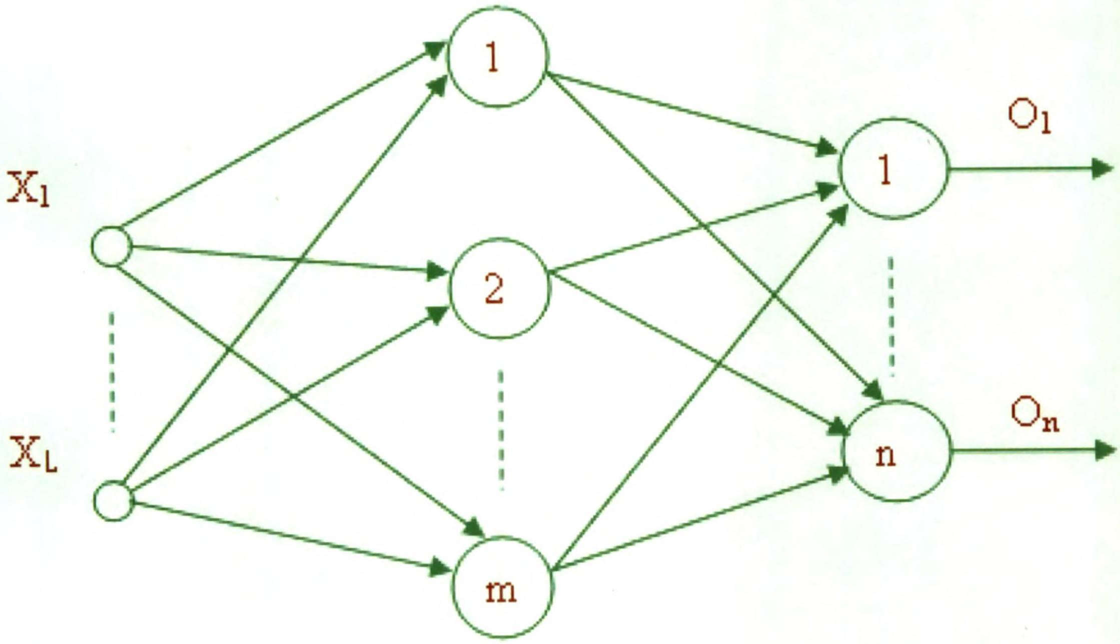
Chapter 1

Introduction



Chapter 1

INTRODUCTION

1.1 Introduction

In this chapter we elaborate upon the motivating factors that influence this work, the literature review and the outline and organization of the dissertation.

1.2 Motivation & Scope

"Artificial Neural Networks" (ANN) is one of the pillars of Soft Computing, the heuristic approach that allows to add intelligence to the dumb entities like computers. In today's world of automation, one is interested in making systems smart and intelligent so that the human intervention is as less as possible. One such application arena is 'Control Systems'.

Controllability problem is one of the fundamental problems in 'Control System theory' in which we are looking for a Steering Control that drives a system from a given initial state to a desired final state in a prescribed time interval. It was first introduced by Kalman [44]. Observability is another property associated with dynamical system. It is the dual of controllability. In observability we completely determine the state of the system by using the observations.

For linear systems, numerous necessary and sufficient conditions for controllability are established for finite and infinite dimensional systems. However, the nonlinear system such as semilinear needs to be analyzed for various types of nonlinearity. Though there are results to check whether a system is controllable or not, but not much algorithms are available in literature for the actual computation of the steering controls, which is very important in automated applications.

In this thesis, our main objective is to implement Artificial Neural Networks (ANN) algorithm for the actual computation of steering controls, both for linear and non-linear systems. This is an application of the Artificial Neural Network as a function approximator. The use of Artificial Neural Networks as a function approximator, resulted from the following facts:

- Cybenko [22] and Hornik [33] proved that the muiltilayred Neural Network is universal approximator. It can approximate any continuous function defined on compact set.
- The Back-propagation algorithm used for the training of the feedforward Neural Networks with the hidden layers.

The Neural Networks implementation of controller is tempting because it can be implemented as Very Large Scale Integrated (VLSI) circuits for the automated systems.

In the real practical situations when there is the lack of the model for the system, the ANN can be trained with the help of Input-Output pairs without fitting any kind of model to the system. The role of Neural Network as a controller can be extended to the adaptive ones when they are integrated with the plant. In this case, they train themselves when they encounter new I/O pairs.

The theory of Neural Networks has got extensive attention recently and has been proved to be a very efficient tool in many of real life applications like Speech Recognition, Pattern Recognition [82], Data Mining, Time Series Prediction [20], Biosciences etc.

1.3 Literature Review

In 1960's Kalman [44] introduced the concept of controllability for finite dimensional linear systems, which is subsequently extended by many researchers to nonlinear systems (refer Joshi and George [36], Sontag [71]) and to infinite dimensional systems (refer Triggiani [75], Pritchard and El Jai [60]). In 1989, Cybenko [22] has proved the universal approximation property of feed-forward Neural Networks. This property has been further investigated by Hornik [33], Mhaskar [52] and others. Sontag [70], Narendra and Levin [57] have proved results related to the controllability and observability property for recurrent neural network and their ability to act as controllers for the dynamical systems.

The dynamics of Hopfield Neural Networks (HNN), a type of Recurrent Networks has been found to be that of a control system and thus we study controllability, observability and stability properties of Hopfield type Neural Networks. Meyer-Base [51] has studied the hyperstability of such Hopfield Model.

Further, we extend the controllability results by Narendra and Levin [57] for the semilinear systems. For the implementation of controllers for linear and semilinear systems, we make use of multilayered feed-forward neural networks with Back-propagation learning algorithm. We also investigate the continuous time semilinear system for controllability results and simulate steering controllers for them using Neural Networks.

In our investigation of controllability property of semilinear dynamical systems, we use various tools from functional analysis. Functional analysis has been firmly established as one of the fundamental disciplines of pure mathematics. It serves as the analytical tool for applied mathematics and their realizations in various sciences. It has greatly stimulated the growth of control theory. Though there has been a considerable development in the theory of nonlinear functional analysis, not much of it has been devoted as an application to System Theory. In our work, we make use of tools of analysis like Banach fixed point theorem, Schauder's fixed point theorem, Grownwal's inequality, Inverse function theorem, Implicit function theorem etc. to investigate the existence of steering controls and computation of steering controllers.

1.4 Dissertation & Organization

In our thesis, we take up following problems:

Problem 1 : Stability, Controllability and Observability of Hopfield Type Neural Networks

After obtaining a mathematical model of Hopfield Type Neural Networks as continuous semilinear dynamical control system we analyze its qualitative properties like existence and uniqueness of solution, stability, controllability, observability, etc.

The dynamical system representing Hopfield type Neural Networks, which is a type of recurrent Neural Network, with n neurons is given by (refer A. Meyer-Base [51])

$$\mu_i \frac{dx_i(t)}{dt} = -x_i(t) + \sum_{j=1}^{n} W_{ij} f_j(x_j(t)) + I_i(t)$$

$$y_i = \sum_{j=1}^{n} C_{ij} x_j(t) \text{ for } i = 1, 2, ..., n$$

where $x_i(t)$ is a state of the i^{th} neuron at time t, W_{ij} is the connecting strength or the synaptic weight between the i^{th} and j^{th} neurons, $\mu_i > 0$ is the time constant, $I_i(t)$ is the external input applied to the i^{th} neuron at time t, f_i's are the activation (transfer) functions, C_{ij} are the ij^{th} element of the observation matrix.

The above equation, using state space representation can be written as

$$\frac{dx}{dt} = Ax + Bu + HF(x) \tag{1.4.1}$$

$$y = Cx \tag{1.4.2}$$

where, $A = \begin{bmatrix} -a_1 & 0 & \ldots & 0 \\ 0 & -a_2 & \ldots & 0 \\ \vdots & & & \\ 0 & 0 & & -a_n \end{bmatrix}$ and B is the diagonal matrix of order n given as $diag(a_1, a_2, ..., a_n)$ and $a_i = \frac{1}{\mu_i}$ for $i = 1, 2, ..., n$.

$$H = \begin{bmatrix} 0 & a_1 W_{12} & \ldots & a_1 W_{1n} \\ a_2 W_{21} & 0 & \ldots & a_2 W_{2n} \\ \vdots & & & \\ a_n W_{n1} & a_n W_{n2} & & 0 \end{bmatrix}; F(x) = \begin{bmatrix} f_1(x_1) \\ f_2(x_2) \\ \vdots \\ f_n(x_n) \end{bmatrix}; C = [C_{ij}]$$

where, $x = [x_1, x_2, ..., x_n]^T$; $u(t) = [I_1(t), I_2(t), ..., I_n(t)]^T; y = [y_1, y_2, ..., y_n]^T$.

The dynamical system represented by (1.4.1) is semilinear system. Existence and uniqueness of solution of (1.4.1) has been proved by using generalized Banach Contraction principle. The asymptotic stability and BIBO stability of the system is proved using the Gronwal's inequality. The system is shown to be controllable under suitable assumptions on the nonlinear functions like Lipschitz continuity and boundedness conditions. In all the results related to HNNs, the activation function which gives rise to nonlinearity is assumed to be sigmoidal which satisfies boundedness and Lipschitz continuity. Under similar assumptions the observability of the system is also proved.

Problem 2 : ANN based Steering Control of Semilinear Continuous Time System

We consider the continuous semilinear system given by the differential equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + f(x(t), u(t))$$
$$x(0) = x_0 \tag{1.4.3}$$

where, $A(t)_{n\times n}$ is the system matrix, which is assumed to be continuous for all t, $B(t)_{n\times m}$ is the control matrix. The state $x(t) \in X \subseteq R^n$, $u(t) \in U \subseteq R^m$ is the control input to the system and $f : R^n \times R^m \to R^n$ is the nonlinear function which is Lipschitz continuous and bounded.

Complete controllability of the continuous time semilinear system is established for the Lipschitz continuous and bounded nonlinearity (refer Joshi and George [36]). For the continuous semilinear system, we implement the steering control using Neural Networks in the phase manner as follows:

- Firstly, we simulated the 1-step Neural Network controller for the continuous time linear system using the mathematical definition for the minimum norm controller.
- Next, the n-step controller for the linear system is simulated using Neural Networks.
- Finally, the steering control for the semilinear dynamical system is computed and implemented using Neural Networks using the coupled iterative equations given below starting with arbitrary $(x^0(t), u^0(t))$

$$u^{n+1}(t) = B^* e^{A^*(t_f - t)} W^{-1}(0, t_f) \left(x_f - e^{At_f} x_0 - \int_0^{t_f} e^{A(t_f - s)} f(x^n(s), u^n(s)) ds \right) \tag{1.4.4}$$

and

$$x^{n+1}(t) = e^{At} x_0 + \int_0^t e^{A(t-\tau)} B u^n(\tau) d\tau + \int_0^t e^{A(t-\tau)} f(x^n(\tau), u^n(\tau)) d\tau \tag{1.4.5}$$

The convergence of the above iterative scheme is guaranteed by Banach Contraction principle.

In the next phase, all the above mentioned controllers are implemented using Neural Networks considering only the I/O data. Hence, the computation of controllability Grammian and its inverse is eliminated and the capability of Neural Network to act as controller is explored.

As an application of Neural Networks controller for semilinear dynamical system we took up the chemical subprocess: mixing tank, of the process of synthesis of Ethyl Acetate.

To produce Ethyl Acetate, Ethanol and Acetic Acid are required in 1:1 mole proportion but, practically it is mixed in the ratio 2:3 say hence, some portion of Acetic Acid remains unused which is recycled into the process. The fresh Acetic Acid and recycled Acetic Acid are fed in the mixing tank with the concentration C_1, C_2 at the rate $Q_1(t)$ and $Q_2(t)$ respectively, which are continuously mixed by the steering rod. The outflow from the mixing tank is at a rate $Q(t)$ with concentration $C(t)$. It is assumed, that steering causes perfect mixing so that the concentration of the solution (Acetic Acid) in the tank is uniform throughout and is same as that of the flow coming out of the tank. Also it is assumed that the density remains constant. Let $V(t)$ be the volume of Acetic Acid in the tank at time t. Then the mass balance and mole balance equations are (refer Gopal [30]):

$$\frac{dV(t)}{dt} = Q_1(t) + Q_2(t) - Q(t)$$

$$\frac{d(C(t)V(t))}{dt} = C_1 Q_1(t) + C_2 Q_2(t) - C(t)Q(t) \tag{1.4.6}$$

where, C_1, C_2 are the concentration and $Q_1(t)$ and $Q_2(t)$ are the flow rates of the influent. The outflow $Q(t)$, is characterized by the turbulent flow relation

$$Q(t) = k\sqrt{h(t)} = k\sqrt{\frac{V(t)}{A_c}} \tag{1.4.7}$$

where, $h(t)$ is the head of the liquid in the tank, A_c is the cross sectional area of the tank and k is a constant.

The above equation can be put in the standard form:

$$\dot{x}(t) = Bu(t) + f(x(t), u(t)) \tag{1.4.8}$$

where,

$$x(t) = \begin{bmatrix} V(t) \\ C(t) \end{bmatrix}, u(t) = \begin{bmatrix} Q_1(t) \\ Q_2(t) \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

And $f(x(t), u(t))$ is given as:

$$f(x(t), u(t)) = \begin{bmatrix} -k\sqrt{\frac{V(t)}{A_c}} \\ \frac{(C_1 - C(t))Q_1(t)}{V(t)} + \frac{(C_2 - C(t))Q_2(t)}{V(t)} \end{bmatrix}$$

Equation (1.4.8) represents semilinear dynamical system in which the nonlinearity f satisfies the Lipschitz continuity and boundedness conditions required for the local controllability around the equilibrium (x_0, u_0), the desired steady, concentration and flow rate. For the realization of controller for the automated plant we train a feed-forward neural network with the suitable architecture. The simulation results are found to be promising.

Problem 3 : ANN based Steering Control of Semilinear Discrete Time System

We study controllability properties of a discrete-time semilinear system given by

$$x(k+1) = F(k)x(k) + G(k)u(k) + f(x(k), u(k)) \tag{1.4.9}$$

$$x(0) = x_0 \tag{1.4.10}$$

where, $F(k)_{nxn}, G(k)_{n\times m}$ are time dependent matrices and $F(k)$ is non-singular for all k. The state $x(k) \in X \subseteq R^n$, $u(k) \in U \subseteq R^m$ is the control input to the system and $f : R^n \times R^m \to R^n$ is the nonlinear function. For the system (1.4.9) we obtain the steering control by implementing Neural Networks algorithms.

In [57], Narendra and Levin analyzed controllability of discrete nonlinear systems, using similar techniques we extend the results for the discrete time semilinear systems. In many real life applications we often encounter semilinear systems as approximations of highly nonlinear systems. For the semilinear system, (1.4.9) we show the existence of the controller in the neighborhood of the stable equilibrium in the open loop form as well as feedback form. The controller thus established brings the system to the desired final states in the neighborhood of the initial stable equilibrium states in n-steps. Finally, using contraction the controllable states are then extended to almost complete state space.

Implementation of the steering controllers, in the different forms, for system (1.4.9) is done using feed-forward Neural Networks algorithms in which the training is done using simply the I/O pairs.

Problem 4 : ANN implementation of Zonal Controller for Parabolic Semilinear Dynamical System

We consider one dimensional parabolic heat equation given by the form

$$\frac{\partial y}{\partial t} - \Delta y = \sum_{i=1}^{p} g_i(x)u_i(t) + f(y) \text{in } \Omega \times (0,T)$$

$$y(x,0) = 0 \text{ in } \Omega$$

$$y(\xi,t) = 0 \text{ in } \partial\Omega \times (0,T) \quad (1.4.11)$$

where, Ω is a bounded domain in R. Equation (1.4.11) represents a system excited by p zone controls $(\Omega_i, g_i)_{1\leq i\leq p}$ with $g_i \in L^2(\Omega_i)$ and $\Omega_i \subset \Omega$. We shall assume that $\Omega_i \cap \Omega_j = \phi$ for $i \neq j$.

Define the linear operator $A : E = L^2(\Omega) \to L^2(\Omega)$ by $Az = \Delta z$ with $D(A) = H_0^1(\Omega) \cap H^2(\Omega)$ and $B : [u_1, u_2, ..., u_p] \to \sum_{i=1}^{p} g_i u_i$. Using these definitions, the equation (1.4.11) can be written as the following abstract differential equation on infinite dimensional space E

$$\dot{z} = Az + Bu + F(z),\ 0 < t \leq T,\ z(0) = 0 \quad (1.4.12)$$

where, $u \in L^2(0,T;U), B \in L(U,E)$ with $U = R^p$, F is a nonlinear operator defined through f. Here, A generates a strongly continuous semigroup $(S(t))_{t\geq 0}$ on E and B is a bounded linear map.

For the system (1.4.12), suppose that (ϕ_m) are eigenfunctions of A with corresponding eigenvalues (λ_m) of multiplicity one. Let $A_n = diag\{\lambda_1, \lambda_2, ..., \lambda_n\}$, $B_n \in L(R^p, R^n)$, $B_n = (B_{ij})\ 1 \leq i,j \leq n$ with

$$B_{ij} = < g_j, \phi_i >_{L^2(\Omega_j)} .$$

Let $z_n \in R^n$ be the finite dimensional approximation of z in state space $L^2(\Omega)$ and F_n be the finite dimensional projection of F onto R^n. Then a finite dimensional approximation of the system (1.4.12) is given by

$$\dot{z}_n = A_n z_n + B_n u + F_n(z_n)$$
$$z_n(0) = 0. \tag{1.4.13}$$

The finite dimensional theory is used to obtain steering control for the system, and which is implemented using feed-forward ANN.

Problem 5 : Controllability of Parabolic Semilinear Dynamical System Using Spatial discretization and its ANN Implementation

The one dimensional semilinear parabolic differential equation investigated for the control of temperature distribution is given by

$$\frac{\partial U}{\partial t} = \alpha \frac{\partial^2 U}{\partial x^2} - \zeta(U - U_\infty) + g(U), \ 0 \le x \le L, \ t > 0 \tag{1.4.14}$$

where, $U = U(x,t)$ is the state of the system representing the temperature distribution along the bar, here t is time and x is the longitudinal coordinate measured from one end. U_∞ is the temperature of the surroundings used as control input. Using another variable $y = U - U_L$ the equation (1.4.14) becomes,

$$\frac{\partial y}{\partial t} = \alpha \frac{\partial^2 y}{\partial x^2} - \zeta y + \zeta y_\infty(t) + f(y) \tag{1.4.15}$$

with the homogeneous boundary and initial conditions $(\partial y/\partial x)(0,t) = 0$, $y(L,t) = 0$ and $y(x,0) = 0$. The system (1.4.15) is analyzed for controllability property by converting it into system of ordinary differential equations using spatial discretization technique. The spatial discretization of system is done by dividing the spatial variable x into n equal parts of length Δx, giving us

$$\frac{dy_i}{dt} = -(2\sigma + \zeta)y_i + \sigma(y_{i-1} + y_{i+1}) + \zeta y_\infty + f(y_i), \ i = 1, 2, ..., n+1 \tag{1.4.16}$$

where $\sigma = \alpha/\Delta x^2$, $i = 1$ indicates the end $x = 0$ and $i = n+1$ indicates the right end $x = L$ of the rod. The boundary conditions used at two ends are $y_0 = y_1$ and $y_{n+1} = 0$, respectively.

The equation (1.4.16) can be put in the form

$$\frac{dy}{dt} = Ay + Bu + f(y) \tag{1.4.17}$$

where, $yt) = [y_1, y_2, ..., y_n]^T \in R^n$ and $u(t) = y_\infty \in R$.

Also, the evolution and control matrices are given as

$$A = \begin{bmatrix} -(\sigma+\zeta) & \sigma & 0 & \cdots & 0 \\ \sigma & -(2\sigma+\zeta) & \sigma & & \cdots \\ 0 & \ddots & \ddots & \ddots & \\ \vdots & & & & \\ 0 & \cdots & 0 & \sigma & -(2\sigma+\zeta) \end{bmatrix} \in R^{n\times n}$$

$B = \zeta[1, ..., 1]^T \in R^n$ and $f = (f_1, f_2, ..., f_n)$ where, $f_i = f(y_i)$. Here the boundary conditions are applied so as to make A non-singular.

System (1.4.15) is analyzed for boundary control as well as distributed control, which are simulated using feed-forward Neural Network.

The thesis consists of seven chapters. The out-line of the thesis with brief contents is as follows :

Chapter 1 gives the overall introduction to the thesis. In Chapter 2, we give the preliminaries required in the work viz. brief about Neural Networks, necessary concept of control systems theory and nonlinear functional analysis.

In Chapter 3, we investigate the stability, controllability and observability properties of Hopfield Neural Network (HNN). The mathematical model of HNN is semilinear dynamical system, in which the nonlinearity depends upon the type of activation function, which is used in the nodes of the network. We assume that the activation function is of the sigmoidal type (e.g. $\frac{1}{1+e^{-\lambda x}}$). In this chapter, to establish the solution of the dynamical system depicting the HNN and its asymptotic and BIBO stability, controllability and observability, we have employed various tools from functional analysis. In fact, many NNs are found to be represented by similar form.

In Chapter 4, we investigate the controllability of the continuous time semilinear dynamical systems. We give the ANN implementation results for the linear and nonlinear dynamical systems. To support the theory, we consider the subprocess, mixing tank of the process of synthesis of Ethyl Acetate, for which the Neural Network controller is also simulated.

In Chapter 5, we study the local controllability of semilinear discrete time dynamical system using the inverse mapping theorem and the implicit mapping theorem from analysis. The nonlinearity in the system is assumed to be Lipschitz continuous. In the later section, we show how the domain of controllable states can be extended to the whole state space using contraction. Numerical illustration is given to substantiate the theory. Also, the simulation results of the NN controller using MATLAB are incorporated.

Chapter 6 deals with the zonal control of semilinear parabolic dynamical system. The analysis is done by converting the partial differential equations into ordinary differential equations in suitable infinite dimensional spaces. Later on, after projecting the abstract system on finite dimensional spaces, we obtain steering control and its ANN implementation is carried out.

In Chapter 7, the parabolic dynamical system is again investigated for establishing the controllability, this time by using the technique of spatial discretization. We convert the system into a system of ordinary differential equation and obtain a steering control, both in the distributed as well as boundary control cases. These controllers are finally simulated and implemented using ANN.

1.5 Summary

This chapter gives the overview of the complete dissertation after giving the influential factors and a note on the existing work for it. In the next chapter, we give the brief note on the Artificial Neural Networks (ANN) and highlight upon the preliminaries required for the development of the work.

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