CHAPTER - 3

MODEL DEVELOPMENT

CHAPTER - III

3.1 INTRODUCTION

The theoretical and empirical models presently available for the prediction of flow rate and tubing head pressure for a surface choke system showed a large deviation while used for the prediction of choke performance in Gandhar field. This may be due to the fact that most of the models have been tested with air- water or air-kerosene systems. Most of the models have assumed polytropic expansion of gas phase and the change in specific volume has been calculated using energy balance equations. For a flow process involving changes in pressure and volume, it is preferable to consider the changes in enthalpy while deriving an expression for mass flux. In this chapter a theoretical model, developed on the basis of enthalpy balance, for multiphase flow through a choke is presented.

3.2 THEORETICAL CONSIDERATION

The flow of fluid through a choke is basically the flow of fluid through a straight hole nozzle. For a given upstream pressure of the choke, reduction in the downstream pressure below a particular value does not result in any increase in the mass flow rate of the fluid. This pressure is known as the critical pressure and the mass flow rate as critical mass flow rate. The ratio between the critical pressure and the upstream pressure is known as critical pressure ratio. The critical mass flow rate per unit cross sectional area of the choke is known as critical mass flux.

3.3 DERIVATION OF THEORETICAL MODEL

The derivation of any relationship for flow through a choke will involve an expression for equation of continuity in terms of fluid specific volume and velocity and another relating the gas liquid ratio of the fluid with pressure.

The following assumptions are made for the derivation of the multiphase choke equation

- **flow is steady and one dimensional**
- **flow of fluid is isentropic and frictionless**
- **the fluid consists of liquid dispersed in the form of droplets in a continuous gas phase**
- **the liquid phase is incompressible**
- **at a given point all phases are at same temperature and pressure**
- **potential head across the choke is neglected**

The fluid mass flux through a tube can be expressed by the following equation :

$$G = \frac{v}{V} \qquad \text{.....................(3.1)}$$

For isentropic flow

$$PV^k = \text{Constant} \qquad \text{.....................(3.2)}$$

For flow through choke

$$P_1 V_1^k = P_2 V_2^k \qquad \text{.....................(3.3)}$$

or

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{(1/k)} \qquad \text{.....................(3.4)}$$

Denoting the ratio of downstream to upstream pressure as F_p, equation (3.4) can be written as

$$V_2 = V_1 \left(\frac{1}{F_p}\right)^{1/k} \qquad \text{.....................(3.5)}$$

The change in enthalpy for isentropic flow is given as :

$$dh = V\ dp \qquad \text{.....................(3.6)}$$

Integrating equation (3.6) and substituting V_2 from equation (3.5)

$$h_1 - h_2 = V_1\ P_1 \left(\frac{k}{k-1}\right)\left(1 - F_p\right)^{(k-1)/k} \qquad \text{.....................(3.7)}$$

Further, under steady state conditions the change in enthalpy is related to the throat velocity by the following equation

$$v_2 = (2\ g_c\ (h_1 - h_2))^{0.5} \qquad \text{.....................(3.8)}$$

Combining equations (3.1), (3.7) and (3.8) the mass flux can be written as

$$G = \frac{2 g_c P_1 k}{V_1 (k-1)} F_p^{2/k} \left(1 - F_p\right)^{\frac{(k-1)}{k}} \qquad \text{.....................(3.9)}$$

At critical flow conditions,

$$\frac{dG}{dF_p} = 0 \qquad \text{.....................(3.10)}$$

Differentiating equation (3.9) and applying critical flow conditions, the critical pressure ratio becomes

$$F_p = \left(\frac{2}{k+1}\right)^{\frac{k}{(k-1)}} \qquad \text{.....................(3.11)}$$

From equations (3.9) and (3.11) and replacing F_p by F_p^* in equation 3.9 the critical mass flux can be expressed as :

$$G^* = \left(k\left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{k-1}\right)} g_c \left(\frac{p_1}{V_1}\right)\right)^{0.5} \qquad \text{.....................(3.12)}$$

The critical mass flow rate through a choke can be obtained by multiplying G* by throat area (A_c) and introducing the choke flow coefficient (C_d).

$$W^* = C_d A_c \left(k\left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{k-1}\right)} g_c \left(\frac{p_1}{V_1}\right)\right)^{0.5} \qquad \text{.....................(3.13)}$$

To adopt equation (3.13) for two phase flow, the appropriate value of k, and the specific volume of fluid at upstream pressure should be used in place of specific volume of gas.

The free gas to liquid ratio at a given temperature and pressure can be related to phase volumes as follows,

$$R(P,T) = \frac{V_1 - V_L}{V_L} \qquad \text{.....................(3.14)}$$

where, V_1 = voulme of mixture in ft^3.
V_L = volume of liquid in ft^3

Rearranging equation 3.14 the specific volume of fluid can be given as,

$$V_1 = V_L(1 + R(P,T)) \qquad \text{.....................(3.15)}$$

Assuming no slip at the throat of the choke, the specific volume of liquid can be written by the following equation suggested by Ashford [9]

$$V_L = \frac{B_o + F_{wo}}{\rho_o + \frac{\rho_g R_s}{5.615} + F_{wo}\rho_w} \qquad \text{.....................(3.16)}$$

where, B_o = formation voulmefactor.
F_{wo} = volume fraction of water to oil
ρ_o = density of oil in Lb/ft^3
ρ_w = density of water in Lb/ft^3

The constant 5.615 has been introduced for converting barrels into cubic foot since the solution gas oil ratio (R_s) is conventionally represented in cubic foot of gas per barrel of oil

The free gas liquid ratio at a given condition can also be written as,

$$R(P,T) = \frac{(R_p - R_s) P_{sc} T_1 Z_1}{5.615 P_1 T_{sc}} \qquad \text{.....................(3.17)}$$

Substituting the values of V_L and R(P,T) in equation (3.15)

$$V_1 = \frac{(B_o + F_{wo})}{\rho_o + \frac{\rho_s R_s}{5.615} + F_{wo}\rho_w} \left(1 + \frac{(R_s - R_p) P_{sc} T_1 Z_1}{5.615 T_{sc} P_1}\right) \qquad \text{.....................(3.18)}$$

Substituting the value of V_1 in equation (3.13), the critical mass flow rate through a choke is written as

$$W^* = C_d A_c \left[k\left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{k-1}\right)} g_c \left(\frac{P_1\left(\rho_o + \frac{\rho_s R_s}{5.615} + F_{wo}\rho_w\right)}{\left(B_o + F_{wo}\right)\left(1 + \frac{\left(R_s - R_p\right) P_{sc}\, T_1\, Z_1}{5.615\, T_{sc}\, P_1}\right)} \right) \right]^{0.5}$$

.....................(3.19)

However, the mass flow rate and the volume flow rate is related by the following equation suggested by Ashford [8]

$$\frac{W}{q_L} = \frac{\left(\rho_o + \frac{\rho_g R_s}{5.615} + F_{wo}\rho_w\right)}{\left(B_o + F_{wo}\right)\left(1 + \frac{\left(R_s - R_p\right) P_{sc}\, T_1\, Z_1}{5.615\, T_{sc}\, P_1}\right)}$$

....(3.20)

The volumetric flow rate in terms of measurable quantities as suggested by Ashford [8] can be written as,

$$q_L = q_o\left(B_o + F_{wo} + \frac{\left(R_p - R_s\right)}{5.615}\frac{P_{sc}\, T_1\, Z_1}{T_{sc}\, P_1}\right)$$

.....................(3.21)

Substituting equation (3.21) in equation (3.20) and rearranging to get oil flow rate (q_o)

$$W / q_o = \left[\frac{\rho_o + \frac{\rho_g R_s}{5.615} + F_{wo}\rho_w}{\left(B_o + F_{wo}\right)\left(1 + \frac{\left(R_s - R_p\right) P_{sc} T_1\, Z_1}{5.615\, T_{sc}\, P_1}\right)} \right] \times \left[B_o + F_{wo} + \frac{\left(R_p - R_s\right) P_{sc}\, T_1\, Z_1}{5.615\, T_{sc}\, P_1} \right]$$

............................(3.21a)

$$q_o = \frac{W\left(B_o + F_{wo}\right)\left(1 + \frac{R_p - R_s}{5.615} \; \frac{P_{sc}\, T_1\, Z_1}{T_{sc}\, P_1}\right)}{\left(\rho_o + \frac{\rho_g\, R_s}{5.615} + F_{wo}\rho_w\right)\left(B_o + F_{wo} + \frac{\left(R_p - R_s\right) P_{sc}\, T_1\, Z_1}{5.615\, T_{sc}\, P_1}\right)} \qquad \text{..................(3.22)}$$

Replacing W by W* in the equation (3.22) , the critical oil flow rate through the choke can be written as,

$$q_o^* = \frac{C_d\, A_c \left[k\left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{k-1}\right)} g_c P_1 \left(\frac{\rho_o + \frac{\rho_g R_s}{5.615} + F_{wo}\rho_w}{\left(B_o + F_{wo}\right)\left(1 + \frac{\left(R_s - R_p\right) P_{sc}\, T_1\, Z_1}{5.615 T_{sc}\, P_1}\right)} \right) \right]^{0.5}}{\left(\rho_o + \frac{\rho_g R_s}{5.615} + F_{wo}\rho_w\right)\left(B_o + F_{wo} + \frac{\left(R_p - R_s\right) P_{sc}\, T_1\, Z_1}{5.615 T_{sc}\, P_1}\right)} \left[\left(B_o + F_{wo}\right)\left(1 + \frac{\left(R_p - R_s\right)}{5.615} \frac{P_{sc} T_1 Z_1}{T_{sc} P_1}\right)\right]$$

....(3.23)

Rearranging and substituting tubing head pressure P_{tf} for P_1 , we get

$$q_o^* = \frac{C_d\, A_c \left[k\left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{k-1}\right)} g_c\, P_{tf}\, A\, B\left(1 + C\right) \right]^{0.5}}{A\left(B + C\right)} \qquad \text{.....................(3.24)}$$

where $$A = \rho_o + \frac{\rho_g\, R_s}{5.615} + F_{wo}\rho_w \qquad \text{.....................(3.25)}$$

$$B = B_o + F_{wo} \qquad \text{.....................(3.26)}$$

$$C = \frac{\left(R_p - R_s\right)}{5.615} + \frac{P_{sc}}{T_{sc}} \frac{T_1\, Z_1}{P_{tf}} \qquad \text{....................(3.27)}$$

P_{tf} = tubing head pressure in lb/ft^2

R_s & R_p = Gas liquid ratios, scf/bbl

A_c = Area of choke in ft^2

q_o^* = Oil flow rate , ft^3/sec

Introducing 86400/5.615 to convert flow rate from ft^3/sec to Bbls/day and 144 to convert P_{tf} to lb/ft^2 from psi, equation (3.24) becomes

$$Q = \frac{86400\ C_d\ A_c \left[k\left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{k-1}\right)} g_c\ 144 P_{tf}\ A\, B\,(1+C) \right]^{0.5}}{5.615 A\,(B+C)} \quad \text{.....................(3.28)}$$

$$Q = (184648.26)\frac{C_d\ A_c \left[k\left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{k-1}\right)} g_c\ P_{tf}\ A\, B\,(1+C) \right]^{0.5}}{A\,(B+C)} \quad \text{.................(3.29)}$$

and equation (3.27 becomes)

$$C = \frac{(R_p - R_s)}{5.615} + \frac{P_{sc}}{T_{sc}}\frac{T_1\ Z_1}{144 P_{tf}} \quad \text{...........................(3.30)}$$

Where Q is in barrels per day and P_{tf} , tubing head pressure, is in psi.