model by adding a nonlinear function $f(y(x,t))$ in the above equation (6.2.5) to get

$$\frac{\partial y(x,t)}{\partial t} = \alpha \frac{\partial^2 y(x,t)}{\partial t^2} + \sum_{i=1}^{p} g_i(x)u_i(t) + f(y(x,t)) \tag{6.3.17}$$

with the initial and boundary conditions as

$$y(x,0) = 0,\ x \in (0,L)$$

$$y(0,t) = y(L,t) = 0,\ t > 0$$

Our aim is to implement ANN controller for the system (6.3.17).

The system such as

$$\frac{\partial y(x,t)}{\partial t} = \alpha \frac{\partial^2 y(x,t)}{\partial t^2} + B(t)u(t) + f(y(x,t)) \tag{6.3.18}$$

is studied for approximate controllability using the finite dimensional projection technique (refer Anil [3]).

In his results he has proved the fact that the controller for the projected semilinear system does approximate the steering control for the infinite dimensional systems, provided the corresponding linearized system is controllable and the nonlinear function in the system (6.3.18) satisfies the Lipschitz condition.

Thus, the zonal control of the (6.3.17) can be established under the assumption that the nonlinear functions involved in the system (6.3.17) satisfies Lipschitz condition, along with the corresponding linear part being approximately controllable as shown in the earlier section.

As in the linear case, the system (6.3.17) can be put in the abstract form:

$$\dot{z} = Az + Bu + F_z(z),\ 0 < t < T,\ z(0) = z_0 \tag{6.3.19}$$

with F_z being a nonlinear operator defined on $L^2(\Omega)$ defined via f.

Finite Dimensional Approximation - Semilinear Parabolic System :

Let the finite dimensional projection system (6.3.17) be given by

$$\dot{y_n} = A_n y_n + B_n u + F_n(y_n) \tag{6.3.20}$$

$$y_n(0) = 0.$$

where, F_n is the finite dimensional projection of f.
For the infinite dimensional system (6.3.17), suppose that (ϕ_m) are eigenfunctions of A with corresponding eigenvalues (λ_m) of multiplicity one. Let $A_n = diag\{\lambda_1, \lambda_2, ..., \lambda_n\}$

Then for all $x > 0$, the projected semigroup S_n is given by

$$S_n(x) \in L(R^n), S_n(x) = (S_{ij})\ 1 \leq i, j \leq n$$

and

$$S_{ij} = \begin{cases} e^{\lambda_i x} & \text{if } 1 \leq i = j \leq n \\ 0 & \text{if } 1 \leq i \neq j \leq n \end{cases} \tag{6.3.21}$$

Also,

$$B_n \in L(R^p, R^n),\ B_n = (B_{ij})\ 1 \leq i, j \leq n$$

and

$$B_{ij} = < g_j, \phi_i >_{L^2(\Omega_j)}$$

Set

$$M_n = \int_0^T S_n(T-\tau) B_n B_n^* S_n^*(T-\tau) d\tau \tag{6.3.22}$$

Then $M_n \in R^{n \times n}$ and has elements

$$M_{ij} = \int_{k=1}^{p} B_{ik} B_{jk} \frac{e^{(\lambda_i + \lambda_j)T} - 1}{\lambda_i + \lambda_j} \tag{6.3.23}$$

The controllability hypothesis implies the existence of

$$V_n = M_n^{-1} \in R^{n\times n} \tag{6.3.24}$$

The minimum norm controller for the approximated system is given by

$$U_{nj}^*(t) = \sum_{k=1}^{n} B_{kj} e^{\lambda_k (T-t)} (\sum_{m=1}^{n} V_{km} y_{dm} - \sum_{r=1}^{n} e^{\lambda_k (T-r)} F_r(y_r)) \tag{6.3.25}$$

where, y_{dm} is the m^{th} component of the n-dimensional projection of the desired temperature distribution z_d.

Now by using Banach Contraction Principle we obtain the following theorem. The proof follows easily as show in Chapter 4.

THEOREM 6.3.1 *Suppose that the nonlinear function f is Lipschitz continuous with Lipschitz constant sufficiently small. Then the projected system (6.3.20) is controllable.*

In the next section, we implement the ANN controller for parameter values as stated in the simulation for the linear version along with the nonlinear function $f(y(x,t)) = sin(y(x,t))/6$. This function satisfies the conditions of the above theorem.

ANN Zonal Controller for Semilinear System :

To demonstrate the ANN zonal controller for the semilinear system we continue with all the values as in the discrete linear case along with the nonlinear function $f_n = sin(y)/6$ in the system (6.3.20).

The multilayered feedforward neural network with the architecture $NN^2_{5,10,40}$ was trained to act as 4-step zonal controller. It is 4-step steering control as the coupled equations given below for generating the I/O pairs converged in 4 iterations.

The I/O pairs for the training are derived by using the definitions for the state and controller for the discrete semilinear form.

$$u^{i+1}(k+1) = U_n^{-1} \left[x_1 - F^n x_0 - \sum_{j=1}^{n} F^{n-j} f(x^i, u^i) \right] \tag{6.3.26}$$

and

$$x^{i+1}(k+1) = F^n x_0 + \sum_{j=1}^{n} F^{n-j} G u(k) + \sum_{j=1}^{n} F^{n-j} f(x^i, u^i) \tag{6.3.27}$$

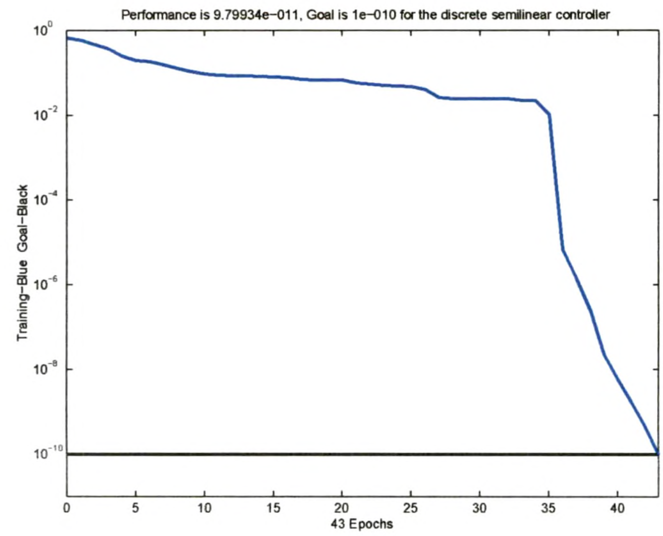


Figure 6.6: The convergence of the training for four step ANN zonal controller.

where,

$$F = e^{A \times ST}$$

and

$$G = \int_0^{ST} e^{A} \tau B d\tau.$$

with the sampling rate $ST = 0.2$ seconds.

The training for the 4-step controller converged in 43 epoches as shown in the Figure 6.6.

The signal of the 4-step ANN zonal controller for the initial state [.1 .2 .3 .4 .5] is 40-dimensional control vector. This signal takes the initial state to the desired state

$$\begin{bmatrix} 0.3870 & -0.0484 & 0.0143 & -0.0060 & 0.0031 \end{bmatrix}$$

the fourier coefficients of the temperature profile $z_d = x(1 - x^2)$ similar to as in the linear case. The final temperature profile achieved due to ANN zonal controller, is shown in the Figure 6.7. The temperature profile after fourth step matches the desired temperature distribution, as shown in the Figure 6.8.

For details see the MATLAB program NNgDZONAL_nl.m in Appendix-A.

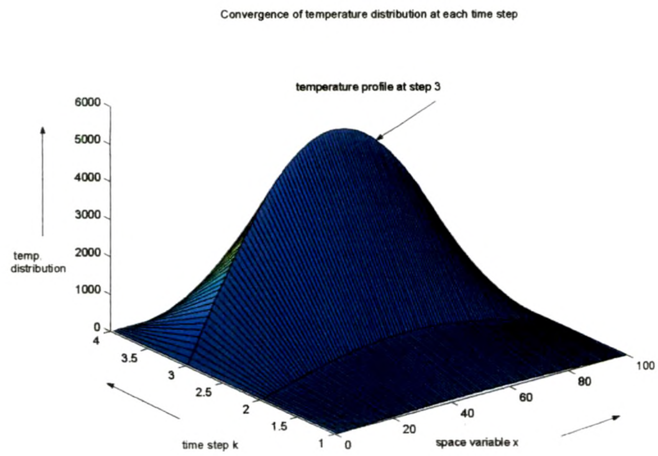


Figure 6.7: The evolution of the temperature profile due to the 4-step ANN zonal controller.

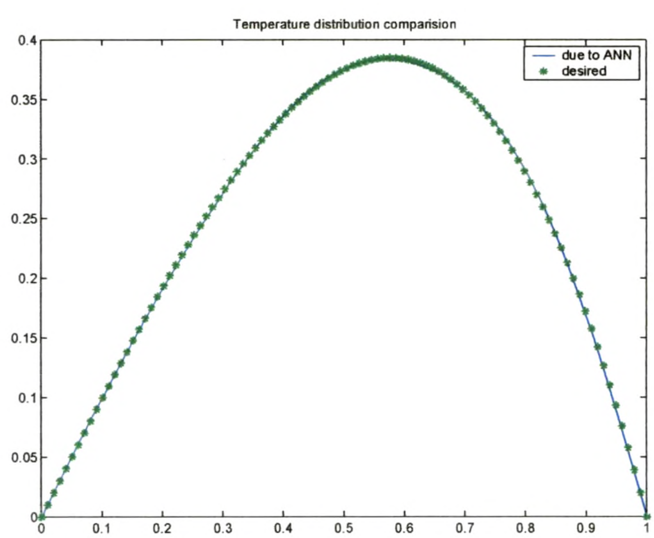


Figure 6.8: The temperature steered by the 4-step ANN zonal controller to the final temperature profile matches the desired temperature distribution.

6.4 Summary

In this chapter we developed the Artificial Neural Networks zonal controller for the linear and semilinear systems. The p-zone controller for the parabolic systems is derived and implemented neural networks. To implement ANN based controller, we first approximate the infinite-dimensional system into finite dimensional projection by using the orthonormal basis of the linear operator involved in the system.

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