CHAPTER-IV

NEGATIVE SEQUENCE & ZERO SEQUENCE COMPENSATOR

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4.1 INTRODUCTION

This chapter presents a novel and online method, which is also a novel contribution of this research work for detecting negative & zero sequence voltages and its compensation. It is based on Instantaneous Active Reactive Power (IARP) theory. Using this concept, negative sequence and zero sequence components are computed online. The α-β transformation is real unlike the complex transformation matrix in case of symmetrical components. So implementation of α-β analysis on line is simple. This theory is verified through simulation and experimentation results.

This chapter describes the effect of negative sequence & zero sequence on electric distribution systems mainly induction motor & energy saved due to compensation.

Modern power systems are three-phase systems that can be balanced or unbalanced and will have mutual coupling between the phases. In many instances, the analysis of these systems is performed using what is known as "per-phase analysis." In this chapter, more generally applicable approach to system analysis known as "symmetrical components" will be described. The concept of symmetrical components was first proposed for power system analysis by C.L. Fortescue in a classic paper devoted to consideration of the general N-phase case (1918). The effect of negative & zero sequence components can be reduced using negative & zero sequence compensator. New method was introduced to detect negative sequence along with zero sequence components. This method detects the negative sequence components instantaneously and on line. After detecting the negative & zero sequence these signal are converted into three reference signals which are given to voltage source inverter to nullify the effect of negative and zero sequence voltage components in the distribution network.

4.2 GENERATION OF NEGATIVE SEQUENCE & ZERO SEQUENCE

4.2.1 VOLTAGE IMBALANCE

Voltage imbalance (also called voltage unbalance) is sometimes defined as the maximum deviation from the average of the three-phase voltages or currents, divided by the average of the three-phase voltages or currents, expressed in percent. Imbalance is more rigorously defined in the standards [A17]-[A20] using symmetrical components. The ratio of either the negative- or zero sequence components to the positive-sequence component can be used to specify the percent unbalance. The most recent standards11 specify that the negative-sequence method be used. Figure 4.2.1-1 shows an example of negative sequence component to the positive-sequence component in current for a 24 hours trend of imbalance on an industrial feeder.

The primary source of voltage unbalances is single-phase loads on a three-phase circuit. Voltage unbalance can also be the result of blown fuses in one phase of a three-phase capacitor bank. Severe voltage unbalance (greater than 5 percent) can result from single-phasing conditions.

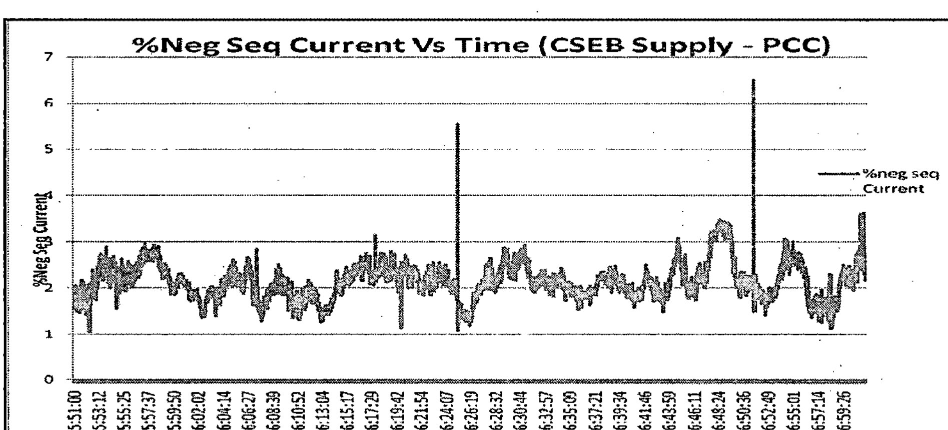


Figure 4.2.1-1: % Negative Sequence Current for an industrial feeder

4.2.2 HARMONIC PHASE SEQUENCES

Power engineers have traditionally used symmetrical components to help describe three-phase system behavior. The three-phase system is transformed into three single-phase systems that are much simpler to analyze. The method of symmetrical components can be employed for analysis of the system's response to harmonic currents provided care is taken not to violate the fundamental assumptions of the method. The method allows any unbalanced set of phase currents (or voltages) to be transformed into three balanced sets. The *positive-sequence* set contains three sinusoids displaced 120° from each other, with the normal A-B-C phase rotation (e.g., 0°, -120°, 120°). The sinusoids of the *negative-sequence* set are also displaced 120°, but have opposite phase rotation (A-C-B, e.g., 0°, 120°, -120°). The sinusoids of the *zero sequence* are in phase with each other (e.g., 0°, 0°, 0°).

In a perfect balanced three-phase system, the harmonic phase sequence can be determined by multiplying the harmonic number h with the normal positive-sequence phase rotation. For example, for the second harmonic, h=2, hence 2 is multiplied with normal phase rotation i.e. 2 x (0, -120°, 120°) or (0°, 120°, -120°), which is the negative sequence. For the third harmonic, h = 3, hence 3 is multiplied with normal phase rotation i.e. 3 x (0°, -120°, 120°) or (0°, 0°, 0°), which is the zero sequence. Phase sequences for all other harmonic orders can be determined in the same fashion. Since a symmetrical distorted waveform in power systems contains only odd-harmonic components, only odd-harmonic phase sequence rotations are summarized here:

- Harmonics of order h = 1, 7, 13, are generally positive sequence.
- Harmonics of order h = 5, 11, 17, are generally negative sequence.

■ Triplens (h = 3, 9, 15,...) are generally zero sequence. Impacts of sequence harmonics on various power system components are detailed in section 1.9

4.2.3 TRIPLEN HARMONICS

As previously mentioned, triplen harmonics are the odd multiples of the third harmonic (h = 3, 9, 15, 21,...). They deserve special consideration because the system response is often considerably different for triplens than for the rest of the harmonics. Triplens become an important issue for grounded-wye systems with current flowing on the neutral. Two typical problems are overloading the neutral and telephone interference. One also hears occasionally of devices that mis-operate because the line-to-neutral voltage is badly distorted by the triplen harmonic voltage drop in the neutral conductor. For the system with perfectly balanced single-phase loads illustrated in Figure 4.2.3-1, an assumption is made that fundamental and third-harmonic components are present. Summing the currents at node N, the fundamental current components in the neutral are found to be zero, but the third-harmonic components are 3 times those of the phase currents because they naturally coincide in phase and time. Transformer winding connections have a significant impact on the flow of triplen harmonic currents from single-phase nonlinear loads. Two cases are shown in Figure 4.2.3-2. In the wye-delta transformer (top), the triplen harmonic currents are shown entering the wye side. Since they are in phase, they add in the neutral. The delta winding provides ampere-turn balance so that they can flow, but they remain trapped in the delta and do not show up in the line currents on the delta side. When the currents are balanced, the triplen harmonic currents behave exactly as zero-sequence currents, which is precisely what they are. This type of transformer connection is the most common employed in utility distribution substations with the delta winding connected to the transmission feed. Using grounded-wye windings on both sides of the transformer (bottom) allows balanced triplens to flow from the low-voltage system to the high-voltage system unimpeded. They will be present in equal proportion on both sides. Many loads in the United States are served in this fashion. Some important implications of this related to power quality analysis are

1. Transformers, particularly the neutral connections, are susceptible to overheating when serving single-phase loads on the wye side that have high third-harmonic content.

2. Measuring the current on the delta side of a transformer will not show the triplens and, therefore, not give a true idea of the heating the transformer is being subjected to.

3. The flow of triplen harmonic currents can be interrupted by the appropriate isolation transformer connection. Removing the neutral connection in one or both wye windings blocks the flow of triplen harmonic current.

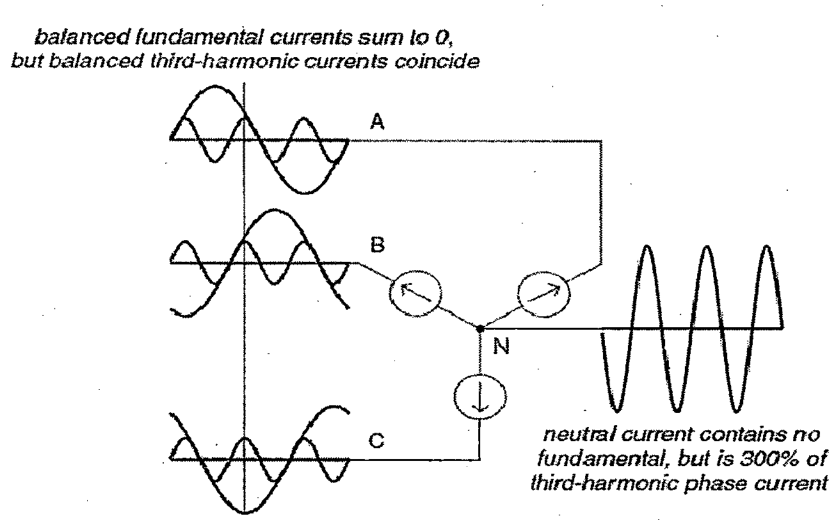


Figure 4.2.3-1: High neutral currents in circuits serving single-phase nonlinear loads.

There is no place for ampere-turn balance. Likewise, a delta winding blocks the flow from the line. One should note that three-legged core transformers behave as if they have a "phantom" delta tertiary winding. Therefore, a wye-wye connection with only one neutral point grounded will still be able to conduct the triplen harmonics from that side. These rules about triplen harmonic current flow in transformers apply only to *balanced* loading conditions. When the phases are not balanced, currents of normal triplen harmonic frequencies may very well show up where they are not expected. The normal mode for triplen harmonics is to be zero sequence. During imbalances, triplen harmonics may have positive or negative sequence components, too. One notable case of this is a three-phase arc furnace. The furnace is nearly always fed by a delta-delta connected transformer to block the flow of the zero sequence currents as shown in Figure 4.2.3-3. Thinking that third harmonics are synonymous with zero sequence, many engineers are surprised to find substantial third-harmonic current present in large magnitudes in the line current.

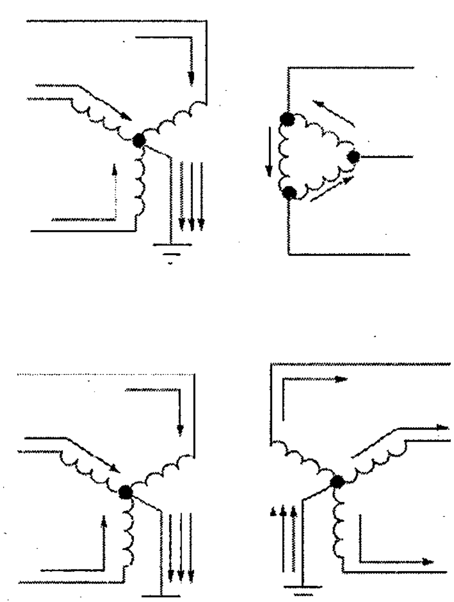


Figure 4.2.3-2: Flow of third-harmonic current in three-phase transformers.

However, during scrap meltdown, the furnace will frequently operate in an unbalanced mode with only two electrodes carrying current. Large third-harmonic currents can then freely circulate in these two phases just as in a single-phase circuit. However, they are not zero-sequence currents. The third-harmonic currents have equal amounts of positive- and negative-sequence currents. But to the extent that the system is *mostly* balanced, triplens mostly behave in the manner described.

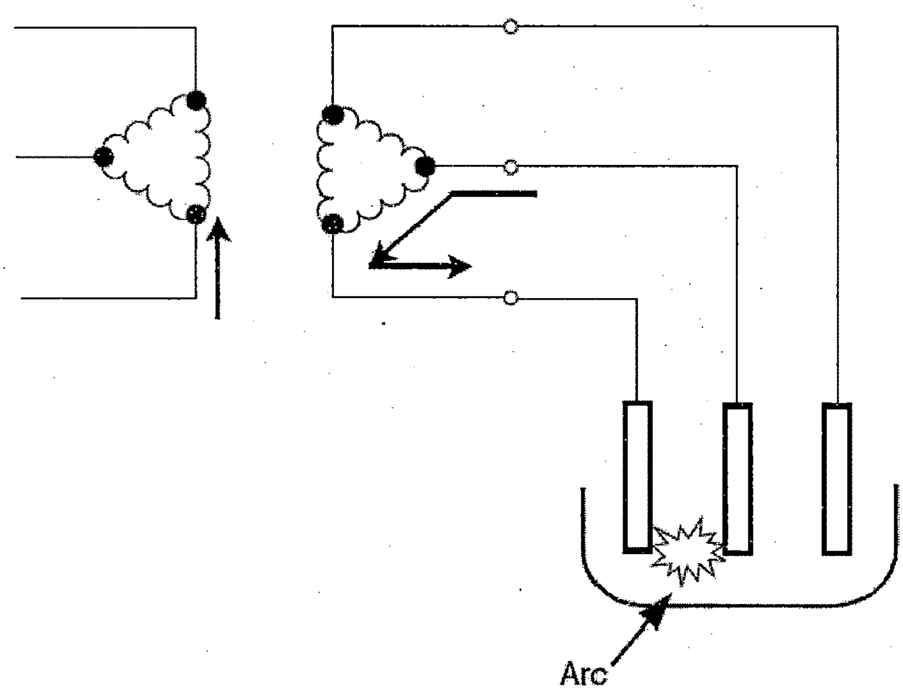


Figure 4.2.3-3: Arc furnace operation in an unbalanced mode allows triplen harmonics to reach the power system despite a delta connected transformer.

4.3 ANALYSIS OF VOLTAGE UNBALANCE PROBLEM

Induction motors have been used in the past mainly in applications requiring a constant speed because conventional methods of their speed control have either been expensive or highly inefficient. Variable speed applications have been dominated by DC drives. Availability of thyristors, power transistors, IGBT and GTO has allowed the development of variable speed induction motor drives. The main drawback of de motors is the presence of commutator and brushes, which require frequent maintenance and make them unsuitable for explosive and dirty environments. On the other hand, induction motors, particularly squirrel-cage are rugged, cheaper, lighter, smaller, more efficient, require lower maintenance and can operate in dirty and explosive environments. Although variable speed induction motor drives are generally expensive than dc drives, they are used in a number of applications such as fans, blowers, mill run-out tables, cranes, conveyers, traction etc. because of the advantages of induction motors. Other dominant applications are underground and underwater installations, and explosive and dirty environments.

Use of 3-ϕ induction motor (IM) in various industries is almost 80% of their total drive requirements. Performance of these induction motors is affected by the unbalance in bus voltage caused by negative sequence currents, which heat up the rotor and generate Braking Torque. Due to the very low value of negative sequence impedance offered by the induction motor as compared to its positive sequence impedance, a very small negative sequence voltage component in the input may give rise to considerable negative sequence current causing significance deterioration in the performance of induction motor.

Therefore even low unbalance factor (ratio of negative sequence to positive sequence) in input voltage has to be carefully looked into. Voltage unbalances occur quite often in distribution systems. Figure 4.3-1 shows the schematic of this conventional induction motor drive and the effect of input voltage unbalance.

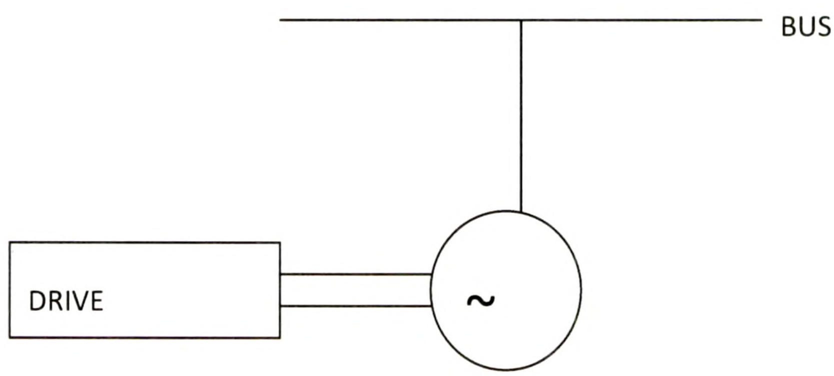


Figure 4.3-1(a): Schematic of conventional induction motor drive

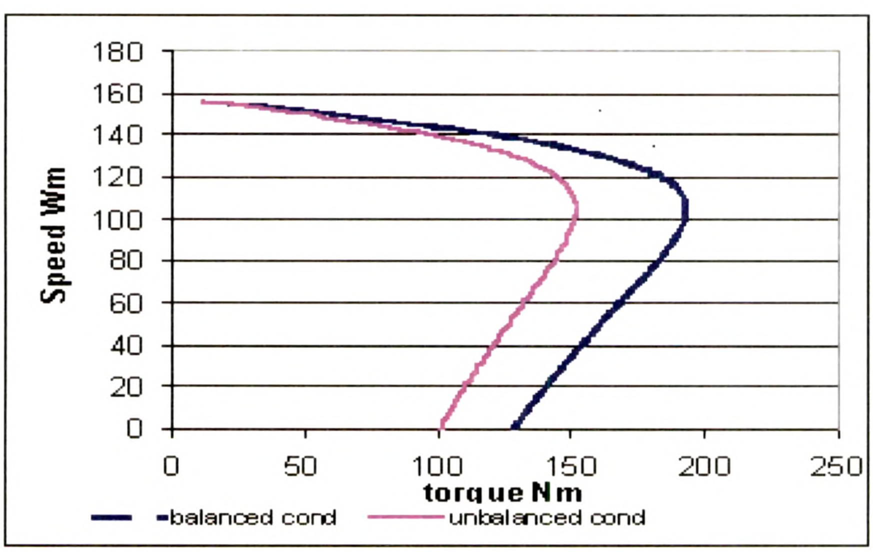


Figure 4.3-1(b): Torque v/s speed characteristic with balanced and unbalanced condition

4.4 SYMMETRICAL COMPONENTS FOR POWER SYSTEMS ANALYSIS

The case for per-phase analysis can be made by considering the simple three-phase system illustrated in Figure 4.4-1. The steady-state circuit response can be obtained by solution of the three loop equations presented in Equation (4.4-1a) through (4.4-1c). By solving these loop equations for the three line currents, Equation (4.4-2a) through (4.4-2c) are obtained. Now, if completely balanced source operation (the impedances are defined to be balanced) is assumed, then the line currents will also form a balanced three-phase set. Hence, their sum, and the neutral current, will be zero. As a result, the line current solutions are as presented in Equation (4.4-3a) through (4.4-3c).

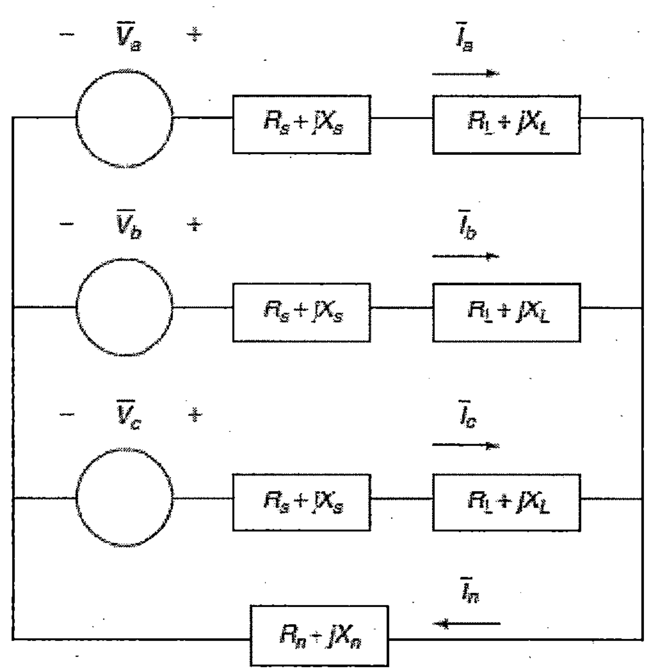


Figure 4.4-1: A simple three-phase system

$$\bar{V}_a - \bar{I}_a (R_S + jX_S) - \bar{I}_a (R_L + jX_L) - \bar{I}_n (R_n + jX_n) = 0 \tag{4.4-1a}$$

$$\bar{V}_b - \bar{I}_b (R_S + jX_S) - \bar{I}_b (R_L + jX_L) - \bar{I}_n (R_n + jX_n) = 0 \tag{4.4-1b}$$

$$\bar{V}_c - \bar{I}_c (R_S + jX_S) - \bar{I}_c (R_L + jX_L) - \bar{I}_n (R_n + jX_n) = 0 \tag{4.4-1c}$$

$$\bar{I}_a = \frac{\bar{V}_a - \bar{I}_n (R_n + jX_n)}{(R_s + R_L) + j(X_s + X_L)} \tag{4.4-2a}$$

$$\bar{I}_b = \frac{\bar{V}_b - \bar{I}_n (R_n + jX_n)}{(R_s + R_L) + j(X_s + X_L)} \tag{4.4-2b}$$

$$\bar{I}_c = \frac{\bar{V}_c - \bar{I}_n (R_n + jX_n)}{(R_s + R_L) + j(X_s + X_L)} \tag{4.4-2c}$$

$$\bar{I}_a = \frac{\bar{V}_a}{(R_s + R_L) + j(X_s + X_L)} \tag{4.4-3a}$$

$$\bar{I}_b = \frac{\bar{V}_b}{(R_s + R_L) + j(X_s + X_L)} \tag{4.4-3b}$$

$$\bar{I}_c = \frac{\bar{V}_c}{(R_s + R_L) + j(X_s + X_L)} \tag{4.4-3c}$$

The circuit synthesis of Equation (4.4-3a) through (4.4-3c) is illustrated in Figure 4.4-2. Particular notice should be taken of the fact the response of each phase is independent of the other two phases. Thus, only one phase need be solved, and three-phase symmetry may be applied to determine the solutions for the other phases.

This solution technique is the per-phase analysis method. If one considers the introduction of an unbalanced source or mutual coupling between the phases in Figure 4.4-1, then per-phase analysis will not result in three decoupled networks as shown in Figure 4.4-2. In fact, in the general sense, no immediate circuit reduction is available without some form of reference frame transformation. The symmetrical component transformation represents such a transformation, which will enable decoupled analysis in the general case and single-phase analysis in the balanced case.

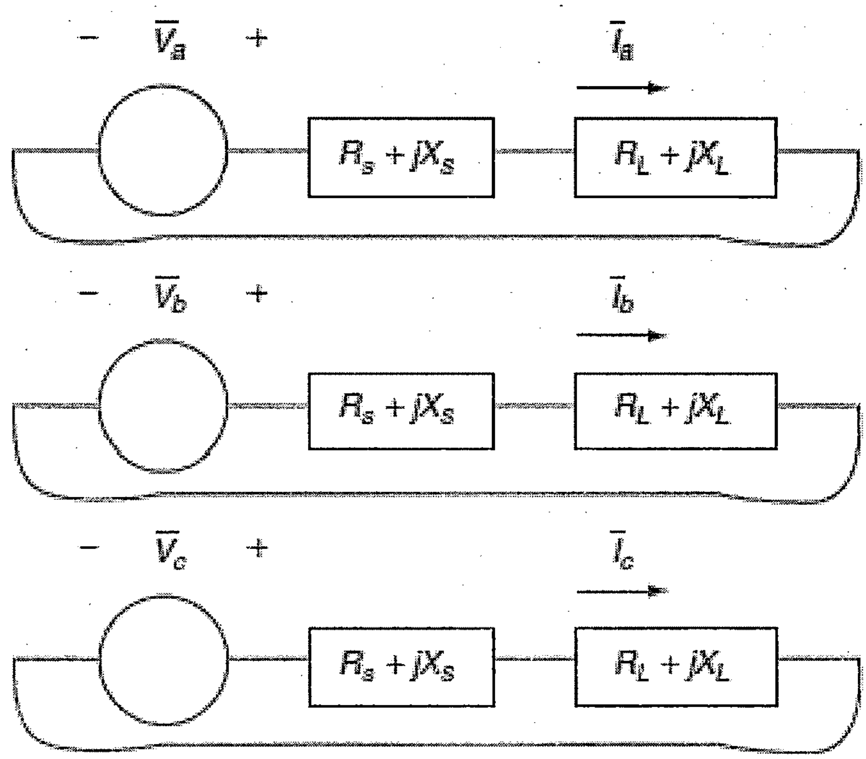


Figure 4.4-2: Decoupled phases of the three phase system

4.4.1 FUNDAMENTAL DEFINITIONS

4.4.1.1 VOLTAGE AND CURRENT TRANSFORMATION

To develop the symmetrical components, [J11]-[J14] let us first consider an arbitrary (no assumptions on balance) three-phase set of voltages as defined in Equation (4.4.1.1-1a) through (4.4.1.1-1c). Note that it could just as easily be considering current for the purposes at hand, but voltage was selected arbitrarily. Each voltage is defined by a magnitude and phase angle. Hence, six degrees of freedom is available to fully define this arbitrary voltage set.

$$\bar{V}_a = V_a \angle \theta_a \quad (4.4.1.1\text{-}1a)$$

$$\bar{V}_b = V_b \angle \theta_b \quad (4.4.1.1\text{-}1b)$$

$$\bar{V}_c = V_c \angle \theta_c \quad (4.4.1.1\text{-}1c)$$

Each of the three given voltages can be represented as the sum of three components as illustrated in Equation (4.4.1.1-2a) through (4.4.1.1-2c). For now, these components are considered to be completely arbitrary except for their sum. The 0, 1 and 2 subscripts are used to denote the zero, positive and negative sequence components of each phase voltage, respectively. Examination of Equation (4.4.1.1-2a-c) reveals that 6 degrees of freedom exist on the left-hand side of the equations while 18 degrees of freedom exist on the right-hand side. Therefore, for the relationship between the voltages in the a-b-c frame of reference and the voltages in the 0-1-2 frame of reference to be unique, the right-hand side of Equation (4.4.1.1-2) must be considered.

$$\bar{V}_a = \bar{V}_{a0} + \bar{V}_{a1} + \bar{V}_{a2} \quad (4.4.1.1\text{-}2a)$$

$$\bar{V}_b = \bar{V}_{b0} + \bar{V}_{b1} + \bar{V}_{b2} \quad (4.4.1.1\text{-}2b)$$

$$\bar{V}_c = \bar{V}_{c0} + \bar{V}_{c1} + \bar{V}_{c2} \quad (4.4.1.1\text{-}2c)$$

It is begin by forcing the a_0, b_0, and c_0 voltages to have equal magnitude and phase. This is defined in Equation (2.6). The zero sequence components of each phase voltage are all defined by a single magnitude and a single phase angle. Hence, the zero sequence components have been reduced from 6 degrees of freedom to 2.

$$\bar{V}_{a0} = \bar{V}_{b0} = \bar{V}_{c0} \equiv \bar{V}_0 = V_0\angle\theta_0 \quad (4.4.1.1\text{-}3)$$

Second, the a_1, b_1, and c_1 voltages are forced to form a balanced three-phase set with positive phase sequence. This is mathematically defined in Equation (4.4.1.1-4a-c). This action reduces the degrees of freedom provided by the positive sequence components from 6 to 2.

$$\bar{V}_{a1} = \bar{V}_1 = V_1\angle\theta_1 \quad (4.4.1.1\text{-}4a)$$

$$\bar{V}_{b1} = V_1\angle(\theta_1 - 120^0) = \bar{V}_1.1\angle-120^0 \quad (4.4.1.1\text{-}4b)$$

$$\bar{V}_{c1} = V_1\angle(\theta_1 + 120^0) = \bar{V}_1.1\angle+120^0 \quad (4.4.1.1\text{-}4c)$$

And finally, the a_2, b_2, and c_2 voltages are forced to form a balanced three-phase set with negative phase sequence. This is mathematically defined in Equation (4.4.1.1-5a-c). As in the case of the positive sequence components, the negative sequence components have been reduced from 6 to 2 degrees of freedom.

$$\bar{V}_{a2} = \bar{V}_2 = V_2\angle\theta_2 \quad (4.4.1.1\text{-}5a)$$

$$\bar{V}_{b2} = V_2\angle(\theta_2 + 120^0) = \bar{V}_2.1\angle+120^0 \quad (4.4.1.1\text{-}5b)$$

$$\bar{V}_{c2} = V_2\angle(\theta_2 - 120^0) = \bar{V}_2.1\angle-120^0 \quad (4.4.1.1\text{-}5c)$$

Now, the right and left hand sides of Equation (4.4.1.1-2a) through (4.4.1.1-2c) each have 6 degrees of freedom. Thus, the relationship between the symmetrical component voltages and the original phase voltages is unique. The final relationship is presented in Equation (4.4.1.1-6a) through (4.4.1.1-6c). Note that the constant "a" has been defined as indicated in Equation (4.4.1.1-7).

$$\bar{V}_a = \bar{V}_0 + \bar{V}_1 + \bar{V}_2 \qquad (4.4.1.1\text{-}6a)$$

$$\bar{V}_b = \bar{V}_0 + \bar{a}^2\bar{V}_1 + \bar{a}\bar{V}_2 \qquad (4.4.1.1\text{-}6b)$$

$$\bar{V}_c = \bar{V}_0 + \bar{a}\bar{V}_1 + \bar{a}^2\bar{V}_2 \qquad (4.4.1.1\text{-}6c)$$

$$\bar{a} = 1\angle 120^0 \qquad (4.4.1.1\text{-}7)$$

Equation (4.4.1.1-6) is more easily written in matrix form, as indicated in Equation (4.4.1.1-8) in both expanded and compact form. In Equation (4.4.1.1-8), the [T] matrix is constant, and the inverse exists. Thus, the inverse transformation can be defined as indicated in Equation (4.4.1.1-9). The over tilde (˜) indicates a vector of complex numbers.

$$\begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix} \begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

$$\tilde{V}_{abc} = [\bar{T}]\,\tilde{V}_{012} \qquad (4.4.1.1\text{-}8)$$

$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix}$$

$$\tilde{V}_{012} = [\bar{T}]^{-1} \tilde{V}_{abc} \qquad (4.4.1.1\text{-}9)$$

Equations (4.4.1.1-10) and (4.4.1.1-11) define an identical transformation and inverse transformation for current.

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix} \begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix}$$

$$\tilde{I}_{abc} = [\bar{T}]\, \tilde{I}_{012} \qquad (4.4.1.1\text{-}10)$$

$$\begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix}$$

$$\tilde{I}_{012} = [\bar{T}]^{-1} \tilde{I}_{abc} \qquad (4.4.1.1\text{-}11)$$

4.4.1.2 IMPEDANCE TRANSFORMATION

In order to assess the impact of the symmetrical component transformation on systems impedances, Figure 4.4.1.2-1 was considered. Note that the balanced case has been assumed. Kirchhoff's Voltage Law for the circuit dictates equations Equation (4.4.1.2-1a -c), which are written in matrix form in Equation (4.4.1.2-2) and even more simply in Equation (4.4.1.2-3).

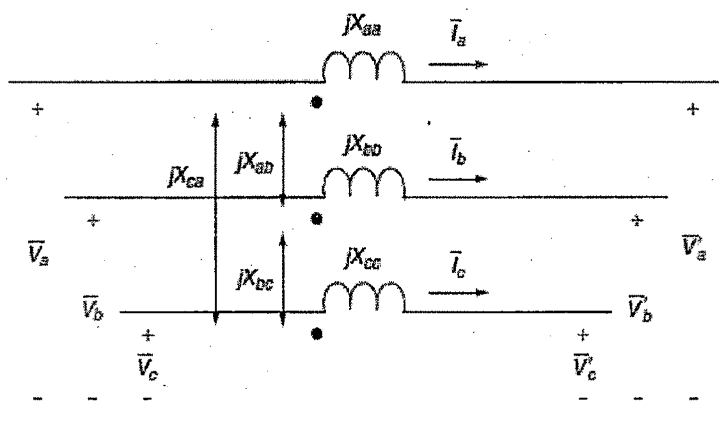


Figure 4.4.1.2-1: Mutually Coupled Series Impedances

$$\bar{V}_a - \bar{V}_a' = jX_{aa}\bar{I}_a + jX_{ab}\bar{I}_b + jX_{ca}\bar{I}_c \qquad (4.4.1.2\text{-}1a)$$

$$\bar{V}_b - \bar{V}_b' = jX_{ab}\bar{I}_a + jX_{bb}\bar{I}_b + jX_{bc}\bar{I}_c \qquad (4.4.1.2\text{-}1b)$$

$$\bar{V}_c - \bar{V}_c' = jX_{ca}\bar{I}_a + jX_{bc}\bar{I}_b + jX_{cc}\bar{I}_c \qquad (4.4.1.2\text{-}1c)$$

$$\begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} - \begin{bmatrix} \bar{V}_a' \\ \bar{V}_b' \\ \bar{V}_c' \end{bmatrix} = j \begin{bmatrix} X_{aa} & X_{ab} & X_{ca} \\ X_{ab} & X_{bb} & X_{bc} \\ X_{ca} & X_{bc} & X_{cc} \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} \qquad (4.4.1.2\text{-}2)$$

$$\bar{V}_{abc} - \bar{V}_{abc}' = [\bar{Z}_{abc}]\,\bar{I}_{abc} \qquad (4.4.1.2\text{-}3)$$

Multiplying both sides of Equation (4.4.1.2-3) by $[T]^{-1}$ yields Equation (4.4.1.2-4). Then, substituting Equation (4.4.1.1-9) and (4.4.1.1-10) into the result leads to the sequence equation presented in Equation (4.4.1.2-5). The equation is written strictly in the 012 frame reference in Equation (4.4.1.2-6) where the sequence impedance matrix is defined in Equation (4.4.1.2-7).

$$[\bar{T}]^{-1}\tilde{V}_{abc} - [\bar{T}]^{-1}\tilde{V}'_{abc} = [\bar{T}]^{-1}[\bar{Z}_{abc}]\,\tilde{I}_{abc} \tag{4.4.1.2-4}$$

$$\tilde{V}_{012} - \tilde{V}'_{012} = [\bar{T}]^{-1}[\bar{Z}_{abc}]\,[\bar{T}]\,\tilde{I}_{012} \tag{4.4.1.2-5}$$

$$\tilde{V}_{012} - \tilde{V}'_{012} = [\bar{Z}_{012}]\,\tilde{I}_{012} \tag{4.4.1.2-6}$$

$$[\bar{Z}_{012}] = [T]^{-1}\,[\bar{Z}_{abc}]\,[\bar{T}] = \begin{bmatrix} \bar{Z}_{00} & \bar{Z}_{01} & \bar{Z}_{02} \\ \bar{Z}_{10} & \bar{Z}_{11} & \bar{Z}_{12} \\ \bar{Z}_{20} & \bar{Z}_{21} & \bar{Z}_{22} \end{bmatrix} \tag{4.4.1.2-7}$$

4.4.1.3 POWER CALCULATIONS

The impact of the symmetrical components on the computation of complex power can be easily derived from the basic definition. Consider the source illustrated in Figure 4.4.1.3-1. The three-phase complex power supplied by the source is defined in Equation (4.4.1.3-1). The algebraic manipulation to Equation (4.4.1.3-1) is presented, and the result in the sequence domain is presented in Equation (4.4.1.3-2) in matrix form and in Equation (4.4.1.3-3) in scalar form.

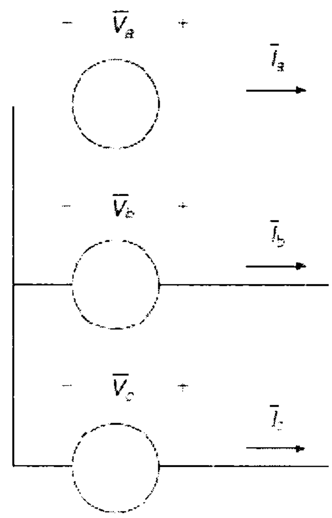


Figure 4.4.1.3-1: Three-Phase Wye Connected Source

$$S_{3\emptyset} = \bar{V}_a \bar{I}_a^* + \bar{V}_b \bar{I}_b^* + \bar{V}_c \bar{I}_c^* = \tilde{V}_{abc}^T \tilde{I}_{abc}^* \qquad (4.4.1.3\text{-}1)$$

$$S_{3\emptyset} = \tilde{V}_{abc}^T \tilde{I}_{abc}^* = \{[\bar{T}]\,\tilde{V}_{012}\}^T \{[\bar{T}]\,\tilde{I}_{012}\}^*$$

$$= \tilde{V}_{012}^T [\bar{T}]^T [\bar{T}]^* \tilde{I}_{012}^*$$

$$[\bar{T}]^T [\bar{T}]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{S}_{3\emptyset} = 3\, \bar{V}_{012}^{T}\, \bar{I}_{012}^{*} \tag{4.4.1.3-2}$$

$$\bar{S}_{3\emptyset} = 3\, \{\bar{V}_0\, \bar{I}_0^{*} + \bar{V}_1\, \bar{I}_1^{*} + \bar{V}_2\, \bar{I}_2^{*}\} \tag{4.4.1.3-3}$$

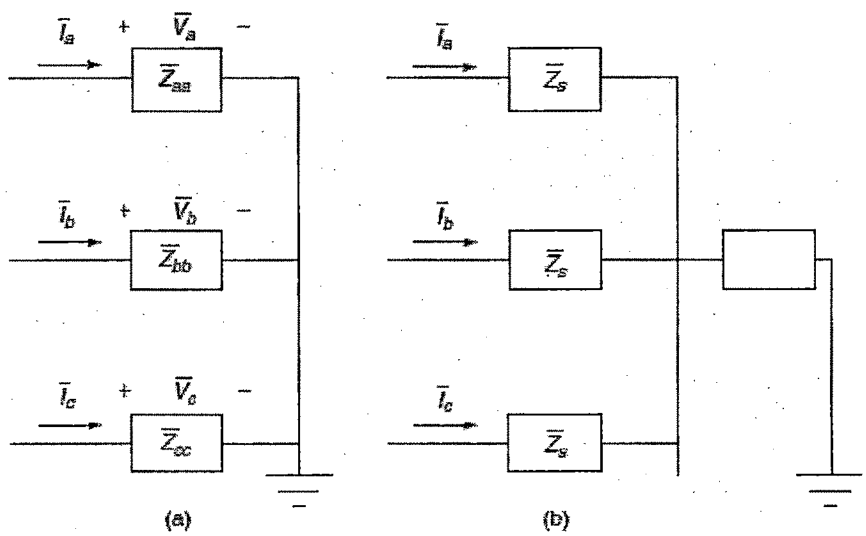


Figure 4.4.1.3-2: Three-Phase Impedance Load Model

Note that the nature of the symmetrical component transformation is not one of power invariance, as indicated by the multiplicative factor of 3 in Equation (4.4.1.3-3). However, this will prove useful in the analysis of balanced systems, which will be seen later. Power invariant transformations do exist as minor variations of the one defined herein. However, they are not typically employed, although the results are just as mathematically sound.

4.4.2 SUMMARY OF THE SYMMETRICAL COMPONENTS IN THE GENERAL THREE-PHASE CASE

The general symmetrical component transformation process has been defined in this section. Table 4.4.2-1 is a short form reference for the utilization of these procedures in the general case (i.e., no assumption of balanced conditions).

Table 4.4.2-1 Summary of the Symmetrical Components in the General Case

Quantity	Transformation Equations: abc ⇒ 012	Transformation Equations: 012 ⇒ abc
Voltage	$\begin{bmatrix}\overline{V}_0\\\overline{V}_1\\\overline{V}_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & \overline{a} & \overline{a}^2\\1 & \overline{a}^2 & \overline{a}\end{bmatrix}\begin{bmatrix}\overline{V}_a\\\overline{V}_b\\\overline{V}_c\end{bmatrix}$	$\begin{bmatrix}\overline{V}_a\\\overline{V}_b\\\overline{V}_c\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\1 & \overline{a}^2 & \overline{a}\\1 & \overline{a} & \overline{a}^2\end{bmatrix}\begin{bmatrix}\overline{V}_0\\\overline{V}_1\\\overline{V}_2\end{bmatrix}$
	$\tilde{V}_{012} = \left[\overline{T}\right]^{-1}\tilde{V}_{abc}$	$\tilde{V}_{abc} = \left[\overline{T}\right]\tilde{V}_{012}$
Current	$\begin{bmatrix}\overline{I}_0\\\overline{I}_1\\\overline{I}_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & \overline{a} & \overline{a}^2\\1 & \overline{a}^2 & \overline{a}\end{bmatrix}\begin{bmatrix}\overline{I}_a\\\overline{I}_b\\\overline{I}_c\end{bmatrix}$	$\begin{bmatrix}\overline{I}_a\\\overline{I}_b\\\overline{I}_c\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\1 & \overline{a}^2 & \overline{a}\\1 & \overline{a} & \overline{a}^2\end{bmatrix}\begin{bmatrix}\overline{I}_0\\\overline{I}_1\\\overline{I}_2\end{bmatrix}$
	$\overline{I}_{012} = \left[\overline{T}\right]^{-1}\overline{I}_{abc}$	$\overline{I}_{abc} = \left[\overline{T}\right]\overline{I}_{012}$
Impedance	$\left[\overline{Z}_{012}\right] = \left[\overline{T}\right]^{-1}\left[\overline{Z}_{abc}\right]\left[\overline{T}\right]$	
Power	$\overline{S}_{3\phi} = \overline{V}_a\overline{I}_a^* + \overline{V}_b\overline{I}_b^* + \overline{V}_c\overline{I}_c^* = \tilde{V}_{abc}^T\tilde{I}_{abc}^*$	
	$\overline{S}_{3\phi} = 3\left\{\overline{V}_0\overline{I}_0^* + \overline{V}_2\overline{I}_2^* + \overline{V}_3\overline{I}_3^*\right\} = 3\tilde{V}_{012}^T\tilde{I}_{012}^*$	

Application of these relationships defined in Table 4.4.2-1 will enable the power system analyst to draw the zero, positive and negative sequence networks for the system under study. These networks can then be analyzed in the 0-1-2 reference frame, and the results can be easily transformed back into the a-b-c reference frame.

Table 4.4.2-2 Summary of the Symmetrical Components in the Balanced Case

	Transformation Equations	
Quantity	abc ⇒ 012	012 ⇒ abc
Voltage	Positive Phase Sequence: $\begin{bmatrix}\overline{V}_0\\\overline{V}_1\\\overline{V}_2\end{bmatrix}=[\overline{T}]^{-1}\begin{bmatrix}\overline{V}_a\\\overline{V}_b\\\overline{V}_c\end{bmatrix}=\begin{bmatrix}0\\\overline{V}_a\\0\end{bmatrix}$ Negative Phase Sequence: $\begin{bmatrix}\overline{V}_0\\\overline{V}_1\\\overline{V}_2\end{bmatrix}=[\overline{T}]^{-1}\begin{bmatrix}\overline{V}_a\\\overline{V}_b\\\overline{V}_c\end{bmatrix}=\begin{bmatrix}0\\0\\\overline{V}_a\end{bmatrix}$	Positive Phase Sequence: $\begin{bmatrix}\overline{V}_a\\\overline{V}_b\\\overline{V}_c\end{bmatrix}=[\overline{T}]\begin{bmatrix}\overline{V}_0\\\overline{V}_1\\\overline{V}_2\end{bmatrix}=\begin{bmatrix}\overline{V}_1\\\overline{a}^2\overline{V}_1\\\overline{a}\overline{V}_1\end{bmatrix}$ Negative Phase Sequence: $\begin{bmatrix}\overline{V}_a\\\overline{V}_b\\\overline{V}_c\end{bmatrix}=[\overline{T}]\begin{bmatrix}\overline{V}_0\\\overline{V}_1\\\overline{V}_2\end{bmatrix}=\begin{bmatrix}\overline{V}_2\\\overline{a}\overline{V}_2\\\overline{a}^2\overline{V}_2\end{bmatrix}$
Current	Positive Phase Sequence: $\begin{bmatrix}\overline{I}_0\\\overline{I}_1\\\overline{I}_2\end{bmatrix}=[\overline{T}]^{-1}\begin{bmatrix}\overline{I}_a\\\overline{I}_b\\\overline{I}_c\end{bmatrix}=\begin{bmatrix}0\\\overline{I}_a\\0\end{bmatrix}$ Negative Phase Sequence: $\begin{bmatrix}\overline{I}_0\\\overline{I}_1\\\overline{I}_2\end{bmatrix}=[\overline{T}]^{-1}\begin{bmatrix}\overline{I}_a\\\overline{I}_b\\\overline{I}_c\end{bmatrix}=\begin{bmatrix}0\\0\\\overline{I}_a\end{bmatrix}$	Positive Phase Sequence: $\begin{bmatrix}\overline{I}_a\\\overline{I}_b\\\overline{I}_c\end{bmatrix}=[\overline{T}]\begin{bmatrix}\overline{I}_0\\\overline{I}_1\\\overline{I}_2\end{bmatrix}=\begin{bmatrix}\overline{I}_1\\\overline{a}^2\overline{I}_1\\\overline{a}\overline{I}_1\end{bmatrix}$ Negative Phase Sequence: $\begin{bmatrix}\overline{I}_a\\\overline{I}_b\\\overline{I}_c\end{bmatrix}=[\overline{T}]\begin{bmatrix}\overline{I}_0\\\overline{I}_1\\\overline{I}_2\end{bmatrix}=\begin{bmatrix}\overline{I}_1\\\overline{a}\overline{I}_1\\\overline{a}^2\overline{I}_1\end{bmatrix}$
Impedance	$[\overline{Z}_{012}]=[\overline{T}]^{-1}[\overline{Z}_{abc}][\overline{T}]=\begin{bmatrix}\overline{Z}_s+2\overline{Z}_m+3\overline{Z}_n & 0 & 0\\0 & \overline{Z}_s-\overline{Z}_m & 0\\0 & 0 & \overline{Z}_2-\overline{Z}_m\end{bmatrix}$	
Power	$\overline{S}_{3\phi}=\overline{V}_a\overline{I}_a^*+\overline{V}_b\overline{I}_b^*+\overline{V}_c\overline{I}_c^*=3\overline{V}_a\overline{I}_a^*$ $\overline{S}_{3\phi}=\{\overline{V}_0\overline{I}_0^*+\overline{V}_1\overline{I}_1^*+\overline{V}_2\overline{I}_2^*\}=\begin{cases}3\overline{V}_1\overline{I}_1^*\ \textit{positive ph. seq}\\3\overline{V}_2\overline{I}_2^*\ \textit{negative ph. seq}\end{cases}$	

4.4.3 INSTANTANEOUS SYMMETRICAL COMPONENTS THEORY FOR COMPENSATION

A three-phase, four-wire compensated system is shown in Figure 4.4.3-1, where the three-phase load may be unbalanced and nonlinear, while the supply voltage may be unbalanced and distorted. A shunt APF (or compensator) and the load are connected at the PCC. For the sake of illustrating the concept, the compensator is considered to be ideal and it is comprised of three ideal current sources as shown in the Figure 4.4.3-1.

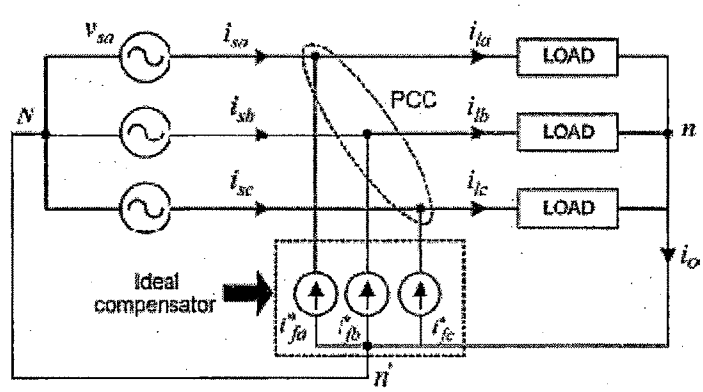


Figure 4.4.3-1: Schematic of Three-Phase, Four-Wire Compensated System

Let the unbalanced and distorted voltages be represented by

$$v_{sa}(t) = \sum_{n=1}^{k} V_{ma\,n} \sin(n\omega t + \emptyset_{va\,n}) \quad (4.4.3\text{-}1a)$$

$$v_{sb}(t) = \sum_{n=1}^{k} V_{mb\,n} \sin(n\omega t + \emptyset_{vb\,n}) \quad (4.4.3\text{-}1b)$$

$$v_{sc}(t) = \sum_{n=1}^{k} V_{mb\,n} \sin(n\omega t + \emptyset_{vc\,n}) \quad (4.4.3\text{-}1c)$$

The subscript "s" stands for supply, "a", "b", "c" for the three phases notation, "m" for maximum or peak value and "n" for the harmonic number. The term "k" is the order of maximum harmonic considered in the supply voltages.

Let us denote instantaneous positive, negative, and zero sequence voltages for phase-a by $v_{sa}^{+}(t)$, $v_{sa}^{-}(t)$ and $v_{sa}^{0}(t)$, respectively. These sequence voltages are expressed using symmetrical transformation as the following:

$$\begin{bmatrix} v_{sa}^{0}(t) \\ v_{sa}^{+}(t) \\ v_{sa}^{-}(t) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} v_{sa}(t) \\ v_{sb}(t) \\ v_{sc}(t) \end{bmatrix} \tag{4.4.3-2}$$

Similarly, the instantaneous load currents can be resolved into its instantaneous zero, positive, negative sequence components as

$$\begin{bmatrix} i_{la}^{0}(t) \\ i_{la}^{+}(t) \\ i_{la}^{-}(t) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} i_{la}(t) \\ i_{lb}(t) \\ i_{lc}(t) \end{bmatrix} \tag{4.4.3-3}$$

Here "a" is a complex operator equal to $e^{j2\pi/3}$. The aforementioned sequence components are denoted by boldface letters as they are complex quantities as a function of time. The zero sequence components are however not complex quantities, still they are represented by bold faced letters to maintain uniformity in equations. In a balanced distorted system with a fundamental frequency of ω and for a non-negative integer n, the harmonics of the order of $3n+1$, $3n+2$, and $3n+3$ follow the positive, negative, and zero sequences, respectively [J22]. Then$v_{sa}^{+}(t)$, $v_{sa}^{-}(t)$ and $v_{sa}^{0}(t)$ in (4.4.3-2), have $3n+1$, $3n+2$, and $3n+3$ harmonics, respectively. But in the unbalanced distorted system$v_{sa}^{+}(t)$, $v_{sa}^{-}(t)$ and $v_{sa}^{0}(t)$ have all of the harmonic orders. The balanced steady state harmonic components can be determined by using the following expressions. In general, for the harmonic order, it is written as follows:

$$V_{sa(3n+1)}^{+} = \frac{\sqrt{2}}{T}\int_{t1}^{t1+T} v_{sa}^{+}(t)e^{-j((3n+1)\omega t-\pi/2} \tag{4.4.3-4}$$

where $n = 0, 1, 2, \ldots \ldots k$. The term is an arbitrary instant and T is the time period of a cycle. The aforementioned integration is carried out using a moving average filter to have a fast response of the compensator. From the expression just shown $\left|V^{+}_{sa(3n+1)}\right|$ and $\angle V^{+}_{sa(3n+1)}$ can be obtained. The balanced voltage harmonics are given by

$$V^{-}_{sa(3n+2)} = \frac{\sqrt{2}}{T}\int_{t1}^{t1+T} v^{-}_{sa}\,(t)e^{-j((3n+2)\omega t-\pi/2} \qquad (4.4.3\text{-}5)$$

From (4.4.3-5), $\left|V^{-}_{sa(3n+2)}\right|$ and $\angle V^{-}_{sa(3n+2)}$ can be obtained. Similarly, the $3n+3$ voltage harmonics can be obtained as follows:

$$V^{0}_{sa(3n+3)} = \frac{\sqrt{2}}{T}\int_{t1}^{t1+T} v^{0}_{sa}\,(t)e^{-j((3n+3)\omega t-\pi/2} \qquad (4.4.3\text{-}6)$$

Using (4.4.3-6), $\left|V^{0}_{sa(3n+3)}\right|$ and $\angle V^{0}_{sa(3n+3)}$ are computed.

Thus, the balanced quantities in the case of unbalanced and distorted supply voltages can be given by (4.4.3-7)–(4.4.3-9), shown at the next page, where $n = 0,1,2,\ldots\ldots k.$ in (4.4.3-7)–(4.4.3-9).

$$v^{+}_{sa(3n+1)}\,(t) = \sqrt{2}\left|V^{+}_{sa(3n+1)}\right| \sin\left((3n+1)\omega t) + \angle V^{+}_{sa(3n+1)}\right) \qquad (4.4.3\text{-}7a)$$

$$v^{+}_{sb(3n+1)}(t) = \sqrt{2}\left|V^{+}_{sa(3n+1)}\right| \sin\left((3n+1)\omega t - 2\pi/3) + \angle V^{+}_{sa(3n+1)}\right) \qquad (4.4.3\text{-}7b)$$

$$v^{+}_{sc(3n+1)}(t) = \sqrt{2}\left|V^{+}_{sa(3n+1)}\right| \sin\left((3n+1)\omega t + 2\pi/3) + \angle V^{+}_{sa(3n+1)}\right) \qquad (4.4.3\text{-}7c)$$

$$v^{-}_{sa(3n+2)}\,(t) = \sqrt{2}\left|V^{-}_{sa(3n+2)}\right| \sin\left((3n+2)\omega t) + \angle V^{-}_{sa(3n+2)}\right) \qquad (4.4.3\text{-}8a)$$

$$v^{-}_{sb(3n+2)}(t) = \sqrt{2}\left|V^{-}_{sa(3n+2)}\right| \sin\left((3n+2)\omega t - 2\pi/3) + \angle V^{-}_{sa(3n+2)}\right) \qquad (4.4.3\text{-}8b)$$

$$v^{-}_{sc(3n+2)}(t) = \sqrt{2}\left|V^{-}_{sa(3n+2)}\right| \sin\left((3n+2)\omega t + 2\pi/3) + \angle V^{-}_{sa(3n+2)}\right) \qquad (4.4.3\text{-}8c)$$

$$v^{0}_{sa(3n+3)}\,(t) = \sqrt{2}\left|V^{0}_{sa(3n+3)}\right| \sin\left((3n+3)\omega t) + \angle V^{0}_{sa(3n+2)}\right) \qquad (4.4.3\text{-}9a)$$

$$v^{0}_{sb(3n+3)}(t) = \sqrt{2}\left|V^{0}_{sa(3n+3)}\right| \sin\left((3n+3)\omega t - 2\pi/3) + \angle V^{0}_{sa(3n+2)}\right) \qquad (4.4.3\text{-}9b)$$

$$v^{0}_{sc(3n+3)}(t) = \sqrt{2}\left|V^{0}_{sa(3n+3)}\right| \sin\left((3n+3)\omega t + 2\pi/3) + \angle V^{0}_{sa(3n+2)}\right) \qquad (4.4.3\text{-}9c)$$

The three-phase balanced set of voltages in the case of unbalanced and distorted supply voltages can be written as

$$v'_{sa}(t) = \sum_{n=0}^{k} \left(v^{+}_{sa(3n+1)}(t) + v^{-}_{sa(3n+2)}(t) + v^{0}_{sa(3n+3)}(t) \right) \qquad (4.4.3\text{-}10a)$$

$$v'_{sb}(t) = \sum_{n=0}^{k} \left(v^{+}_{sb(3n+1)}(t) + v^{-}_{sb(3n+2)}(t) + v^{0}_{sb(3n+3)}(t) \right) \qquad (4.4.3\text{-}10b)$$

$$v'_{sc}(t) = \sum_{n=0}^{k} \left(v^{+}_{sc(3n+1)}(t) + v^{-}_{sc(3n+2)}(t) + v^{0}_{sc(3n+3)}(t) \right) \qquad (4.4.3\text{-}10c)$$

Now v'_{sa}, v'_{sb} and v'_{sc} form balanced quantities for the available distorted and unbalanced supply voltages.

Let the source currents after compensation be i_{sa}, i_{sb} and i_{sc} in phases *a*, *b* and *c* respectively, as shown in Figure 4.4.3-1. In order to meet the requirements of load compensation, the following three conditions are to be satisfied. The first condition is that the neutral current after compensation must be zero. Therefore

$$i_{sa} + i_{sb} + i_{sc} = 0 \qquad (4.4.3\text{-}11)$$

The second objective of compensation is that the reactive power delivered from the source is controlled by the phase angle between the positive sequence voltage and current, hence

$$\angle\left(v^{+}_{sa}(t)\right) = \angle\left(i^{+}_{sa}(t)\right) + \phi^{+} \qquad (4.4.3\text{-}12)$$

This implies that

$$\angle(v_{sa} + a v_{sb} + a^2 v_{sc}) = \angle(i_{sa} + a i_{sb} + a^2 i_{sc}) + \phi^{+}$$

Simplifying the equation that was just shown leads to

$$(v_{sb} - v_{sc} - 3\gamma v_{sa})i_{sa} + (v_{sc} - v_{sa} - 3\gamma v_{sb})i_{sb} + (v_{sa} - v_{sb} - 3\gamma v_{sc})i_{sc} = 0$$

(4.4.3-13)

Where $\gamma = \tan\emptyset^{+}/\sqrt{3}$ and $\emptyset^{+}$ is the angle between the instantaneous phasors $\boldsymbol{v}_{sa}^{+}$ and $\boldsymbol{i}_{sa}^{+}$. In order to control the reactive power from the source through the negative sequence quantities, the following condition should be met:

$$\angle(\boldsymbol{v}_{sa}^{-}(\boldsymbol{t})) = \angle(i_{sa}^{-}(t)) + \emptyset^{-} \quad (4.4.3\text{-}14)$$

This gives

$$(v_{sc} - v_{sb} - 3\beta v_{sa})i_{sa} + (v_{sa} - v_{sc} - 3\beta v_{sb})i_{sb} + (v_{sb} - v_{sa} - 3\beta v_{sc})i_{sc} = 0$$

(4.4.3-15)

where $\beta = \tan\emptyset^{-}/\sqrt{3}$ and $\emptyset^{-}$ is the angle between the instantaneous phasors $\boldsymbol{v}_{sa}^{-}$ and $\boldsymbol{i}_{sa}^{-}$. By keeping both $\gamma = 0$ and $\beta = 0$ means that the reactive power supplied from the source through the positive sequence and negative sequence components of voltages and currents is zero. Alternatively, the positive sequence currents and negative sequence currents are in phase with the positive sequence voltages and negative sequence voltages, respectively. It results in the following equation:

$$(v_{sb} - v_{sc})i_{sa} + (v_{sc} - v_{sa})i_{sb} + (v_{sa} - v_{sb})i_{sc} = 0 \quad (4.4.3\text{-}16)$$

The third condition is that the source should supply the average load power P_{lavg} i.e., $v_{sa}i_{sa} + v_{sb}i_{sb} + v_{sc}i_{sc} = P_{lavg}$ (4.4.3-17)

By solving (4.4.3-11), (4.4.3-16), and (4.4.3-17), the reference source currents for compensation are obtained as follows:

$$i_{sa} = \frac{v_{sa} - v_{sa}^0}{\Delta} P_{lavg} \quad (4.4.3\text{-}18a)$$

$$i_{sb} = \frac{v_{sb} - v_{sa}^0}{\Delta} P_{lavg} \quad (4.4.3\text{-}18b)$$

$$i_{sc} = \frac{v_{sc} - v_{sa}^0}{\Delta} P_{lavg} \quad (4.4.3\text{-}18c)$$

Where $v_{sa}^0 = (1/3) \sum_{j=a,b,c} v_{sj}$ and $\Delta = \sum_{j=a,b,c} v_{sj}^2 - 3\,(v_{sa}^0)^2$

For the balanced sinusoidal conditions, is zero. For the supply voltages with unbalanced fundamental and unbalanced harmonics, it is not zero. Even for balanced distorted supply voltages, it is nonzero because of the triplen harmonics. By knowing the reference source currents, the APF reference currents can be obtained as shown

$$i_{fa}^* = i_{la} - i_{sa} = i_{la} - \frac{v_{sa} - v_{sa}^0}{\Delta} P_{lavg} \quad (4.4.3\text{-}19a)$$

$$i_{fb}^* = i_{lb} - i_{sb} = i_{lb} - \frac{v_{sb} - v_{sa}^0}{\Delta} P_{lavg} \quad (4.4.3\text{-}19b)$$

$$i_{fc}^* = i_{lc} - i_{sc} = i_{lc} - \frac{v_{sc} - v_{sa}^0}{\Delta} P_{lavg} \quad (4.4.3\text{-}19c)$$

While deriving the reference compensator currents in (4.4.3-19), in general, it is not assumed that supply voltages should be balanced and sinusoidal. However, the compensator meets the requirements as follows.

- There is no neutral current after compensation.
- The source supplies average load power and the rest of the load power is supplied by the compensator.

4.5 INDUCTION MOTOR ANALYSIS

Three-phase induction motors are of two types: squirrel-case and wound-rotor. In squirrel-cage, the rotor consists of longitudinal conductor-bars shorted by circular connectors at the two ends while in wound-rotor motor, the rotor also has a balanced three-phase distributed winding having same poles as stator winding. However, in both, stator carries a three-phase balanced distributed winding.

4.5.1 PER PHASE EQUIVALENT CIRCUIT OF MOTOR

Per-phase equivalent circuit of a three-phase induction motor is shown in Figure 4.5.1-1(a). *R'* and X' are the stator referred values of rotor resistance *R*, and rotor reactance *X*. Slip is defined by

$$s = \frac{\omega_{ms} - \omega_m}{\omega_{ms}} \qquad (4.5.1\text{-}1)$$

where ω_m and ω_{ms} are rotor and synchronous speeds, respectively. Further

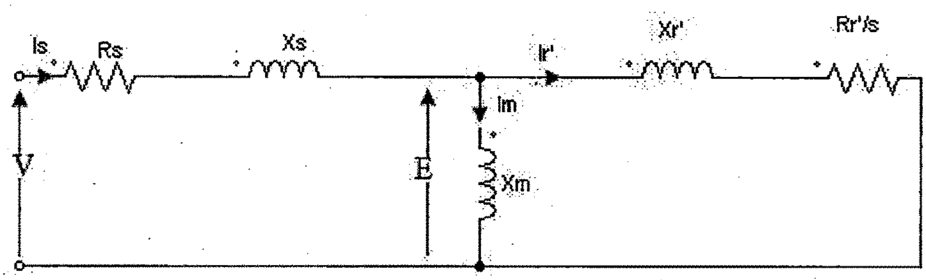
$$\omega_{ms} = \frac{4\Pi f}{p}\ rad/sec \qquad (4.5.1\text{-}2)$$

Where f and *p* are supply frequency and number of poles, respectively.

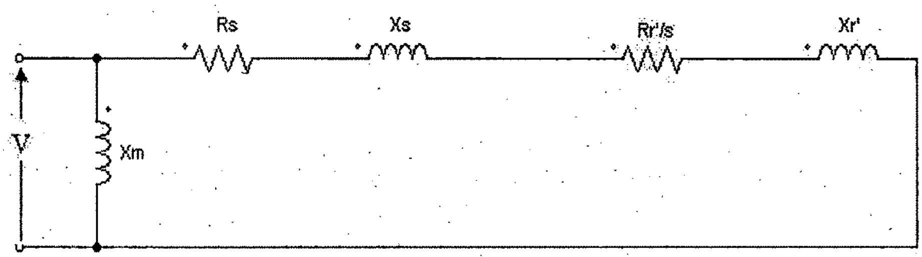
Since, stator impedance drop is generally negligible compared to terminal voltage V, the equivalent circuit can be Simplified to that shown in Figure 4.5.1-1(b).

Also from Equation (4.5.1-1)

$$\omega_m = \omega_{ms}(1-s) \qquad (4.5.1\text{-}3)$$



(a)



(b)

Figure 4.5.1-1: Per-Phase Stator Referred Equivalent Circuits of An Induction Motor

From Figure 4.5.1-1(b);

$$\bar{I}_p' = \frac{V}{\left(R_s + \frac{R_r'}{s}\right) + j\,(X_s + X_r')} \qquad (4.5.1\text{-}4)$$

Power transferred to rotor (or air-gap power)

$$P_g = 3\, I_r'^2\, R_r' \,/\, s \qquad (4.5.1\text{-}5)$$

Rotor copper loss is

$$p_{cu} = 3\, I_r'^2\, R_r' \qquad (4.5.1\text{-}6)$$

Electrical power converted into mechanical power

$$P_m = P_g - p_{cu} = 3\, I_r'^2\, R_r' \left(\frac{1-s}{s}\right) \tag{4.5.1-7}$$

Torque developed by motor

$$T = P_m \,/\, \omega_m \tag{4.5.1-8}$$

Substituting from Equations (4.5.1-3)and (4.5.1-7) yields

$$T = \frac{3}{\omega_{ms}}\, I_r'^2\, \frac{R_r'}{s} \tag{4.5.1-9}$$

Substituting from Equation (4.5.1-4) gives

$$T = \frac{3}{\omega_{ms}} \left[\frac{V^2\, R_r'/s}{\left(R_s + \frac{R_r'}{s}\right)^2 + (X_s + X_r')^2}\right] \tag{4.5.1-10}$$

A comparison of Equations (4.5.1-5) and (4.5.1-9) suggests that

$$T = P_g \,/\, \omega_{ms} \tag{4.5.1-11}$$

Motor output torque at the shaft is obtained by deducting friction windage and *core-loss* torques from the developed torque. The developed torque is a function of slip only (Equation 4.5.1-10). Differentiating *T* (4.5.1-10) with respect to s and equating to zero gives the slip for maximum torque

$$s_m = \pm \frac{R_r'}{\sqrt{R_s^2 + (X_s + X_r')^2}} \tag{4.5.1-12}$$

Substituting from Equation (4.5.1-12) into (4.5.1-10) yields an expression for maximum torque

$$T_{max} = \frac{3}{2\omega_{ms}} \left[\frac{V^2}{R_s \pm \sqrt{R_s^2 + (X_s + X_r')^2}}\right] \tag{4.5.1-13}$$

Maximum torque is also known as breakdown torque. While it is independent of rotor resistance, s_m is directly proportional to rotor resistance.

The natures of speed-torque and speed-rotor current characteristics are shown in Figure 4.3-1(b). Both rotor-current and torque are zero at synchronous speed. With decrease in speed, both increase, While torque reduces after reaching breakdown value, the rotor-current continues to increase, reaching a maximum value at zero speed, Drop in speed from no load to full load depends on the rotor resistance. When rotor resistance is low, the drop is quite small, and therefore, motor operates essentially at a constant speed. The breakdown torque is a measure of short-time torque overload capability of the motor.

Motor runs in the direction of the rotating field. Direction of rotating field and therefore motor speed can be reversed by reversing the phase sequence. Phase sequence can be reversed by interchanging any two terminals of the motor.

4.5.2 OPERATION WITH UNBALANCED SOURCE VOLTAGES AND SINGLE PHASING

As Supply voltage may sometimes become unbalanced, it is useful to know the effect of unbalanced voltages on motor performance. Further, motor terminal voltage may be unbalanced intentionally for speed control or starting as described later. A three-phase set of unbalanced voltages (V_a, V_b, and V_c) can be resolved into three-phase balanced positive sequence (V_p), negative sequence (V_n) and zero sequence (V_o) voltages, using symmetrical component relations as mentioned in section 4.4:

Motor performance can be calculated for positive and negative sequence voltages separately. Resultant performance is obtained by the principle of superposition by assuming motor to be a linear system. Positive sequence voltages produce an air-gap flux wave which rotates at synchronous speed in the forward direction. For a forward rotor speed w_m' slip *s* is given by Equation (4.5.1-1). For positive sequence voltages, equivalent circuits are same as shown in Figure 4.5.1-1, except that *V* is replaced by V_p. The positive sequence rotor current and torque are obtained by replacing *V by* V_p in Equations (4.5.1-4) and (4.5.1-10). Thus

$$I'_{rp} = \frac{V_p}{\left(R_s + \frac{R'_r}{s}\right) + j\,(X_s + X'_r)}$$

$$T_p = \frac{3}{\omega_{ms}} \left[\frac{V_p^2\, R'_r/s}{\left(R_s + \frac{R'_r}{s}\right)^2 + (X_s + X'_r)^2} \right] \qquad (4.5.2\text{-}1)$$

Negative sequence voltages produce an air-gap flux wave which rotates at synchronous speed in the reverse direction. The slip is

$$s_n = \frac{-\omega_{ms} - \omega_m}{-\omega_{ms}}$$

Substitution from Equation (4.5.1-3) gives

$$s_n = (2 - s) \qquad (4.5.2\text{-}2)$$

Again, equivalent circuits of Figure 4.5.1-1 are applicable when *s* is replaced by (2 - *s)* or s_n and *V are* replaced by V_n. Proceeding as in Sec. 4.5.1, following expressions are obtained for rotor current and torque:

$$I'_{rn} = \frac{V_n}{\left(R_s + \frac{R'_r}{2-s}\right) + j\,(X_s + X'_r)}$$

$$T_n = -\frac{3}{\omega_{ms}}\left[\frac{V_n^2\, R'_{r}/(2-s)}{\left(R_s + \frac{R'_r}{2-s}\right)^2 + (X_s + X'_r)^2}\right] \qquad (4.5.2\text{-}3)$$

Torque has a negative sign because for negative sequence voltages the synchronous speed is $(-\omega_{ms})$. The rms rotor current and torque are given by

$$I'_r = \left(I'^2_{rp} + I'^2_{rn}\right)^{1/2} \qquad (4.5.2\text{-}4)$$

$$T = T_p + T_n$$

$$= \frac{3}{\omega_{ms}}\left[\frac{V_p^2\, R'_{r}/s}{\left(R_s + \frac{R'_r}{s}\right)^2 + (X_s + X'_r)^2} - \frac{V_n^2\, R'_{r}/(2-s)}{\left(R_s + \frac{R'_r}{2-s}\right)^2 + (X_s + X'_r)^2}\right] \qquad (4.5.2\text{-}5)$$

Positive sequence and negative sequence speed-torque characteristics are shown in Figure 4.5.2-1(a). *Single phasing (when supply to anyone phase fails) is the extreme case of unbalancing, when* $V_p = V_n$. At zero speed, *s* is also equal to s_n *consequently* starting torque is zero. Speed-torque curves for single phasing are shown in Figure 4.5.2-1(b). Interaction between positive sequence air-gap flux wave and positive sequence rotor currents produce positive sequence torque T_p. Negative sequence torque T_n is produced due to interaction between negative sequence flux wave and negative sequence rotor currents.

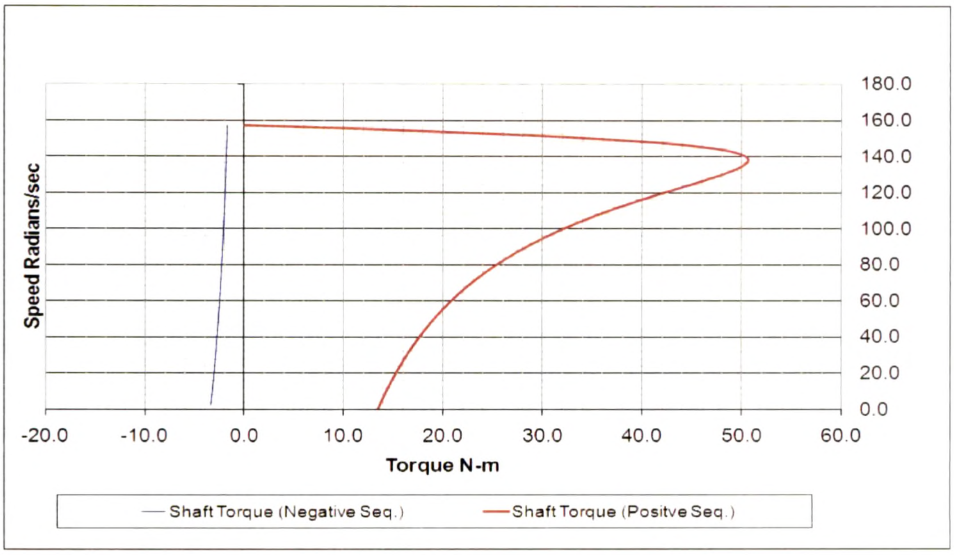


Figure 4.5.2-1(a): Speed-Torque Curves of An Induction Motor With Unbalance Stator Voltages

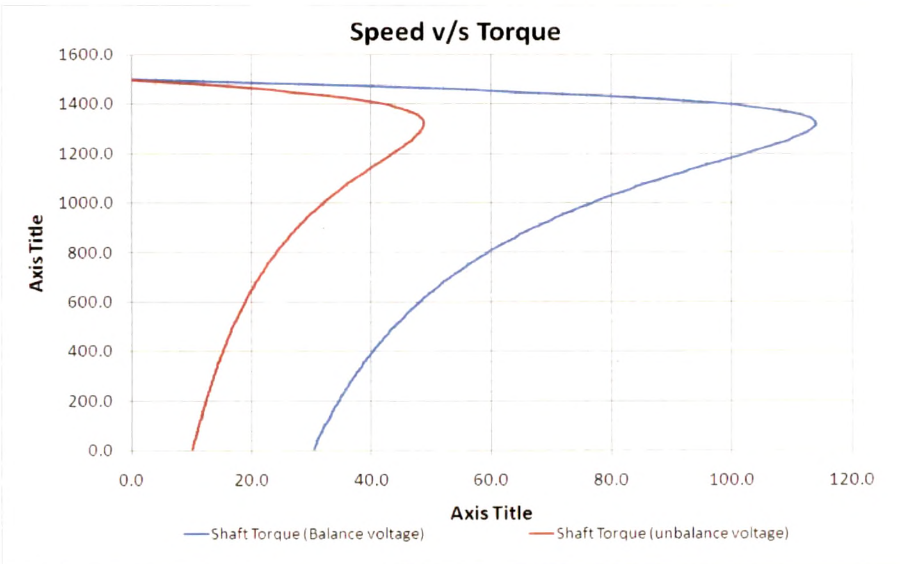


Figure 4.5.2-1(b): Speed-Torque Curves Of An Induction Motor With Unbalance Stator Voltages & Balance Stator Voltage For Single Phasing

Interactions between positive sequence flux wave and negative sequence rotor currents, and negative sequence flux wave and positive sequence rotor currents, *also* produce torques. However, these torques are pulsating in nature with zero average values. The pulsating torques cause vibrations which reduce the life of motor and produce hum.

Equations (4.5.2-5) and (4.5.2-4) suggest that while the torque is reduced, copper losses (and also core losses) are increased. Thus, the unbalanced operation substantially reduces the motor torque capability and efficiency. To prevent burning of the motor, it is not allowed to run for a prolonged period when the unbalance in voltages is more than 5%. For the same reason, motor is disconnected from the source whenever single phasing occurs, unless the single phasing is always accompanied by a light load.

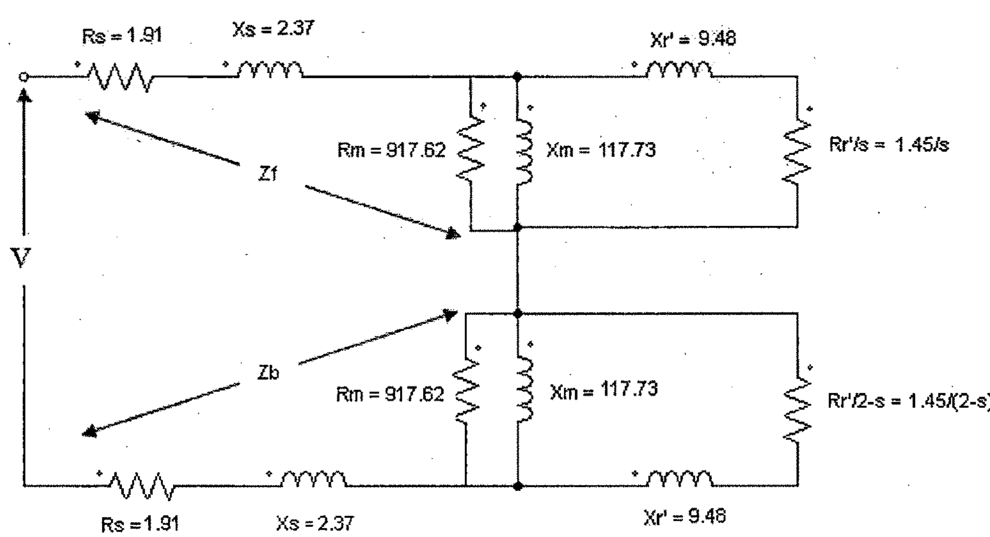


Figure 4.5.2-2: Per phase Equivalent circuit of Induction motor for calculation of speed torque characteristics under unbalance stator voltage condition.

From equivalent circuit of Figure 4.5.2-2, one can obtain speed-torque curve for the motor. This curve and the load speed-torque curve are plotted on a graph. Intersection provides the values of steady state speed and torque. As a sample motor current and torque for a slip of 0.035 is calculated in Annexure-III.

4.5.3 ANALYSIS OF INDUCTION MOTOR FED FROM NON-SINUSOIDAL VOLTAGE SUPPLY

When fed from an inverter or cyclo-converter, the motor terminal voltage is non-sinusoidal but it has half-wave symmetry. A non-sinusoidal waveform can be resolved into fundamental and harmonic components using Fourier analysis. Because of half-wave symmetry only odd harmonics will be present. The harmonics can be divided into positive sequence, negative sequence and zero sequence. The harmonics, which have the same phase sequence as that of fundamental are called positive sequence harmonics. The harmonics having phase sequence opposite to fundamental are called negative sequence harmonics. The harmonics, which have all three-phase voltages in phase, are called zero sequence harmonics.

Consider the fundamental phase voltage components V_{AN} = Vl sin wt, V_{BN}=Vl sin(wt-2π/3) and V_{CN} = Vl sin(wt-4π/3) with the phase sequence ABC. The corresponding 5th and 7th harmonic phase voltages are

$$V_{AN} = V_5 \sin 5\omega t$$

$$V_{BN} = V_5 \sin 5(\omega t - 2\Pi/3) = V_5 \sin(5\omega t - 4\Pi/3)$$

$$V_{CN} = V_5 \sin 5(\omega t - 4\Pi/3) = V_5 \sin (5\omega t - 2\Pi/3)$$

and

$$V_{AN} = V_7 \sin 7\omega t$$

$$V_{BN} = V_7 \sin 7(\omega t - 2\Pi/3) = V_7 \sin (7\omega t - 2\Pi/3)$$

$$V_{CN} = V_7 \sin 7(\omega t - 4\Pi/3) = V_7 \sin (7\omega t - 4\Pi/3)$$

The above equations show that 7th harmonic has the phase sequence A-B-C, which is the same as that of fundamental. Hence it is a positive sequence harmonic. The 5th harmonic has a phase sequence A-C-B, hence it is a negative sequence harmonic. It can be shown that the harmonic voltages and currents of the order m = $6k + 1$ (where k is an integer) are of positive sequence and harmonic voltages of the order m = $6k$ - I are of negative sequence. Similarly it can be shown that harmonics of the order m = $3k$ are of zero sequence. A positive sequence harmonic m will produce a rotating field, which moves in the same direction as the fundamental at a speed m times that of the fundamental field. Similarly rotating field produced by a negative sequence harmonic III will move in the direction opposite to the fundamental at III times its speed. Zero sequence components do not produce a rotating field.

For fundamental component the equivalent circuits of Figure 4.5.1-1 will be applicable. For any mth harmonic, equivalent circuit will be as shown in Figure 4.5.3-1(a). Each reactance has been increased by a factor III. Due to the skin effect resistances will also be increased several times. Slip s_m for the mth harmonic is given by

$$s_m = \frac{m\omega_{ms} \mp \omega_m}{m\omega_{ms}}$$

Negative sign is applicable to harmonics which produce forward rotating fields and the positive sign to those which produce backward rotating fields. Since s_m is close to unity, resistance (R_m'/s_m) has a small value. As reactance are very large compared to resistance, equivalent circuit of Figure 4.5.3-1(a) can be replaced by the simplified circuit of Figure 4.5.3-1(b). When fed from a semiconductor converter, it can be shown that the net torque produced by harmonics is close to zero.

In view of this motor torque can be evaluated from equivalent circuits of Figure 4.5.3-1(b), using Equation (4.5.1-10), where V is the fundamental component of supply voltage.

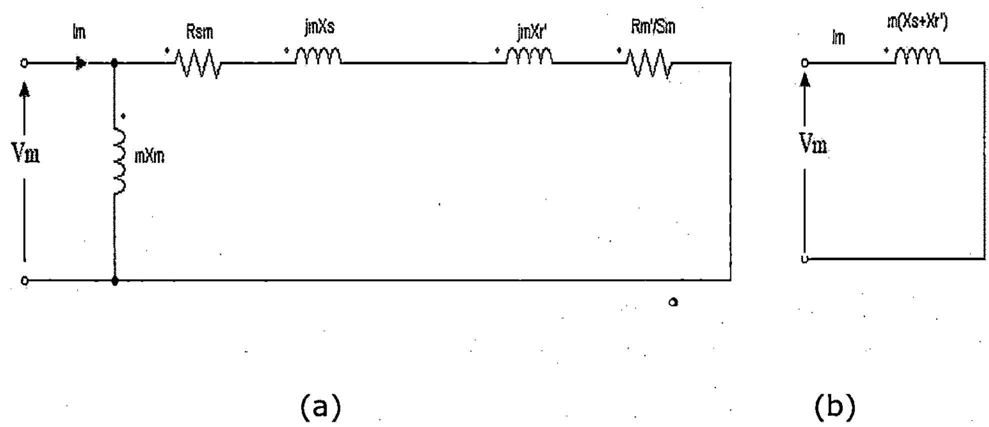


Figure 4.5.3-1: Harmonic Equivalent Circuits of an Induction Motor

Fundamental component of rotor current is obtained from Equation (4.5.1-4) and the m[th] harmonic current is calculated from Figure 4.5.3-1(b) as

$$I_m = \frac{V_m}{mX} \tag{4.5.3-1}$$

where $X = X_s + X_r'$.

Generally, supply will have odd harmonics. When stator is star-connected triplen harmonics (third harmonic and its multiples) will not flow. The rms motor current I_{rms} will then be

$$I_{rms}^2 = I_s^2 + \sum_{m=5,7,11,\ldots} I_m^2 \tag{4.5.3-2}$$

When motor is delta-connected, triplen harmonics will circulate in delta, but will not flow in the source. The source current therefore can be obtained by multiplying I_{rms}, given by Equation (4.5.3-2) by $\sqrt{3}$. The rms motor phase current will be obtained by

$$I_{rms}^2 = I_s^2 + \sum_{m=3,57\ldots} I_m^2 \tag{4.5.3-3}$$

For a given motor torque and power, rms current flowing through the motor has a higher value. Further due to skin effect harmonic rotor resistance has higher value. Therefore presence of harmonics, increase the copper loss substantially. Core losses are also increased by harmonics. Because of increase in losses, motor has to be derated in the sense that the power output that can be obtained from machine for the same temperature rise has to be smaller. The efficiency is also reduced due to increase in losses.

Another important effect of non-sinusoidal supply is the production of pulsating torques due to interaction between the rotating field produced by one harmonic and rotor current of another harmonic. Harmonic 5, 7, 11 and 13 are major contributors of torque pulsations. 5^{th} harmonic produces backward rotating field whereas 7^{th} harmonic produces forward rotating field. Therefore, relative speed between the field produced by the fundamental and 5^{th} and 7^{th} harmonics is six time the speed of fundamental. Consequently, torque pulsations produced due to the interaction of 5^{th} and 7^{th} harmonic currents and the fundamental rotating field has a frequency six times the fundamental. It can be similarly shown that harmonics 11 and 13 produce torque pulsations whose frequency is 12 times the fundamental. When motor supply frequency is not very low, the frequency of torque pulsations is large enough to be filtered out by motor inertia. Consequently the torque pulsations do not have significant effect on motor speed, although they do increase noise and reduce motor life due to vibrations. However, when motor supply frequency is low, these torque pulsations cause pulsations in speed. The motor then does not move smoothly but have jerky motion.

4.6 SOLUTION TO NEGATIVE SEQUENCE & ZERO SEQUENCE PROBLEM

This section presents a novel and online method, which is also a novel contribution of this research work for detecting negative & zero sequence voltages and its compensation. It is based on instantaneous Active Reactive power theory. Using this concept, negative sequence and zero sequence components are computed online. The α-β transformation is real unlike the complex transformation matrix in case of symmetrical components. So implementation of α-β analysis on line is simple. The theory is verified through simulation and experimentation results.

The section also analyzes theoretically the effect of negative sequence voltages on induction motors, energy saved due to compensation.

4.6.1 ELECTROMAGNETIC COMPENSATOR:

Figure 4.6.1-1 shows the schematic connection diagram for this system. It requires another 3ϕ squirrel cage motor (much smaller in capacity) whose rotor is mounted in the same shaft and whose stator windings are connected with reverse phase sequence. Stator windings of both the motors are electrically connected in series with the system voltage. The theory behind this method is that the second motor with negative sequence winding connection will offer large impedance to the negative sequence currents in the input lines (caused by the mains voltage unbalance) and low impedance to the positive sequence currents. Thus this connection will reduce the magnitude of negative sequence currents in the stator of the main motor for the same input voltage unbalance factor. Improvement in the performance of the main motor by this method has been reported in reference [J1].

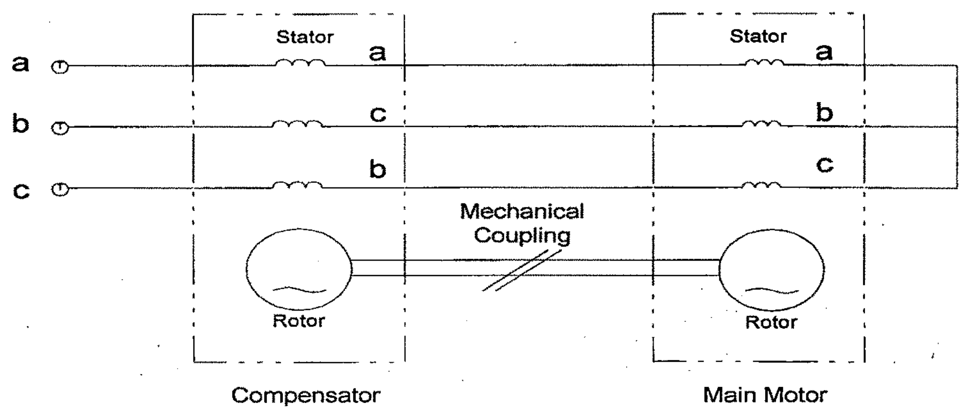


FIGURE 4.6.1-1: Electromagnetic Compensator

The drawback of this method in addition to the cost of another motor is that the stator winding of the second motor which is connected in series with the stator of the main motor while reducing the magnitude of negative sequence current also reduces the positive sequence voltage applied to the main motor since the series winding has some impedance offered to the flow of positive sequence currents. Since the output of induction motor is approximately proportional to the square of the voltage applied, this method results in reduced output of the motor while improving its efficiency due to reduction in negative sequence torque. Further the positive sequence currents flowing through the stator windings of the second motor generate the breaking torque on the rotor reduces the net torque developed by main motor. It is very difficult to retrofit this type of compensator. Also this type of compensator is applicable for motors only.

4.6.2 ELECTRONIC COMPENSATOR

The above problem can be solved by the use of electronic compensator connected in series with the stator of main motor, which will compensate only the negative sequence voltage in the mains, and the positive sequence voltage remains unaffected. This compensator can be used for other types of loads also which are sensitive to negative sequence voltages. This chapter describes the principle of this electronic compensator.

Electronic negative sequence compensator is a voltage source inverter [J2]-[J3] whose output is changed through pulse width modulation (PWM). PWM method utilizes a reference signal (one for each phase), which is compared with a high frequency triangular wave, and the points of intersection of both these signals decide the time of switching on and off the devices connected to that particular phase. The output of PWM inverter follows the reference signal used.

If the reference signal is derived from the negative sequence voltage with opposite polarity then the negative sequence compensator voltages obtained from the PWM inverter which are connected in series with the main motor windings, will cancel the negative sequence component voltages presents in the mains applied voltage.

4.7 CONTORLLING OF NEGATIVE SEQUENCE COMPENSATOR

To control Voltage source inverter, in series with load for negative & zero sequence compensation, reference signal having information of negative sequence & zero sequence of the source is required. For extracting negative sequence and zero sequence from the bus voltage a novel technique using instantaneous active reactive power theory were used. After extracting the signal, the extracted signal was used as reference signal to voltage source inverter along with PI controller to control VSC so that it compensate the negative & zero sequence converter.

4.7.1 GENERATION OF REFERENCE SIGNALS:

It is well known that the average power flow due to positive sequence voltage and negative sequence current is zero

$$(V_m \sin(\omega t) I_m \sin(\omega t) + (V_m \sin(\omega t - 120) I_m \sin(\omega t + 120)) + (V_m \sin(\omega t + 120) I_m \sin(\omega t - 120)) = 0 \quad (4.7.1\text{-}1)$$

Similarly the negative sequence voltage will generate nonzero power with negative sequence current.

Hence this concept is used for generating the reference signal for negative sequence compensator using instantaneous Active reactive power theory [J4]-[J8].

To implement this theory, digitally generate reference unit rms amplitude 3-phase negative sequence voltages in synchronization with supply voltage v_a, v_b and v_c as follows.

$$v_{ar} = \sqrt{2}\ \sin(\omega t)$$

$$v_{br} = \sqrt{2}\,\sin\left(\omega t + {}^{2\pi}/_{3}\right)$$

$$v_{cr} = \sqrt{2}\,\sin\left(\omega t - {}^{2\pi}/_{3}\right) \qquad (4.7.1\text{-}2)$$

Convert these reference voltages to α-β-0 reference frame through the transformation matrix as below.

$$\begin{bmatrix} v_{0r} \\ v_{\alpha r} \\ v_{\beta r} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_{ar} \\ v_{br} \\ v_{cr} \end{bmatrix} \qquad (4.7.1\text{-}3)$$

Let v_a, v_b and v_c be the phase to neutral unbalanced three phase voltages of the input power supply. It is assumed that these are of fundamental frequency and free of harmonics. These voltages are sensed through voltage sensor and converted into α-β-0 component through the transformation matrix as below.

$$\begin{bmatrix} v_{0} \\ v_{\alpha} \\ v_{\beta} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_{a} \\ v_{b} \\ v_{c} \end{bmatrix} \qquad (4.7.1\text{-}4)$$

Now the following calculation is performed using these α-β-0 components.

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} v_{0r} & 0 & 0 \\ 0 & v_{\alpha r} & v_{\beta r} \\ 0 & -v_{\beta r} & v_{\alpha r} \end{bmatrix} \begin{bmatrix} v_{0} \\ v_{\alpha} \\ v_{\beta} \end{bmatrix} \qquad (4.7.1\text{-}5)$$

p_o is zero because e_{ro} is zero and p & q can have dc and ac components as function of time.

In the instantaneous p-q theory DC components in p & q are responsible for the fundamental positive sequence or negative sequence power & this depends upon the reference voltage signal. If reference voltage v_{ra}, v_{rb}, v_{rc} are the positive sequence then DC components in the power p & q reflects the positive sequence power otherwise if reference voltage v_{ra}, v_{rb}, v_{rc} are of negative sequence, then DC components in p & q reflects the fundamental negative sequence power.

In Equation (4.7.1-5), the DC components are due to negative sequence voltages in the bus.

So DC components are filtered out using low pass filter to get the p_{dc} and q_{dc}. Negative sequence component is computed using the following transformation.

$$\begin{bmatrix} v_{\alpha n} \\ v_{\beta n} \end{bmatrix} = \begin{bmatrix} v_{\alpha r} & v_{\beta r} \\ -v_{\beta r} & v_{\alpha r} \end{bmatrix}^{-1} \begin{bmatrix} -p_{dc} \\ -q_{dc} \end{bmatrix}$$

$$\begin{bmatrix} v_{\alpha n} \\ v_{\beta n} \end{bmatrix} = \frac{1}{(v_{\alpha r}{}^2 + v_{\beta r}{}^2)} \begin{bmatrix} v_{\alpha r} & -v_{\beta r} \\ v_{\beta r} & v_{\alpha r} \end{bmatrix} \begin{bmatrix} -p_{dc} \\ -q_{dc} \end{bmatrix} \quad (4.7.1\text{-}6)$$

$v_{\alpha n}$ and $v_{\beta n}$ are the α, β components of negative sequence voltage of the supply, which can be transformed back to three-reference voltages for PWM voltage source inverter.

$$\begin{bmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -v_0 \\ v_{\alpha n} \\ v_{\beta n} \end{bmatrix} \quad (4.7.1\text{-}7)$$

Where v_o is the zero sequence voltage component of the three phase bus voltages.

In the above manner, the instantaneous values of negative sequence voltages (including zero sequence components) of phase 'a' phase 'b' and phase 'c' are computed.

When once these three reference signals are obtained they are fed to the voltage source inverter as reference signals to generate corresponding voltages at the output of voltage source inverter which will completely eliminate the negative sequence and zero sequence component voltages in the mains supply and will not affect its positive sequence component voltages which is the problem with the previously proposed electromagnetic compensator [J1]. Further there will be no additional braking torque on the rotor due to the positive sequence currents flowing through negative sequence compensator, as was the case with the other compensator. So, the mechanical output will not be reduced. The active energy input to negative sequence compensator is negligible since its negative sequence output voltage sees positive sequence currents flowing to the main motor and so the negative sequence compensator output is only reactive in nature. Only very little amount of power loss takes place in the inverter and transformer. Further more positive sequence voltage applied to the motor can be improved to its rated voltage by adding a portion of the fundamental positive sequence mains voltage to the reference signals of the negative sequence compensator scheme. Hence further improvement in the effective torque of the motor is also possible.

4.7.2 RESULTS:

The simulation studies are carried out to predict the performance of the proposed new theory using MATLAB-SIMULINK-POWER SYSTEM BLOCK. For simulation of negative sequence compensator a voltage source inverter whose output is changed through pulse width modulation (PWM) is used. PWM method utilizes a reference signal (one for each phase), which is compared with a high frequency triangular wave, and the points of intersection of both these signals decide the time of switching on and off the devices connected to that particular phase. Block diagram for simulation is shown in Figure 4.7.2.1-1. Negative sequence condition is simulated using two methods.

One method is by adding balance three phase negative sequence voltage to the balance three-phase positive sequence voltage (here zero sequence components is absent) and second method is through unbalancing the supply voltage itself which can have zero sequence component.

4.7.2.1 SIMULATION RESULTS & WAVEFORMS

The simulation results for both the methods are shown in Figure 4.7.2.1-2(a) to Figure 4.7.2.1-2(d). In second method, by unbalancing the supply voltage if the zero sequence components are ignored for calculation of the three phase compensating signals, then the output remains still unbalance due to the presence of the zero sequence components in the system. This is also verified through simulation.

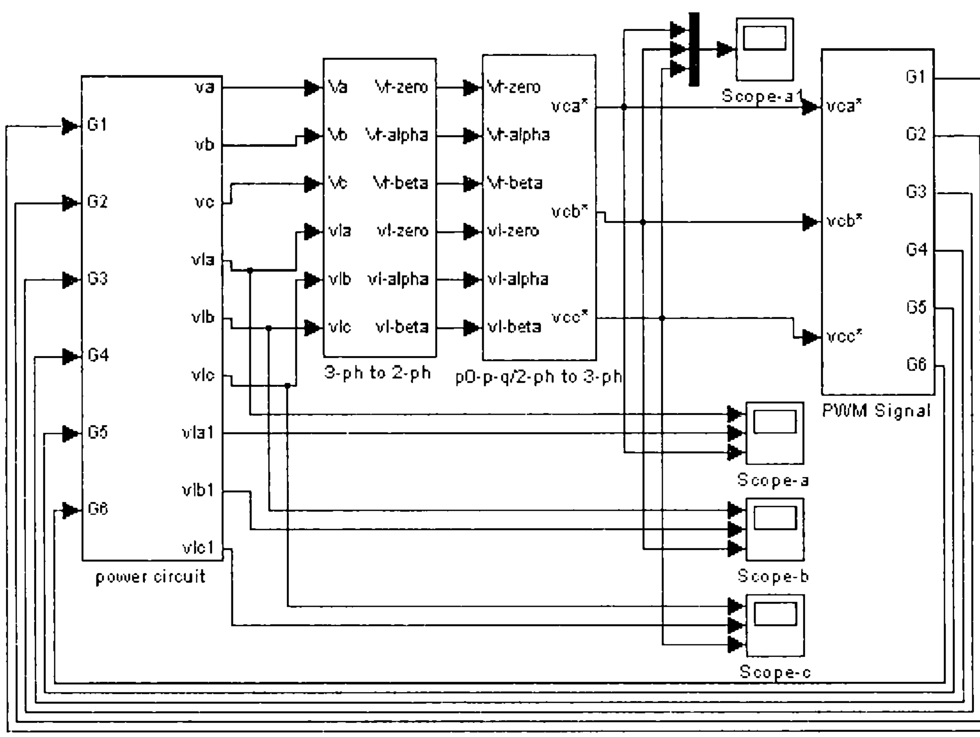


Figure 4.7.2.1-1: Block Diagram for Simulation

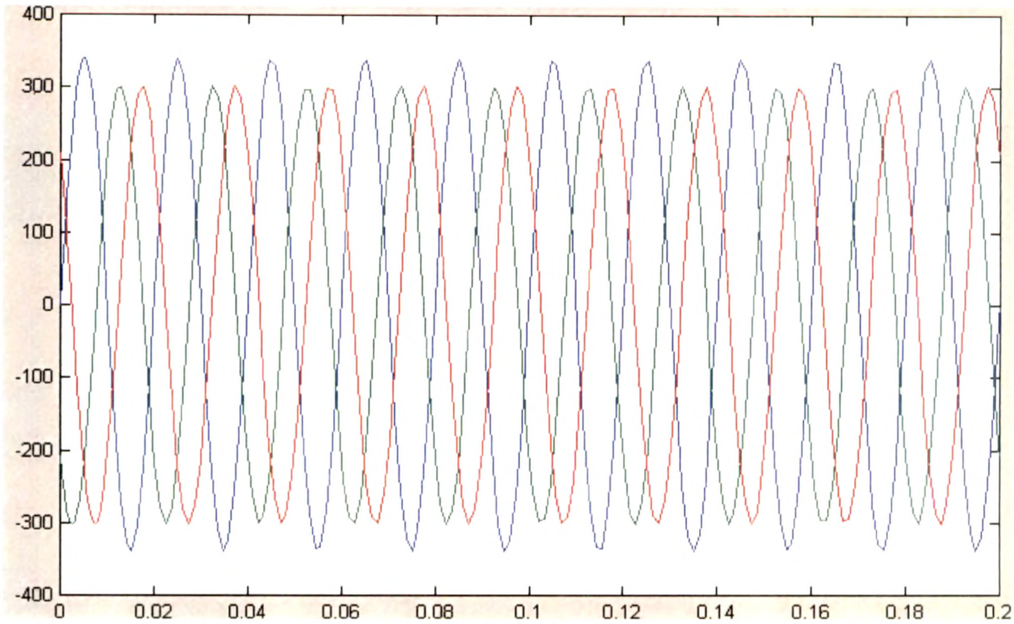


Figure 4.7.2.1-2(a)**:** Unbalanced Input Voltages

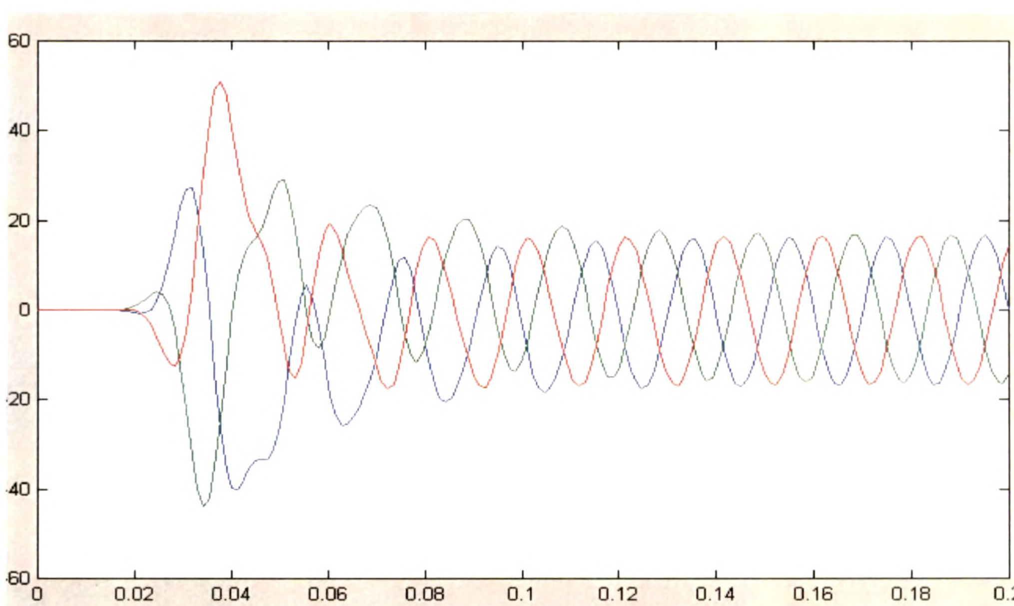


Figure 4.7.2.1-2(b)**:** Compensating Reference Voltages Without Accounting Zero Sequence Components.

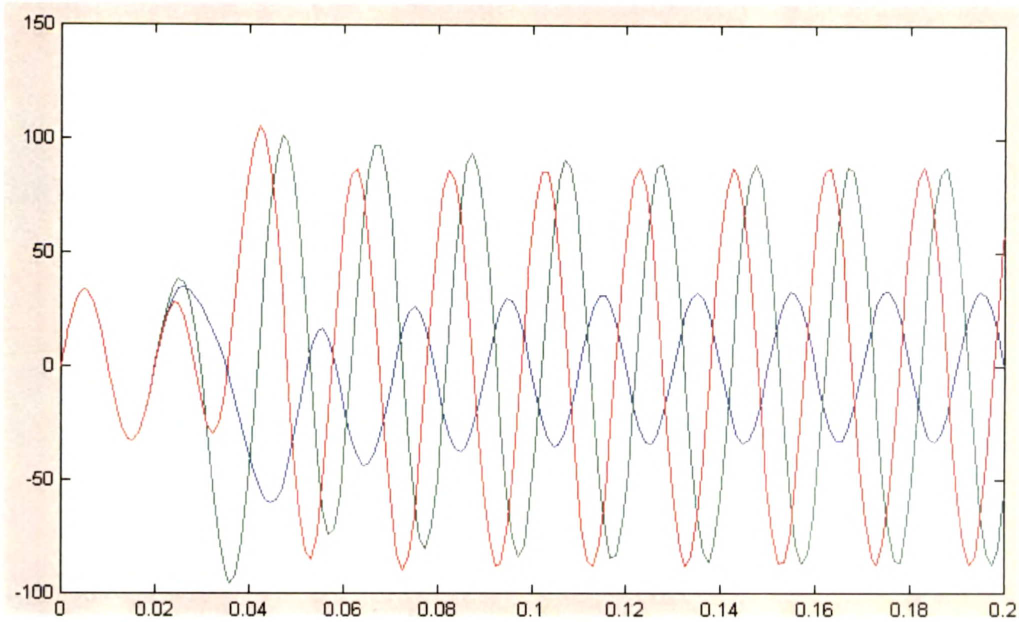


Figure 4.7.2.1-2(c): Compensating Reference Voltages With Accounting Zero Sequence Components

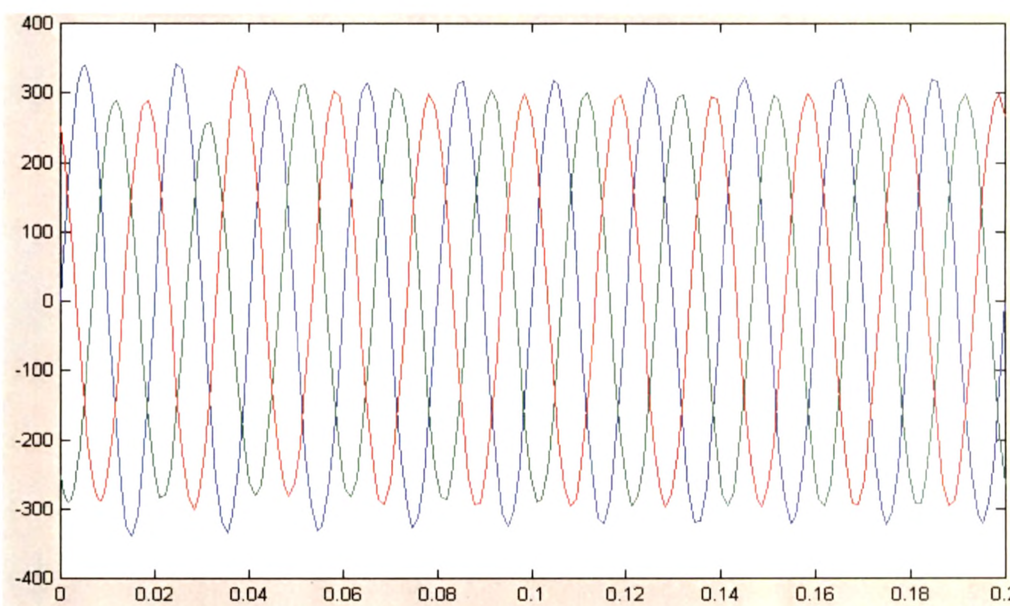


Figure 4.7.2.1-2(d): Load Voltages After Compensation Without Accounting Zero Sequence Components

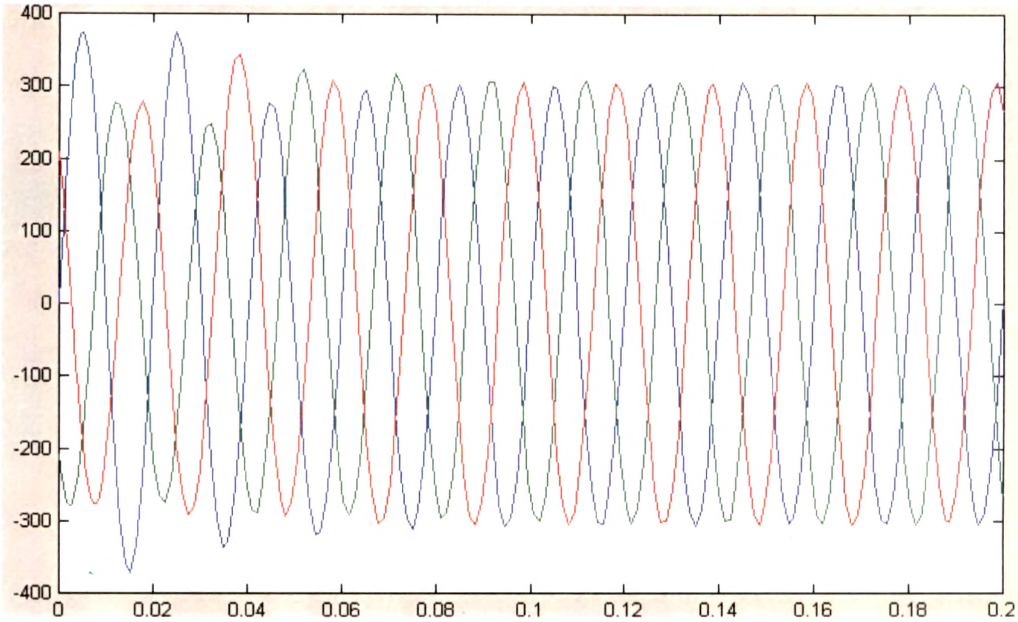


Figure 4.7.2.1-2(e): Load Voltages After Compensation With Accounting Zero Sequence Components

4.7.2.2 EXPERIMENTAL SETUP & WAVEFORMS:

The experimentation set up is prepared for detecting and compensating the negative sequence voltage in unbalance supply voltage. Schematic of experimental set up is given in Figure 4.7.2.2-1.

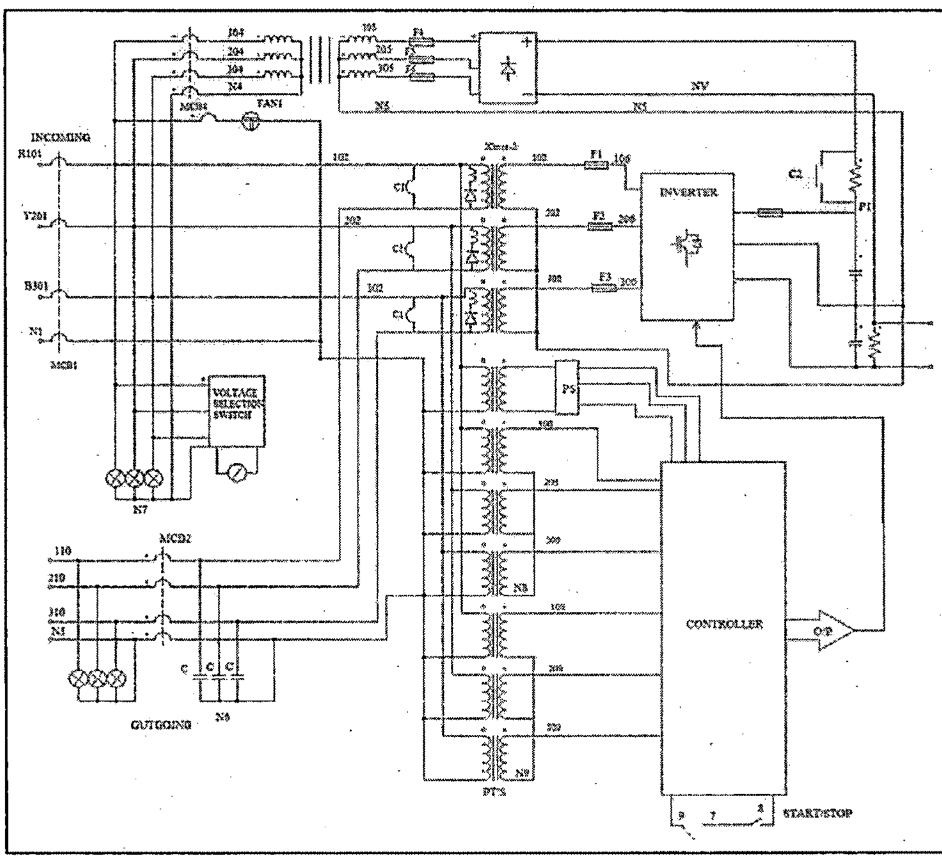


Figure 4.7.2.2-1: Schematic Diagram of Experimental Setup

The experiments set up were design for 12 amp 415 volt input supply voltage. The power circuit, as shown in figure, mainly consists of Inverter Bridge, high frequency transformer for injecting the voltage in between supply and load, LC filter. For bridge inverter, the IGBT used are SEMIKRON make SKM50 GB123 (50 Amps, 1200 V, dual module, with anti-parallel diode) for three phase application three such IGBT dual module were used. For DC bus capacitor C1, C2 = 6900 μF (4700 μF, 450 VDC Capacitor two in series and such three branch connected in parallel ALCON make with center point of each branch common) are used.

The high frequency transformer plays important role in the series compensation. The secondary of this transformer is connected in series between the source and the linear load. Rating of the transformer is decided based on the maximum unbalance to be compensated by negative sequence compensator. Considering this as based the rating of the high frequency transformer [J10] are fixed as follows:

Voltage ratings	:0-110-150//0-110-150volts per phase
Current rating	:15 amps per phase
Configuration	: three phase
VA ratings	:2250 VA per phase
Insulation Level	:3 kV
Leakage reactance	:10%.
Pri Turns	:120
Sec Turns	:132
Pri	: 8 Sq. mm
Sec	: 8 Sq. mm
Flux density	: 1.45 T,
Current density	: 1.87 A / mm^2

The high frequency isolation transformer is designed for high leakage reactance to filter out the effect of switching frequency. The frequency of switching is kept 4.8 kHz. The PWM signals (generated as a result of comparison of triangular wave with reference compensating voltages) are fed as gate signals to the IGBTs through the driver circuit. The Driver card is design to drive the IGBT using Mitsubishi hybrid gate Driver IC = M57962L so that it can be placed near the IGBT gate to reduce the parasitic impedance & to minimize the delay. Thus the output of the inverter contains the compensating fundamental voltage components plus the switching frequency. As the switching frequency component is not required, it is filtered out using the tuned LC filter. The value of the capacitor used in LC filter at the output of the negative sequence compensator as shown in the Figure are 0.47 μF, 2 kV.

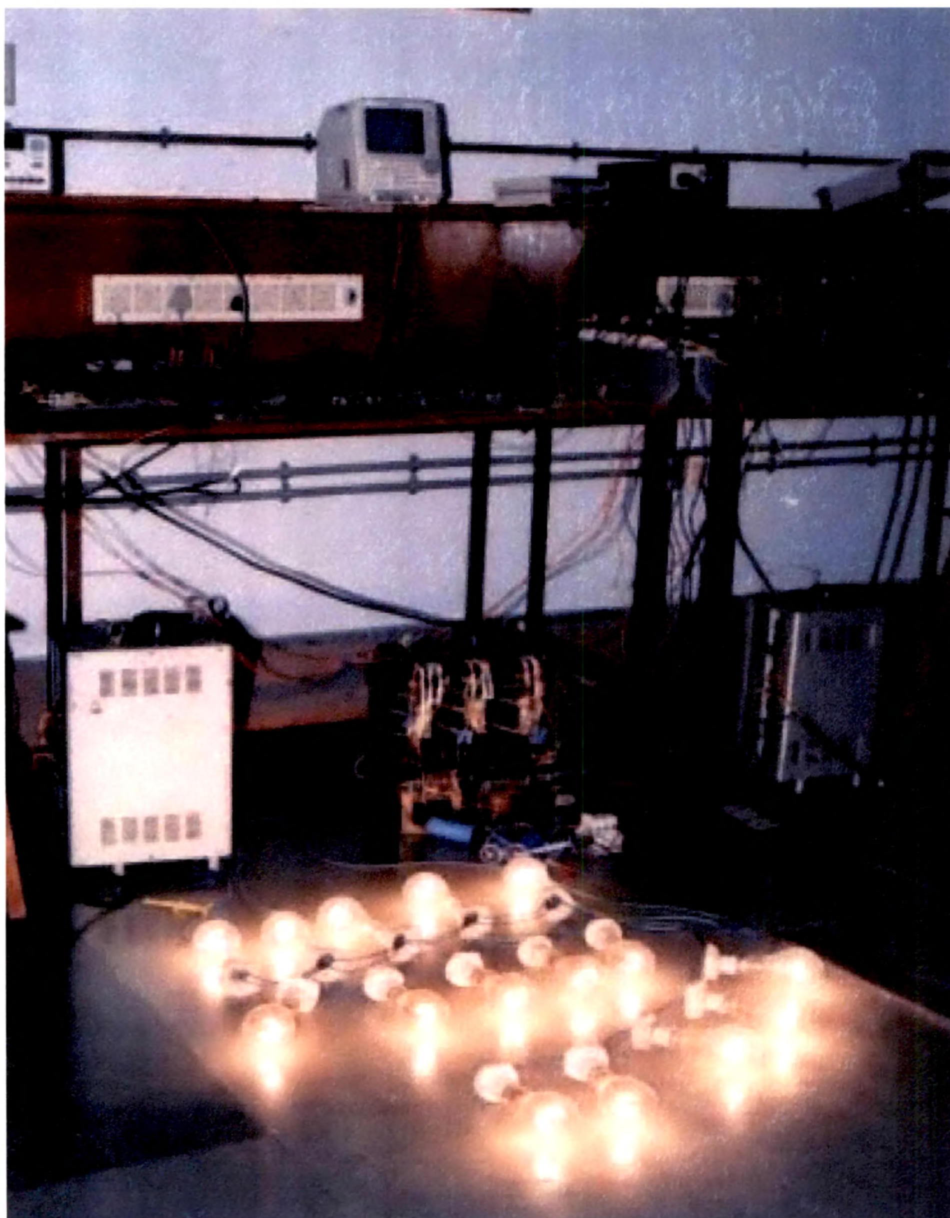


Figure 4.7.2.2-2: Experimental Set-Up

The technique explained in this paper is implemented using an analog circuit. The multiplication and division of the signal is done using low cost analog multiplier AD633 which is LASER trimmed 10 V scaling reference. Unbalance voltage is generated through three single-phase variacs. The unbalance voltages are sensed through PTs and negative sequence reference voltages are derived. The experimentation setup with control using analog IC, power circuit using IGBT, high frequency transformer, capacitor and lamp load is shown in Figure 4.7.2.2-2 and the experimental wave form are shown in the Figure 4.7.2.2-3 to Figure 4.7.2.2-10

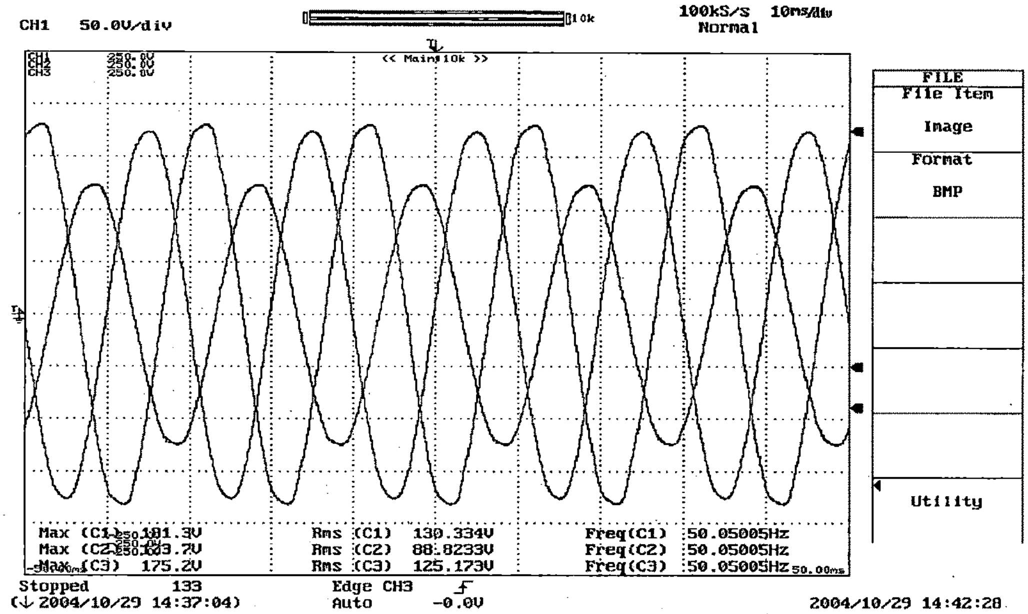


Figure 4.7.2.2-3: Unbalance Input Voltages

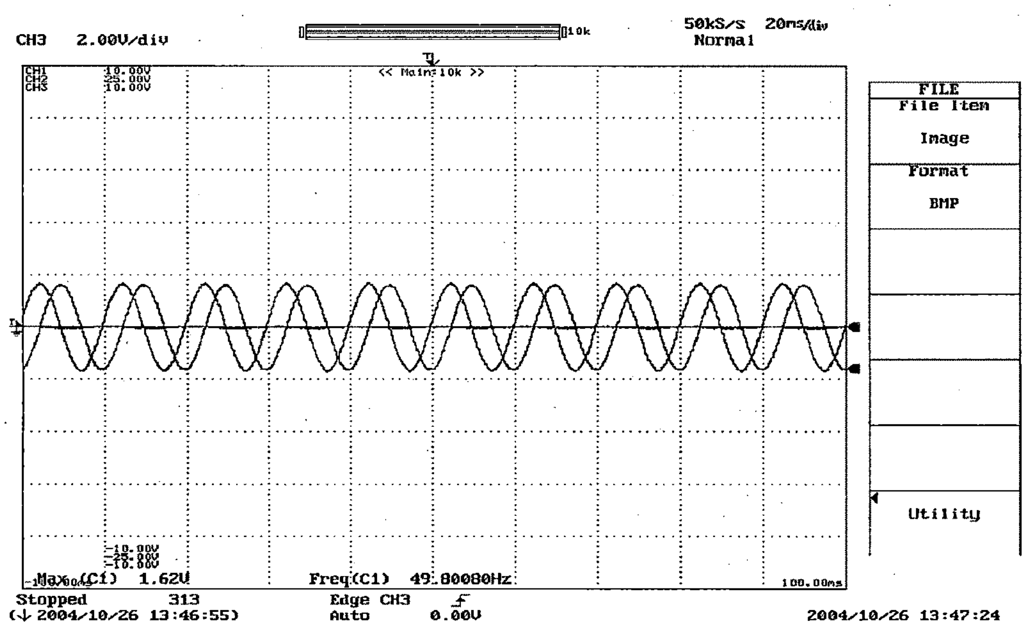


Figure 4.7.2.2-4: $V_{\alpha R}$ & $V_{\beta R}$ for Reference Input Voltages

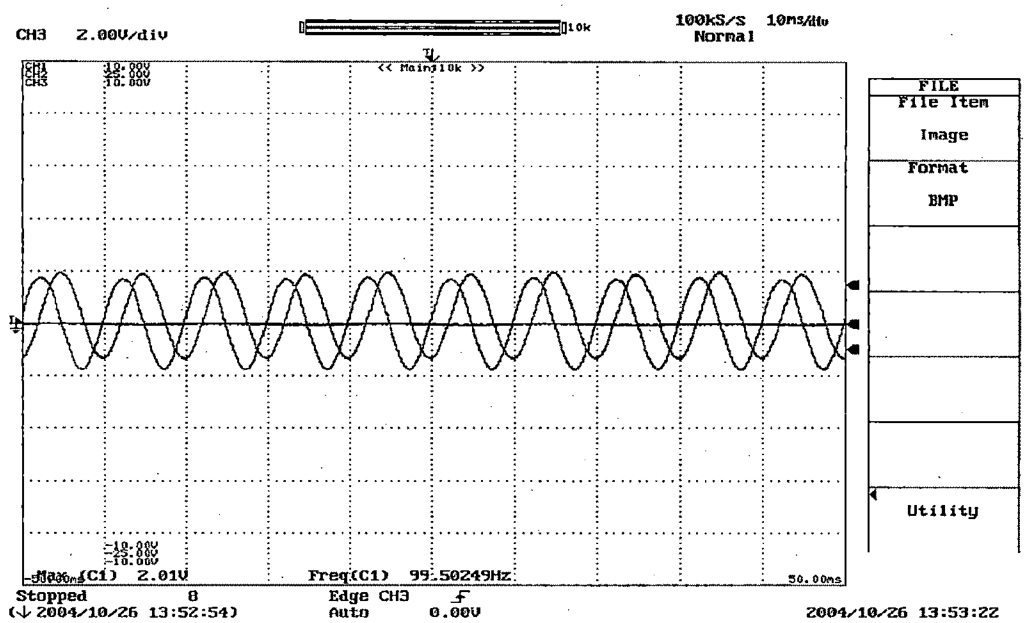


Figure 4.7.2.2-5: V_α & V_β for Input Voltages to Be Sensed

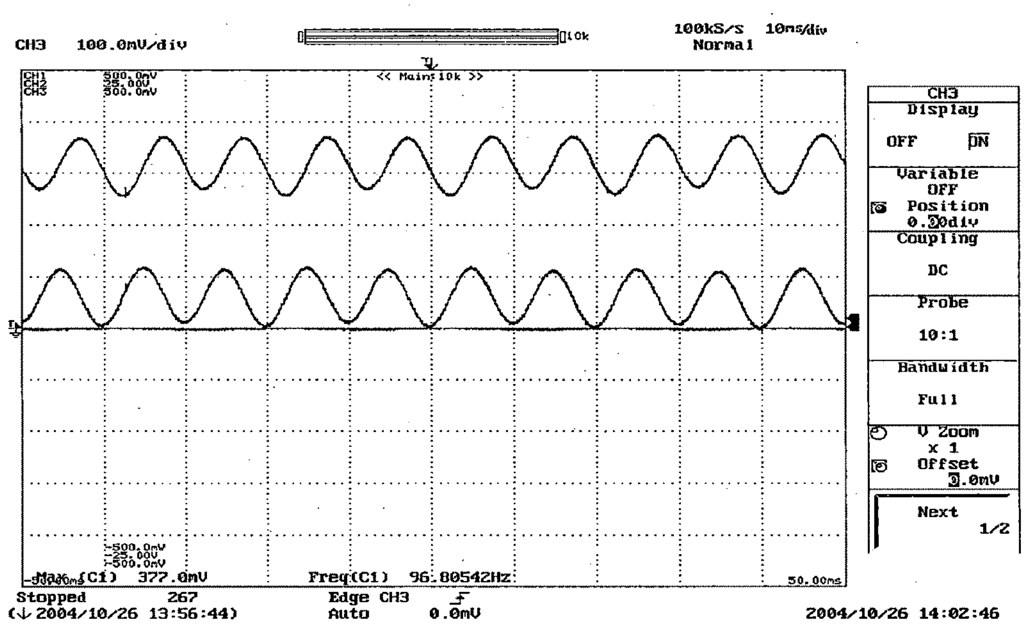


Figure 4.7.2.2-6: Active Power p & Reactive Power q In α-β Reference Frame

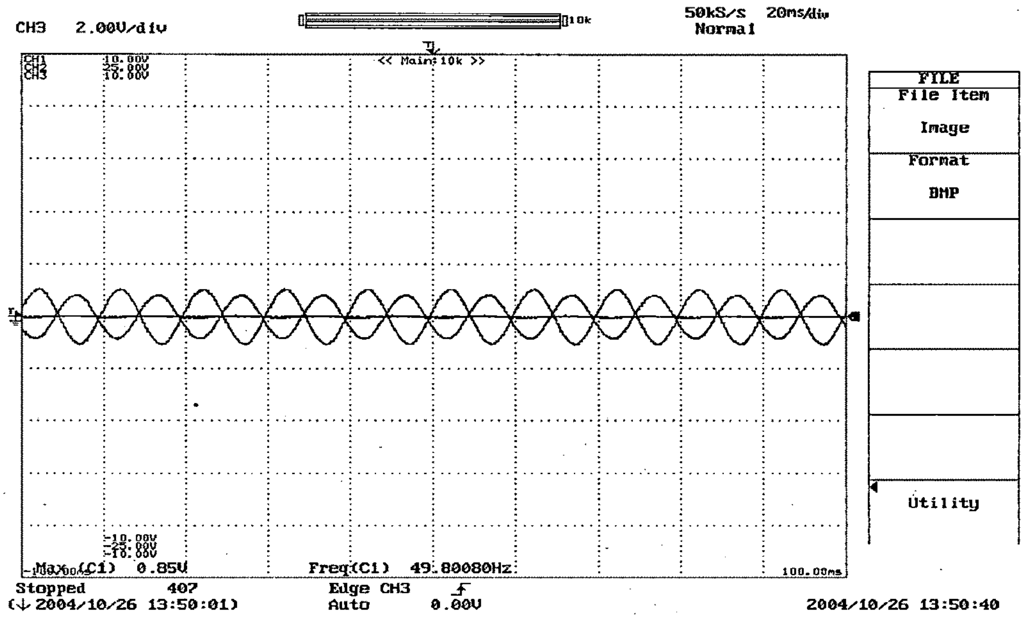


Figure 4.7.2.2-7: p_{ac} & q_{ac} in α-β Reference Frame after Filtration

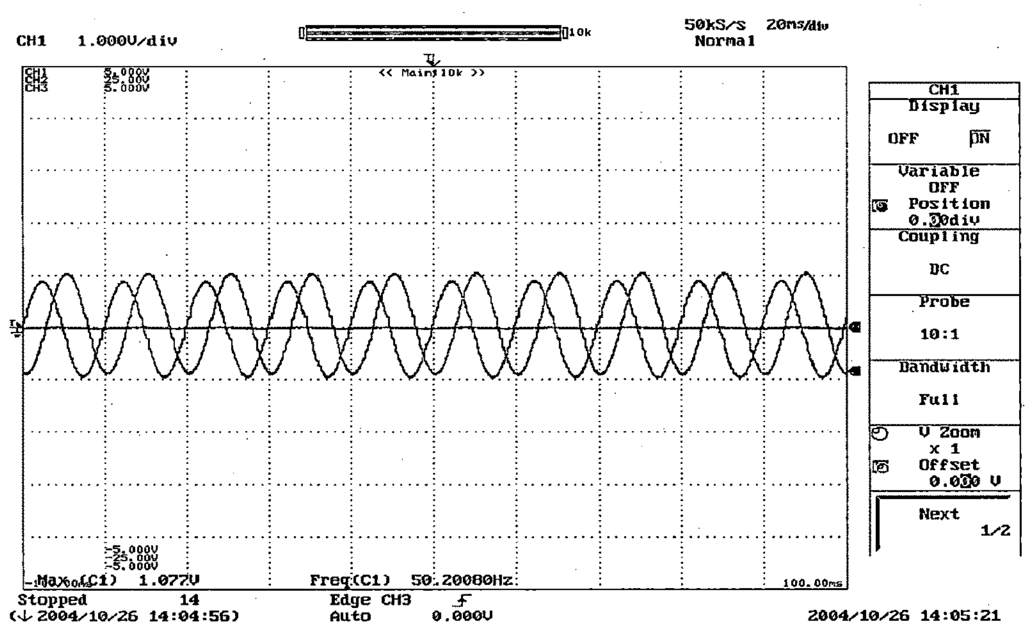


Figure 4.7.2.2-8: $V_{\alpha n}$ & $V_{\beta n}$ in α-β Reference Frame after Filtration

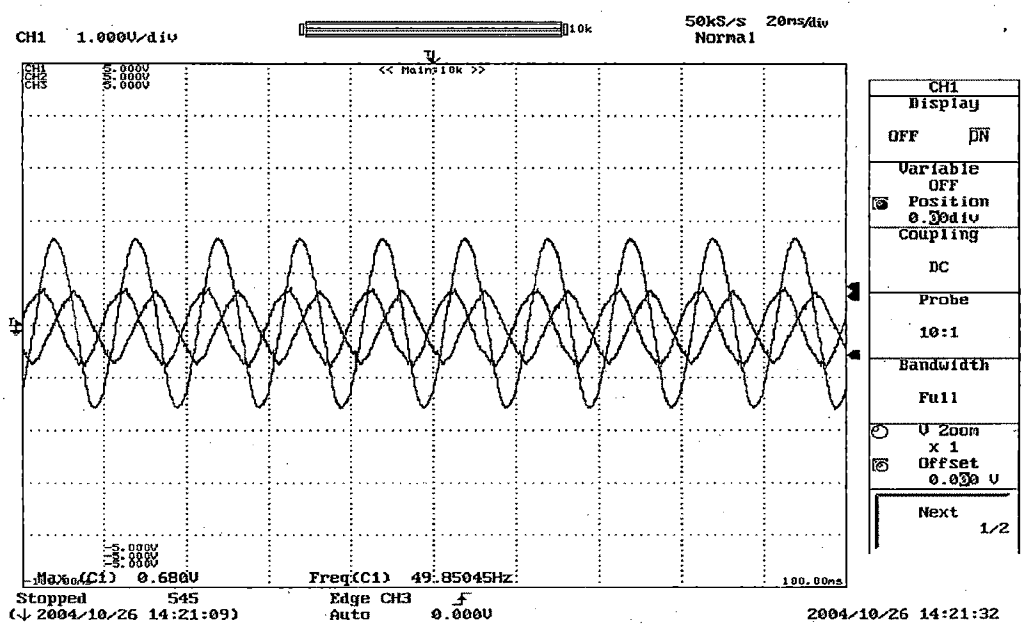


Figure 4.7.2.2-9: Compensating Reference Voltages

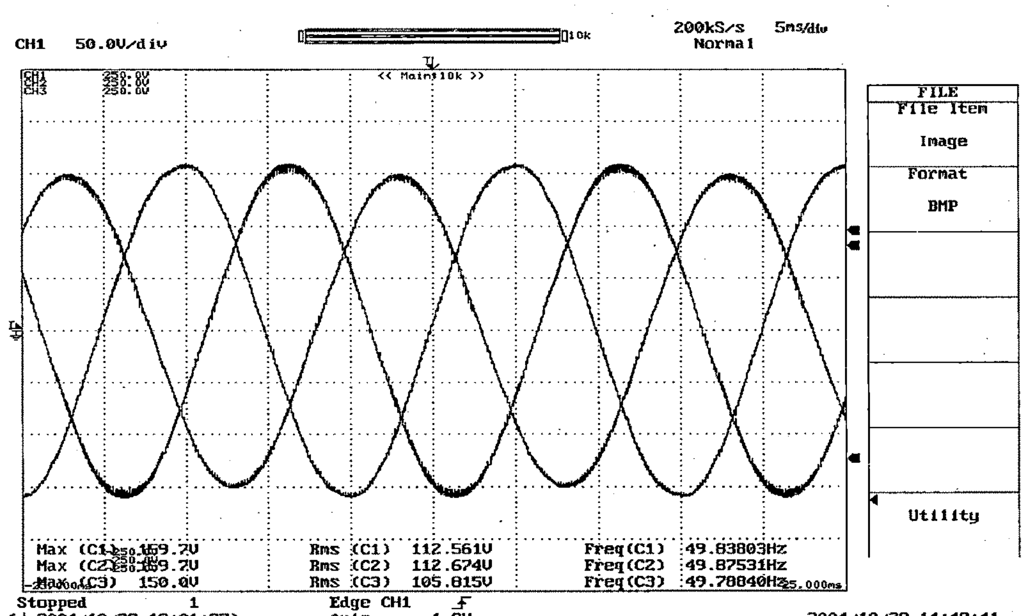


Figure 4.7.2.2-10: Three Phase Balance Load Voltage after Compensation

4.8 CONCLUSION:

Simulation results as well as experimentation results show that this new technique detect online negative & zero sequence component and compensate the same using voltage source inverter whose output changes through PWM techniques. Main advantage of detecting the negative & zero sequence component using instantaneous α-β-0 theory is that this can be easily implemented using DSP or analog circuit and no complex algebra is required which is the case using symmetrical components even with instantaneous symmetrical components as discussed in section 4.4.3 of this chapter. Also online instantaneous negative & zero sequence parameter of the bus voltage can be detected using this method. Negative sequence compensation is shown to be desirable for improved performance of 3-ϕ induction motor. The losses reduces due to compensation of negative sequence & zero sequence for 10 kW, 3 phase, 415 Volts, 50 Hz induction motor are given in following table:

	Input power in Watts	output Power in Watts	Total Losses in Watts	% Effi. of motor	Total Saving in Watts	% saving of input power
with balance voltage	11312	9822	1038	87		
with Unbalance voltage	10986	6858	3823	62	2785	28%