CHAPTER V

THEORETICAL DEVELOPMENTS

5.1 GENERAL

Rock masses are characterised by presence of discontinuities. Movement along the discontinuities govern the behaviour of the rock mass. Hence mechanical behaviour of a rock mass is nothing but its sliding behaviour. Study of friction is therefore of great importance in rock machanics. The classical concept of friction is developed for metals. As a result most of the investigations on friction were based on the frictional behaviour of either flat or non-interlocking rough surfaces. Discontinuities in rock mass are rough and interlocking. The classical concept of friction is therefore not directly applicable to rock joints. During sliding along discontinuities volume change takes place. This phenomenon of dilation must be considered while evaluating sliding processes.

5.2 MECHANISM OF SLIDING IN JOINTED ROCKS

5.2.1 The basic laws of friction are :

(i) Tangential force is directly proportional to the normal force.

(ii) Coefficient of friction is independent of contact area.

(iii) Static coefficient of friction is greater than kinetic friction.

(iv) Coefficient of friction is independent of sliding speed.

5.2.2 Resistance against relative movement along the planes of separation in a rock mass is generally referred to as friction. In this context, the term friction is used for relative movement along pre-existing planes. Different research workers have put forward different views regarding phenomenon of friction in rocks. Kragelskii (1965) explained friction in terms of lifting of microasperities over each other. Bowden and Tabor (1967) explained friction as a result of overcoming the forces of molecular attraction between the two solids. Some researchers visualized friction arising from the deformation of certain amount of material which is penetrated on one solid by the asperity of the other solid, whereas a composite theory represents friction as resulting from interlocking of the surface roughness and lifting of microasperities over each other. Byerlee (1966) considered that tips of asperities which are subjected to a normal force, crush to a certain extent under the action of the normal force, and on application of shear force, local tensile stresses develop on the tip of the asperities which exceed the tensile strength. Byerlee's theory, however, does not take into account interlocking of asperities. Patton (1966) studied the influence of asperities and the phenomenon of interlocking of asperities on the failure envelopes. He postulated that failure envelope has to be represented by two straight lines. Inclination of the upper or the secondary portion is very close to ϕ_r - the residual strength - and that of primary portion is very close to (ϕ_μ + i) where ϕ_μ is the basic friction angle and i is the angle of inclination of asperity. Einstein et al (1970) explained the influence of asperities and the phenomenon of interlocking in rock friction. According to them the two surfaces will normally be not in plane contact but will interlock where certain portions are in tip to tip contact but major portions will be staggered (Fig. 5.1). At small to medium values of normal load the asperities slide over each other and the shear resistance can be represented by the equation,

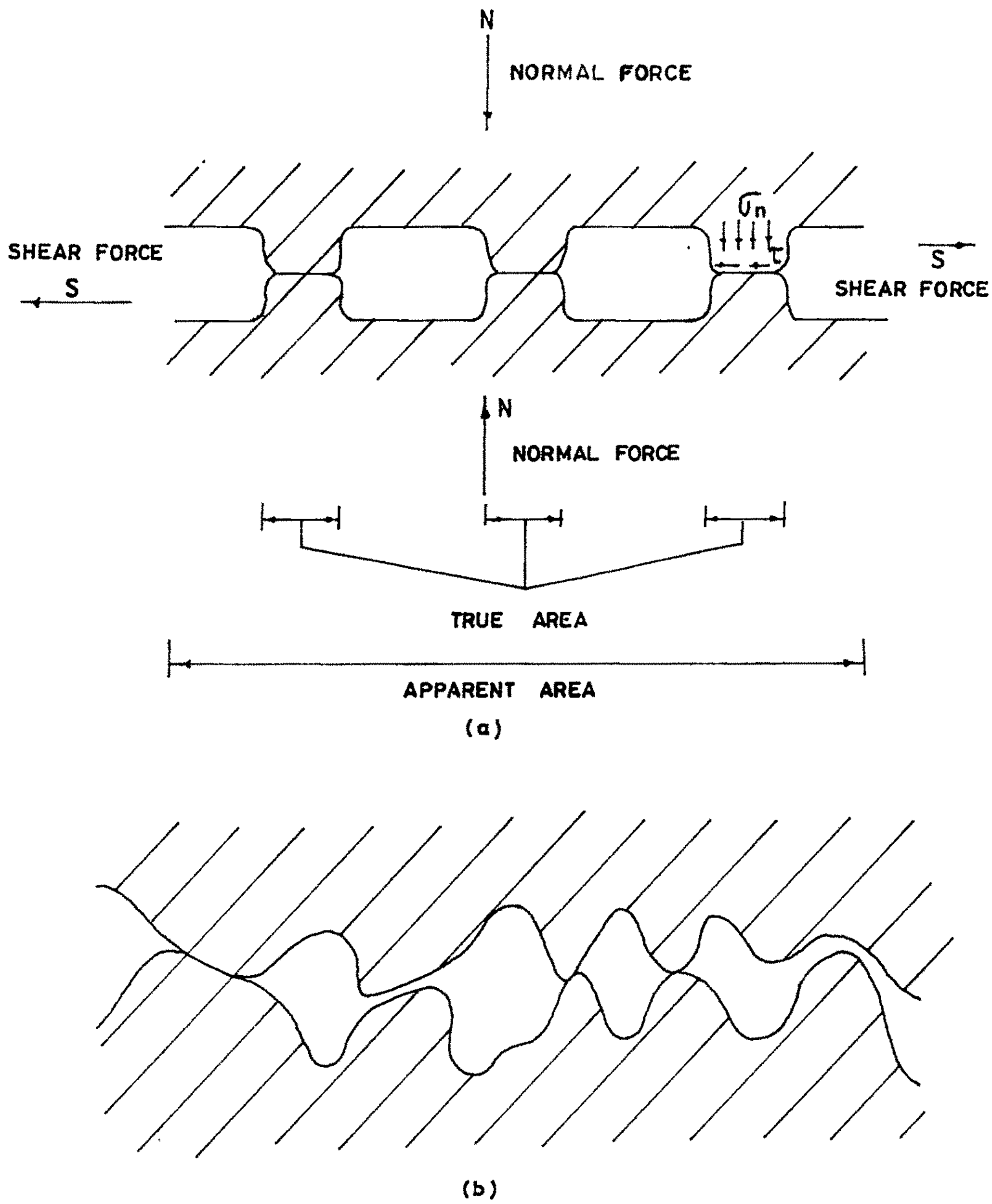

FIG: 5·1 (a) CONTACT OF ASPERITIES (b) INTERLOCKING OF ASPERITIES

(Einstein, Bruhn and Hirschflld 1970)

$$S = N \tan (\phi_{\mu} + i) \qquad (5.1)$$

After a pair of interlocked asperities have ridden over upto a certain level the stresses in the asperities will reach the strength of the asperity and the asperity will shear off at this level. This stage is represented by the equation,

$$S = K + N \tan \phi \qquad (5.2)$$

The riding over of the asperities gives rise to changes in the value of deformation at right angles to the direction of application of shear force which has been termed as dilatancy. A schematic representation of dilatancy is shown in Fig. 5.2. According to Rowe, Bardon and Lee (1964) the shear force S may be divided into 3 components,

$$S = S_1 + S_2 + S_3 \qquad (5.3)$$

where, S_1 = shear force component due to external work done in dilating against the external force N.

S_2 = shear force component due to additional internal work done in friction due to dilatancy.

S_3 = shear force component due to work done in internal friction if the specimen does not change in volume in shear.

Ladyani and Archambault (1969) carried the argument further stating that in shearing along an irregular rock surface, there is the fourth component which occurs as a result of the shearing of the teeth and the value of this component (S_4) may be determined by assuming that all the teeth are sheared off at the base. Thus,

$$S_4 = AK + N \tan \phi_o \qquad (5.4)$$

where, A is the total projected area of the teeth at the plane

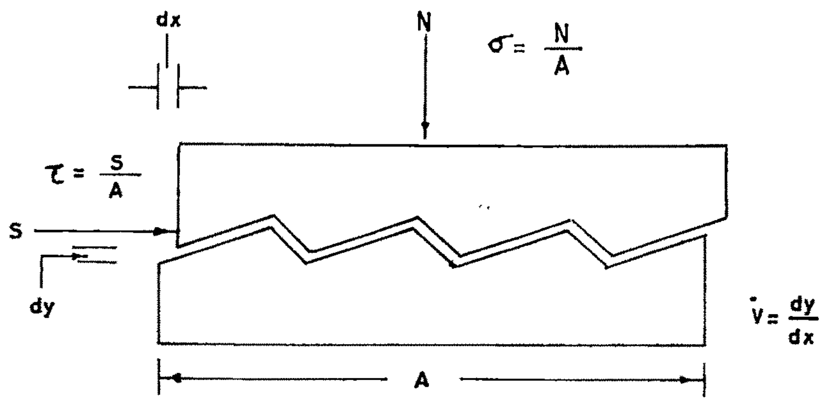

FIG: 5·2 SCHEMATIC REPRESENTATION OF DILATANCY

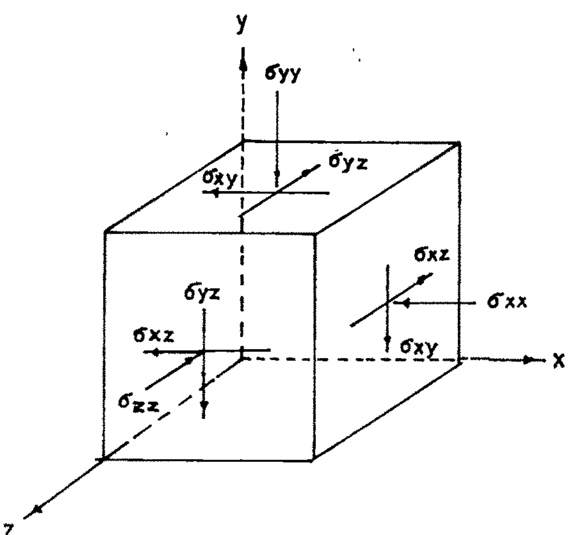

FIG: 5·3 SIGN CONVENTION FOR STRESS – COMPRESSION POSITIVE of shear and K and Øo are the Coulomb shear strength parameters for rock substance.

5.2.3 Thus, it is seen that sliding of rock blocks over pre-existing joints is different from sliding of one body over the other. There is no dilation or volume change when two bodies slide on plane surfaces. This type of behaviour can be represented by a saint venant body, whereas, in sliding of rock blocks over joints, volume change (dilation) is involved due to presence of asperities (roughness) on the sliding surfaces. As a consequence of the roughness of the surfaces, the contact area between the two surfaces along a joint is always discrete, i.e. it occurs at individual points. These points in contact are deformed on application of normal load. This increases the number of discrete contact points. Depending upon the angle of inclination of the asperities (i.e. shape of asperities), the distribution of the asperity height and mechanical properties of the material, some of the asperities shall be deformed elastically, plastically or crushed and hence the area of contact will go on changing non-linearly with increase in load. It is therefore not possible to apply the classical concept of friction in rock joints.

5.3 PHENOMENON OF VOLUME CHANGE DURING SHEARING

5.3.1 Only recently, measurement of volume changes associated with rock disintegration became of particular interest in rock mechanics. Experiments have indicated that volume increases with progressive failure. This effect is generally called dilation, dilatation or dilatancy. Under low confining pressure, dilation may reach a considerable amount. It is believed that it results from crack development during increasing axial deformation. In case of sliding along a joint, the riding over of the asperities gives rise to changes in the value of normal deformation which is termed as dilatancy. The term dilatancy is used to indicate thickening of a joint, that is an increase in the separation of the two joint blocks. Dilation can also be looked upon as a consequence of change in structure of the sliding surfaces. Dilatancy relates normal strains to shear strains. A dilatant joint tested in direct shear set up under condition of constant normal deformation will have a higher friction angle than one tested under constant normal stress. This leads to the introduction of dilatancy in analysis through an adjustment of friction angle. However, dilatancy also affects the amount of deformation of a joint and hence a stress analysis will not be realistic merely by adjusting the friction angle. The model of sliding of jointed rock should therefore include the volume change or dilational parameter.

5.3.2 In order to derive some basic equations relating stresses and strains, Poisson's ratio and principal strains etc. the sign convention for stresses shown in Fig. 5.3 is used. As compression is more common than tension in Geotechnical problems, normal stress components are considered positive when they are compressive. Similar convention is adopted for strains. It will be noted that there are two conventions for shear strains, the γ's and the ε's, the former are called the engineering shear strains. They are most useful for experimental work. The later are called the tensorial shear strains and are more useful for theoretical derivations. From Fig. 5.4 it can be seen that the engineering definition of shear strain includes some rotation but the tensorial definition describes a plane deformation. The reason for defining a tensorial strain is that the components of complete set of strains then become a tensor, which obeys all the transformation laws. It has invariants, principal values and so on. The two strains are related as

$$\gamma_{ij} = 2\,\varepsilon_{ij} \qquad (5.5)$$

5.3.3 In analyzing the conventional direct shear test and

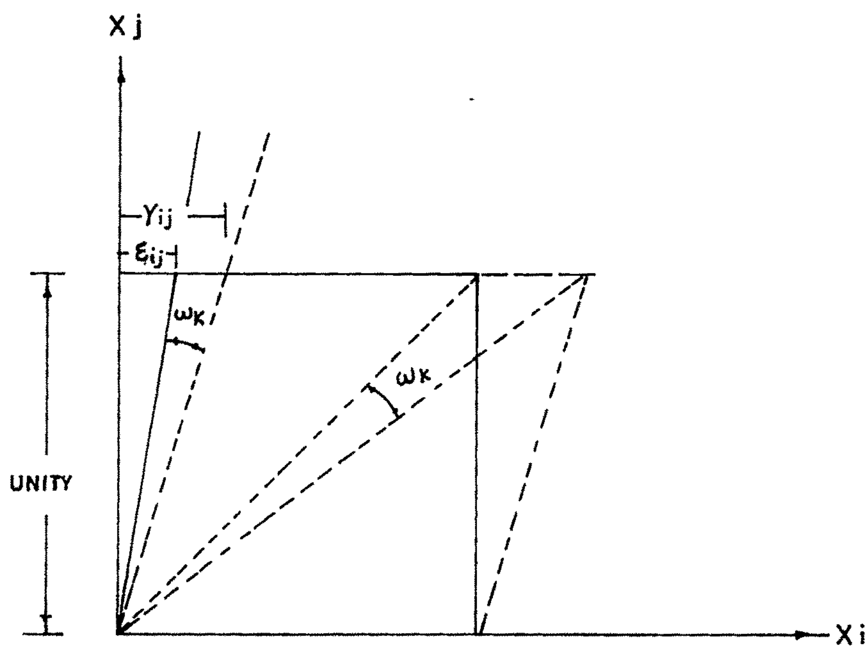

FIG 5.4 ENGINEERING AND TENSORIAL STRAINS

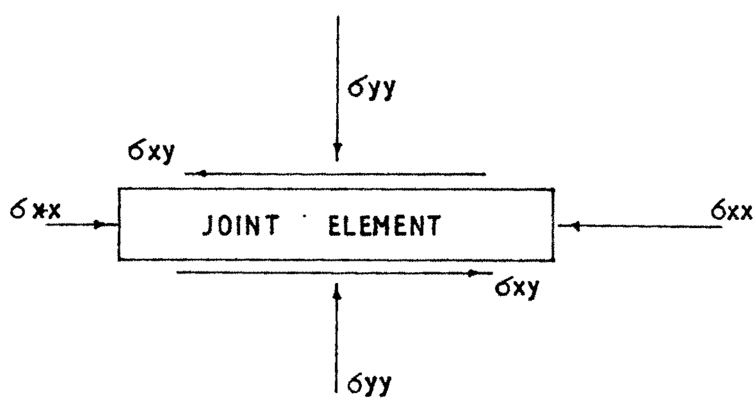

FIG: 5.5 PRESENTATION OF STRESSES ON A JOINT ELEMENT IN DIRECT SHEAR TEST for the purpose of defining parameters for a joint element the stresses and strains in the joint plane are only required. Such a joint element in plane stress condition is shown in Fig. 5.5. Using the sign convention shown in Fig. 5.3, the normal stress σ_{yy} is considered positive, σ_{xx} is zero and σ_{xy} is positive. Similarly ε_{yy} is considered positive if compressional and negative if dilational (increase in volume) ε_{xx} is zero and ε_{xy} is positive.

The usual way of presenting dilational parameter is in the form of ratio of vertical (normal) displacement to horizontal (shear) displacement. However, this is not invariant. It is, therefore, proposed to express it in the form of ratio between principal strains. For a 2D plane stress joint element shown in Fig. 5.5, the principal strains can be arrived at as under :

$$\varepsilon_1 = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2} \qquad (5.6)$$

$$\varepsilon_2 = \varepsilon_{zz} = 0 \qquad (5.7)$$

$$\varepsilon_3 = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2} \qquad (5.8)$$

For the 2D joint element shown in Fig. 5.5, $\varepsilon_{xx} = 0$. Therefore, the above equations reduce to,

$$\varepsilon_1 = \frac{\varepsilon_{yy}}{2} + \sqrt{\left(\frac{\varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2} \quad \text{and} \qquad (5.9)$$

$$\varepsilon_3 = \frac{\varepsilon_{yy}}{2} - \sqrt{\left(\frac{\varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2} \qquad (5.10)$$

The ratio of principal strains for a continuum in classical mechanics terminology is known as Poisson's ratio ν. Similarly, ratio of principal strains for a joint element worked out from above equations is designated as Poisson's ratio for the joint element. Thus,

$$\nu \text{ for a joint element} = \frac{\varepsilon_3}{\varepsilon_1} = \frac{\varepsilon_{yy}/2 - \sqrt{(\varepsilon_{yy}/2)^2 + (\varepsilon_{xy})^2}}{\varepsilon_{yy}/2 + \sqrt{(\varepsilon_{yy}/2)^2 + (\varepsilon_{xy})^2}} \quad (5.11)$$

It is obvious from equation 5.11 that the value of Poisson's ratio (ν) for a joint element shown in Fig. 5.5 is less than unity under compressional mode, it approaches unity when the joint just strats dilating ($\varepsilon_{yy} = 0$) and it is more than unity under dilational (opening) mode.

5.3.4 For the purpose of working out invariants of stresses and strains following definitions are followed :

(a) Invariant of stress tensor :

$$J_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = tr(\sigma) \quad (5.12)$$

$$J_2 = 1/2(I_1^2\sigma - 2I2\sigma) = 1/2\ tr(\sigma)^2 \quad (5.13)$$

$$= 1/2[(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})^2 - 2(\sigma_{xx}.\sigma_{yy} - \sigma_{xy}^2 + \sigma_{yy}.\sigma_{zz} - \sigma_{yz}^2 + \sigma_{xx}.\sigma_{zz} - \sigma_{zx}^2)] \quad (5.14)$$

$$\therefore J_2 = 1/2[(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + 2\sigma_{xy}^2 + 2\sigma_{yz}^2 + 2\sigma_{zx}^2)] \quad (5.15)$$

$$\therefore J_2 = \frac{\sigma_{xx}^2}{2} + \frac{\sigma_{yy}^2}{2} + \frac{\sigma_{zz}^2}{2} + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2 \quad (5.16)$$

For 2D joint element of Fig. 5.5,

$$\sigma_{xx} = \sigma_{zz} = 00 \quad \text{and} \quad \sigma_{yz} = \sigma_{zx} = 0$$

$$\therefore J_1 = \sigma_{yy} \quad \text{and} \quad (5.17)$$

$$\therefore J_2 = \frac{\sigma_{yy}^2}{2} + \sigma_{xy}^2 \quad (5.18)$$

$$\text{Thus,} \quad \frac{J_2}{J_1^2} = \frac{\sigma^2_{yy}/2 + \sigma_{xy}^2}{\sigma_{yy}^2} \tag{5.19}$$

$$\therefore \quad \frac{J_2}{J_1^2} = 1/2 + \frac{\sigma^2_{xy}}{\sigma_{yy}^2} \tag{5.20}$$

$$\therefore \quad \frac{J_2}{J_1^2} = 0.5 + \tan^2 \phi \tag{5.21}$$

$$\therefore \quad \frac{J_2}{J_1^2} - 0.5 = \tan^2 \phi \tag{5.22}$$

Similar equations can be developed for strains

$$\text{where,} \quad I_1 = \varepsilon_{yy} = \varepsilon_{vol.} \tag{5.23}$$

$$I_2 = \frac{\varepsilon_{yy}^2}{2} + \varepsilon_{xy}^2 \quad \text{and} \tag{5.24}$$

$$\frac{I_2}{I_1^2} = 0.5 + \frac{\varepsilon_{xy}^2}{\varepsilon_{yy}^2} \tag{5.25}$$

In other words sliding takes place when J_2/J_1^2 approaches a particular value. Sliding (J_2/J_1^2) is dependent on characteristics of sliding surfaces. If the surfaces are perfectly plane, then sliding will take place without any volume change, i.e. rigid plastic movement (ideal Saint Venant body). Equation 5.22 will therefore yield basic friction value ϕ_μ. If the surfaces are not plane which is the case with rock joints, then sliding will be associated with volume change. Therefore Saint Venant body is required to be modified. Equation 5.22 will then yield a value of (ϕ_μ + i). The implication of volume change would be that the ratio J_2/J_1^2 will increase depending on the surface roughness.

Volumetric strain is defined as,

$$\varepsilon_v = I_1$$

$$= \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \quad (5.26)$$

Therefore, for the joint element of Fig. 5.5, volumetric strain,

$$\varepsilon_v = I_1 = \varepsilon_{yy} \quad (5.27)$$

5.3.5 From the above discussion it is clear that as the shear stress on a joint increases, both Poisson's ratio for the joint and the volumetric strain increases. Thus, it can be said that sliding phenomenon is associated with change in Poisson's ratio of the joint. Thus, dilatancy can be represented by state of Poisson's ratio of the joint. The volume change is a consequence of resistance to shearing. Basic friction is the resistance to shearing without volume change. During sliding on rough surfaces there occurs a volume change due to the resistance beyond the basic friction. Hence this volume change is due to inherent surface imperfections and resultant surface imperfections.

5.4 CONSTITUTIVE RELATIONSHIP

5.4.1 A constitutive law represents mathematical model that describes behaviour of a material. In other words it is a mathematical model that can permit reproduction of the observed response of a continuous medium. Establishment of constitutive equations can be based on the experimental observations or on physical theories of molecular behaviour. The first approach can impart physical significance in engineering and physical sciences. In formation of a constitutive law the first stage is the identification of the relevant constitutive variables for a given material. Once such variables or parameters are identified, it is necessary to know the relationships among these variables.

5.4.2 Stresses and strains are connected through stress-strain laws which are also known as constitutive laws. A simple constitutive law is Hooke's law defining linear elastic behaviour which can be written as :

$$\sigma = E\varepsilon \tag{5.28}$$

when, is the stress, the strain and E the response parameter commonly known as the Young's modulus. In general form the stress - strain relation is

$$\{\sigma\} = [C]\{\varepsilon\} \tag{5.29}$$

where, [C] is the stress-strain matrix. For a linear elastic and isotropic material it is generally composed of two parameters, Young's modulus E and Poisson's ratio . Varieties of linear and non linear stress-strain relations are used in geotechnical engineering. The basic relations between stress and strain are those of linear elasticity. For most numerical purposes, it is most useful to write the stresses and strains as vectors :

$$\{\sigma\}^T = \{\sigma xx \quad \sigma yy \quad \sigma zz \quad \sigma xy \quad \sigma yz \quad \sigma zx\} \tag{5.30}$$

$$\text{and}\{\varepsilon\}^T = \{\varepsilon xx \quad \varepsilon yy \quad \varepsilon zz \quad \varepsilon xy \quad \varepsilon yz \quad \varepsilon zx\} \tag{5.31}$$

When plane stress or plane strain conditions apply, these vectors can be reduced to :

$$\{\sigma\}^T = \{\sigma xx \quad \sigma yy \quad \sigma xy\} \tag{5.32}$$

$$\text{and}\{\varepsilon\}^T = \{\varepsilon xx \quad \varepsilon yy \quad \varepsilon xy\} \tag{5.33}$$

Since there are 6 independent components of stress and 6 of strain, 36 coefficients are needed to relate them linearly in the most general way. However, considering the energy stored in a strained linearly elastic body one can show that the coefficients must form a symmetric array. This reduces the number of independent terms to 21. The stress-stain matrix [C] of equation 5.29 will have a form :

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ \text{Sym.} & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix} \tag{5.34}$$

For a plane stress or plane strain condition the number of independent terms reduces to 6. Therefore the [C] matrix becomes,

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ \text{Sym.} & & C_{33} \end{bmatrix} \tag{5.35}$$

The most useful form of [C] matrix for a 3D problem is,

$$[C] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \tag{5.36}$$

For conditions of plane stress

$$[C] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \qquad (5.37)$$

and for condition of plane strain

$$[C] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \qquad (5.38)$$

The relation of equation 5.29 can also be written in the inverse form :

$$\{\varepsilon\} = [D]\{\sigma\} \qquad (5.39)$$

where, [D] for a plane stress case is

$$[D] = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \qquad (5.40)$$

and for a plane strain case

$$[D] = \frac{1+\nu}{E} \begin{bmatrix} 1-\nu & -\nu & 0 \\ -\nu & 1-\nu & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad (5.41)$$

In the formulation of this constitutive law, E and are considered as constants.

5.4.3 Geotechnical materials and especially jointed rocks do not exhibit linear behaviour. Hence in order to represent Geotechnical problems realistically it is imperative that the non-linearity in material behaviour has to be introduced.

Various available schemes for defining the constitutive behaviour of such geotechnical non-linear materials can be divided into three main groups (i) representation of given stress-strain curve by using curve-fitting methods, interpolation, or mathematical functions, (ii) non-linear elasticity theories, and (iii) plasticity theories.

The simplest type of nonlinear relation is the bilinear one illustrated in Fig. 5.6. The material has initial moduli [Ci] until the stress reaches a yield value σy, after which the moduli is changed to [Cy]. The usual way to develop bilinear relations is to change the Young's modulus from an initial value Ei to a yielded value Ey. Alternatively the nonlinear curve can be divided into a number of linear curves leading to the so-called multilinear or piecewise linear models (Fig. 5.7). This method gives satisfactory and reliable answers as does the use of mathematical functions such as polynomials, hyperbolas - parabolas and splines. An advantage of the use of mathematical functions is that we need only a few parameters to describe the curves. Use of mathematical functions essentially constitutes the piecewise linear approach.

The widely used function for simulation of stress strain curves in finite element analysis is the Hyperbolic relation illustrated in Fig. 5.8, which can be stated in equation form as :

$$\sigma = \frac{\varepsilon}{b + a\varepsilon} \qquad (5.42)$$

OR

$$\frac{\varepsilon}{\sigma} = b + a\varepsilon$$

where, $\frac{i}{b}$ is the initial Young's modulus and $\frac{1}{a}$ is the compressive strength.

The Ramberg - Osgood function can provide an alternative simulation procedure. This model can be expressed as,

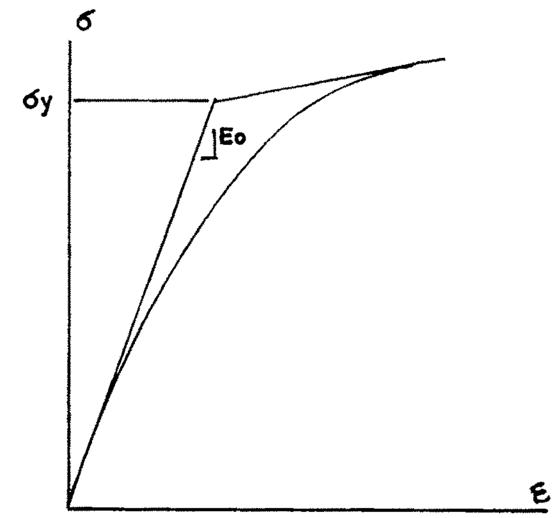

FIG 5.6 BILINEAR MODEL FOR NONLINEAR MATERIAL

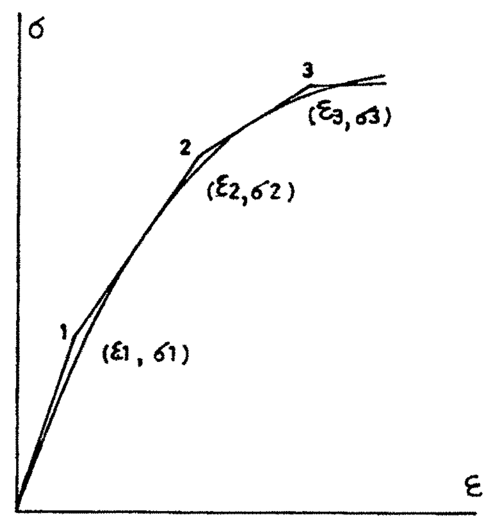

FIG: 5·7 MULTILINEAR MODEL FOR NONLINEAR MATERIAL

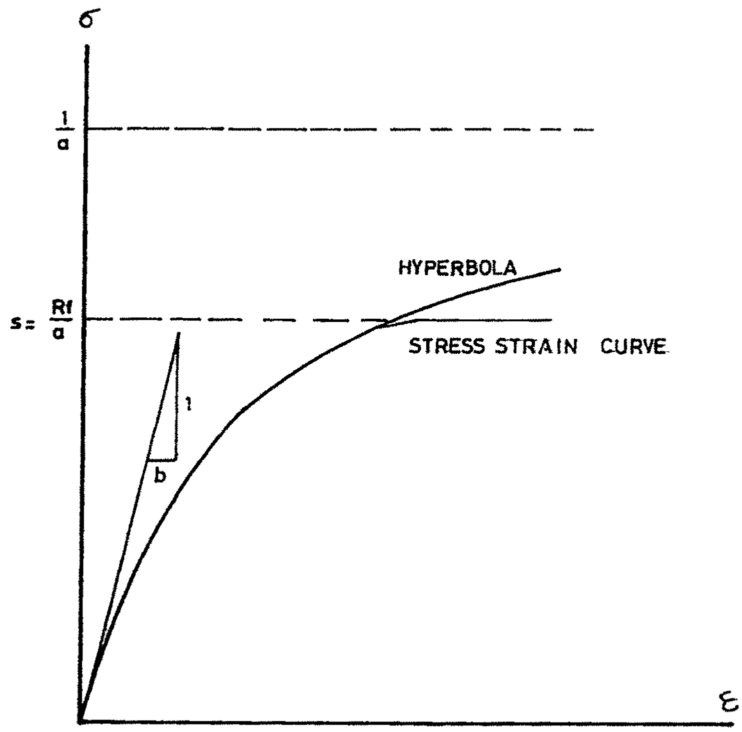

FIG 5·8 HYPERBOLIC MODEL

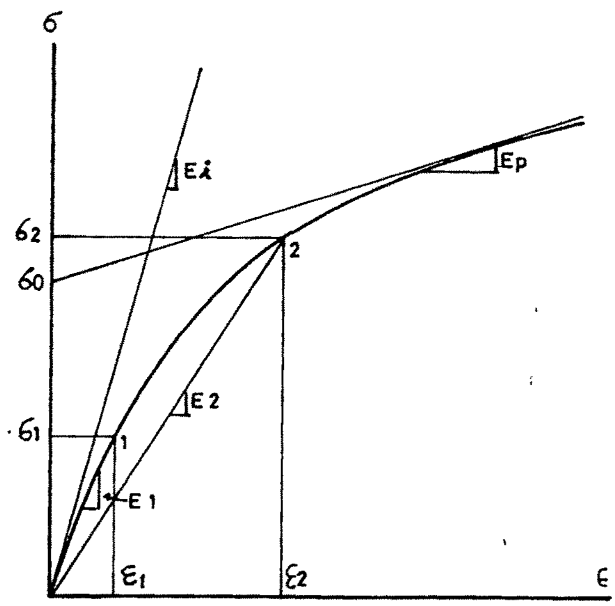

FIG: 5·9 RAMBERG OSGOOD MODEL

$$\varepsilon = \frac{\sigma}{E_i} + \lambda \left(\frac{\sigma}{E_i}\right)^m \qquad (5.43)$$

where, $$\lambda = \left(\frac{1}{\theta_2} - 1\right) \left(\frac{\sigma 2}{E_i}\right)^{1-m}$$

Here m is an exponent defining the shape of the curve and θ_2 is the ratio E2/E1 (Fig. 5.9).

The approach of spline function uses cubic and bicubic splines for simulation of a set of stress - strain data (Fig. 5.10). The spline function provides continuous first and second derivatives and hence is formed to provide better simulation of curves compared with that given by hyperbola.

If the stress - strain data are summarized by a set of (n + 1) pairs of stress - strain data, it is easy to fit an n^{th} order polynomial through the points.

The stress - strain laws based on the generalized Hooke's law represent the lowest order of the higher order elasticity models that have been used in Geotechnical Engineering. It is possible to employ higher order elastic or hyperelastic and hypoelastic constitutive laws for description of the behaviour of geologic materials allowing incorporation of a number of factors that cannot be accounted for by the piecewise linear approximation discussed earlier. The hyperelastic models rely on finding constitutive laws by differentiation of a strain - energy function. Different orders of hyperelastic models can be obtained by retaining different order terms. In the hyperelastic law, the stresses are expressed as functions of strains whereas in hypoelastic formulation rate of stress is expressed as a function of rate of deformation.

The hyperelastic and hypoelastic models require number of material parameters to be determined from representative

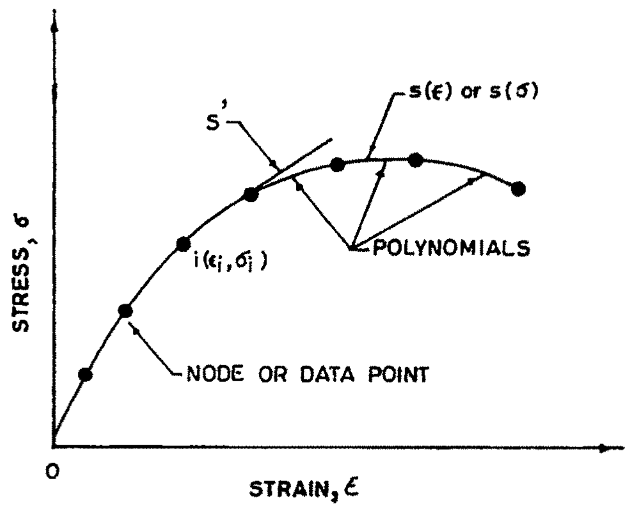

FIG 5·10 SPLINE FUNCTION MODEL

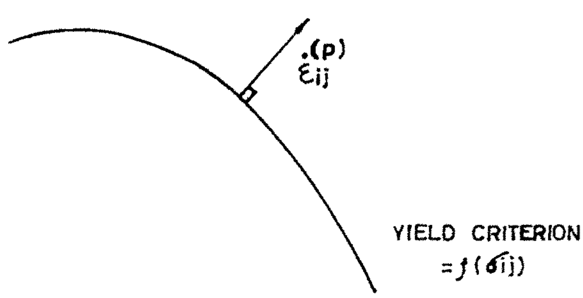

FIG: 5·11 ASSOCIATED FLOW RULE FOR PLASTICITY laboratory tests. It usually requires curve fitting and regression analysis to determine the parameters from a set of laboratory tests. Often the question of uniqueness arises as it is possible to fit more than one set of parameters to a set of laboratory data. Therefore further research in hyperelastic and hypoelastic approaches is needed before they can be reliably applied.

5.4.4 All the constitutive relations described so far relate stress directly to the strain even though they may be expressed in the form of an instantaneous or tangent modulus. The relations arising from plasticity theory are usually incremental, i.e. the stresses and strains are related entirely by their incremental or differential behaviour. A central concept in plasticity theory is the theory of plastic potential and the associated flow rule. This states that when a material is in the plastic state, the differential increments of strain are proportional to the outward normal to the yield criterion. Fig. 5.11 illustrates this. The rule can also be expressed by stating that the strain increments are proportional to the gradient of the yield criterion,

$$\dot{\varepsilon}_{ij}^{(P)} = \lambda \frac{df}{d\sigma_{ij}} \qquad (5.44)$$

In this equation λ is a factor of proportionality, and $\dot{\varepsilon}_{ij}^{(P)}$ represents the plastic strain increment. Although the dot connotes a time derivative (or rate) and this term is often called the strain rate, it is not really a rate because no time derivative is involved. It is instead a differential increment. The nonfrictional perfect plasticity criterion ignores the fact that the geotechnical materials do have frictional components to their shear strengths. Drucker-Prager suggested an incremental stress - strain law based on the frictional criteria as under :

$$\{d\sigma\} = [C^{ep}]\{d\varepsilon\} \qquad (5.45)$$

where, $[C^{ep}]$ is the elastoplastic constitutive matrix expanded in Table 5.1. In the frictional criteria with associated flow rules, yielding actually occurs well below the failure envelope of the Mohr Coulomb equation. To overcome this difficulty capped yield models have been proposed. The theory has been subject to extensive research. Desai et al (1986) have proposed a Hierarchical concept for the development of constitutive models to account for various factors that influence behaviour of geologic materials. It permits evolution of models of progressively higher grades from the basic model representing isotropic hardening with associative behaviour. Factors such as non-associativeness, induced anisotropy due to friction, cyclic loading and softening are introduced as corrections or perturbations to the basic model.

5.5 A NEW APPROACH INCORPORATING DILATANCY

5.5.1 A New Constitutive Relationship

It is seen from the above discussion that constitutive laws for geologic materials have been defined in number of ways following different concepts. However, very few of them allow inclusion of dilatancy. Such models are very complex and need determination of a number of material parameters which make them unwieldy. On the other hand it is also seen that during sliding on rock joints having asperities, structural changes take place continuously which are represented by changes in Poisson's ratio of the joint element. Thus Poisson's ratio can be considered as a dilation parameter for formulation of a constitutive relationship. A new approach, directly incorporating this dilation parameter in terms of Poisson's ratio is therefore developed and proposed. This approach is simple and accomodates both changes in Poisson's ratio and associated changes in Young's modulus in stages.

TABLE 5.1 : STRESS-STRAIN MATRIX FOR DRUCKER-PRAGER MATERIAL MODEL

$$
\begin{Bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{23} \\ d\sigma_{13} \end{Bmatrix}
= 2G
\begin{bmatrix}
1-T_1\sigma_{11}-R_1 & -(T_1\sigma_{22}+R_1) & -(T_1\sigma_{33}+R_1) & -(T_1\sigma_{12}) & -(T_1\sigma_{23}) & -(T_1\sigma_{13}) \\
-(T_2\sigma_{11}+R_2) & 1-T_2\sigma_{22}-R_2 & -(T_2\sigma_{33}+R_2) & -(T_2\sigma_{12}) & -(T_2\sigma_{23}) & -(T_2\sigma_{13}) \\
-(T_3\sigma_{11}+R_3) & -(T_3\sigma_{22}+R_3) & 1-T_3\sigma_{33}-R_3 & -(T_3\sigma_{12}) & -(T_3\sigma_{23}) & -(T_3\sigma_{13}) \\
-T_1\sigma_{12} & -T_2\sigma_{12} & -T_3\sigma_{12} & 1/2-C\sigma_{12}^2 & -C\sigma_{12}\sigma_{23} & -C\sigma_{12}\sigma_{13} \\
-T_1\sigma_{23} & -T_2\sigma_{23} & -T_3\sigma_{23} & -C\sigma_{12}\sigma_{23} & 1/2-C\sigma_{23}^2 & -C\sigma_{13}\sigma_{23} \\
-T_1\sigma_{13} & -T_2\sigma_{13} & -T_3\sigma_{13} & -C\sigma_{12}\sigma_{13} & -C\sigma_{13}\sigma_{23} & 1/2-C\sigma_{13}^2
\end{bmatrix}
\begin{Bmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\varepsilon_{33} \\ d\varepsilon_{12} \\ d\varepsilon_{23} \\ d\varepsilon_{13} \end{Bmatrix}
$$

Where, $T_a = A + C\sigma_{aa}$ and $R_a = A\sigma_{aa} + B$; $a = 1, 2, 3$

$$A = \frac{h}{P'K}$$

$$P' = \frac{\sqrt{J_{2D}}}{K}\,(1 + 9\alpha^2 K/G)$$

$$h = \frac{-K\,(P'-1)}{6\alpha\sqrt{J_{2D}}} = -\left(\frac{3\alpha K}{2G} + \frac{J_1}{6\sqrt{J_{2D}}}\right)$$

$$B = \frac{2h^2\sqrt{J_{2D}}}{P'K} - \frac{3\nu K}{E} = \frac{2h^2}{1+9\alpha^2 K/G} - \frac{3\nu K}{E}$$

$$C = \frac{1}{2KP'\sqrt{J_{2D}}}$$

Following constitutive relationship for joint is proposed to be used for a 2D plane stress case :

$$\{\varepsilon\} = [D]\{\sigma\} \tag{5.46}$$

where,

$$[D] = \frac{1}{E}\begin{bmatrix} 1 & 0 & 0 \\ 0 & (1-\nu^2) & (1-\nu) \\ 0 & 0 & (1+\nu) \end{bmatrix}$$

This relationship differs from that given by equation 5.40. It indicates that normal strain is not only dependent on σyy but is also dependent on σxy. As the strain vector includes the tensorial shear strain εxy instead of engineering shear strain γxy the term in the place D_{33} in the [D] matrix is $(1+\nu)$ instead of $2(1+\nu)$ as given in equation 5.40. The relation given by equation 5.46 can also be given in the inverse form as :

$$\{\sigma\} = [C]\{\varepsilon\} \tag{5.47}$$

where,

$$[C] = E\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{1-\nu^2} & \frac{-1}{(1+\nu)^2} \\ 0 & 0 & \frac{1}{1+\nu} \end{bmatrix}$$

This [C] matrix is, however, not symmetrical and therefore may not be suitable for application in Finite Element Analysis. Following alternative [D] matrix is therefore proposed to be used for plane stress case.

$$[D] = \frac{1}{E} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2D(1-\nu)(1+2\nu) & 0 \\ 0 & 0 & 1+ \end{bmatrix} \quad (5.48)$$

Inverse of this matrix can be written as,

$$[C] = E \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2D(1-\nu)(1+2\nu)} & 0 \\ 0 & 0 & \frac{1}{1+\nu} \end{bmatrix} \quad (5.49)$$

In the above matrix D is a parameter defined as,

$$D = \frac{i_{av}}{i_o} \quad (5.50)$$

where, i_o = average angle of asperity, and

i_{av} = average dilation angle under given normal stress observed during the direct shear test.

The [C] matrix given in equation 5.49 is symmetric and therefore, acceptable in FEM analysis. It is also easy and simple to conduct a direct shear test on sample of rock joint and obtain relation between Poisson's ratio ν and shear stress σxy. Values of shear modulus G and Young's modulus E can be obtained from following relationships, at different shear stresses σxy

$$G = \frac{\sigma xy}{\gamma xy} = \frac{\sigma xy}{2\varepsilon xy} \quad (5.51)$$

$$E = 2(1+\nu)G \quad (5.52)$$

It is also conveniently possible to measure average angle of asperity (i_o) by a profilometer and to evaluate the value of average dilation angle i_{av} from a direct shear test from following relationship,

$$i_{av} = \tan^{-1}\left(\frac{\text{Total vertical displacement (dv)}}{\text{Total horizontal displacement (du)}} \right)$$

The parameters namely, ν , G and E have to be evaluated for different stages of shear loading σxy. The nature of shear stress (σxy) shear strain (γxy) relationship is likely to be nonlinear. Because of this nonlinearity the value of G is changing from stage to stage. As the shear test proceeds the value of ν is also changing as discussed earlier. Value of ν is less than unity under compressional mode, it tends to unity when the joint is just tending to dilate and it becomes more than unity when the joint dilates. It goes on increasing beyond unity till the joint continues to dilate and thereafter it again falls back to unity. Because of the variation in the values of G and ν, value of E also goes on changing as the shear stress σxy changes.

Thus having determined the parameters as stated above at different stages of the test, it is possible to formulate [C] matrix given by equation 5.47 or [D] matrix given by equation 5.48 for different stages of shear loading and use it to predict displacements, strains or stresses under given conditions using an appropriate Finite Element Programme.

5.5.2 A New Joint Element

A new joint element is proposed to be used for application in Finite Element Method analysis of jointed rocks. The joint element is considered as a solid element like other solid elements. It has a small initial thickness. Thus it differs from the joint element proposed by Goodman which is having initial zero thickness. The constitutive matrix of the joint element directly includes a dilation parameter in terms of Poisson's ratio.

5.6 CONSTRAINT OF THE CONSTITUTIVE RELATIONSHIP AND ITS PRACTICAL RELEVANCE

5.6.1 Constraint

The constituent relationship proposed for a joint element incorporates an approximation of nonlinear relationship between xy and xy into a piecewise linear form. This may introduce some errors. In the FEM analysis it is proposed to consider joint element similar to other solid elements but having very small thickness. Thus the aspect ratio of the joint element would be very high as compared to that for the adjoining elements. This may also introduce some errors, which may not be of engineering significance.

5.6.2 Practical Relevance

With a view to verify the practical relevance of the proposed constitutive relationship for the joint element, it is proposed to analyse some typical cases. It is also proposed to study a case of underground opening with the proposed constitutive relationship.

5.7 IMPLEMENTATION OF THE CONSTITUENT RELATIONSHIP

In order to verify the constituent relationship proposed in two alternative forms, it is proposed to implement it to the test data to be generated in the present laboratory investigation. It is also proposed to implement it to the test data generated by other research workers, during laboratory and in situ testing. With a view to verify its applicability in Finite Element Analysis, it is proposed to implement it in the Finite Element Analysis of the test samples of laboratory and in situ tests.

5.8 CONCLUDING REMARKS

From the theoretical developments it is obvious that in the present state of art, there is no simple and yet realistic approach for the analysis of jointed rocks which directly incorporates effect of dilatancy in the constitutive relationship. A constitutive relationship is therefore developed incorporating a dilation parameter in terms of Poisson's ratio. It is proposed to apply it to the laboratory and field testing data and to some typical cases. The results will be presented to verify efficiency and efficacy of the proposed relationship.