Chapter 7

Conclusions and Future Outlook

In this thesis the statistical Spectral Averaging Theory in Large Shell Model Spaces (SAT-LSS) for state, expectation value and strength densities is explored , developed further (for strength densities) and applied (for calculating level densities and β-decay rates) to establish that it is a viable interacting particle theory for understanding, analyzing and predicting statistical properties of nuclei. To this end several studies are carried out in the thesis.

In SAT-LSS, (i) using a significant unitary group decomposition under which a one plus two-body nuclear hamiltonian decomposes into orthogonal effective one-body part $\mathbf{h}$ and irreducible two-body part $\mathbf{V}$ ($H \rightarrow \mathbf{h} + \mathbf{V}$); (ii) applying the CLT's locally and (iii) decomposing the shell model space into distant non-interacting S - subspaces (for light nuclei S denotes $\hbar\omega$ excitation), the state densities take a convolution form, $I^H = \sum_S I^{\mathbf{h},S} \otimes \rho_G^{\mathbf{V},S}[E]$ where $I^{\mathbf{h}}$ is renormalized NIP state density and ρ_G is a normalized Gaussian density. Similar convolution forms are derived for spin-cutoff and occupancy densities. Firstly, systematic studies of two important aspects of the level density theory are carried out in the thesis and they are: (a) a part of the interaction that produces non-NIP like shifts of the single particle energies, that is neglected in the theory, is conclusively demonstrated (using norms of operators and considering large number of effective interactions) to be indeed small all across the periodic table; (b) moment methods for constructing locally smoothed forms for NIP state, spin-cutoff and occupancy densities which are one of the convolution factors in the IP theory, are comprehensively tested for the three densities and also for their S - decompositions in an example of 12 protons and 12 neutrons in a large shell model space consisting of 16 orbits (s, p, ds, fp, sdg and $1h_{11/2}$) with $0-4$ $\hbar\omega$ excitations. All the technical details of SAT-LSS are so far worked out in pn formalism and these results are employed in the above studies. In future it will be useful to develop the complete formalism in isospin form.

For a first systematic analysis of experimental data using the convolution forms for state and spin-cutoff densities, in a given region of the periodic table, fp-shell nuclei are chosen as the example. The reason for this choice is that there is significant amount of experimental data (at various energies) for several fp-shell nuclei. In addition, nuclei in this region of the periodic table have acquired new significance recently as some of them play an important role in nuclear astrophysics problems, double β-decay, problem of quenching of GT sum rule strengths etc. The calculations, using surface delta interaction with strength G, are carried out in large spaces by including configurations upto $2\hbar\omega$ excitations and by employing eight spherical orbits (ds, fp, $1g_{9/2}$) for the five nuclei ^{55}Mn, ^{56}Fe, ^{59}Co, ^{60}Co and ^{60}Ni and by extending for obvious reasons, to ten spherical orbits (ds, fp, $1g_{9/2}$, $2d_{5/2}$, $1g_{7/2}$) for the three $A > 60$ nuclei ^{62}Ni, ^{63}Cu and ^{65}Cu. In general the convolution form for the densities is seen to provide an excellent representation of the observed total level densities and spin-cutoff factors. The calculations determine a magnitude parameter of the interaction - the strength G which is found to be $\sim 20/A\ MeV$ for all the nuclei is compatible with the values deduced from low-energy spectroscopy. It will be interesting to extend the analysis using SAT-LSS to ds-shell nuclei and for these nuclei considerable amount of experimental data are available in literature [Ba-77]. In addition it will be useful to carry out calculations with two parameter SDI, P + Q.Q interaction and interactions derived from microscopic theories etc. and also construct fixed parity densities with two reference energies; these studies require new experimental data. All these studies will shed further light on the applicability of SAT-LSS in analyzing level density data.

Using some plausible arguments it was shown by French et al [Fr-88b] that the bivariate strength density $\mathbf{I}^{H}_{\mathcal{O}}(E_i, E_f)$ for a transition operator $\mathcal{O}$ takes a convolution form, $\mathbf{I}^{H}_{\mathcal{O}}(E_i, E_f) = \mathbf{I}^{\mathbf{h}}_{\mathcal{O}} \otimes \rho^{\mathbf{V}}_{BIV-\mathcal{G};\mathcal{O}}[E_i, E_f]$. Leaving aside the question of determining the five parameters of the spreading bivariate Gaussian due to $\mathbf{V}$, one needs good methods for constructing the NIP density $\mathbf{I}^{\mathbf{h}}_{\mathcal{O}}$ in large spaces. A complete formalism for constructing the exact NIP strength densities using spherical orbits is worked out for one-body transition operators. For rapid construction of the bivariate NIP densities and also to carry out S-decomposition, trace propagation formulas for bivariate moments M_{rs} with $r + s \leq 2$ are derived for unitary orbit partial densities which determine the bivariate Gaussian representation for the partial densities. In a large space numerical example with $GT(\beta^-)$ transition operator, the superposition of unitary orbit partial bivariate Gaussian densities is seen to give a good representation of the exact NIP strength densities. Trace propagation formulas for

M_{rs} with $r+s \leq 4$ are also derived in m-particle scalar spaces, which are useful for many purposes. The formalism developed for NIP strength densities for one-body transition operator is applied in the thesis to construct GT strength densities. In addition one can apply the formalism directly for a systematic study of electromagnetic (for example $E2$ and $M1$) transition strength sum rules and distributions. Although the present exercise is restricted to one-body transition operators, the methods and results extend to the case of half-body (single nucleon transfer) and two-body transition operators (relevant for TRNI studies). It is important to work out the technical details in these cases and also develop methods that yield approximate values (the methods that yield exact values will obviously be cumbersome to be used in practice) for higher order bivariate cumulants (K_{rs} with $r + s \geq 3$). With these developments and similarly for the bivariate spreading function with angular momentum incorporated, the bivariate convolution form can be employed commonly in analyzing wide variety of strength density data.

A method to calculate β-decay rates at finite temperature is developed by writing the expression for the rates explicitly in terms of bivariate GT strength densities ($\mathbf{I}^{H}_{\mathcal{O}(GT)}$) and state densities of parent nucleus besides having the usual phase space factors. The method developed in this thesis for constructing NIP strength densities is applied together with a plausible procedure for constructing the GT bivariate spreading Gaussian due to interactions. Recently there is considerable interest in evaluating β-decay rates for $A > 60$ fp-shell nuclei as these rates play a very important role in determining the structure of the core of massive supernova stars and hence on their subsequent evolution towards gravitational collapse and supernova explosion phases. Therefore as an example, using our method (which is based on SAT-LSS), β-decay rates for

$^{61,62}Fe$ and $^{62-64}Co$ isotopes are calculated at presupernova matter densities $\rho = 10^7 - 10^8$ gm/cc, temperatures $T = (3-5) \times 10^9$ ^oK and for various values of the electron fraction Y_e. The results of the β-decay rates calculations demonstrate that one can use the present method, with interactions, for meaningful calculations of β-decay rates. As a biproduct, the calculations produced a value (via observed β-decay half lifes), for the bivariate correlation coefficient $\bar{\zeta}$ between the hamiltonian and the GT operator to be $\bar{\zeta} \sim 0.67$. In addition an expression for the GT NEWSR strength is also deduced. It should be mentioned that the GT strength densities constructed using interactions in SAT-LSS can be used in electron capture rates calculations and with this it is possible to generate detailed and complete tabulation of β-decay and electron capture rates for various (ρ, T, Y_e) values for all neutron exceess fp-shell nuclei relevant for presupernova evolution calculations. Then they can be used, for example as carried out recently by Aufderheide et al [Au-94] using a tabulation they generated, in steller evolution codes and study the influence of the calculated rates on the evolution of presupernova stars. This study is beyond the scope of the present thesis. In addition, the GT strength densities have important applications in solar neutrino experiments [Od-86]. The present exercise which represents a first application, with interactions, of the bivariate convolution form for strength densities, with the bivariate densities constructed interms of partial bivariate moments, in nuclear astrophysics problems together with earlier applications to the problem of deriving bound on TRNI in nucleon-nucleon force [Fr-88b] and single particle transfer strengths [Po-91] show that the strength density formalism given by SAT-LSS is a viable tool for studying a wide variety of nuclear physics and nuclear astrophysics problems.

The studies carried out in the thesis provide a better understanding of SAT-LSS and establish that this theory provides a powerful formalism for analyzing and applying statistical properties of atomic nuclei. Some of the future studies in SAT-LSS should include:

1) A viable method/model, for incorporating $S = 0 \oplus 2 \oplus 4 \oplus ....$ mixing effects into the convolution forms for state, spin-cutoff, occupancy, GT and other strength densities etc. should be developed.

2) It is plausible to argue that [Fr-94] SAT-LSS is a natural theory in the 'quantum chaotic' domain of complex nuclei — good understanding of this aspect is called for. Although for many observables SAT-LSS extends below the region of onset of chaos (i.e. below the reference energy) it is essential to develop good methods for incorporating effects due to non-chaotic states (collective effects may be incorporated via further convolutions or otherwise).

3) One important result of the level density analysis of fp-shell nuclei is that they determine from the resonance (chaotic) domain data a value for a magnitude parameter of nuclear hamiltonian. Similarly the β-decay half lifes determine a value for the bivariate correlation coefficient between H and GT operators. The information thus deduced is found to be compatible with those deduced from low energy spectroscopy. *Thus it appears that the complex spectroscopy in the chaotic domain gives information about nuclear hamiltonian and opens up the subject of information propagation in complex systems.* This aspect of SAT-LSS requires further study.

4) With SAT-LSS one is in a position to deal, in a realistic approach with

problems in nuclear astrophysics. Much of the future of SAT-LSS in applications side, appear to be in carrying out large scale applications in nuclear astrophysics where one needs level densities, partition functions, orbit occupancies, β-decay rates etc.

5) IP strength density formalism with partitioning should be studied in more detail and this may pave way for avoiding some of the approximations that are used at present.

6) A usable theory for incorporating angular momentum in strength densities, perhaps in terms of trivariate and tetravariate distributions, should be worked out.

7) With SAT-LSS one has a powerful "statistical mechanical" method for dealing with large shell model spaces (the scope of complete shell model matrix diagonalization is extended recently from ds-shell [Br-88] to fp-shell upto $A = 48$ nuclei [Ca-94; see also Na-94]), there are very few alternative approaches available for the same. Comparisons with the recently introduced Monte Carlo method for the shell model [Ko-94d] may give new insights into SAT-LSS.

8) Various problems in nuclear structure at finite temperature [Al-91, Al-93 and references therein], pre-equilibrium reactions, heavy ion physics etc. can be studied using SAT-LSS and the principles/methods therein. Studies in these areas will enhance the scope of SAT-LSS.