Chapter VII

Findings, Discussions and Suggestions

CHAPTER VII
FINDINGS, DISCUSSIONS AND SUGGESTIONS

7.1 Introduction

This chapter gives the entire study in a capsule form. The findings of each tool and each objective is also discussed. The chapter ends with suggestions for further researches.

7.2 Summary

NPE (1986) views mathematics as a vehicle to train the pupil to think logically, Plato (trans by Lee, 1987) says that it draws the mind upwards and Newson (1951) felt that mathematics must be a part of the curriculum if wise decision-making citizenery were to be produced. These are the ultimate goals of teaching mathematics. However, the irony is the high rate of failure in mathematics reported in Fourth Survey of Research in Education and a separate section on this area in the Fifth Survey of Educational Research. It is not only found at the national level but even at international level. Holt (1964) holds that there are certain bits and pieces of the vast body of human knowledge, that everyone should know. Also, that one should be protected from a diet of unbroken failure. Hence one finds very high goals in teaching of mathematics, high rate of failure in mathematics and some amount of mathematics essential for successful living. In order to ensure all acquire the benefits of learning mathematics and not remain juggling with numbers and symbols, it is necessary to monitor the progress and remedy the errors. Repeated failure dampens the motivation and hinders acquisition of higher concepts. This present study attempted to diagnose and remedy mathematical backwardness among eighth standard students in Ponda, Goa. The Fifth Survey has reported only three such studies, even though Fourth Survey identified them in the research gaps. It becomes more relevant since no such study has been done in Goa. Ponda, one of the eleven talukas in Goa, of an area approximately three hundred and thirty sq. kms, with forty one schools, was selected for the study. Cluster sampling technique was used to select the samples. The study

consisted of standardisation, diagnosis, case study and remedial programme. Below is given the sample for each stage:

A. Standardisation
   i. Pilot study – three hundred fifty students from six schools
   ii. Administration of the find version – three hundred and seventy seven students from ten schools
   iii. Administration of the standard test for norms – five hundred and eighty six students from fourteen schools
   iv. Administration of the standard test - three hundred and fifty eight students from ten schools

B. Diagnosis – One hundred and sixty students from four schools

C. Case study and Remedial Programme – Nine cases from two schools.

The sample for standardisation was chosen from the forty one schools by cluster sampling technique. The sample for diagnosis were only students those found below thirty percentile. From the four schools chosen for diagnosis, two were chosen for case study. Based on their score on diagnostic test and feasibility of coming for the remedial programme nine cases were chosen.

The standardized mathematics achievement test was constructed based on seventh standard mathematics text book. Only topics from algebra and arithmetic were included. The final version consisted of forty eight items. Experts, researchers, mathematics teachers and guide were consulted during the course of test construction. Data collection was done over a period of one and a half years. Diagnostic test was constructed based on the error areas in the standardised test, other diagnostic tests, research evidences, opinion of teachers. The test consisted of one hundred and nine items. Data collection was done in fifteen days. The case study and remedial programme was completed in one month time. In addition to the diagnostic test, the home background and other details questionnaire was administered on the same sample selected for diagnostic test. The questionnaire was also constructed by the investigator. Interviews were taken of all the cases and their parents, also of twenty

randomly chosen mathematics teachers. The interview schedules were constructed by the investigator. Academic records, intelligence tests scores were also collected of all the cases.

Several research studies, Indian and foreign, have contributed towards this present study. Several books have also influenced the thought-process of the investigator. Bishop (1988) calls mathematics as a pan-human phenomenon. Mathematical knowledge like any other knowledge is invented through social and inter-personal interaction. According to Thom (1973) the real goal of mathematical education is that of developing appropriate mathematical meaning through learner oriented mathematical activities, communication and negotiation. Studies by Tzeng Shwu – Rong (1987), Wangu and Thomas (1995), Sumangala (1995), Kalamaros (1991), Thampurati (1994) found that attitude towards mathematics influenced mathematics achievement. Lack of basic concepts was one of the factors of low achievement in mathematics was found by Manika (1983), Rastogi (1983), Sashidharan (1992), Ashar (1972), Gupta (1972). When errors were diagnosed and remedied pupils progressed well in mathematics was the claim by S.I.E. Guj (1969), Rastogi (1983), Bharadwaj (1987), Das and Barua (1968), Kapur and Rosario (1992).

Studies on mathematics achievement by Smith (1999), Waggon (2000), Ridlon (1999), Bellisio (1999), Dupree (1999), Rose (1991) found certain elements in the teaching-learning of mathematics. They were: student's attitude towards themselves, student's decisions, their language and representations, student's beliefs, notations used by children, students' construction of mathematical knowledge.

Kalamaros (1991), Tzeng, Shwu-Rong (1987), Karen (1988), Greg (1998), Siebert (2000) Aviles (1989), tried to investigate into the attributes of low achievement in mathematics, effect of neglect of errors, attitude of teachers towards mathematics knowledge and belief of mathematics teachers.

7.3 Findings

7.3.1 Findings based on Standardization

The findings from the standardization can be given as follows:

i. Pilot study (a) item difficulty, (b) item discrimination

The item analysis was done in order to eliminate test items not found suitable. Items with difficulty level between forty percent and seventy percent was included in the final version. Items that got eliminated were rational numbers, variations, decimal numbers, and scientific notations. All these items were found to be very difficult. Items which are very easy and very difficult cannot discriminate. Out of ninety three items only forty eight could be retained. The discriminating power of the test items retained in the final version ranged between 0.33 to 0.57. It showed that the items had high discriminating power. Even though a wide range of item difficulty was taken, many items got eliminated. However the length of the test did not affect the reliability of the test.

ii. Administration of the final version

Reliability (a) KR 21

(b) Split half

The reliability of the test was 0.8723 and 0.79 as established by Spearman-Brown's Split-half method and KR21 method, respectively. Thse values showed that the test was reliable.

iii. Administration for establishing norms

a) Norms for entire sample

b) Norms for boys

c) Norms for girls

The means for the entire sample, boys, girls, were 9.6642, 9.506, 9.763 respectively. The medians for the entire sample, boys, girls were 8.653, 8.944, 8.484 respectively. Means were greater than medians showing positive skewness. The scores were crowded over the lower end of the scale. The

ninety percentile showed the highest score was 18.275 which was not even fifty percent of forty eight.

iv. Administration for selecting sample for diagnostic test

a) Descriptive statistics for entire sample

b) Descriptive statistics for boys

c) Descriptive statistics for girls.

The descriptive statistics were drawn for this sample also. However, they were selected with respect to the norms. All those students scoring below 5.864 (i.e. thirty percentile according to the norms) were fit to be included in the diagnosis. However, only students of four schools from the fourteen schools, were selected for diagnosis. The means for the entire sample, boys,. girls were 7.952, 7.919, 7.992 respectively, which was much lower than the norms itself. The highest score of the entire sample was only 16.269 much lower then the norm.

7.3.2 Findings based on Diagnostic Test

The diagnostic test was constructed to find the type of errors and the content area where majority of errors lie. The test items with the percentage of correct responses is given in list (Table 5.2.6.1). The type of errors have been discussed in 5.6. Errors were found in division, addition of variables, simplification, recognizing variables, addition of fractions, subtraction of fractions, identifying odd and even integers, multiplication of binomial with a binomial, solving linear equations.

The type of errors showed that the students did not have understanding of mathematical language i.e., the use of symbols, place value, placing of numbers. It seemed that they were "juggling with numbers" Dienes (1970). The most striking example was, $a + a = a^2$, $a \times a = a^2$. It was found as a common error. It did not make any difference to the students whether the operation was '+' addition or 'x' multiplication. There was similar error of overlooking mathematical symbols between '÷' division and 'x' multiplication. The inability to distinguish between constant and

variable, was unexpected. The inability to observe the rule of mathematical operations for eg. in subtraction: subtracting subtrahend from minuend, was surprising. These were not minor mistakes. They pointed towards a totally disfigured picture of mathematical basics. The error of $2^6 = 3$ showed that students had no understanding of the concept of indices, i.e., even the symbolic representation Distinguishing between rational numbers and integers was one of the items with least correct responses. In such a situation how would the students comprehend difference between addition of rational numbers and integers. The addition of rational numbers with different denominations was one of the item with least correct responses.

All the items involving indices showed thoroughly faulty conceptualisation of law of indices, i.e., adding of indices, multiplication of indices, taking common index. Bracket expansion in case of binomial, binomial expansion was very poor. All the terms were not involved. $(2a + 1)(3a + 1) = 6a^2 + 2$ and $(a + b)^2 = a^2 + b^2$; $(ab)^2$ showed partial multiplication and even faulty multiplication. The concept of equation and LHS = RHS was the least known. Even a simple linear equation like, if $x + 1 = 0$ then $x = 0$ or 1 showed the students had no clarity about LHS = RHS. How to interpret the errors if $2x = 1$ then $x = 3$ and if $a + 1 = 2$ then $a = 3a$. This would be considered as the worst violation of the rules of equation.

The percentages of correct responses on diagnostic test (Table 5.2.6.2) reflect the areas of backwardness. To make the task even easier (Table 5.2.6.2), gives the test items, with least correct responses. While the test items on basic skills in arithmetic do not feature much in this list, a few do appear. Division seem to be a tough operation. Idea of inverse was also very difficult. Adding of variables, identifying variables, were the most difficult areas of basics in algebra. Identification of rational numbers adding fractions with. Similar denominator with negative sign, adding subtracting and dividing fraction where very few could response correctly. Multiplying indices with integer as lease was found to be far more difficult than indices with variable as base. Identifying odd integers was more difficult than identifying variables. Multiplying binomial with a binomial was found to be very

difficult. Big surprises were with linear equations where even fill in the blanks were not responded correctly.

7.3.3. Findings based on Case Study

The purpose of doing case study was to understand the students backward in mathematics. It brought forth the feelings opinions, unfulfilled desires, needs, of the students. The case study included interviews of the students' parents. The interviews of the students and parents are given in 6.6. The students were by and large happy with their schools. They preferred strict and humorous, type of teachers. They lamented over the fact that teachers neglected them and failed to show any appreciation. Particularly, very few of them could recollect any favourite mathematics teacher. They felt the large number of students in the class was the main problem. Preference was shown for teachers who have patience to give explanation again and again. Their lack of interest was only superficial. It was due to backwardness, neglect by teachers. The backward students expressed the need for revision of previous topics, checking of homework, paying attention to backward students. The students had very vague idea about their backwardness. They were not very sure about their difficulties and errors. In fact, they felt previous topics were not essential any more. After attending the Remedial Programme they felt previous topics need to be thoroughly revised. The study habits of the cases revealed, except three, all resorted to rote memorise especially definitions and formulas, even the intellectually average. Some even rote memorised solved problems. All took tuitions in mathematics except two. The investigator felt that the cases had not developed proper study habits. Verification, enquiry were not known to them. All they knew was to ask for approval from the teacher and copy down from the chalk board or from somewhere. They never even paused to find out where they went wrong, what was the problem, what is the answer. Their main look was for the 'correct answer'. The parents' interviews of the cases showed dissatisfaction with the school system. They all had the common complain of rebellious behaviour of their wards as soon as they stepped into high school. Typical adolescent behaviour characteristics were listed down by the parents. Only Deepak and Gayatri could get some help from parents for their studies. Other cases could get only moral support and monetory support from parents. Manthan,

Purva, Chetana came from lower middle class. It was very assuring to see parents investing into education even though they had to put in hard labour.

The case study revealed the rampant low achievement in mathematics (refer Table 6.7.1). The eye-ball test was all that was needed to conclude the low achievement in mathematics. Hence in eighth standard except three students, all others were promoted with grace marks. The academic records show years of very low achievement in mathematics. This shows that these students taken up in the case study could never scored even upto thirty five percent, in majority of the cases. It points towards proliferation of neglect of errors and lack of insistence or motivation to score high. The opinion of the students in the case study about their mathematics teachers was not very positive. They had one mathematics teacher on an average worth remembering. They all had an aversion for mathematics except Gayatri and Deepak. The expectations of students from mathematics teacher and the kind of teacher behaviours appreciated by the students, were posing questions about prevalent teacher practices. They were antigonised by the indifference shown by the teachers, the text-book language used for explanation, lack of sensitivity to their performance, lack of humour.

7.3.4 Findings based on Remedial Programmes

The remedial programme revealed the hurdles in understanding of mathematics. The symbols, language and syntax of mathematical expressions were vaguely known. Meaning of equations were absent. What became evident was there was a need for individual attention, which does not necessarily mean teaching one student at a time. Checking their note books, asking about their doubts, giving them challenges, asking them to find the responses themselves, appreciating them, maintaining punctuality and high standards. These elements were the highlights of the remedial programme. They were not planned consciously to be included, but became the features, gradually.

The cases showed improvement in terms of attitude and performance. The pre-test post-test scores on the diagnostic test showed reduction of errors. The areas where

majority of them had shown improvement were: (i) linear equations (on a few test items, (ii) multiplication using indices, (iii) monomial-binomial multiplication, (iv) bracket expansion, (v) recognising bases and powers.

These improvements have come about in fifteen sessions of the remedial programme. The verbal expression of the method in a simple language (refer 6.5.3) enhanced their understanding. Another feature of the remedial programme which promoted learning was peer explanation. They found some of their friends to be very slow to understand and hence resisted from explaining them. The investigator found it surprising to find the remedial programme groups, engrossed in doing mathematics of seventh standard for even three hours. They did not show signs of boredom or restless, except at the end of the sessions. They did come regularly and punctually irrespective of the distance. For every topic, they were asked to frame problems and to solve them. This also helped in enhancing concept clarity. This exercise brought to light the source of errors. Lack of knowledge about mathematical symbols, equality, place values, variables, even and odd numbers, use of brackets. The remedial programme made them more aware of their errors and helped them to overcome some of them. More than the errors being reduced and improvement in achievement what they valued was that they began to like the subject.

7.3.5 Findings based on Teachers' Interviews

The interviews of twenty randomly chosen mathematics teachers (refer 6.8) were analysed. The responses for the major questions were categorised and analysed. The common views and unique views, that emerged are given below:

I. ***Ability of backward students to understand explanation by teachers***

The opinion seemed to be about the language of instruction, need for repetition of explanation need for individual attention.

II. ***Regularity in homework by backward students***

The experiences were sharply different. There seemed to be a need of threat to get homework done. Some teachers have found their own unique

verification schedule – like having leaders to check and keep record of culprits, getting homework done in the class itself, asking questions based on homework. There were extremes like not giving homework at all, since they copy or because teacher cannot correct it. While there are teachers – who ask backward students to do homework problems in the board or give personal explanation to the student or check their homework personally.

III. Attempt by backward students to improve

The general impression was the backward students feel shy and do not approach. Another factor being tuitions. The teacher has to take the initiative, in all cases.

IV. Parents' contribution towards improving backward students

By and large parents do not take active role. They are happy by sending students to tuition classes. They give entire responsibility to the teachers and express their inability to control their wards. Illiteracy is a factor in some cases.

V. Experience of trying to improve the backward students

Each teacher seemed to have devised a method. There was a need to motivate, show concern, to create interest in mathematics. Personal efforts of the teacher is found to be prominent rather than any curricular provision. While some were negative and had given up efforts.

VI. Methods adopted for algebra teaching

Different teachers had different perceptions. This decided the prerequisites to the revised before the topic. While some believed in rote, some insisted on building the concepts and some elaborated in making the environment conducive for teaching. However, inductive method seems to be prevalent.

VII. Special approach for the backward students

Individual attention was the crucial need. Gaining confidence of the students and being approachable and friendly was the most essential element. While some aimed for students to just get through the exams, other went to moulding them.

VIII. Text-books

There was a dissatisfaction with the text-books. It was found to be not useful for students. Presentation was not according to the age level of the learner and topics were not arranged sequentially. Need for a workbook was opined. While there was not much against the NCERT text-books, it was felt the omissions and dilutions done by the board made the text-book less challenging, paucity of problems were also voiced. Language was also an impediment at for backward students. However a common comment remained regarding no link between standard eighth and ninth text-book.

IX. Special characteristics of backward students

A major special characteristic was the great interest exhibited in sports and cultural activities. They haul been found exceptional in these fields. Lack of urge for learning, inability to understand, have been also found. Credit has been given for their behaviour and their respect for teachers.

X. Topics found to be difficult by the backward students

Most of the comments contained topics from geometry, from different standards, mensuration – II, theorems, riders. Trignometry, cuberoots, square roots, surds, factorisation, linear equations and word problems were the oft-mentioned topics.

7.3.6 Findings Based on Questionnaire Responses

The questionnaire responses by the students backward in mathematics (refer 5.5) brought forth, the background of the backward students. Also their difficulties with mathematics, mathematics teaching. Language was a major hurdle for the

students though only twenty eight percent felt need for explanation in vernacular language. However, word problems were found to be difficult. There was a demand for more explanation and repetition of explanation. All these evidences lead to the need for simplification of the language used to explain. The home background of the backward students showed about sixty four percent belonged to large families and approximately sixty two percent had father doing white collar job. The responses show that students received encouragement from parents and found sufficient time to study mathematics. Thirty two percent spend one hour per day to study mathematics at home. More than one hour was found to be even lesser. Keeping the TV on some times while studying was found in forty six point eighty-three percent and writing while studying mathematics was prevalent among approximately seventy six percent. (Writing 41%, reading and writing 35%). How they manage TV viewing and writing and how much could they concentrate, is doubtful. The feelings associated with studying mathematics was generally found to be healthy i.e. happy, sometimes boring and sometimes nervous. This was a but surprising for the investigator. It was expected to be always boring and sleepy. Rote memorizing was rampant, (of solved problems). Among the changes suggested by the students were more than one period for mathematics, making students find correct answer, asking students to solve problems by their own methods. These suggestions were unexpected since the sample was of backward students. This runs contrary to the notion that students want to get correct answers from teachers. There were suggestions for more time and easy questions in the examination for backward students.

7.4 Discussions

In this section the investigator has weaved in ideas from research studies, official documents and findings of the present study itself. The discussions offer a continuous flow of thoughts interspersed with references. To give lee way to the thoughts without interruption, compartmentalization has been avoided. Recurrence of discussions under various contexts makes it more thought provoking.

The present study found severe backwardness in mathematics achievement among eighth standard students in Ponda, Goa. This was evident from the mean score

of 9.642 on the standardised achievement test of maximum of forty eight marks. This was restricted to topics on algebra and arithmetic of standard seven. Similar findings of low achievement in high school were cited by Indian and foreign studies. [Sharma (1978), Sashidharan (1992), Kasat (1991), Jain and Burad (1988), Rastogi (1983), Jain (1979), Bhirud (1975), Ashar (1972), S.I.E. Guj (1969), Sjostrom (2000), Winter (1991), Lee (1999), Aviles (1989)].

This showed that mathematical backwardness was an epidemic and also universal. Fifth survey of Educational Research commented about the difficulties found in algebra. Studies like Sharma (1978), Ashar (1972), Bhirud (1975), Sjostrom (2000), Dominguez (2001), Bellisio (1999) found various factors for low achievement, types of errors, difficulties students have with algebra. The need for algebra was amply emphasised and elaborated by Kinney and Purdy (1960). However, (Table 5.2.6.2) shows the test items with least correct responses, belonged to Basics in algebra, Indices, Binomial – Binomial multiplication, Linear Equations. The types of common errors in 5.7 reveals the paucity of even basic knowledge of arithmetic at eighth standard. Algebra is introduced as early as standard six in Goa. The photocopies of few pages from the sixth standard mathematics text book is given in (Appendix-J). 'Variables' and 'literals', both terms are used in chapter five where it is introduced. What is puzzling is that the use of variables for generalizations, begins in chapter two itself. The cramped and brief presentation along with the haphazard sequence can create misconception leading to rote memorization. The more pertinent point here is, how much of mathematics at a particular age level. The basis has to be consciously decided, on depending on research evidences, cognitive level of the learner. Any exercise that overlooks these criteria perpetuates misconceptions, as found by the present study. ***Sarangapani (1990) found that concepts were arranged logically rather than psychologically. Sashidharan (1992) found that students were not cognitively prepared to learn mathematics even at standard ten. The present study found the students to be deficient in the basics of algebra and arithmetic essential to learn eighth standard syllabus***. The mean scores of the entire sample, boys and girls were found to be low. The causes may be faulty teaching method, lack of diagnosis and remediation. However, it would be beneficial to perceive this

problem from the point of view of objectives, psychological/ cognitive preparedness as put forward by cognitive psychologists. The cases selected for the case study, were either intellectually average or intellectually below average. This factor also may have been an impediment for low mathematical achievement. The emphasis made here is, about the suitability of the concepts vis-à-vis the age of the learner and the objectives of teaching algebra. Much thought has to be given to how much of algebra and the purpose. After studying algebra the student will be able to, according to Kinney and Purdy (1960) for eg: 'explain the purpose for using symbols in mathematics', 'knows the purpose for representing relations as formulas', 'solve a formula for different variables.' ***If the common errors (refer 5.6) are examined, one would find it difficult to comprehend what the students know after studying arithmetic and basic algebra for three years. There is a need to check whether the objectives of teaching algebra are achieved.*** This becomes more essential when high rate of failure is observed in high school in mathematics, particularly in algebra as found by Chel (1990), Jain and Burad (1988), Kasat (1991), Jain 1979), Sjostrom (2000). Fourth Survey of Research in Education, also reported about the large number of students failing in mathematics as per N.C.E.R.T. Survey. Holt (1964) says that one should protect a child from a diet of unbroken failure. He also says that of all the human knowledge, certain amount, that can be called essential, must be known by all. Hence, there needs to be a demarcation between the essentials and advanced in the mathematics syllabus. The need for diagnosis and remediation of mathematical backwardness has been found world over and even at college level by Duncan (2000), Winter (1991), Dominguez (2001). Studies have found improvement in mathematics achievement after a remedial programme. [Rastogi (1983), S.I.E. Guj (1969), Bharadwaj (1987),, Das and Barua (1968)].

The present study also found improvement in mathematics achievement, change in attitude towards mathematics even after a short remedial programme of one month. (refer 7.3.4) The reduction in the number of errors from the pre-test to the post-test was remarkable taking into consideration their intelligence, scores on standardised test, scores on diagnostic test. ***The perpetuation of mathematical***

deficiencies without any intervention brings about a kind of ignorance about mathematics.

The mathematical backwardness, high rate of failure in mathematics, lack of remediation, nurtures a mathematically deficient citizenery. Going back to the objectives of teaching mathematics by National Curriculum Framework (2000), NPE (1986), Kinney and Purdy (1960) it seems the purpose of teaching mathematics has not been served. Errors have been neglected and promotion policy catapulated the students along with their backwardness. This resulted into a situation where students did not know their errors/ difficulties. This was evident from the questionnaire response analysis.

They identified the topics as 'easy' in which there were least correct responses (refer Table 5.2.6.2) and absurd common errors. (refer 5.7). ***The common errors revealed the rampant misconceptualisations. They again point towards the uninterrupted negligence of deficiencies and the accumulation of higher concepts. Mathematics being a heirarchical subject, errors in the understanding lower concepts hampers the conceptualisation of higher concepts [Sashidharan (1992), Manika (1983), Rastogi (1983), Sarangapani (1990)].***

Diagnostic test responses illustrated the extent and areas of backwardness, (refer 7.3.2). Among the areas of backwardness were fractions, division, indices, binomial – binomial multiplication, linear equations. These areas constitute a major part of algebra and mathematics in general. It is the ignorance about this backwardness, that is more surprising. The teachers would readily pass the buck to teacher in lower classes. ***Somewhere in the rigamarole the student is blamed for lack of intelligence, interest and at the same time given ample protection from any failure by the promotion policy. It seems there is no ned to correct the errors. Even with it they pass standard tenth with flying colours, which is the ultimate goal.***

The need for remedial programme and the purposefulness of diagnosis was equally voiced by teachers (refer 6.8) and students involved in case study (refer 6.6).

The hurdles however, were time availability, accomodation, place of stay/ distance. Inspite of all these impediments many teachers have attempted to rectify errors in their own way (refer 6.8). It would be hence beneficial to provide them with a diagnostic test developed by the investigator or any such diagnostic test. It is futile to blatantly blame the teachers who have been trained barely to teach. This brings the discussion to teacher education. Very few studies could be cited due to paucity. [Mohapatra (1990), Siebert (2000), Dandapani (1992).] The crux of the matter is the emphasis during teacher education. Instead of filling the student-teacher with theories of learning and methods and techniques of teaching, an investigation into their conceptualisations, beliefs is also essential. ***Besides appropriate training during teacher education, upgradation of knowledge and assistance, during school teaching is of vital importance***. The reasons for backwardness, measures to tackle it, purpose of teaching mathematics, creating positive attitude towards mathematics. Such inclusions are essential in the education of a mathematics teacher. The interviews with mathematics teachers revealed, teaching of mathematics and its associated activities were found to highly individual-dependent. While some have developed their own productive way of functioning even in dire situations, others have taken safe refuge in the students' lack of interest and inability. Lack of co-operation from the parents, lack of space and time to conduct' remedial classes, need for separate syllabus for backward students, were strongly voiced. There seem to be a scale of initiatives and willingness on which the teachers could be placed, with majority, on the higher level. Also, the investigator felt that each one was trying to tackle unprecedented, grave, challenging, cognitive and managerial problem single-handed. There is a need for periodic updating, replenishing. Most of the teachers showed dissatisfaction with the text-book in terms of arrangement of content. However, the collective wisdom of the mathematics teachers could be utilised rather than a few chosen ones. Every revision or change of text-book, should be preceded by an opinion survey of the consumers (teachers and students). More conscious efforts towards grading and maintaining two channels would serve the idea of avoiding some topics found difficult by backward students. Need for more problems for practice and homework, can be met by developing workbooks with graded problems. Teachers could be also provided with brief idea about the approach, the perspective and the

objectives, of the text-book. ***The teachers need to be provided with assistance in terms of new approaches, methods, strategies for handling backward students. The state institutes, teacher educators, researchers could contribute towards the problem, rather than leaving it to the wisdom of the mathematics teacher alone.*** The insights from various researches have to be utilised while designing teacher education programmes for mathematics.

The intelligence tests revealed that the cases were intellectually below average except a few. Their academic records where not very impressive and they did not have very highly educated parents (refer 6.6). Rajyaguru (1991), Nagalakshmi (1996), Kasat (1991), Shah (1985) found lack of parents' attention and parental qualification highly related to mathematical achievement. Verma (1996), Srivastava (1993), Jain (1979), Gupta et.al. (1993), Stella et.al. (1995), found intelligence to be a factor related to mathematical achievement. The present study would to some extent agree that these factors, apart from other factors, have been influential in the mathematical backwardness of the cases. The investigator had been to the homes of all the cases and seen their living condition. Hence the investigator can make a general comment that except in the case of Deepak, Gayatri, no other cases benefitted from parents qualification (refer 6.6). Only Chetana, Deepak, Manthan and Nilesh were found to be intellectually average on SPM. However, among those who showed significant improvement on the diagnostic test scores were Jatin, Ashwin who were found to be below average in intellectual capacity. Hence, the investigator was not able to conclude as far as intelligence playing a major role. During the Remedial Programme Purva, Jatin, Gayatri used to show faster understanding. Investigator felt mathematics education should aim for meaningful learning through learner oriented mathematical activities, communication and negotiation. Also, understanding of teaching and learning mathematics as a model of enculturation rather than a model of transmitting knowledge. ***These ideas emerged from views on a paradigm shift from culture-free context-free absolute truth to mathematical concepts constructed by learners and also from a depersonalised learning to meaningful and more realistic and culturally relevant mathematics. In short, making the learning of mathematics relevant and enjoyable.*** This paradigm shift has been advocated by Thom (1973), Bishop ((1995),

Mellin – Olsen (1987), Bauersfeld (1995), Saxe and Bermudez (1992), Vygotsky (1978). ***What is essential is to verify (a) whether students enter high school with basics of arithmetic and algebra (b) whether all the mathematics that is being taught is required for all citizenery (c) whether the content matches with the cognitive readiness of the students.*** This would mean rethinking in terms of syllabus for each level, relevance of the content, and cognitive suitability. However, with the need for mathematically literate citizenery is increasing, it becomes imperative to ensure continuous monitoring of mathematical backwardness. This has been reiterated by Winter (1991). The findings of the case study (refer 7.3.3) shows the low achievement in mathematics. ***The low achievement bears witness to the lack of official will to improve achievement and neglect of errors. The unseen academic blunder due to administrative provision is the proliferation of lack of basic concepts. Students smoothly sail from fifth to eighth standard even with marks as low as eighteen out of hundred.*** What arrests the attention is, the amount of mathematics, such students would have in order to understand eighth standard mathematics. The students have never felt efforts to pass. Hence, never even knew of their backwardness.

Efforts in India have been largely towards change in syllabus to improve educational standards. Indian studies reflect the theoretical inclination. Foreign studies on the other hand try to find out children's thinking pattern or teachers' beliefs, train parents to assist in doing homework, even a learning theory for mathematics learning. In fact, they may even come out with a solution. It was enlightening to know about different perspectives. It is not to undermine Indian studies altogether. What impressed was the different perspective and the approach to the problem. Studies like Duncan (2000) finding relationship between mathematics preparation and mathematics skills of college entering students, try to find whether school mathematics is learnt sufficiently. Such studies are found to be more meaningful. Bellisio (1999) tried to find if students' written notations match the expressions of their ideas in natural language. He found the notations students invented helped them keep track of their ideas and organize their data. Studies such as these try to go over to make mathematics closer to child's thought process. Study by

Smith (1999) also tried to find role of language and representations in childrens' mathematical reasoning. Winter (1991) tried to teach mathematics through art which helped reluctant learners to comprehend the contribution of mathematics to the world culture, everyday life. These studies give a new dimension to the teaching-learning of mathematics, novel ideas to tackle low achievement and lack of motivation. Among the Indian studies Saranagapani (1990) analysed curriculum design of the N.C.E.R.T. primary mathematics series and found the concepts were sequenced logically rather than psychologically. ***There was high level of algorithmisation which may help children in coping with what has to be learnt, but which will also impede conceptualisation, as it rules out the scope of children inventing conceptual links.*** This study is very significant in the present context when students at large are found with misconceptions in basics of arithmetic and algebra. Mohapatra (1990) found teachers gave least importance to objectives like development of discipline, determination, a sense of proportion. They were found to be more conservative as far as objectives were concerned. They emphasised on the fundamentals mathematical operations, development of mathematical skills and resisted intrusion of new topics. Students, were found to be more pragmatic in their approach and considered mathematics to be a utilitarian subject. This study revealed the conflict between the aims of students and teachers. The study brings to light teachers' beliefs which decides their teaching itself. In the present context, constructivism is found to be more humane way of learning, enhancing, understanding among even reluctant learners. Studies such as these will bring forth the crucial areas that need a paradigm shift. It would be futile do cosmetic changes, while the core issues remain untouched. Hence the high rate of failure in mathematics, lack of basics of arithmetic, lack of positive attitude towards mathematics could be accounted to lack of understanding of children's mental representation, purpose of learning mathematics, rote memorization. To add to these woes the absence of monitoring of students' errors. ***The Education Commission (1964-66) has identified backwardness due to two reasons. One of them being underachievement to perform up to the level of one's intelligence. Commission points out that when pupils are unable to profit from education then it is a wastage of educational facilities and of human resources.*** Further, Commission marks it as a great concern and suggested diagnosis, collation of data to have a total

appraisal of the situation and indicate lines of remedial treatment. Aim of such remedial measures should be to correct the basic errors, raising the attainment level in the subject or subjects, re-establishing the confidence in oneself and to succeed, creating interest and motivation in his studies.

Hence, the Education Commission (1964-66) has given very explicitly about the backwardness and its handling. It gave buttress to the present study, in terms of the methodology and the emphasis laid on developing a positive attitude towards mathematics.

From the diagnostic test errors, standardized test responses certain areas/ topics of mathematics have emerged as difficult ones. Education Commission (1964-66) in one of its recommendations for mathematics syllabus at secondary stage has emphatically suggested elimination of out-dated, material from syllabus such as simplification, factorisation, the finding of HCF, LCM, cut down on identities, solution of triangles, heights and distances. One can find that these topics have remained as they are in the syllabus. Over the years the syllabi have increased in volume in the attempt to increasing the standards. It is time to wake up to the alarming situation where the high goals of mathematics are far from achieved rather the subject itself becoming a nightmare. The errors identified in the diagnostic test reponses (refer 5.6) show the kind of errors. The errors based on place value mathematical operations sign convention, use of brackets equality leave very little hope for mathematical teaching-learning. The very core of mathematics being relationship between entities and the symbolic representation. These errors have been found in eighth standard. These errors are a result of years of neglect. The academic records of the students in the case study show the habitual, low achievement and uninterrupted promotion (refer 6.6) All concerned in the field of education have to inevitably address this grave situation. A subject meant to train to think and reason logically, universally accepted as essential element, of school curriculum, now renders large number of students as either failures or backward.There is a need to think of what Holt (1964) states as bits and pieces of human knowledge found essential for all citizenery. Also avoiding a continuous diet of failure. ***This brings to a***

point of streamlining the mathematics syllabus, making it relevant vis-à-vis for general and specific purpose, monitoring and rectifying mathematical backwardness. These steps would usher in new life into the corpse of mathematics education, making the Education Commission (1964-66) recommendations see a new day. A much less thought of vital issue is the wastage of educational facilities and human resource. Providing equal educational opportunities becomes null and void when it leads to failure or low achievement. In no way it adds up to the development of the country.

The present study shows low achievement in mathematics and prevelance of misconceptualisations at eighth standard. The errors that have been identified were found to be rampant and having a long history. There has to be a rethinking about what students learn from mathematics and how it benefits them. ***The educational system has to fulfill its moral responsibility of ensuring the minimum skills and knowledge of mathematics to lead a productive life. In order to do this it has to revamp its teaching methods examination and promotion policy***. Periodic monitoring of the progress of the students and replenishing of the deficiencies has to be done on a war-footing.

According to NCTM (1998), the basis for Standard 2000 (set of standards for the mathematics curriculum) are the set of core beliefs about students and learning mathematics: (I) every student deserves an excellent programme of instruction in mathematics that challenges them to achieve at the high level required for productive citizenship and employment. ***(ii) Learning mathematics is maximised when teachers focus on mathematical thinking and reasoning. (iii) Learning mathematics is enhanced when content is placed in context and connected to other subject areas***.

The ideas voiced by Thom (1973), Bishop (1995), Bauersfeld (1995), Saxe and Bermudez (1992), Vygotsky (1978), Education Commission (1964-66), N.C.T.M. (1998) seen to be converging. The chanting is all about making mathematics learning beneficial for productive citizenery, achieving the goal of 'train the pupil to think and

reason logically' and make mathematics learning as a process of enculturation, making it meaningful by giving culturally relevant experiences.

These views though developed in different contexts, different time, different cultures seem to aim at the utility of mathematics for the learner.

7.5 Suggestions

The research scenario of mathematics education is not very different from other subjects in terms of the boom of research activity period i.e., 1970 onwards. Mathematics, being at the core of the educational process, Nesher (2000), needs to be more researched. Fourth Survey of Research in Education and Fifth Survey of Educational Research have very elaborate by discussed about the research scenario of mathematics education. The investigator attempts to put forth certain suggestions for research and for mathematics education by itself. After having a glimpse of the world of mathematics education and the backwardness therein, investigator found it imperative to give suggestions pertaining to: teaching of mathematics, curriculum, diagnosis and remediation, teacher education, inservice education, test book making.

7.5.1 Teaching of Mathematics

The general trend found in teaching of mathematics, based on teachers' interviews and students' interviews, was the traditional style being followed. Emphasis on drill and rote was clearly evident. Students who were good at this, liked the subject, and were found to be scoring high. Teaching of mathematics has to bring in variations with respect to the topic and emphasis should be on understanding of the method. There should be discussion about the errors committed rather than mere correction.

A paradigm shift from rote memorizing, drill and practice to constructing knowledge based on one's own experience. Making mathematics relevant to the learner. Taking into account learners' thinking, representations, notations. In the context of mathematics education Mellin – Oslen (1987) stresses the importance of student experiences of mathematical activity rather than more instruction in

mathematical rules and procedures. No more authoritative role but facilitator and learners will be the meaning maker, Bishop (1985). According to Thom (1973) the real problem which confronts mathematics teaching is not that of rigour, but the problem of development of meaning. Thus, emphasis would be on learner thinking mathematics with one's own real world situations resulting in different meanings for different individuals.

Teachers should make efforts to reduce teacher-dependent learning situations. Instruction based on rules, memorisation, algorithmic presentation which concentrated on correct answers and neglected cognitive thought processes should be avoided. The teachers need to reflect upon the objectives drawn for teaching of mathematics. Rather than teacher being the centre of the knowledge and a necessary component for successful task completion, student should become confident in their skills, transfer their skills to novel situations, apply higher level thinking to novel situation without teacher help. Teacher has to review one's own teaching. Whether sufficient time is spent on student responses, how much of student independence is being encouraged, are the student becoming active participants, how much time is spent on single student during classwork. Prolonged periods of assistance from the teacher deprived students of their opportunity and reinforce helplessness. Independent learning is the goal of education. ***The present study found independent and challenging learning situation was appreciated even by the students who were backward in mathematics. Grieb and Easley (1984) found independence in learning to be critical for capturing the meaning of mathematics in both high level tasks and in the applications to the real world.***

7.5.2 Diagnosis and Remediation

The low achievement of the sample on the standardised mathematics achievement test and the common errors together point towards the need to diagnose the mathematical backwardness among students at various levels. It also reveals the cumulative effect of the promotion policy prevalent (refer 1.4). The only criteria seems to be the tenth standard results, where all lacunae are nullified. Hence, diagnosis at tenth standard to reveal the backwardness would be an eye-opener.

This has be recommended by Education Commission (1964-66). One fails to find it implemented even though low achievement, backwardness, high rate of failures has infested school education. Diagnosis and remediation must be the part of the school curriculum like evaluation. Teachers should be provided with diagnostic tests and also trained to construct diagnostic tests. Remedial Programmes are not mere repetition of classwork. They ask for making the students aware of their backwardness, motivating them, making the subject interesting, clarifying the basic concepts. The biggest hurdle identified for remedial programme is accomodation in the school within the school hours. Another hurdle the time factor. In order to tackle these problems remedial programme could be a joint venture of two mathematics teachers. Either alternatively between regular teaching and remedial teaching. It could be even planned for first few months of the academic year or few months after the result.

The remedial programme in the present study revealed that giving individual attention, not in terms of giving the correct answer, but enquiring about their doubts, checking their notebooks, challenging them, appreciating them, did bring out better attitude among the backward students. Hence, these elements need to be included in any remedial programme instead of repetition of classwork and half-hearted approach. Expressing the methods in a language understood by the students rather than text-book language, promoting peer explanation, asking students to frame problems and solve them, made the remedial programme exciting and challenging. These features are essential in a remedial programme for mathematics.

Enquiry into inability to achieve must be done on a regular basis. All individuals will not need high level of mathematics for their professional or personal life. Hence there could be general and specialised mathematics syllabi. As the learner moves from one standard to another he/she should be equipped with at least minimum skills, knowledge. This must be ensured by the evaluation system. Even if the learner is promoted by grace, he/she does possess, the essentials to acquire further knowledge.

7.5.3 Teacher Education

The interviews with the teachers (refer 6.8) show that the teachers find out their own methods, strategies to handle various challenges, problems in classroom teaching. Teacher has to take initiative to improve the backward students, to check homework, in addition to frequent text-book changes, burgeoning class size. The teacher education programmes seem to be ignoratn about any real life situation a teacher has to face. Moreover, teachers still carry the idea of teacher-centred, text-book based, exam-oriented mathematics teaching. Teacher education programmes for mathematics methods needs revamping, in terms of methods, strategies and techniques in view of large class size, need for concept clarity, handling backwardness, using of new technology in teaching, especially computers, activity based teaching.

The concept of mathematics teacher education needs to undergo change. The idea of teacher being the authority, to whom the students look upon for the approval or disapproval of their interpretation of the mathematical tasks. He/she should act as negotiator of meanings constructed by the students. Mathematics learning is to be considered as a process of both individual and social construction of meaning.

Teacher must have an awareness about the impact of culture on mathematics learning. They need to be aware of how their teaching contributes not just to the mathematical development of their pupils but also the mathematical development in their culture. Hence, teacher education courses should be modified to include about culture, society relationship between language and thought and history of evolution of mathematical concepts. In order to equip the teacher about the new concept of mathematics teacher real life illustrations of such mathematics teaching could be cited. A brief idea about problems, issues of mathematics education and research evidences could also be an inclusion. Mere theoretical inputs needs to be avoided. Hands-on experience of innovative ideas should be provided.

7.5.4 Inservice Education

Teachers interviews revealed very clearly that the day-to-day problems, of handling students, teaching various new topics, handling students of different abilities, were done by teachers according to their own judgement. This is where inservice programme have to play a vital role. Pre-service programmes cannot give a in-depth comprehensive input. It has to be replenished frequently. Teachers need to be encouraged to experiment with new innovative techniques. In this regards teacher education colleges need to render a helping hand. In order to keep inservice teachers abreast with new strategies, to know about the problems they face, there needs to be regular updating. There are many agencies existing precisely to do this, may be, S.C.E.R.T., S.I.E., mathematics teachers association, teacher education colleges. Inservice programmes have to be need based, with regular follow-up and monitoring. Research based innovations and action research should be the order of the day for mathematics teachers. Incentives could be given to exemplary performance. Dissemination of ideas should be an essential component of inservice.

7.5.5 Textbook Making

Textbook making has to based on the criteria for textbook making. This may not be novel idea. The state of the textbooks speak otherwise. The carelessness in text-book making can be seen, for example in standard six mathematics text book. The clientele of the text-books are students. It should speak to them. Surveys are done by various agencies about existing textbooks. The questionnaire response analysis show that only about twenty five percent students were reading mathematics text-book and mainly to check the method. According to the teachers based on their interviews, the text-books were not useful to the students. Need for a work book was also opined. Presentation was not according to the age level of the learner and topics were not arranged sequentially. It was also felt that the language was not suitable for backward students. It is very surprising how the new text-books are made without taking into consideration the existing woes of teachers and students. Text-books-cum-work book could be also thought of for students. What happened to the innovative ideas given is not known. Textbooks for students must be in an lucid manner and self-explanatory. It should be accompanied with a workbook having graded problems.

There should be problems of varying difficulty levels, for the backward, the average and above average. Even if the student remains absent he/she should able to make up by oneself. There could be provision within the textbook, to check backwardness. Having a chapter on revision in the beginning of the text book which remains like a vestigial organ, does not serve the purpose. It has to be a part of the regular classwork, monitored by the teacher, followed by remedial programme. Teachers should be provided with reference books rather than textbooks. They should be also given the objectives of teaching the topics in an explicit manner. It would go a long way if guidelines regarding teaching of various topics is also provided. These could be modified by the teacher. There are teachers guides existing but they merely collect dust.

The questionnaire responses show word problems as a difficult topic and the interviews of mathematics teachers also show the need of simplified language and even use of vernacular language to explain. Hence, the language used in the text-book could be made more student-friendly. This was tried out in the remedial programme (refer 6.5.3). The explanation and the algorithm were made as simple as possible. What the students should look out for in a given problem and then what should be done. This was given for each topic. This helped the students in understanding the method better. Infact students began to coin the statements themselves.

7.5.6 Suggestions for Similar Studies

i. Research studies exclusively for locating the errors and finding their causes.

ii. Use of various strategies, technologies can be experimented in the remedial programme.

iii. Details about the personality characteristics of the case study subjects can be studied. Their potentials, interests could be explored. An attempt could be made to introduce mathematics through their interest areas, wherever possible.

iv. Diagnostic tests on specific areas could be constructed and detailed errors could be identified.

7.5.7 Suggestions for Further Research

Researches are being done in mathematics education without much focus. There is no serious thought given to the research gaps identified by the surveys. Whether the mathematics taught at various levels matches with the cognitive readiness of the learners needs to researched. Innovations based on paradigm shift, from drill methods to constructing one's own knowledge. Such ideas could be experimented upon. Mathematics education finds itself caught up with text-books, teachers, students. Involving parents in the education process is essential. Research on training parents to assist students could be also thought of. This will help parents to contribute towards their ward's education in a better way. Mathematics curriculum in the school vis-à-vis various professions, whether it serves the purpose can be another area of research. Mathematics essential of taking up any profession and that required for very specialized careers could be also bifurcated. This could be an area of research to truncate redundantness and backwardness in Mathematics. Students' thinking notations, representations in mathematics learning and relating it to mathematical language are some new areas of research.

Teaching mathematics through other subjects, studying belief system of the teachers, knowledge base of mathematics teachers, have not been addressed by many a researchers in India. Looking into the necessity of the various topics in mathematics curriculum and elimination of out-dated topics, world-wide practices in this regard, recommendations of commissions and committees, could be an interesting area of research.

The investigator, in the process of the study itself, found many more areas where research could be carried out.

- Developing teaching strategies for various concepts, with gradual advancement in higher classes can be researched upon.
- Investigating about the concept formation of concepts with maximum difficulty, what are the various sources of errors, how do errors concretise, would be useful.

- Longitudinal research studies about the students backward in mathematics will help in a big way to know the manner in which backwardness set in and how it can be avoided.
- Cross-sectional research studies about the teaching methods adopted for specific concepts can reveal reasons for lacunae in concept formation.
- Use of technology to enhance motivation among students backward in mathematics can be meaningful in the present technology-based world.
- The achievement of the objectives of mathematics have been seldom verified. This could be researched at various levels.